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Instructor's and Solutions Manual
to accompany

Vector Mechanics for Engineers, Statics

Ninth Edition

Volume 2, Chapters 6–10

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Instructor's and Solutions Manual, Volume 2 to accompany
VECTOR MECHANICS FOR ENGINEERS, STATICS, NINTH EDITION
Ferdinand P. Beer, E. Russell Johnston, Jr., David F. Mazurek, and Elliot Eisenberg

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TO THE INSTRUCTOR

As indicated in its preface, *Vector Mechanics for Engineers: Statics* is designed for the first course in statics offered in the sophomore year of college. New concepts have, therefore, been presented in simple terms and every step has been explained in detail. However, because of the large number of optional sections which have been included and the maturity of approach which has been achieved, this text can also be used to teach a course which will challenge the more advanced student.

The text has been divided into units, each corresponding to a well-defined topic and consisting of one or several theory sections, one or several Sample Problems, a section entitled *Solving Problems on Your Own*, and a large number of problems to be assigned. To assist instructors in making up a schedule of assignments that will best fit their classes, the various topics covered in the text have been listed in Table I and a suggested number of periods to be spent on each topic has been indicated. Both a minimum and a maximum number of periods have been suggested, and the topics which form the standard basic course in statics have been separated from those which are optional. The total number of periods required to teach the basic material varies from 26 to 39, while covering the entire text would require from 41 to 65 periods. If allowance is made for the time spent for review and exams, it is seen that this text is equally suitable for teaching a basic statics course to students with limited preparation (since this can be done in 39 periods or less) and for teaching a more complete statics course to advanced students (since 41 periods or more are necessary to cover the entire text). In most instances, of course, the instructor will want to include some, but not all, of the additional material presented in the text. In addition, it is noted that the text is suitable for teaching an abridged course in statics which can be used as an introduction to the study of dynamics (see Table I).

The problems have been grouped according to the portions of material they illustrate and have been arranged in order of increasing difficulty, with problems requiring special attention indicated by asterisks. We note that, in most cases, problems have been arranged in groups of six or more, all problems of the same group being closely related. This means that instructors will easily find additional problems to amplify a particular point which they may have brought up in discussing a problem assigned for homework. A group of problems designed to be solved with computational software can be found at the end of each chapter. Solutions for these problems, including analyses of the problems and problem solutions and output for the most widely used computational programs, are provided at the instructor s edition of the text’s website:
http://www.mhhe.com/beerjohnston.

To assist in the preparation of homework assignments, Table II provides a brief description of all groups of problems and a classification of the problems in each group according to the units used. It should also be noted that the answers to all problems are given at the end of the text, except for those with a number in italic. Because of the large number of problems available in both systems of units, the instructor has the choice of assigning problems using SI units and problems using U.S. customary units in whatever proportion is found to be most desirable for a given class. To illustrate this point, sample lesson schedules are shown in Tables III, IV, and V, together with various alternative lists of assigned homework problems. Half of the problems in each of the six lists suggested in Table III and Table V are stated in SI units

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and half in U.S. customary units. On the other hand, 75% of the problems in the four lists suggested in Table IV are stated in SI units and 25% in U.S. customary units.

Since the approach used in this text differs in a number of respects from the approach used in other books, instructors will be well advised to read the preface to *Vector Mechanics for Engineers,* in which the authors have outlined their general philosophy. In addition, instructors will find in the following pages a description, chapter by chapter, of the more significant features of this text. It is hoped that this material will help instructors in organizing their courses to best fit the needs of their students. The authors wish to acknowledge and thank Amy Mazurek of Williams Memorial Institute for her careful preparation of the solutions contained in this manual.

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DESCRIPTION OF THE MATERIAL CONTAINED IN *VECTOR MECHANICS FOR ENGINEERS: STATICS*, Ninth Edition

Chapter 1 Introduction

The material in this chapter can be used as a first assignment or for later reference. The six fundamental principles listed in Sec. 1.2 are introduced separately and are discussed at greater length in the following chapters. Section 1.3 deals with the two systems of units used in the text. The SI metric units are discussed first. The base units are defined and the use of multiples and submultiples is explained. The various SI prefixes are presented in Table 1.1, while the principal SI units used in statics and dynamics are listed in Table 1.2. In the second part of Sec. 1.3, the base U.S. customary units used in mechanics are defined, and in Sec. l.4, it is shown how numerical data stated in U.S. customary units can be converted into SI units, and vice versa. The SI equivalents of the principal U.S. customary units used in statics and dynamics are listed in Table 1.3.

The instructor's attention is called to the fact that the various rules relating to the use of SI units have been observed throughout the text. For instance, multiples and submultiples (such as kN and mm) are used whenever possible to avoid writing more than four digits to the left of the decimal point or zeros to the right of the decimal point. When 5-digit or larger numbers involving SI units are used, spaces rather than commas are utilized to separate digits into groups of three (for example, 20 000 km). Also, prefixes are never used in the denominator of derived units; for example, the constant of a spring which stretches 20 mm under a load of 100 N is expressed as 5 kN/m, not as 5 N/mm.

In order to achieve as much uniformity as possible between results expressed respectively in SI and U.S. customary units, a center point, rather than a hyphen, has been used to combine the symbols representing U.S. customary units (for example, 10 lb · ft); furthermore, the unit of time has been represented by the symbol s, rather than sec, whether SI or U.S. customary units are involved (for example, 5 s, 50 ft/s, 15 m/s). However, the traditional use of commas to separate digits into groups of three has been maintained for 5-digit and larger numbers involving U.S. customary units.

Chapter 2 Statics of Particles

This is the first of two chapters dealing with the fundamental properties of force systems. A simple, intuitive classification of forces has been used: forces acting on a particle (Chap. 2) and forces acting on a rigid body (Chap. 3).

Chapter 2 begins with the parallelogram law of addition of forces and with the introduction of the fundamental properties of vectors. In the text, forces and other vector quantities are always shown in bold-face type. Thus, a force **F** (boldface), which is a vector quantity, is clearly distinguished from the magnitude *F* (italic) of the force, which is a scalar quantity. On the blackboard and in handwritten work, where bold-face lettering is not practical, vector quantities can be indicated by underlining. Both the magnitude and the direction of a vector quantity must be given to completely define that quantity. Thus, a force **F** of magnitude *F* = 280 lb, directed upward to the right at an angle of 25° with the horizontal, is indicated as **F** = 280 lb $\measuredangle$ 25° when printed or as $\underline{F}$ = 280 lb $\measuredangle$ 25° when handwritten. Unit vectors i and j are introduced in Sec. 2.7, where the rectangular components of forces are considered.

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In the early sections of Chap. 2 the following basic topics are presented: the equilibrium of a particle, Newton's first law, and the concept of the free-body diagram. These first sections provide a review of the methods of plane trigonometry and familiarize the students with the proper use of a calculator. A general procedure for the solution of problems involving concurrent forces is given: when a problem involves only three forces, the use of a force triangle and a trigonometric solution is preferred; when a problem involves more than three forces, the forces should be resolved into rectangular components and the equations $\Sigma F_x = 0$, $\Sigma F_y = 0$ should be used.

The second part of Chap. 2 deals with forces in space and with the equilibrium of particles in space. Unit vectors are used and forces are expressed in the form $\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} = F\lambda$, where **i**, **j**, and **k** are the unit vectors directed respectively along the *x*, *y*, and *z* axes, and λ is the unit vector directed along the line of action of **F**.

Note that since this chapter deals only with particles or bodies which can be considered as particles, problems involving compression members have been postponed with only a few exceptions until Chap. 4, where students will learn to handle rigid-body problems in a uniform fashion and will not be tempted to erroneously assume that forces are concurrent or that reactions are directed along members.

It should be observed that when SI units are used a body is generally specified by its mass expressed in kilograms. The weight of the body, however, should be expressed in newtons. Therefore, in many equilibrium problems involving SI units, an additional calculation is required before a free-body diagram can be drawn (compare the example in Sec. 2.11 and Sample Probs. 2.5 and 2.9). This apparent disadvantage of the SI system of units, when compared to the U.S. customary units, will be offset in dynamics, where the mass of a body expressed in kilograms can be entered directly into the equation $\mathbf{F} = m\mathbf{a}$, whereas with U.S. customary units the mass of the body must first be determined in $\text{lb} \cdot \text{s}^2/\text{ft}$ (or slugs) from its weight in pounds.

Chapter 3
Rigid Bodies:
Equivalent Systems of Forces

The principle of transmissibility is presented as the basic assumption of the statics of rigid bodies. However, it is pointed out that this principle can be derived from Newton s three laws of motion (see Sec. 16.5 of *Dynamics*). The vector product is then introduced and used to define the moment of a force about a point. The convenience of using the determinant form (Eqs. 3.19 and 3.21) to express the moment of a force about a point should be noted. The scalar product and the mixed triple product are introduced and used to define the moment of a force about an axis. Again, the convenience of using the determinant form (Eqs. 3.43 and 3.46) should be noted. The amount of time which should be assigned to this part of the chapter will depend on the extent to which vector algebra has been considered and used in prerequisite mathematics and physics courses. It is felt that, even with no previous knowledge of vector algebra, a maximum of four periods is adequate (see Table I).

In Secs. 3.12 through 3.15 couples are introduced, and it is proved that couples are equivalent if they have the same moment. While this fundamental property of couples is often taken for granted, the authors believe that its rigorous and logical proof is necessary if rigor and logic are to be demanded of the students in the solution of their mechanics problems.

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In Sections 3.16 through 3.20, the concept of equivalent systems of forces is carefully presented. This concept is made more intuitive through the extensive use of *free-body-diagram equations* (see Figs. 3.39 through 3.46). Note that the *moment* of a force is either not shown or is represented by a green vector (Figs. 3.12 and 3.27). A red vector with the symbol ↺ is used only to represent a *couple*, that is, an actual system consisting of two forces (Figs. 3.38 through 3.46). Section 3.21 is optional; it introduces the concept of a *wrench* and shows how the most general system of forces in space can be reduced to this combination of a force and a couple with the same line of action.

Since one of the purposes of Chap. 3 is to familiarize students with the fundamental operations of vector algebra, students should be encouraged to solve all problems in this chapter (two-dimensional as well as three-dimensional) using the methods of vector algebra. However, many students may be expected to develop solutions of their own, particularly in the case of two-dimensional problems, based on the direct computation of the moment of a force about a given point as the product of the magnitude of the force and the perpendicular distance to the point considered. Such alternative solutions may occasionally be indicated by the instructor (as in Sample Prob. 3.9), who may then wish to compare the solutions of the sample problems of this chapter with the solutions of the same sample problems given in Chaps. 3 and 4 of the parallel text *Mechanics for Engineers*. It should be pointed out that in later chapters the use of vector products will generally be reserved for the solution of three-dimensional problems.

Chapter 4
Equilibrium of Rigid Bodies

In the first part of this chapter, problems involving the equilibrium of rigid bodies in two dimensions are considered and solved using ordinary algebra, while problems involving three dimensions and requiring the full use of vector algebra are discussed in the second part of the chapter. Particular emphasis is placed on the correct drawing and use of free-body diagrams and on the types of reactions produced by various supports and connections (see Figs. 4.1 and 4.10). Note that a distinction is made between hinges used in pairs and hinges used alone; in the first case the reactions consist only of force components, while in the second case the reactions may, if necessary, include couples.

For a rigid body in two dimensions, it is shown (Sec. 4.4) that no more than three independent equations can be written for a given free body, so that a problem involving the equilibrium of a single rigid body can be solved for no more than three unknowns. It is also shown that it is possible to choose equilibrium equations containing only one unknown to avoid the necessity of solving simultaneous equations. Section 4.5 introduces the concepts of statical indeterminacy and partial constraints. Sections 4.6 and 4.7 are devoted to the equilibrium of two- and three-force bodies; it is shown how these concepts can be used to simplify the solution of certain problems. This topic is presented only after the general case of equilibrium of a rigid body to lessen the possibility of students misusing this particular method of solution.

The equilibrium of a rigid body in three dimensions is considered with full emphasis placed on the free-body diagram. While the tool of vector algebra is freely used to simplify the computations involved, vector algebra does not, and indeed cannot, replace the free-body diagram as the focal point of an equilibrium problem. Therefore, the solution of every sample problem in this section begins with a reference to the drawing of a free-body diagram. Emphasis is also

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placed on the fact that the number of unknowns and the number of equations must be equal if a structure is to be statically determinate and completely constrained.

Chapter 5
Distributed Forces:
Centroids and Centers of Gravity

Chapter 5 starts by defining the center of gravity of a body as the point of application of the resultant of the weights of the various particles forming the body. This definition is then used to establish the concept of the centroid of an area or line. Section 5.4 introduces the concept of the first moment of an area or line, a concept fundamental to the analysis of shearing stresses in beams in a later study of mechanics of materials. All problems assigned for the first period involve only areas and lines made of simple geometric shapes; thus, they can be solved without using calculus.

Section 5.6 explains the use of differential elements in the determination of centroids by integration. The theorems of Pappus-Guldinus are given in Sec. 5.7. Sections 5.8 and 5.9 are optional; they show how the resultant of a distributed load can be determined by evaluating an area and by locating its centroid. Sections 5.10 through 5.12 deal with centers of gravity and centroids of volumes. Here again the determination of the centroids of composite shapes precedes the calculation of centroids by integration.

It should be noted that when SI units are used, a given material is generally characterized by its density (mass per unit volume, expressed in kg/m^3), rather than by its specific weight (weight per unit volume, expressed in N/m^3). The specific weight of the material can then be obtained by multiplying its density by g = 9.81 m/s^2 (see footnote, page 222 of the text).

Chapter 6
Analysis of Structures

In this chapter students learn to determine the internal forces exerted on the members of pin-connected structures. The chapter starts with the statement of Newton s third law (action and reaction) and is divided into two parts: (*a*) trusses, that is, structures consisting of two-force members only, (*b*) frames and machines, that is, structures involving multiforce members.

After trusses and simple trusses have been defined in Secs. 6.2 and 6.3, the method of joints and the method of sections are explained in detail in Sec. 6.4 and Sec. 6.7, respectively. Since a discussion of Maxwell’s diagram is not included in this text, the use of Bow’s notation has been avoided, and a uniform notation has been used in presenting the method of joints and the method of sections.

In the method of joints, a free-body diagram should be drawn for each pin. Since all forces are of known direction, their magnitudes, rather than their components, should be used as unknowns. Following the general procedure outlined in Chap. 2, joints involving only three forces are solved using a force triangle, while joints involving more than three forces are solved by summing x and y components. Sections 6.5 and 6.6 are optional. It is shown in Sec. 6.5 how the analysis of certain trusses can be expedited by recognizing joints under special loading conditions, while in Sec. 6.6 the method of joints is applied to the solution of three-dimensional trusses.

It is pointed out in Sec. 6.4 that forces in a simple truss can be determined by analyzing the truss joint by joint and that joints can always be found that involve only two unknown forces. The method of sections

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(Sec. 6.7) should be used (*a*) if only the forces in a few members are desired, or (*b*) if the truss is not a simple truss and if the solution of simultaneous equations is to be avoided (for example, Fink truss). Students should be urged to draw a separate free-body diagram for each section used. The free body obtained should be emphasized by shading and the intersected members should be removed and replaced by the forces they exerted on the free body. It is shown that, through a judicious choice of equilibrium equations, the force in any given member can be obtained in most cases by solving a single equation. Section 6.8 is optional; it deals with the trusses obtained by combining several simple trusses and discusses the statical determinacy of such structures as well as the completeness of their constraints.

Structures involving multiforce members are separated into frames and machines. Frames are designed to support loads, while machines are designed to transmit and modify forces. It is shown that while some frames remain rigid after they have been detached from their supports, others will collapse (Sec. 6.11). In the latter case, the equations obtained by considering the entire frame as a free body provide necessary but not sufficient conditions for the equilibrium of the frame. It is then necessary to dismember the frame and to consider the equilibrium of its component parts in order to determine the reactions at the external supports. The same procedure is necessary with most machines in order to determine the output force **Q** from the input force **P** or inversely (Sec. 6.12).

Students should be urged to resolve a force of unknown magnitude and direction into two components but to represent a force of known direction by a single unknown, namely its magnitude. While this rule may sometimes result in slightly more complicated arithmetic, it has the advantage of matching the numbers of equations and unknowns and thus makes it possible for students to know at any time during the computations what is known and what is yet to be determined.

Chapter 7
Forces in Beams and Cables

This chapter consists of five groups of sections, all of which are optional. The first three groups deal with forces in beams and the last two groups with forces in cables. Most likely the instructor will not have time to cover the entire chapter and will have to choose between beams and cables.

Section 7.2 defines the internal forces in a member. While these forces are limited to tension or compression in a straight two-force member, they include a shearing force and a bending couple in the case of multiforce members or curved two-force members. Problems in this section do not make use of sign conventions for shear and bending moment and answers should specify which part of the member is used as the free body.

In Secs. 7.3 through 7.5 the usual sign conventions are introduced and shear and bending-moment diagrams are drawn. All problems in these sections should be solved by drawing the free-body diagrams of the various portions of the beams.

The relations among load, shear, and bending moment are introduced in Sec. 7.6. Problems in this section should be solved by evaluating areas under load and shear curves or by formal integration (as in Probs. 7.87 and 7.88). Some instructors may feel that the special methods used in this section detract from the unity achieved in the rest of the text through the use of the free-body diagram, and they may wish to omit Sec. 7.6. Others will feel that the study of shear and bending-moment diagrams is incomplete without this

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section, and they will want to include it. The latter view is particularly justified when the course in statics is immediately followed by a course in mechanics of materials.

Sections 7.7 through 7.9 are devoted to cables, first with concentrated loads and then with distributed loads. In both cases, the analysis is based on free-body diagrams. The differential-equation approach is considered in the last problems of this group (Probs. 7.124 through 7.126). Section 7.10 is devoted to catenaries and requires the use of hyperbolic functions.

Chapter 8
Friction

This chapter not only introduces the general topic of friction but also provides an opportunity for students to consolidate their knowledge of the methods of analysis presented in Chaps. 2, 3, 4, and 6. It is recommended that each course in statics include at least a portion of this chapter.

The first group of sections (Secs. 8.1 through 8.4) is devoted to the presentation of the laws of dry friction and to their application to various problems. The different cases which can be encountered are illustrated by diagrams in Figs. 8.2, 8.3, and 8.4. Particular emphasis is placed on the fact that no relation exists between the friction force and the normal force except when motion is impending or when motion is actually taking place. Following the general procedure outlined in Chap. 2, problems involving only three forces are solved by a force triangle, while problems involving more than three forces are solved by summing *x* and *y* components. In the first case the reaction of the surface of contact should be represented by the resultant **R** of the friction force and normal force, while in the second case it should be resolved into its components **F** and **N**.

Special applications of friction are considered in Secs. 8.5 through 8.10. They are divided into the following groups: wedges and screws (Secs. 8.5 and 8.6); axle and disk friction, rolling resistance (Secs. 8.7 through 8.9); belt friction (Sec. 8.10). The sections on axle and disk friction and on rolling resistance are not essential to the understanding of the rest of the text and thus may be omitted.

Chapter 9
Distributed Forces Moments of Inertia

The purpose of Sec. 9.2 is to give motivation to the study of moments of inertia of areas. Two examples are considered: one deals with the pure bending of a beam and the other with the hydrostatic forces exerted on a submerged circular gate. It is shown in each case that the solution of the problem reduces to the computation of the moment of inertia of an area. The other sections in the first assignment are devoted to the definition and the computation of rectangular moments of inertia, polar moments of inertia, and the corresponding radii of gyration. It is shown how the same differential element can be used to determine the moment of inertia of an area about each of the two coordinate axes.

Sections 9.6 and 9.7 introduce the parallel-axis theorem and its application to the determination of moments of inertia of composite areas. Particular emphasis is placed on the proper use of the parallel-axis theorem (see Sample Prob. 9.5). Sections 9.8 through 9.10 are optional; they are devoted to products of inertia and to the determination of principal axes of inertia.

Sections 9.11 through 9.18 deal with the moments of inertia of masses. Particular emphasis is placed on the moments of inertia of thin plates (Sec. 9.13) and on the use of these plates as differential elements in the computation of moments of inertia

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of symmetrical three-dimensional bodies (Sec. 9.14). Sections 9.16 through 9.18 are optional but should be used whenever the following dynamics course includes the motion of rigid bodies in three dimensions. Sections 9.16 and 9.17 introduce the moment of inertia of a body with respect to an arbitrary axis as well as the concepts of mass products of inertia and principal axes of inertia. Section 9.18 discusses the determination of the principal axes and principal moments of inertia of a body of arbitrary shape.

When solving many of the problems of Chap. 5, information on the specific weight of a material was generally required. This information was readily available in problems stated in U.S. customary units, while it had to be obtained from the density of the material in problems stated in SI units (see the last paragraph of our discussion of Chap. 5). In Chap. 9, when SI units are used, the mass and mass moment of inertia of a given body are respectively obtained in kg and $kg \cdot m^2$ directly from the dimensions of the body in meters and from its density in kg/m^3. However, if U.S. customary units are used, the density of the body must first be calculated from its specific weight or, alternatively, the weight of the body can be obtained from its dimensions and specific weight and then converted into the corresponding mass expressed in $lb \cdot s^2/ft$ (or slugs). The mass moment of inertia of the body is then obtained in $lb \cdot ft \cdot s^2$ (or $slug \cdot ft^2$). Sample Problem 9.12 provides an example of such a computation. Attention is also called to the footnote on page of the 513 regarding the conversion of mass moments of inertia from U.S. customary units to SI units.

Chapter 10
Method of Virtual Work

While this chapter is optional, the instructor should give serious consideration to its inclusion in the basic statics course. Indeed, students who learn the method of virtual work in their first course in mechanics will remember it as a fundamental and natural principle. They may, on the other hand, consider it as an artificial device if its presentation is postponed to a more advanced course.

The first group of sections (Secs. 10.2 through 10.5) is devoted to the derivation of the principle of virtual work and to its direct application to the solution of equilibrium problems. The second group of sections (Secs. 10.6 through 10.9) introduces the concept of potential energy and shows that equilibrium requires that the derivative of the potential energy be zero. Section 10.5 defines the mechanical efficiency of a machine and Sec. 10.9 discusses the stability of equilibrium.

The first groups of problems in each assignment utilize the principle of virtual work as an alternative method for the computation of unknown forces. Subsequent problems call for the determination of positions of equilibrium, while other problems combine the conventional methods of statics with the method of virtual work to determine displacements (Probs. 10.55 through 10.58).

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TABLE I: LIST OF THE TOPICS COVERED IN *VECTOR MECHANICS FOR ENGINEERS: STATICS*

		Suggested Number of Periods		
Sections	**Topics**	**Basic Course**	**Additional Topics**	**Abridged Course to be used as an introduction to dynamics+**
1. INTRODUCTION				
1.1–6	This material may be used for the first assignment or for later reference			
2. STATICS OF PARTICLES				
2.1–6	Addition and Resolution of Forces	0.5–1		0.5–1
2.7–8	Rectangular Components	0.5–1		0.5–1
2.9–11	Equilibrium of a Particle	1		1
2.12–14	Forces in Space	1		1
2.15	Equilibrium in Space	1		1
3. RIGID BODIES: EQUIVALENT SYSTEMS OF FORCES				
3.1–8	Vector Product; Moment of a Force about a Point	1–2		1–2
3.9–11	Scalar Product; Moment of a Force about an Axis	1–2		1–2
3.12–16	Couples	1		1
3.17–20	Equivalent Systems of Forces	1–1.5		1–1.5
*3.21	Reduction of a Wrench		0.5–1	
4. EQUILIBRIUM OF RIGID BODIES				
4.1–4	Equilibrium in Two Dimensions	1.5–2		1.5–2
4.5	Indeterminate Reactions; Partial Constraints	0.5–1		
4.6–7	Two- and Three-Force Bodies	1		
4.8–9	Equilibrium in Three Dimensions	2		2
5. CENTROIDS AND CENTERS OF GRAVITY				
5.1–5	Centroids and First Moments of Areas and Lines	1–2		
5.6–7	Centroids by Integration	1–2		
*5.8–9	Beams and Submerged Surfaces		1–1.5	
5.10–12	Centroids of Volumes	1–2		
6. ANALYSIS OF STRUCTURES				
6.1–4	Trusses by Method of Joints	1–1.5		
*6.5	Joints under Special Loading Conditions		0.25–0.5	
*6.6	Space Trusses		0.5–1	
6.7	Trusses by Method of Sections	1–2		
*6.8	Combined Trusses		0.25–0.5	
6.9–11	Frames	2–3		1–2
6.12	Machines	1–2		0.5–1.5
7. FORCES IN BEAMS AND CABLES				
*7.1–2	Internal Forces in Members		1	
*7.3–5	Shear and Moment Diagrams by FB Diagram		1–2	
*7.6	Shear and Moment Diagrams by Integration		1–2	
*7.7–9	Cables with Concentrated Loads; Parabolic Cable		1–2	
*7.10	Catenary		1	
8. FRICTION				
8.1–4	Laws of Friction and Applications	1–2		1–2
8.5–6	Wedges and Screws	1		
*8.7–9	Axle and Disk Friction; Rolling Resistance		1–2	
8.10	Belt Friction	1		
9. MOMENTS OF INERTIA				
9.1–5	Moments of Inertial of Areas	1		
9.6–7	Composite Areas	1–2		
*9.8–9	Products of Inertia; Principal Axes		1–2	
*9.10	Mohr's Circle		1	
9.11–15	Moments of Inertia of Masses#		1–2	
*9.16–18	Mass Products of Inertia; Principal Axes and Principal Moments of Inertia		1–2	
10. METHOD OF VIRTUAL WORK				
10.1–4	Principle of Virtual Work		1–2	
10.5	Mechanical Efficiency		0.5–1	
10.6–9	Potential Energy; Stability		1–1.5	
	Total Number of Periods	26–39	15–26	14–21

+ A sample assignment schedule for a course in dynamics including this minimum amount of introductory material in statics is given Table V. It is recommended that a more complete statics course, such as the one outlined in Tables III and IV of this manual, be used in curricula which include the study of mechanics of materials.

Mass moments of inertia have not been included in the basic statics course since this material is often taught in dynamics.

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS

Problem Number*		
SI Units	**U.S. Units**	**Problem Description**
		CHAPTER 2: STATICS OF PARTICLES
		FORCES IN A PLANE
		Resultant of concurrent forces
2.1, *4*	2.2, 3	graphical method
2.7, 8	2.5, *6*	law of sines
2.9, 10	2.11, *12*	
2.13	2.14	special problems
2.17, *18*	2.15, 16	laws of cosines and sines
2.19, *20*		
		Rectangular components of force
2.21, *24*	2.22, 23	simple problems
2.26, 27	2.25, 30	more advanced problems
2.*28*, *29*		
2.32, 34	2.31, *33*	Resultant by $\Sigma F_x = 0$, $\Sigma F_y = 0$
2.35, 36	2.37, *38*	
2.39, 40	2.*41*, 42	Select force so that resultant has a given direction
		Equilibrium. Free-Body Diagram
2.43, *44*	2.47, 48	equilibrium of 3 forces
2.45, *46*		
2.51, 52	2.49, *50*	equilibrium of 4 forces
2.55, 56	2.53, *54*	
2.57, 60	2.*58*, 59	find parameter to satisfy specified conditions
2.61, 62	2.63, *64*	
2.65, *66*	2.67, 68	special problems
2.69, *70*		
		FORCES IN SPACE
		Rectangular components of a force in space
2.71, 72	2.*75*, 76	given F, θ, and ϕ, find components and direction angles
2.73, 74	2.77, *78*	
2.79, *80*	2.81, 82	relations between components and direction angles
2.*83*, 84		
2.87, *88*	2.85, 86	direction of force defined by two points on its line of action
2.89, *90*		
2.91, 92	2.*93*, 94	resultant of two or three forces
2.95, 96	2.97, *98*	
		Equilibrium of a particle in space
2.99, *100*	2.103, 104	load applied to three cables, introductory problems
2.101, *102*		
2.107, 108	2.105, *106*	intermediate problems
2.111, 112	2.109, *110*	
2.115, 116	2.113, 114	advanced problems
2.117, 118	2.*119*, *120*	

* Problems which do not involve any specific system of units have been indicated by underlining their number. Answers are not given to problems with a number set in italic type.

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Problem Number* SI Units	U.S. Units	Problem Description
2.121, *122*		problems involving cable through ring
2.123, *124*		
	2.125, 126	special problems
2.131, *132*	2.127, *128*	Review problems
2.135, 136	2.*129*, 130	
2.137, *138*	2.133, 134	
2.C1, C3	2.C2	Computer problems
2.C4	2.C5	
		CHAPTER 3: RIGID BODIES; EQUIVALENT SYSTEMS OF FORCES
		Moment of a force about a point: Two dimensions
3.1, 2	3.5, *6*	introductory problems
3.3, 4	3.7, *8*	
3.9, 10	3.12, 13	direction of a force defined by two points on its line of action
3.*11*, *14*		
3.15		derivation of a formula
3.16, 17	3.*18*	applications of the vector product
		Moment of a force about a point: Three dimensions
3.19	3.*20*	computing **M** = **r** × **F**, introductory problems
3.21, 22	3.24, 26	computing **M** = **r** × **F**, more involved problems
3.*23*, *25*		
3.27, 28	3.*29*, 30	using **M** to find the perpendicular distance from a point to a line
3.31, 32	3.33, *34*	
3.35	3.36	Scalar Product
3.37, *38*	3.39, 40	Finding the angle between two lines
3.41, *42*	3.43, 44	
3.45	3.46	Mixed triple product
3.47, 48	3.51, *52*	Moment of a force about the coordinate axes
3.49, 50	3.53, *54*	
3.55, 56	3.59, *60*	Moment of a force about an oblique axis
3.57, 58	3.61, *62*	
	3.63	
*3.*64*, *65	*3.68, *69	Finding the perpendicular distance between two lines
*3.66, **67*		
3.70, 72	3.71, *73*	Couples in two dimensions
	3.*74*	
3.76	3.75, 77	Couples in three dimensions
3.79, 80	3.78	
3.83, *84*	3.81, 82	Replacing a force by an equivalent force-couple system: two dimensions
3.85, *86*		
3.87, 88	3.89, 90	Replacing a force-couple system by an equivalent force or forces
3.*91*, *92*		
3.93, 94	3.95, *96*	Replacing a force by an equivalent force-couple system: three dimensions
3.97, 98	3.99, *100*	

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Problem Number*		
SI Units	**U.S. Units**	**Problem Description**
3.101, 102	3.104	Equivalent force-couple systems
3._103_		
3._107_	3.105, 106	Finding the resultant of parallel forces: two dimensions
3.111, 112	3.108, 109	Finding the resultant and its line of action: two dimensions
3.115, 116	3.110, 113	
3.118	3._114_, _117_	
3.119, 120	3.121, 122	Reducing a three-dimensional system of forces to a single force-couple system
3.124, 125	3._123_, _126_	
3.127, 128	3.129, 130	Finding the resultant of parallel forces: three dimensions
*3._131_, *_132_		
		Reducing three-dimensional systems of forces or forces and couples to a wrench
*3.133, *_135_	*3.134, *136	axis of wrench is parallel to a coordinate axis or passes through _O_
*3.137		force-couple system parallel to the coordinate axes
*3._139_, 140	*3._138_	general, three-dimensional case
*3.141	*3._142_	special cases where the wrench reduces to a single force
*3.143, *144	*3.145, *146	special, more advanced problems
3.147, 148	3.149, 151	Review problems
3._150_, _154_	3._152_, 153	
3.155, 157	3.156, _158_	
3.C1, C4	3.C2, C3	Computer problems
3.C5	3.C6	

CHAPTER 4: EQUILIBRIUM OF RIGID BODIES

EQUILIBRIUM IN TWO DIMENSIONS

4.2, 3	4.1, 4	Parallel forces
4.5, 6	4._7_, _8_	
4.9, _10_	4.11, 14	Parallel forces, find range of values of loads to satisfy multiple criteria
4.12, _13_		
4.15, _16_	4.17, 18	Rigid bodies with one reaction of unknown direction and one of known direction
4.19, _20_		
4.21, 26	4._22_, 23	
4.27, 28	4.24, _25_	
4.30, _31_	4.29, 33	
4._32_, _34_		
4.37, 38	4.35, 36	Rigid bodies with three reactions of known direction
4.41, 42	4._39_, _40_	
4.43, 46	4.44, _45_	Rigid bodies with a couple included in the reactions
4.49, 50	4.47, _48_	
4.51, 53	4.52, 54	Find position of rigid body in equilibrium
4._55_, _56_	4.57, 58	
4.59	4._60_	Partial constraints, statical indeterminacy

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Problem Number* **SI Units**	 **U.S. Units**	 **Problem Description**
		Three-force bodies
4.61, 62	4.63, 64	simple geometry, solution of a right triangle required
4.66, 67	4.*65*, *68*	simple geometry, frame includes a two-force member
4.69, 70	4.71, 74	more involved geometry
4.*72*, *73*		
4.*76*, 77	4.75, 81	
4.*78*, 79	4.82	
4.80		
4.83, 84	4.*86*, 87	find position of equilibrium
4.85, 88	4.*89*, 90	
		EQUILIBRIUM IN THREE DIMENSIONS
4.92, 93	4.91, 94	Rigid bodies with two hinges along a coordinate axis and
4.*95*, *96*		an additional reaction parallel to another coordinate axis
4.97, 98	4.99, 100	Rigid bodies supported by three vertical wires or by vertical reactions
4.101, 102	4.*103*, *104*	
4.106, 107	4.105, 108	Derrick and boom problems involving unknown tension in two cables
4.109, 110	4.*111*, *112*	
4.113, 114	4.117, 118	Rigid bodies with two hinges along a coordinate axis and an additional
4.*115*, *116*		reaction not parallel to a coordinate axis
4.119, 122	4.120, 121	Problems involving couples as part of the reaction at a hinge
4.125, 126	4.*123*, *124*	Advanced problems
4.129, 130	4.127, 128	
4.*131*, *132*		
4.135, 136	4.133, 134	Problems involving taking moments about an oblique line passing
4.*140*, *141*	4.137, 138	through two supports
	4.139	
4.142, 143	4.*144*, 146	Review problems
4.145, *149*	4.*147*, 148	
4.150, *151*	4.152, 153	
4.C2, C5	4.C1, C3	Computer problems
4.C6	4.C4	

CHAPTER 5: DISTRIBUTED FORCES: CENTROIDS AND CENTERS OF GRAVITY

		Centroid of an area formed by combining
5.1, 2	5.3, 4	rectangles and triangles
5.6, 9	5.5, *7*	rectangles, triangles, and portions of circular areas
	5.*8*	
5.*10*, 12	5.11, 14	triangles, portions of circular or elliptical areas, and areas of analytical functions
5.*13*, *15*		
5.16	5.17	Derive expression for location of centroid
5.*19*	5.18	Find ratio of dimensions so that centroid is at a given point
5.20, *23*	5.21, 22	First moment of an area
5.24, *25*	5.26, *27*	Center of gravity of a wire figure
5.29, 30	5.28, *31*	Equilibrium of wire figures
	5.32, 33	Find dimension to maximize distance to centroid

* Problems which do not involve any specific system of units have been indicated by underlining their number. Answers are not given to problems with a number set in italic type.

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Problem Number* SI Units	U.S. Units	Problem Description
		Use integration to find centroid of
5.35, *36*	5.34	simple areas
5.*38*, 39	5.37	areas obtained by combining shapes of Fig. 5.8A
5.40	5.41	
5.42		parabolic area
	5.*43*, 44	areas defined by different functions over the interval of interest
5.45, 46	*5.*47*	homogeneous wires
*5.48, *49		areas defined by exponential or cosine functions
	5.50, 51	areas defined by a hyperbola
		Find areas or volumes by Pappus - Guldinus
5.52, 53	5.54, *55*	rotate simple geometric figures
5.56, 57		
5.59, 61	5.58, *60*	practical applications
5.*62*, **65*	5.63, 64	
		Distributed load on beams
5.67	5.66	resultant of loading
5.68, 71	5.69, *70*	reactions at supports
5.73	5.*72*	
5.74, *75*	5.76, 77	special problems
5.78, *79*		
		Forces on submerged surfaces
5.80, 82	5.81, 84	reactions on dams or vertical gates
5.83, *86*	5.85	
5.*87*		
5.88, 89	5.90, 91	reactions on non-vertical gates
5.93, 94	5.*92*	
	5.*95*	special applications
		Centroids and centers of gravity of three-dimensional bodies
5.96, 99	5.97, 98	composite bodies formed from two common shapes
5.100, 101	5.103, 104	composite bodies formed from three or more elements
5.*102*, *105*		
5.106, 107	5.108, *109*	composite bodies formed from a material of uniform thickness
5.110, 112	5.111, *113*	
5.114, 115	5.*116*, 117	composite bodies formed from a wire or structural shape of uniform cross section
5.118, 121	5.119, *120*	composite bodies made of two different materials
		use integration to locate the centroid of
5.123, 124	5.122	standard shapes: single integration
5.*126*, *127*	5.125	bodies of revolution: single integration
*5.128, *129		
5.132	*5.130, *131*	special applications: single integration
	5.133, 134	special applications: double integration
5.*135*, *136		bodies formed by cutting a standard shape with an oblique plane: single integration

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Problem Number* **SI Units**	**U.S. Units**	**Problem Description**
5.137, *139*	5.138, 140	Review problems
5.143, *144*	5.141, *142*	
5.147, 148	5.*145*, 146	
5.C2, C3	5.C1, C4	Computer problems
5.C5, C6	*5.C7	

CHAPTER 6: ANALYSIS OF STRUCTURES

TRUSSES

		Method of joints
6.1, 3	6.2, *4*	simple problems
6.6, 7	6.5, *8*	
6.9, 10	6.11, 12	problems of average difficulty
6.*13*, *14*	6.15, 16	
6.19, 20	6.*17*, *18*	more advanced problems
6.23, 24	6.21, 22	
6.*25*, *26*	6.27	
6.28		
6.29	6.30	designate simple trusses
6.31, 32		find zero-force members
6.*33*, *34*		
*6.36, *37	*6.35, *38	space trusses
*6.39, *40	*6.41, *42	
		Method of sections
6.45, 46	6.43, 44	two of the members cut are parallel
6.*47*, *48*		
6.49, 50	6.51, 52	none of the members cut are parallel
6.53, 54	6.57, 58	
6.55, 56	6.*59*, *60*	
6.61, 62	6.*63*, *64*	K-type trusses
6.65, 66	6.67, 68	trusses with counters
6.70, 71	6.69, 72	Classify trusses according to constraints
6.*73*, *74*		

FRAMES AND MACHINES

		Analysis of Frames
6.75, 76	6.77, 78	easy problems
6.81, 82	6.*79*, *80*	
6.83, 84	6.87, 88	problems where internal forces are changed by repositioning a couple or by moving a force along its line of action
6.*85*, *86*	6.89	
6.90		replacement of pulleys by equivalent loadings
6.91, 92	6.93, 94	analysis of frames supporting pulleys or pipes
	6.95, 96	analysis of highway vehicles

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Problem Number*		
SI Units	**U.S. Units**	**Problem Description**
6.*97*, *98*	6.99, 100	analysis of frames consisting of multiforce members
6.101, 102	6.103, 104	
6.*105*, *106*		
6.107, 108	6.109, 110	problems involving the solution of simultaneous equations
6.111, *112*	6.115, 116	
6.113, *114*		
6.119, *120*	6.117, 118	unusual floor systems
	6.*121*	
		Analysis of Machines
6.*124*, *125*	6.122, 123	toggle-type machines
6.126, 127		
6.128		
6.129, 130	6.131, 132	machines involving cranks
6.133, 134		
6.137, 138	6.*135*, *136*	machines involving a crank with a collar
6.139, 140		robotic machines
6.143, 144	6.*141*, 142	tongs
6.146, 148	6.145, *147*	pliers, boltcutters, pruning shears
	6.149, 150	find force to maintain position of toggle
6.151, 154		garden shears, force in hydraulic cylinder
6.*152*, *153*	6.155, 156	large mechanical equipment
	6.*157*, *158*	
6.159, 160	*6.*161*, **162*	gears and universal joints
*6.163		special tongs
6.*165*, 166	6.164, 168	Review problems
6.167, 170	6.*169*, 171	
6.172, *174*	6.*173*, 175	
6.C2, C4	6.C1, C3	Computer problems
6.C6	6.C5	

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Problem Number* SI Units	U.S. Units	Problem Description
		CHAPTER 7: FORCES IN BEAMS AND CABLES
		Internal forces in members
7.3, 4	7.1, 2	simple frames
7.*5*, *6*		
7.7, 8	7.9, 10	curved members
7.13, 14	7.*11*, *12*	
7.15, 16	7.19, 20	frames with pulleys or pipes
7.17, 18		
	7.*21*, *22*	effect of supports
7.23, *25*	7.24, 28	bending moment in circular rods due to their own weight
7.*26*, 27		
		BEAMS
		Shear and bending-moment diagrams using portions of beam as free bodies
7.29, 30	7.31, 32	problems involving no numerical values
	7.*33*, *34*	
7.35, 36	7.*37*, *38*	beams with concentrated loads
7.39, 40	7.41, 42	beams with mixed loads
7.43, 44	7.45, 46	beams resting on the ground
7.*47*, *48*		
7.49, *50*	7.52, 53	beams subjected to forces and couples
7.*51*, 54		
7.55, 56	7.58, 59	find value of parameter to minimize absolute value of bending moment
7.57, *62	7.*60*, *61*	
		Shear and bending-moment diagrams using relations among ω, V, and M
7.63, 64	7.65, 66	problems involving no numerical values
7.*67*, *68*		
7.69, 70	7.73, 74	problems involving numerical values
7.71, 72	7.*75*, *76*	
7.77, 78	7.79, 80	find magnitude and location of maximum bending moment
7.83, 84	7.*81*, *82*	
7.*87*, 88	7.85, 86	Determine V and M by integrating ω twice
7.89, 90	*7.91, *92	Find values of loads for which $\left\|M\right\|_{max}$ is as small as possible
		CABLES
		Cables with concentrated loads
7.93, 94	7.95, 96	vertical loads
7.97, 98	7.*99*, *100*	
7.103, 104	7.101, 102	horizontal and vertical loads
7.*105*, *106*		
		Parabolic cables
7.107, 108	7.109, 110	supports at the same elevation
7.*111*, *112*	7.113, 114	
7.117, 118	7.115, 116	supports at different elevations

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Problem Number*		
SI Units	**U.S. Units**	**Problem Description**
*7.119		Derive analogy between a beam and a cable
7.120, 121	7._122_, _123_	use analogy to solve previous problems
*7.124	*7.125, *126	Derive or use $d^2y/dx^2 = w(x)\ T_0$
		Catenary
7.127, 129	7.128, 130	given length of cable and sag or T_m, find span of cable
7._131_, _132_		
7.133, 135	7.134, 136	given span and length of cable, find sag and/or weight
7.139, 140	7._137_, _138_	given span, T_m, and w, find sag
7.143, 144	7.141, 142	given T_0, w, and sag or slope, find span or sag
7._145_, _146_	*7.147, *148	
*7.151, *152	7._149_, _*150_	special problems
*7.153		
7.154, 155	7.156, 157	Review problems
7.159, _161_	7._158_, 160	
7.164, _165_	7.162, _163_	
7.C2, C4	7.C1, C3	Computer problems
7.C5, C6		
		CHAPTER 8: FRICTION
8.3, 4	8.1, 2	For given loading, determine whether block is in equilibrium and find friction force
8.5, 6	8._8_, _10_	Find minimum force required to start, maintain, or prevent motion
8.7, 9		
8.11, 12		Analyze motion of several blocks
8._13_, _14_		
	8.15, 16	Sliding and/or tipping of a rigid body
8.19, 20	8.17, 18	Problems involving wheels and cylinders
8.21, _22_	8.23, _24_	Problems involving rods
8.26, 28	8.25, 27	Analysis of mechanisms with friction
8.29, _32_	8.30, 31	
8._33_		
8.36, 37	8.34, _35_	Analysis of more advanced rod and beam problems
8.38	8.39, _40_	
8.42, 43	8.41, 45	Analysis of systems with possibility of slippage for various loadings
8.44		
8.48, 49	8.46, 47	Wedges, introductory problems
8._50_, _51_	8.52	
8.53, 54		
8.55		
8.57, 60	8.56, _58_	Wedges, more advanced problems
8.61	8._59_, 62	
*8.64, *65	8.63	
8.66	8._67_, _68_	Square-threaded screws
8.69, 70	8.72, 73	
8._71_, _74_		

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Problem Number* SI Units	U.S. Units	Problem Description
8.76, 77	8.75, *81*	Axle friction
9.78, 79	8.82, *84*	
8.80, *83*	8.85, 86	
8.88, *89*	8.87	
8.91, *92	8.90, 95	Disk friction
*8.*93*, **94*		
8.96, 99	8.97, 98	Rolling resistance
8.100		
		Belt friction
8.102, 103	8.101, 104	belt passing over fixed drum
8.*105*, *106*		
8.107, 108	8.110, 111	transmission belts and band brakes
8.109, 112	8.*113*, *114*	
	8.115	
8.116, 117	8.*118*, *119*	advanced problems
8.122, 123	8.120, 121	
8.126, *127*	8.124, 125	
8.*130*, 131	8.128, 129	derivations, V belts
8.132, *134*	8.133, 136	Review problems
8.135, 139	8.*137*, 138	
8.142, *143*	8.140, *141*	
8.C1, C3	8.C2, C5	Computer problems
8.C4, C6	8.C7	
8.C8		

CHAPTER 9: DISTRIBUTED FORCES: MOMENTS OF INERTIA

MOMENTS OF INERTIA OF AREAS

SI Units	U.S. Units	Problem Description
		Find by direct integration
9.1, 3	9.2, 4	moments of inertia of an area
9.6, 7	9.*5*, *8*	
9.9, 10	9.11, 12	
9.*13*, *14*		
9.15, 17	9.16, 18	moments of inertia and radii of gyration of an area
9.*19*, 9.*20*		
9.21, 23	9.22, 25	polar moments of inertia and polar radii of gyration of an area
9.24, 28	9.*26*, *27*	
*9.29	*9.30	Special problems
		Parallel-axis theorem applied to composite areas to find
9.31, 33	9.32, 34	moment of inertia and radius of gyration
9.*35*, *36*		
9.37, 38	9.39, *40*	centroidal moment of inertia, given I or J
9.41, 42	9.43, 44	centroidal moments of inertia

* Problems which do not involve any specific system of units have been indicated by underlining their number. Answers are not given to problems with a number set in italic type.

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Problem Number* **SI Units**	**U.S. Units**	**Problem Description**
9.45, 46	9.*47, 48*	centroidal polar moment of inertia
		centroidal moments of inertia of composite areas consisting of rolled-steel shapes:
9.49, 51	9.50, 52	symmetrical composite areas
9.53, 55	9.*54*, *56*	singly-symmetrical composite areas (first locate centroid of area)
		Center of pressure
9.57, 58	9.59, *9.60	for end panel of a trough
9.*61*, 9.*62*		for a submerged vertical gate or cover
*9.64	*9.63	used to locate centroid of a volume
*9.65	*9.66	special problems

		Products of inertia of areas found by
9.67, 68	9.69, *70*	direct integration
9.71, 72	9.*73*, 74	parallel-axis theorem
9.75, 78	9.76, *77*	
		Using the equations defining the moments and products of inertia with respect to rotated axes to find
9.79, 80	9.81, 83	$I_{x'}$, $I_{y'}$, $I_{x'y'}$ for a given angle of rotation
9.*82*, *84*		
9.85, 86	9.87, 89	principal axes and principal moments of inertia
9.*88*, *90*		

		Using Mohr's circle to find
9.91, 92	9.93, 95	$I_{x'}$, $I_{y'}$, $I_{x'y'}$ for a given angle of rotation
9.*94*, *96*		
9.97, 98	9.99, 100	principal axes and principal moments of inertia
	9.*101*, *102*	
9.104, 105	9.103, *106	
9.107, 108	9.*109*, *110*	Special problems

MOMENTS OF INERTIA OF MASSES

		Mass moment of inertia
9.111, 114	9.112, 113	of thin plates: two-dimensions
9.115, 116	9.*117*, *118*	
9.119, 122	9.120, 121	of simple geometric shapes by direct single integration
9.*123*, *126*	*9.124, *125	
9.127	9.128	and radius of gyration of composite bodies
9.129, *130*	9.*132*, 133	special problems using the parallel-axis theorem
9.131, 134		
9.135, 136	9.137, 138	of bodies formed of sheet metal or of thin plates: three-dimensions
9.*139*, **140*		
9.141, 144	9.142, *143*	of machine elements and of bodies formed of homogeneous wire
9.145, 148	9.*146*, 147	
		Mass products of inertia
9.149, 150	9.151, 152	of machine elements
9.*153*, *154*		of bodies formed of sheet metal or of thin plates
9.155, 156		
9.157, 159	9.158, 160	of bodies formed of homogeneous wire
9.162	9.161	Derivation and special problem

* Problems which do not involve any specific system of units have been indicated by underlining their number. Answers are not given to problems with a number set in italic type.

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Problem Number* SI Units	U.S. Units	Problem Description
9.165, 166	9.*163*, *164*	Mass moment of inertia of a body with respect to a skew axis
9.169, 170	9.167, 168	
	9.*171*, *172*	
9.173, 174	9.175, 176	Ellipsoid of inertia and special problems
9.*177*, *178*		
*9.179, *180	*9.182, *183	Principal mass moments of inertia and principal axes of inertia
*9.*181*, **184*		
9.185, *186*	9.187, 188	Review problems
9.189, 190	9.191, *192*	
9.195, *196*	9.193, *194*	
9.C3, C5	9.C1, C2	Computer problems
*9.C7, *C8	9.C4, C6	
		CHAPTER 10: METHOD OF VIRTUAL WORK
		For linkages and simple machines, find
10.1, 3	10.2, 4	force or couple required for equilibrium (linear relations among displacements)
10.7, 8	10.*5*, *6*	
10.11, 12	10.9, 10	force required for equilibrium (trigonometric relations among displacements)
10.*13*, *14*		
10.15, 18	10.16, 17	couple required for equilibrium (trigonometric relations among displacements)
10.*19*, *20*	10.21, 22	force or couple required for equilibrium
		Find position of equilibrium
10.24, 25	10.*23*, *27*	for numerical values of loads
10.26, 28		
10.*29*, 32	10.30, 31	linear springs included in mechanism
10.33, *34*	10.36, 37	
10.35, 38		
10.39, 40		torsional spring included in mechanism
10.43, 44	10.*41*, *42*	Problems requiring the use of the law of cosines
	10.45, 46	
10.49, *50*	10.47, *48*	Problems involving the effect of friction
10.51, 52		
		Use method of virtual work to find:
10.53, 54		reactions of a beam
10.*55*	10.*56*	internal forces in a mechanism
	10.57, 58	movement of a truss joint
		Potential energy method used to
10.61, 63	10.*59*, 60	solve problems from Sec. 10.4
10.65, 66	10.*62*, 64	
10.67	10.68	establish that equilibrium is neutral
		find position of equilibrium and determine its stability for a system involving:
10.71, 72	10.69, *70*	gears and drums
10.73, 74		torsional springs
10.*77*, 79	10.*75*, *76*	linear springs

* Problems which do not involve any specific system of units have been indicated by underlining their number.
Answers are not given to problems with a number set in italic type.

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Problem Number* SI Units	U.S. Units	Problem Description
10.80, *82*	10.78, 81	
10.*87*, 88	10.85, 86	
10.83, *84*		two applied forces
		determine stability of a known position of equilibrium for a system with
10.92, 93	10.*89*, 90	one degree of freedom
10.94, 96	10.91, *95*	
*10.*97*, *98	*10.*99*, *100	two degrees of freedom
10.104, *105*	10.101, 102	Review problems
10.106, 110	10.*103*, 107	
10.*111*, 112	10.108, *109*	
10.C2, C3	10.C1, C4	Computer problems
10.C5, C6		
10.C7		

* Problems which do not involve any specific system of units have been indicated by underlining their number. Answers are not given to problems with a number set in italic type.

TABLE III: SAMPLE ASSIGNMENT SCHEDULE FOR A COURSE IN STATICS

This schedule includes all of the material of *VECTOR MECHANICS FOR ENGINEERS: STATICS* with the exception of Sections 3.21, 6.6, 7.7–7.10, 8.7–8.9, and 9.18.

50% OF THE PROBLEMS IN EACH OF THE FOLLOWING LISTS USE SI UNITS AND 50% U.S. CUSTOMARY UNITS

			ANSWERS TO ALL OF THESE PROBLEMS ARE GIVEN IN THE BACK OF THE BOOK				NO ANSWERS ARE GIVEN IN THE BOOK FOR ANY OF THESE PROBLEMS	
Period	Sections	Topics	List 1	List 2	List 3	List 4	List 5	List 6
1	1.1–6	Introduction						
2	2.1–8	Addition and Resolution of Forces	2.7, 15, 27, 37	2.9, 16, 26, 31	2.5, 19, 25, 35	2.11, 17, 30, 32	2.6, 20, 28, 38	2.12, 18, 29, 33
3	2.9–11	Equilibrium of a Particle	2.47, 52, 59, 69	2.48, 51, 63, 65	2.45, 53, 57, 68	2.43, 49, 60, 67	2.46, 54, 58, 70	2.44, 50, 64, 66
4	2.12–14	Forces in Space	2.74, 82, 89, 94	2.71, 81, 87, 97	2.76, 84, 85, 96	2.77, 79, 86, 95	2.78, 83, 90, 98	2.75, 80, 88, 93
5	2.15	Equilibrium in Space	2.103, 111, 114, 123	2.104, 107, 113, 121	2.99, 109, 116, 126	2.101, 105, 117, 125	2.100, 110, 119, 124	2.102, 106, 120, 122
6	3.1–8	Vector Product, Moment of a Force about a Point	3.1, 13, 21, 30	3.3, 12, 22, 33	3.5, 9, 24, 28	3.7, 10, 26, 32	3.8, 14, 25, 29	3.6, 11, 23, 34
7	3.9–11	Scalar Product, Moment of a Force about an Axis	3.39, 50, 59, *66	3.40, 47, 61, *65	3.41, 53, 55, *69	3.37, 51, 56, *68	3.42, 52, 62, *67	3.38, 54, 60, *64
8	3.12–16	Couples	3.70, 81, 87, 99	3.72, 82, 88, 95	3.71, 85, 89, 94	3.75, 83, 90, 93	3.74, 86, 91, 100	3.73, 84, 92, 96
9	3.17–20	Equivalent Systems of Forces	3.105, 111, 121, 127	3.106, 112, 122, 128	3.101, 108, 119, 129	3.102, 109, 120, 130	3.103, 117, 123, *132	3.107, 114, 126, *131
10		EXAM NUMBER ONE						
11	4.1–4	Equilibrium in Two Dimensions	4.2, 14, 19, 24	4.3, 11, 15, 23	4.1, 9, 17, 28	4.4, 12, 18, 21	4.7, 13, 16, 25	4.8, 10, 20, 22
12	4.1–5	Equilibrium in Two Dimensions	4.29, 41, 44, 53	4.33, 42, 47, 51	4.30, 36, 46, 57	4.34, 35, 43, 54	4.32, 40, 45, 56	4.31, 39, 48, 55
13	4.6–7	Two- and Three-Force Bodies	4.61, 74, 77, 87	4.62, 71, 79, 90	4.63, 70, 75, 88	4.64, 69, 81, 83	4.65, 73, 78, 89	4.68, 72, 76, 86
14	4.8–9	Equilibrium in Three Dimensions	4.91, 100, 105, 114	4.94, 99, 108, 113	4.92, 98, 106, 118	4.93, 97, 107, 117	4.95, 104, 111, 116	4.96, 103, 112, 115
15	4.8–9	Equilibrium in Three Dimensions	4.119, 128, 135	4.122, 127, 136	4.120, 130, 138	4.121, 129, 133	4.123, 132, 140	4.124, 131, 141
16	5.1–5	Centroids and First Moments	5.2, 14, 24, 28	5.1, 11, 20, 26	5.4, 12, 22, 30	5.3, 10, 21, 29	5.7, 15, 23, 31	5.8, 13, 25, 27
17	5.6–7	Centroids by Integration	5.37, 40, 58, 59	5.34, 46, 54, 61	5.35, 41, 52, 63	5.39, 44, 53, 64	5.38, 43, 55, *65	5.36, *47, 60, 62
18	5.8–9	Beams and Submerged Surfaces	5.67, 77, 80, 91	5.68, 76, 83, 90	5.66, 74, 81, 89	5.69, 78, 84, 88	5.70, 75, 86, 95	5.72, 79, 87, 92
19	5.10–12	Centroids of Volumes	5.104, 107, 117, 124	5.103, 106, 119, 123	5.101, 111, 114, 125	5.100, 108, 115, 122	5.102, 113, 120, 127	5.105, 109, 116, 126
20		EXAM NUMBER TWO						
21	6.1–5	Trusses: Method of Joints	6.1, 12, 23, 30	6.7, 11, 24, 27	6.2, 10, 21, 29	6.5, 9, 22, 28	6.4, 13, 25, 34	6.8, 14, 26, 33
22	6.7–8	Trusses: Method of Sections	6.43, 54, 68, 71	6.44, 53, 67, 70	6.45, 58, 65, 69	6.46, 57, 61, 72	6.48, 60, 64, 74	6.47, 59, 63, 73
23	6.9–11	Analysis of Frames	6.75, 89, 101, 110	6.76, 88, 102, 109	6.77, 83, 99, 108	6.78, 84, 100, 107	6.79, 85, 105, 114	6.80, 86, 106, 112
24	6.12	Analysis of Machines	6.122, 130, 145, 154	6.123, 134, 142, 151	6.127, 132, 144, 156	6.126, 131, 148, 155	6.124, 136, 141, 153	6.125, 135, 147, 152
25	6.1–12	Review of Chapter 6	6.20, 52, 91, 150	6.51, 92, 149, 166	6.15, 50, 93, 139	6.16, 49, 94, 128	6.17, 47, 97, 158	6.18, 48, 98, 157
26	7.1–2	Internal Forces in Members	7.1, 8, 20, 27	7.2, 14, 19, 23	7.4, 10, 16, 24	7.3, 9, 17, 28	7.5, 11, 21, 26	7.6, 12, 22, 25
27	7.3–5	Beams	7.39, 46, 54, 59	7.35, 45, 49, 58	7.41, 43, 53, 57	7.42, 44, 52, 56	7.38, 48, 50, 61	7.37, 47, 51, 60
28	7.6	Beams	7.65, 70, 79, 90	7.66, 69, 80, 89	7.63, 74, 77, 86	7.64, 73, 78, 85	7.67, 76, 81, 88	7.68, 75, 82, 87
29		EXAM NUMBER THREE						
30	8.1–4	Laws of Friction	8.5, 16, 26, 34	8.9, 15, 28, 39	8.1, 11, 30, 42	8.2, 12, 31, 38	8.8, 14, 32, 35	8.10, 13, 33, 40
31	8.5–6	Wedges and Screws	8.46, 57, 62, 70	8.47, 54, 63, 69	8.48, 56, 60, 73	8.49, 52, 61, 72	8.50, 58, 67, 71	8.51, 59, 68, 74
32	8.10	Belt Friction	8.101, 112, 120, 131	8.104, 109, 121, 126	8.102, 111, 116, 125	8.103, 110, 122, 124	8.105, 113, 118, 127	8.106, 114, 119, 130
33	9.1–5	Moments of Inertia of Areas	9.1, 12, 15, 25	9.7, 11, 17, 22	9.4, 10, 16, 24	9.2, 9, 18, 21	9.8, 13, 19, 27	9.5, 14, 20, 26
34	9.6–7	Composite Areas	9.32, 45, 50, 57	9.34, 41, 52, 58	9.33, 43, 49, 59	9.31, 44, 55, *60	9.35, 47, 56, 62	9.36, 48, 54, 61
35	9.8–9	Product of Inertia	9.75, 83, 86	9.71, 81, 85	9.76, 79, 87	9.74, 80, 89	9.77, 84, 88	9.73, 82, 90
36	9.10	Mohr's Circle	9.93, 103, 107	9.95, 104, *106	9.92, 100, 105	9.91, 99, 108	9.94, 102, 110	9.96, 101, 109
37	9.11–15	Moments of Inertia of Masses	9.111, 121, 135, 147	9.116, 120, 136, 142	9.113, 122, 137, 144	9.112, 119, 138, 141	9.117, 123, 139, 146	9.118, 126, *140, 143
38	9.16–17	Principal Axes	9.151, 159, 167, 174	9.152, 157, 168, 173	9.149, 158, 165, 176	9.150, 160, 166, 175	9.153, 164, 171, 178	9.154, 163, 172, 177
39		EXAM NUMBER FOUR						
40	10.1–5	Virtual Work	10.1, 10, 15, 22	10.8, 9, 18, 21	10.2, 12, 16, 26	10.4, 11, 17, 25	10.5, 14, 19, 27	10.6, 13, 20, 23
41	10.1–5	Virtual Work	10.30, 40, 46, 54	10.31, 38, 45, 53	10.32, 37, 43, 58	10.33, 36, 49, 57	10.29, 41, 48, 55	10.34, 42, 50, 56
42	10.6–9	Potential Energy	10.71, 81, 88, 90	10.74, 78, 83, 91	10.70, 80, 85, 92	10.69, 79, 86, 93	10.75, 82, 87, 95	10.76, 77, 84, 89

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TABLE IV: SAMPLE ASSIGNMENT SCHEDULE FOR A COURSE IN STATICS

This schedule includes all of the material of *VECTOR MECHANICS FOR ENGINEERS: STATICS* with the exception of Sections 3.21, 6.6, 7.7–7.10, 8.7–8.9, and 9.18.

75% OF THE PROBLEMS IN EACH OF THE FOLLOWING LISTS USE SI UNITS AND 25% U.S. CUSTOMARY UNITS

ANSWERS TO ALL OF THESE PROBLEMS ARE GIVEN IN THE BACK OF THE BOOK

Period	Sections	Topics	List 1a	List 2a	List 3a	List 4a
1	1.1–6	Introduction				
2	2.1–8	Addition and Resolution of Forces	2.7, 15, 27, 36	2.9, 16, 26, 34	2.8, 19, 25, 35	2.10, 17, 30, 32
3	2.9–11	Equilibrium of a Particle	2.47, 52, 61, 69	2.48, 51, 62, 65	2.45, 56, 57, 68	2.43, 55, 60, 67
4	2.12–14	Forces in Space	2.74, 82, 89, 92	2.71, 81, 87, 91	2.73, 84, 85, 96	2.72, 79, 86, 95
5	2.15	Equilibrium in Space	2.103, 111, 118, 123	2.104, 107, 115, 121	2.99, 112, 116, 126	2.101, 108, 117, 125
6	3.1–8	Vector Product, Moment of a Force about a Point	3.1, 13, 21, 27	3.3, 12, 22, 31	3.2, 9, 24, 28	3.4, 10, 26, 32
7	3.9–11	Scalar Product, Moment of a Force about an Axis	3.39, 50, 57, *66	3.40, 47, 58, *65	3.41, 48, 55, *69	3.37, 49, 56, *68
8	3.12–16	Couples	3.70, 81, 87, 97	3.72, 82, 88, 98	3.76, 85, 89, 94	3.80, 83, 90, 93
9	3.17–20	Equivalent Systems of Forces	3.105, 111, 124, 127	3.106, 112, 125, 128	3.101, 116, 119, 129	3.102, 115, 120, 130
10		EXAM NUMBER ONE				
11	4.1–4	Equilibrium in Two Dimensions	4.2, 14, 19, 27	4.3, 11, 15, 26	4.5, 9, 17, 28	4.6, 12, 18, 21
12	4.1–5	Equilibrium in Two Dimensions	4.29, 41, 50, 53	4.33, 42, 49, 51	4.30, 37, 46, 57	4.34, 38, 43, 54
13	4.6–7	Two- and Three-Force Bodies	4.61, 74, 77, 85	4.62, 71, 79, 84	4.66, 70, 75, 88	4.67, 69, 81, 83
14	4.8–9	Equilibrium in Three Dimensions	4.91, 100, 109, 114	4.94, 99, 110, 113	4.92, 102, 106, 118	4.93, 101, 107, 117
15	4.8–9	Equilibrium in Three Dimensions	4.119, 128, 135	4.122, 127, 136	4.125, 130, 138	4.126, 129, 133
16	5.1–5	Centroids and First Moments	5.2, 14, 24, 33	5.1, 11, 20, 32	5.6, 12, 22, 30	5.9, 10, 21, 29
17	5.6–7	Centroids by Integration	5.37, 40, 56, 59	5.34, 46, 57, 61	5.35, 42, 52, 63	5.39, 45, 53, 64
18	5.8–9	Beams and Submerged Surfaces	5.67, 77, 80, 94	5.68, 76, 83, 93	5.71, 74, 81, 89	5.73, 78, 84, 88
19	5.10–12	Centroids of Volumes	5.104, 107, 118, 124	5.103, 106, 121, 123	5.101, 112, 114, 125	5.100, 110, 115, 122
20		EXAM NUMBER TWO				
21	6.1–5	Trusses: Method of Joints	6.1, 12, 23, 32	6.7, 11, 24, 31	6.6, 10, 21, 29	6.3, 9, 22, 28
22	6.7–8	Trusses: Method of Sections	6.43, 54, 66, 71	6.44, 53, 62, 70	6.45, 56, 65, 69	6.46, 55, 61, 72
23	6.9–11	Analysis of Frames	6.75, 89, 101, 111	6.76, 88, 102, 113	6.81, 83, 99, 108	6.82, 84, 100, 107
24	6.12	Analysis of Machines	6.122, 130, 143, 154	6.123, 134, 146, 151	6.127, 137, 144, 156	6.126, 133, 148, 155
25	6.1–12	Review of Chapter 6	6.20, 52, 91, 129	6.51, 92, 140, 166	6.19, 50, 93, 139	6.49, 94, 128, 167
26	7.1–2	Internal Forces in Members	7.1, 8, 15, 27	7.2, 14, 18, 23	7.4, 13, 16, 24	7.3, 7, 17, 28
27	7.3–5	Beams	7.39, 46, 54, 55	7.35, 45, 49, 58	7.40, 43, 53, 57	7.36, 44, 52, 56
28	7.6	Beams	7.65, 70, 83, 90	7.66, 69, 84, 89	7.63, 72, 77, 86	7.64, 71, 78, 85
29		EXAM NUMBER THREE				
30	8.1–4	Laws of Friction	8.5, 16, 26, 37	8.9, 15, 28, 36	8.3, 11, 30, 42	8.6, 12, 31, 38
31	8.5–6	Wedges and Screws	8.46, 57, *64, 70	8.47, 54, *65, 69	8.48, 55, 60, 73	8.49, 53, 61, 72
32	8.10	Belt Friction	8.101, 112, 123, 131	8.104, 109, 117, 126	8.102, 108, 116, 125	8.103, 107, 122, 124
33	9.1–5	Moments of Inertia of Areas	9.1, 12, 15, 28	9.7, 11, 17, 23	9.6, 10, 16, 24	9.3, 9, 18, 21
34	9.6–7	Composite Areas	9.32, 45, 53, 57	9.34, 41, 51, 58	9.33, 42, 49, 59	9.31, 46, 55, *60
35	9.8–9	Product of Inertia	9.75, 83, 86	9.71, 81, 85	9.72, 79, 87	9.78, 80, 89
36	9.10	Mohr's Circle	9.93, 103, 107	9.95, 104, *106	9.92, 98, 105	9.91, 97, 108
37	9.11–15	Moments of Inertia of Masses	9.111, 121, 135, 148	9.116, 120, 136, 145	9.115, 122, 137, 144	9.114, 119, 138, 141
38	9.16–17	Principal Axes	9.151, 159, 170, 174	9.152, 157, 169, 173	9.149, 155, 165, 176	9.150, 156, 166, 175
39		EXAM NUMBER FOUR				
40	10.1–5	Virtual Work	10.1, 10, 15, 24	10.8, 9, 18, 28	10.3, 12, 16, 26	10.7, 11, 17, 25
41	10.1–5	Virtual Work	10.30, 40, 51, 54	10.31, 38, 44, 53	10.32, 35, 43, 58	10.33, 39, 49, 57
42	10.6–9	Potential Energy	10.71, 81, 88, 96	10.74, 78, 83, 94	10.72, 80, 85, 92	10.73, 79, 86, 93

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TABLE V: SAMPLE ASSIGNMENT SCHEDULE FOR A COMBINED COURSE IN STATICS AND DYNAMICS

This schedule is intended for a 4-semester-credit-hour course consisting of (1) the abridged Statics course defined in Table I of this manual and (2) the standard Dynamics course defined in Table I of the Dynamics Instructor's Manual. Note that the Statics portion of the course is designed to serve only as an introduction to Dynamics; a more complete coverage of Statics is recommended for curricula including courses in Mechanics of Materials (see Tables III and ff. of the Statics Manual).

			50% OF THE PROBLEMS IN EACH OF THE FOLLOWING LISTS USE SI UNITS AND 50% U.S. CUSTOMARY UNITS					
			ANSWERS TO ALL OF THESE PROBLEMS ARE GIVEN IN THE BACK OF THE BOOK				NO ANSWERS ARE GIVEN IN THE BOOK FOR ANY OF THESE PROBLEMS	
Period	Sections	Topics	List 1	List 2	List 3	List 4	List 5	List 6
1	1.1–6	Introduction						
2	2.1–8	Addition and Resolution of Forces	2.7, 15, 27, 37	2.9, 16, 26, 31	2.5, 19, 25, 35	2.11, 17, 30, 32	2.6, 20, 28, 38	2.12, 18, 29, 33
3	2.9–11	Equilibrium of a Particle	2.47, 52, 59, 69	2.48, 51, 63, 65	2.45, 53, 57, 68	2.43, 49, 60, 67	2.46, 54, 58, 70	2.44, 50, 64, 66
4	2.12–14	Forces in Space	2.74, 82, 89, 94	2.71, 81, 87, 97	2.76, 84, 85, 96	2.77, 79, 86, 95	2.78, 83, 90, 98	2.75, 80, 88, 93
5	2.15	Equilibrium in Space	2.103, 111, 114, 123	2.104, 107, 113, 121	2.99, 109, 116, 126	2.101, 105, 117, 125	2.100, 110, 119, 124	2.102, 106, 120, 122
6	3.1–8	Vector Product, Moment of a Force about a Point	3.1, 13, 21, 30	3.3, 12, 22, 33	3.5, 9, 24, 28	3.7, 10, 26, 32	3.8, 14, 25, 29	3.6, 11, 23, 34
7	3.9–11	Scalar Product, Moment of a Force about an Axis	3.39, 50, 59, *66	3.40, 47, 61, *65	3.41, 53, 55, *69	3.37, 51, 56, *68	3.42, 52, 62, *67	3.38, 54, 60, *64
8	3.12–16	Couples	3.70, 81, 87, 99	3.72, 82, 88, 95	3.71, 85, 89, 94	3.75, 83, 90, 93	3.74, 86, 91, 100	3.73, 84, 92, 96
9	3.17–20	Equivalent Systems of Forces	3.105, 111, 121, 127	3.106, 112, 122, 128	3.101, 108, 119, 129	3.102, 109, 120, 130	3.103, 117, 123, *132	3.107, 114, 126, *131
10		EXAM NUMBER ONE						
11	4.1–4	Equilibrium in Two Dimensions	4.2, 14, 19, 24	4.3, 11, 15, 23	4.1, 9, 17, 28	4.4, 12, 18, 21	4.7, 13, 16, 25	4.8, 10, 20, 22
12	4.1–5	Equilibrium in Two Dimensions	4.29, 41, 44, 53	4.33, 42, 47, 51	4.30, 36, 46, 57	4.34, 35, 43, 54	4.32, 40, 45, 56	4.31, 39, 48, 55
13	6.9–11	Analysis of Frames	6.75, 89, 101, 110	6.76, 88, 102, 109	6.77, 83, 99, 108	6.78, 84, 100, 107	6.79, 85, 105, 114	6.80, 86, 106, 112
14	6.12	Analysis of Machines	6.122, 130, 145, 154	6.123, 134, 142, 151	6.127, 132, 144, 156	6.126, 131, 148, 155	6.124, 136, 141, 153	6.125, 135, 147, 152
15	4.8–9	Equilibrium in Three Dimensions	4.91, 100, 105, 114	4.94, 99, 108, 113	4.92, 98, 106, 118	4.93, 97, 107, 117	4.95, 104, 111, 116	4.96, 103, 112, 115
16	4.8–9	Equilibrium in Three Dimensions	4.119, 128, 135	4.122, 127, 136	4.120, 130, 138	4.121, 129, 133	4.123, 132, 140	4.124, 131, 141
17	8.1–4	Laws of Friction	8.5, 16, 26, 34	8.9, 15, 28, 39	8.1, 11, 30, 42	8.2, 12, 31, 38	8.8, 14, 32, 35	8.10, 13, 33, 40
18	8.5–6	Wedges and Screws	8.46, 57, 62, 70	8.47, 54, 63, 69	8.48, 56, 60, 73	8.49, 52, 61, 72	8.50, 58, 67, 71	8.51, 59, 68, 74
19		EXAM NUMBER TWO						
20	11.2–3	Rectilinear Motion	11.1, 12, 17, 24	11.2,11,18,23	11.3,10,15,22	11.4,9,16,21	11.7,14,19,30	11.8,13,20,29
21	11.4–6	Uniformly Accelerated Motion	11.33, 40, 49, 56	11.34,39,50,55	11.35,42,47,58	11.36,41,48,57	11.37,46,51,60	11.38,45,52,59
22	11.9–12	Curvilinear Motion (Rect. Comps)	11.97, 108, 117, 128	11.98,105,118,127	11.99,108,119,126	11.100,107,120,125	11.103,110,121,132	11.104,109,122,131
23	11.13–14	Curvilinear Motion (Other Comp.)	11.139, 144, 156, 166	11.141,143,157,165	11.137,146,153,170	11.138,145,154,169	11.140,150.159,168	12.142,149,160,167
24	12.1–6	Equations of Motion	12.5, 19, 28, 47	12.6, 17, 29, 47	12.7, 21, 30, 45	8, 20, 31, 44	12.13, 23, 32, 54	12.14, 22, 33, 52
25	12.7–10	Angular Momentum	12.57, 67, 80, 88	12.58, 66, 81, 87	12.55, 69, 78, 91	12.56, 68, 79, 90	12.59, 73, 83, 93	12.69, 72, 85, 92
26		Review						
27		EXAM NUMBER THREE						
28	13.1–5	Work and Energy, Power	13.7, 16, 28, 42	13.8, 15, 29.41	13.5, 18, 26, 40	13.6, 17, 27, 39	13.11, 20, 34, 48	13.12, 19, 37, 47
29	13.6–8	Conservation of Energy	13.58, 63, 72, 80	13.59, 62, 73, 79	13.55, 65, 70, 82	13.56, 64, 71, 81	13.60, 67, 76, 84	13.61, 66, 77, 83
30	13.9	Application to Space Mechanics	13.85, 96, 100, 110	13.86, 95, 101, 109	13.87, 94, 102, 107	13.88, 93, 103, 106	13.91, 99, 104, 113	13.92, 98, 105, 112
31	13.10–11	Impulse and Momentum	13. 119, 130, 139, 150	13. 122, 129, 140, 148	13. 120, 132, 141, 146	13.121, 131, 142, 145	13.125, 138, 143, 154	13.128, 133, 144, 153
32	13.12–15	Impact	13.157, 165, 178, 185	13.158, 164, 179, 184	13.155, 168, 174, 188	13.156, 167, 175, 186	13.161, 172, 181, 189	13.162, 171182, 187
33	14.1–6	Systems of Particles	14.1, 14, 17, 22	14.2, 13, 18, 21	14.3, 10, 15, 24	14.4, 9, 16, 23	14.5, 12, 19, 26	14.6, 11, 20, 25
34	14.7–9	Systems of Particles	14. 33, 38, 49, 52	14. 34, 37, 50, 51	14.31, 40, 45, 54	14.32, 39, 46, 53	14.35, 44, 47, 56	14.36, 43, 48, 55
35		EXAM NUMBER FOUR						
36	15.1–4	Translation, Rotation	15.1, 11, 18, 29	15.2, 10, 19, 28	15.3, 13, 21, 31	15.4, 12, 22, 30	15.7, 15, 20, 35	15.8, 14, 23, 34
37	15.5–6	General Plane Motion	15.38, 51, 55, 65	15.39, 50, 56, 64	15.40, 49, 57, 69	15.41, 48, 58, 68	15.42, 54, 59, 67	15.43, 53, 62, 66
38	15.7	Instantaneous Center	15.75, 85, 89, 98	15.76, 84, 90, 97	15.73, 83, 92, 96	15.74, 82, 93, 95	15.80, 87, 88, 104	15.81, 86, 91, 101
39	15.8–9	Acceleration in Plane Motion	15.115, 121, 127, 139	15.116, 120, 128, 138	15.112, 122, 129, 141	15.113, 122, 130, 140	15.114, 124, 131, 146	15.117, 123, 134, 145
40	15.10–11	Coriolis Acceleration in Plane Motion	15.151, 163, 168, 178	15.152, 162, 169, 177	15.150, 161, 166, 175	15.153, 160, 167, 174	15.154, 159, 172, 180	15.155, 158, 173, 179
41	9.11–15	Moments of Inertia of Masses	9.111, 121, 135, 147	9.116, 120, 136, 142	9.113, 122, 137, 144	9.112, 119, 138, 141	9.117, 123, 139, 146	9.118, 126, *140, 143
42		Review						
43		EXAM NUMBER FIVE						
44	16.1–7	Plane Motion of Rigid Bodies	16.1, 20, 27, 42	16.2, 17, 28, 41	16.3, 16, 25, 40	16.4, 14, 34, 39	16.9, 18, 32, 44	16.10, 15, 33, 43
45	16.1–7	Plane Motion of Rigid Bodies	16.47, 56, 64, 72	16.48, 55, 65, 71	16.50, 58, 63, 70	16.51, 57, 63, 69	16.53, 62, 68, 74	16.54, 61, 68, 73
46	16.8	Constrained Plane Motion	16.78, 85, 102, 110	16.79, 84, 105, 109	16.76, 87, 98, 107	16.77, 86, 100, 106	16.80, 91, 103, 115	16.83, 90, 104, 114
47	16.8	Constrained Plane Motion	16.119, 125, 133, 143	16.121, 124, 136, 142	16.117, 128, 134, 147	16. 118, 127, 135, 146	16.122, 132, 137, 150	16.123, 131, 140, 149
48	17.1–7	Work and Energy	17.4, 20, 24, 40	17.5, 18, 26, 39	17.8, 17, 28, 37	17.11, 16, 29, 36	17.14, 22, 27, 43	17.15, 21, 32, 41
49	17.8–10	Impulse and Momentum	17.54, 71, 79, 89	17.59, 69, 84, 88	17.52, 72, 77, 95	17.60, 70, 81, 90	17.55, 76, 82, 93	17.58, 73, 85, 91
50	17.11–12	Eccentric Impact	17.98, 106, 114, 122	17.100, 105, 117, 121	17.96, 108, 115, 124	17.97, 107, 116, 123	17.103, 111, 118, 130	17.104, 109, 119, 129
51		Review						
52		EXAM NUMBER SIX						
53	19.1–3	Free Vibrations of Particles	19.3, 13, 17, 26	19.5, 12, 18, 25	19.4, 14, 19, 24	19.6, 11, 20, 23	19.8, 16, 21, 29	19.10, 15, 22, 28
54	19.5	Free Vibrations of Rigid Bodies	19.38, 47, 56, 65	19.40, 44, 58, 63	19.37, 46, 55, 66	19.39, 45, 57, 64	19.41, 51, 60, 68	19.42, 48, 62, 67
55	19.6	Energy Methods	19.69, 79, 84, 92	19.71, 78, 85, 90	19.70, 80, 83, 91	19.72, 77, 88, 89	19.74, 82, 86, 95	19.75, 81, 87, 93
56	19.7	Forced Vibrations	19.100, 106, 113, 122	19.101, 105, 115, 121	19.99, 104, 114, 124	19.102, 107, 116, 123	19.103, 111, 117, 126	19.104, 110, 119, 125

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CHAPTER 6

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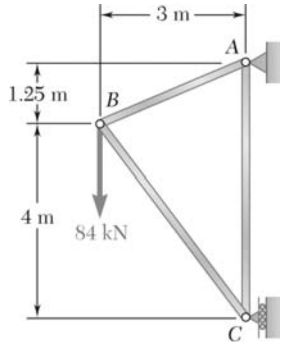
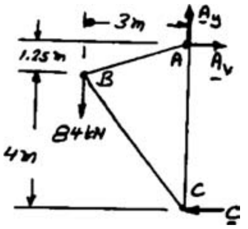


PROBLEM 6.1

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION



$$AB = \sqrt{3^2 + 1.25^2} = 3.25\text{ m}$$

$$BC = \sqrt{3^2 + 4^2} = 5\text{ m}$$

Reactions:

$$+\circlearrowleft \Sigma M_A = 0: \quad (84\text{ kN})(3\text{ m}) - C(5.25\text{ m}) = 0$$

$$\mathbf{C} = 48\text{ kN} \longleftarrow$$

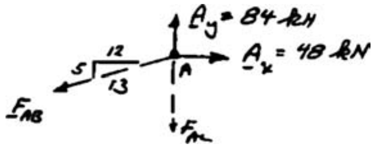
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - C = 0$$

$$\mathbf{A}_x = 48\text{ kN} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y = 84\text{ kN} = 0$$

$$\mathbf{A}_y = 84\text{ kN} \uparrow$$

Joint A:



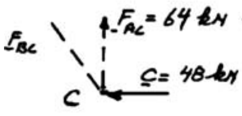
$$\xrightarrow{+} \Sigma F_x = 0: \quad 48\text{ kN} - \frac{12}{13}F_{AB} = 0$$

$$F_{AB} = +52\text{ kN} \qquad F_{AB} = 52\text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 84\text{ kN} - \frac{5}{13}(52\text{ kN}) - F_{AC} = 0$$

$$F_{AC} = +64.0\text{ kN} \qquad F_{AC} = 64.0\text{ kN} \quad T \blacktriangleleft$$

Joint C:



$$\frac{F_{BC}}{5} = \frac{48\text{ kN}}{3} \qquad F_{BC} = 80.0\text{ kN} \quad C \blacktriangleleft$$

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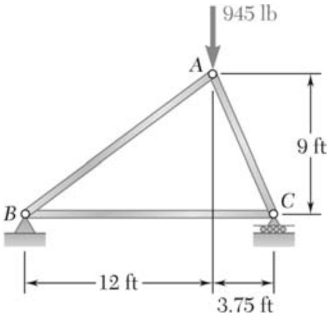
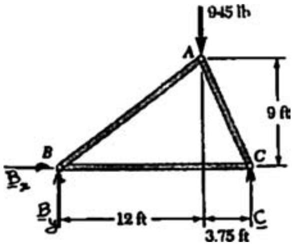


PROBLEM 6.2

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Entire truss



$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

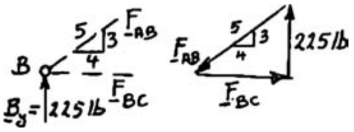
$$+\circlearrowleft \Sigma M_B = 0: \quad C(15.75 \text{ ft}) - (945 \text{ lb})(12 \text{ ft}) = 0$$

$$\mathbf{C} = 720 \text{ lb} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + 720 \text{ lb} - 945 \text{ lb} = 0$$

$$\mathbf{B}_y = 225 \text{ lb} \uparrow$$

Free body: Joint *B*:

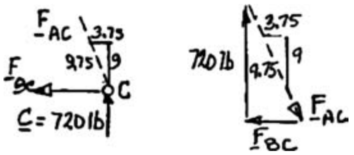


$$\frac{F_{AB}}{5} = \frac{F_{BC}}{4} = \frac{225 \text{ lb}}{3}$$

$F_{AB} = 375 \text{ lb} \quad C$ ◀

$F_{BC} = 300 \text{ lb} \quad T$ ◀

Free body: Joint *C*:



$$\frac{F_{AC}}{9.75} = \frac{F_{BC}}{3.75} = \frac{720 \text{ lb}}{9}$$

$F_{AC} = 780 \text{ lb} \quad C$ ◀

$F_{BC} = 300 \text{ lb} \quad T \quad$ (Checks)

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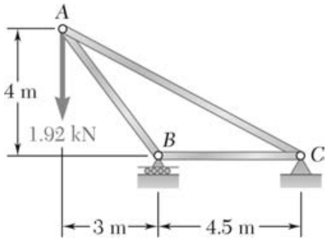
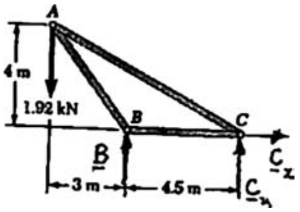


PROBLEM 6.3

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Entire truss



$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0 \quad \mathbf{C}_x = 0$$

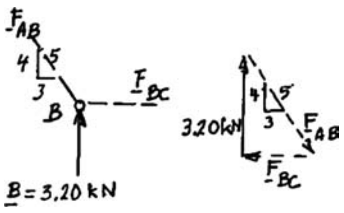
$$+\circlearrowleft \Sigma M_B = 0: \quad (1.92 \text{ kN})(3 \text{ m}) + C_y(4.5 \text{ m}) = 0$$

$$C_y = -1.28 \text{ kN} \quad \mathbf{C}_y = 1.28 \text{ kN} \downarrow$$

$$+\uparrow \Sigma F_y = 0: \quad B - 1.92 \text{ kN} - 1.28 \text{ kN} = 0$$

$$\mathbf{B} = 3.20 \text{ kN} \uparrow$$

Free body: Joint B:

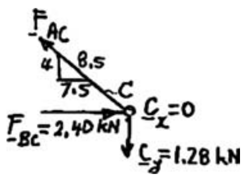


$$\frac{F_{AB}}{5} = \frac{F_{BC}}{3} = \frac{3.20 \text{ kN}}{4}$$

$$F_{AB} = 4.00 \text{ kN} \quad C \blacktriangleleft$$

$$F_{BC} = 2.40 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint C:



$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{7.5}{8.5} F_{AC} + 2.40 \text{ kN} = 0$$

$$F_{AC} = +2.72 \text{ kN} \qquad F_{AC} = 2.72 \text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = \frac{4}{8.5}(2.72 \text{ kN}) - 1.28 \text{ kN} = 0 \text{ (Checks)}$$

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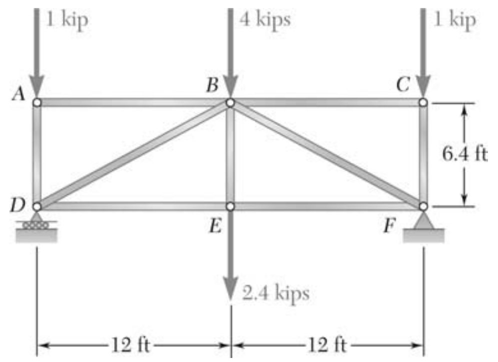
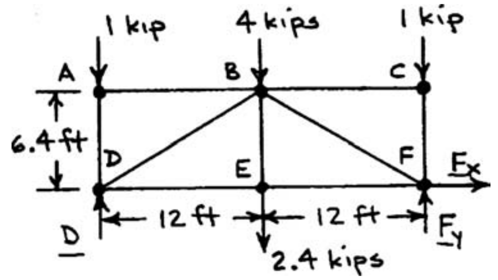


PROBLEM 6.4

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Reactions:



$$+\circlearrowright \Sigma M_D = 0: \quad F_y(24) - (4+2.4)(12) - (1)(24) = 0$$

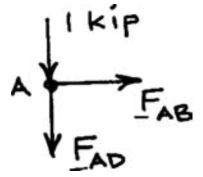
$$\mathbf{F}_y = 4.2 \text{ kips} \uparrow$$

$$\Sigma F_x = 0: \quad \mathbf{F}_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad D - (1+4+1+2.4) + 4.2 = 0$$

$$\mathbf{D} = 4.2 \text{ kips} \uparrow$$

Joint A:

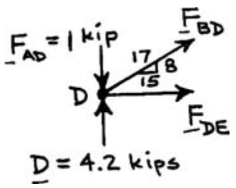


$$\Sigma F_x = 0: \quad F_{AB} = 0 \qquad F_{AB} = 0 \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -1 - F_{AD} = 0$$

$$F_{AD} = -1 \text{ kip} \qquad F_{AD} = 1.000 \text{ kip} \quad C \blacktriangleleft$$

Joint D:



$$+\uparrow \Sigma F_y = 0: \quad -1 + 4.2 + \frac{8}{17}F_{BD} = 0$$

$$F_{BD} = -6.8 \text{ kips} \qquad F_{BD} = 6.80 \text{ kips} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{15}{17}(-6.8) + F_{DE} = 0$$

$$F_{DE} = +6 \text{ kips} \qquad F_{DE} = 6.00 \text{ kips} \quad T \blacktriangleleft$$

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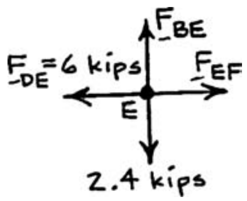
PROBLEM 6.4 (Continued)

Joint E:



$$+\uparrow \Sigma F_y = 0: \quad F_{BE} - 2.4 = 0$$

$$F_{BE} = +2.4 \text{ kips} \qquad F_{BE} = 2.40 \text{ kips} \quad T \blacktriangleleft$$

Truss and loading symmetrical about ℄

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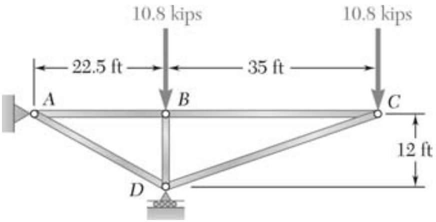
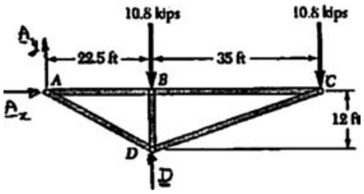


PROBLEM 6.5

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss



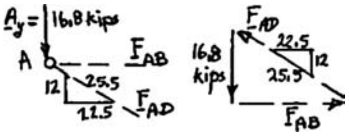
$\xrightarrow{+} \Sigma F_x = 0: \quad \mathbf{A}_x = 0$

$+\circlearrowleft \Sigma M_A = 0: \quad D(22.5) - (10.8 \text{ kips})(22.5) - (10.8 \text{ kips})(57.5) = 0$

$\mathbf{D} = 38.4 \text{ kips} \uparrow$

$\Sigma F_y = 0: \quad \mathbf{A}_y = 16.8 \text{ kips} \downarrow$

Free body: Joint A:

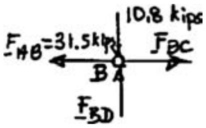


$$\frac{F_{AB}}{22.5} = \frac{F_{AD}}{25.5} = \frac{16.8 \text{ kips}}{12}$$

$F_{AB} = 31.5 \text{ kips} \quad T$ ◀

$F_{AD} = 35.7 \text{ kips} \quad C$ ◀

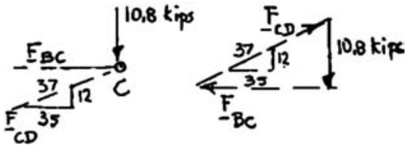
Free body: Joint B:



$\Sigma F_x = 0: \qquad F_{BC} = 31.5 \text{ kips} \quad T$ ◀

$\Sigma F_y = 0: \qquad F_{BD} = 10.80 \text{ kips} \quad C$ ◀

Free body: Joint C:



$$\frac{F_{CD}}{37} = \frac{F_{BC}}{35} = \frac{10.8 \text{ kips}}{12}$$

$F_{CD} = 33.3 \text{ kips} \quad C$ ◀

$F_{BC} = 31.5 \text{ kips} \quad T \quad \text{(Checks)}$

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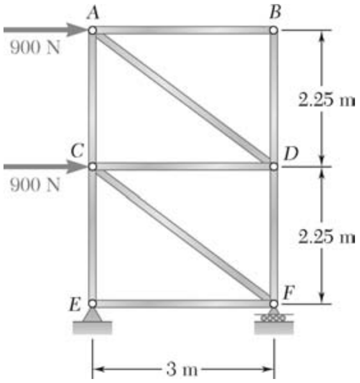
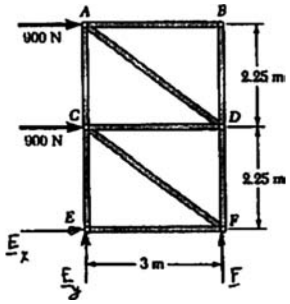


PROBLEM 6.6

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free Body: Truss



$$+\circlearrowleft \Sigma M_E = 0: \quad F(3\text{ m}) - (900\text{ N})(2.25\text{ m}) - (900\text{ N})(4.5\text{ m}) = 0$$

$$\mathbf{F} = 2025\text{ N}\uparrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad E_x + 900\text{ N} + 900\text{ N} = 0$$

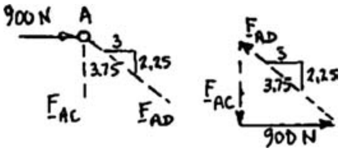
$$E_x = -1800\text{ N} \quad \mathbf{E}_x = 1800\text{ N} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad E_y + 2025\text{ N} = 0$$

$$E_y = -2025\text{ N} \quad \mathbf{E}_y = 2025\text{ N}\downarrow$$

We note that AB and BD are zero-force members: $F_{AB} = F_{BD} = 0$ ◀

Free body: Joint A:

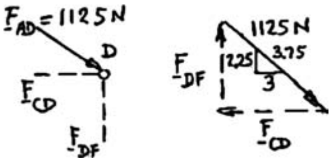


$$\frac{F_{AC}}{2.25} = \frac{F_{AD}}{3.75} = \frac{900\text{ N}}{3}$$

$F_{AC} = 675\text{ N}$ T ◀

$F_{AD} = 1125\text{ N}$ C ◀

Free body: Joint D:

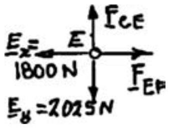


$$\frac{F_{CD}}{3} = \frac{F_{DE}}{2.23} = \frac{1125\text{ N}}{3.75}$$

$F_{CD} = 900\text{ N}$ T ◀

$F_{DF} = 675\text{ N}$ C ◀

Free body: Joint E:



$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{EF} - 1800\text{ N} = 0$$

$F_{EF} = 1800\text{ N}$ T ◀

$$+\uparrow \Sigma F_y = 0: \quad F_{CE} - 2025\text{ N} = 0$$

$F_{CE} = 2025\text{ N}$ T ◀

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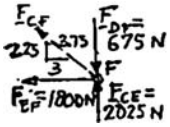
PROBLEM 6.6 (Continued)

Free body: Joint F:



$$+\!\uparrow \Sigma F_y = 0: \quad \frac{2.25}{3.75} F_{CF} + 2025\text{ N} - 675\text{ N} = 0$$

$$F_{CF} = -2250\text{ N} \qquad F_{CF} = 2250\text{ N} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = -\frac{3}{3.75}(-2250\text{ N}) - 1800\text{ N} = 0 \quad \text{(Checks)}$$

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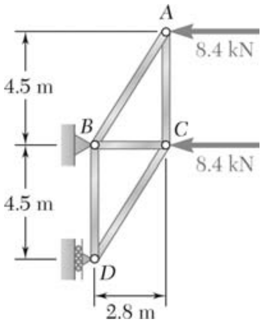
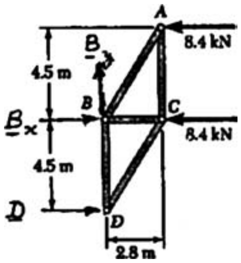


PROBLEM 6.7

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

<u>Free body: Truss</u>



$$+\uparrow \Sigma F_y = 0: \quad \mathbf{B}_y = 0$$

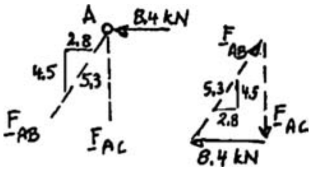
$$+\circlearrowleft \Sigma M_B = 0: \quad D(4.5 \text{ m}) + (8.4 \text{ kN})(4.5 \text{ m}) = 0$$

$$D = -8.4 \text{ kN} \qquad \mathbf{D} = 8.4 \text{ kN} \longleftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x - 8.4 \text{ kN} - 8.4 \text{ kN} - 8.4 \text{ kN} = 0$$

$$B_x = +25.2 \text{ kN} \quad \mathbf{B}_x = 25.2 \text{ kN} \longrightarrow$$

<u>Free body: Joint *A*:</u>

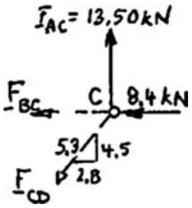


$$\frac{F_{AB}}{5.3} = \frac{F_{AC}}{4.5} = \frac{8.4 \text{ kN}}{2.8}$$

$$F_{AB} = 15.90 \text{ kN} \quad C \blacktriangleleft$$

$$F_{AC} = 13.50 \text{ kN} \quad T \blacktriangleleft$$

<u>Free body: Joint *C*:</u>



$$+\uparrow \Sigma F_y = 0: \quad 13.50 \text{ kN} - \frac{4.5}{5.3} F_{CD} = 0$$

$$F_{CD} = +15.90 \text{ kN} \qquad F_{CD} = 15.90 \text{ kN} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{BC} - 8.4 \text{ kN} - \frac{2.8}{5.3}(15.90 \text{ kN}) = 0$$

$$F_{BC} = -16.80 \text{ kN} \qquad F_{BC} = 16.80 \text{ kN} \quad C \blacktriangleleft$$

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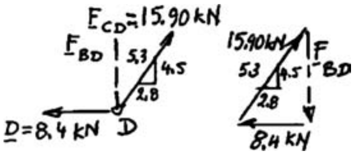
PROBLEM 6.7 (Continued)

Free body: Joint D:



$$\frac{F_{BD}}{4.5} = \frac{8.4 \text{ kN}}{2.8}$$

$F_{BD} = 13.50 \text{ kN} \quad C$ ◀

We can also write the proportion

$$\frac{F_{BD}}{4.5} = \frac{15.90 \text{ kN}}{5.3}$$

$F_{BD} = 13.50 \text{ kN} \quad C$ (Checks)

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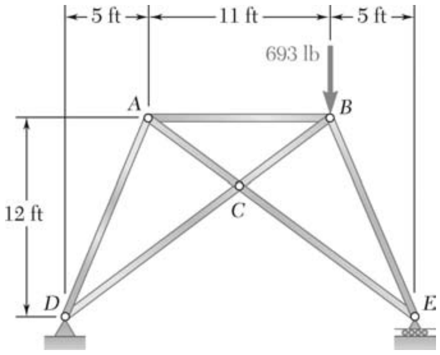
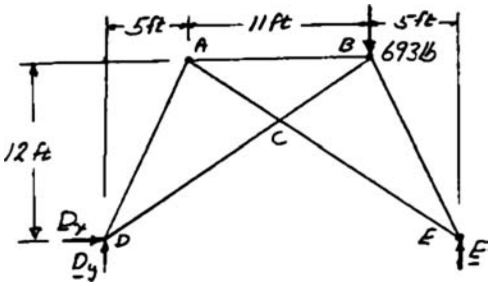


PROBLEM 6.8

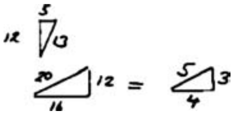
Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION



$$AD = \sqrt{5^2 + 12^2} = 13 \text{ ft}$$

$$BCD = \sqrt{12^2 + 16^2} = 20 \text{ ft}$$



Reactions: $\Sigma F_x = 0: \quad D_x = 0$

$+\circlearrowleft \Sigma M_E = 0: \quad D_y(21\text{ ft}) - (693\text{ lb})(5\text{ ft}) = 0 \qquad \mathbf{D}_y = 165\text{ lb} \uparrow$

$+\uparrow \Sigma F_y = 0: \quad 165\text{ lb} - 693\text{ lb} + E = 0 \qquad \mathbf{E} = 528\text{ lb} \uparrow$

Joint D: $\xleftarrow{+} \Sigma F_x = 0: \quad \frac{5}{13}F_{AD} + \frac{4}{5}F_{DC} = 0 \qquad (1)$

$+\uparrow \Sigma F_y = 0: \quad \frac{12}{13}F_{AD} + \frac{3}{5}F_{DC} + 165\text{ lb} = 0 \qquad (2)$

Solving (1) and (2), simultaneously:

$F_{AD} = -260\text{ lb} \qquad F_{AD} = 260\text{ lb} \quad C$ ◀

$F_{DC} = +125\text{ lb} \qquad F_{DC} = 125\text{ lb} \quad T$ ◀

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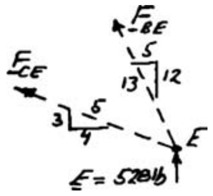
PROBLEM 6.8 (Continued)

Joint E:



$$\overset{+}{\leftarrow} \Sigma F_x = 0: \quad \frac{5}{13}F_{BE} + \frac{4}{5}F_{CE} = 0 \tag{3}$$

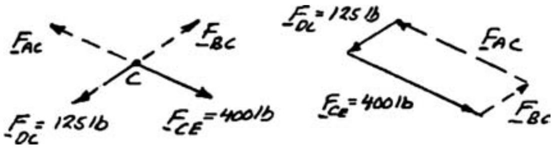
$$+\!\uparrow \Sigma F_y = 0: \quad \frac{12}{13}F_{BE} + \frac{3}{5}F_{CE} + 528 \text{ lb} = 0 \tag{4}$$

Solving (3) and (4), simultaneously:

$$F_{BE} = -832 \text{ lb} \qquad F_{BE} = 832 \text{ lb} \quad C \blacktriangleleft$$

$$F_{CE} = +400 \text{ lb} \qquad F_{CE} = 400 \text{ lb} \quad T \blacktriangleleft$$

Joint C:



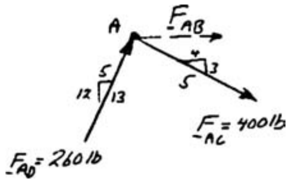
Force polygon is a parallelogram (see Fig. 6.11 p. 209)

$$F_{AC} = 400 \text{ lb} \quad T \blacktriangleleft$$

$$F_{BC} = 125 \text{ lb} \quad T \blacktriangleleft$$

Joint A:

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad \frac{5}{13}(260 \text{ lb}) + \frac{4}{5}(400 \text{ lb}) + F_{AB} = 0$$



$$F_{AB} = -420 \text{ lb} \qquad F_{AB} = 420 \text{ lb} \quad C \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad \frac{12}{13}(260 \text{ lb}) - \frac{3}{5}(400 \text{ lb}) = 0$$

$$0 = 0 \quad \text{(Checks)}$$

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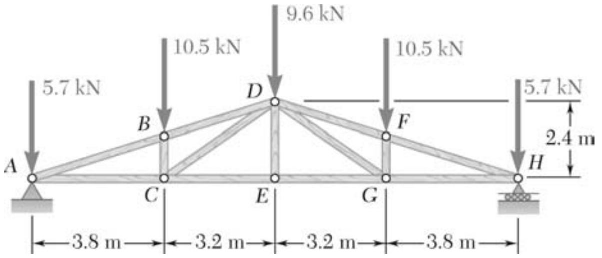
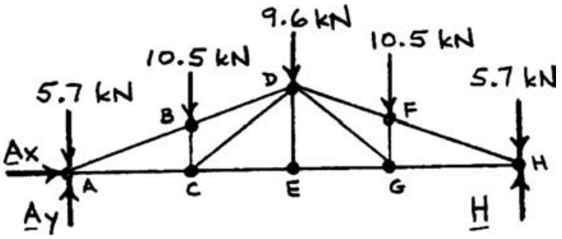


PROBLEM 6.9

Determine the force in each member of the Pratt roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss

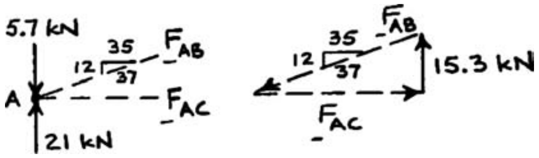


$$\Sigma F_x = 0: \quad A_x = 0$$

Due to symmetry of truss and load

$$A_y = H = \frac{1}{2} \text{ total load} = 21 \text{ kN}\uparrow$$

Free body: Joint A:

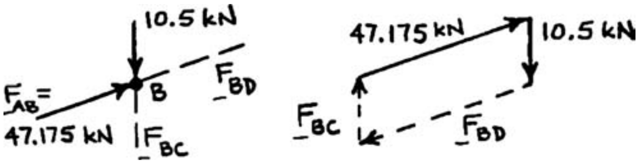


$$\frac{F_{AB}}{37} = \frac{F_{AC}}{35} = \frac{15.3 \text{ kN}}{12}$$

$F_{AB} = 47.175 \text{ kN} \quad F_{AC} = 44.625 \text{ kN}$ $\qquad F_{AB} = 47.2 \text{ kN} \quad C$ ◀

$F_{AC} = 44.6 \text{ kN} \quad T$ ◀

Free body: Joint B:



From force polygon: $\quad F_{BD} = 47.175 \text{ kN}, \quad F_{BC} = 10.5 \text{ kN}$ $\qquad F_{BC} = 10.50 \text{ kN} \quad C$ ◀

$F_{BD} = 47.2 \text{ kN} \quad C$ ◀

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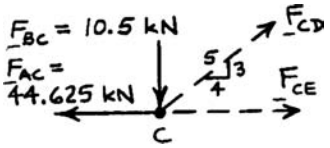
PROBLEM 6.9 (Continued)

Free body: Joint *C*:

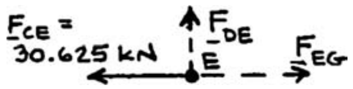


$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{CD} - 10.5 = 0$ $\qquad F_{CD} = 17.50 \text{ kN} \quad T$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} + \frac{4}{5}(17.50) - 44.625 = 0$

$F_{CE} = 30.625 \text{ kN} \qquad F_{CE} = 30.6 \text{ kN} \quad T$ ◀

Free body: Joint *E*:



DE is a zero-force member $\qquad F_{DE} = 0$ ◀

Truss and loading symmetrical about ℄

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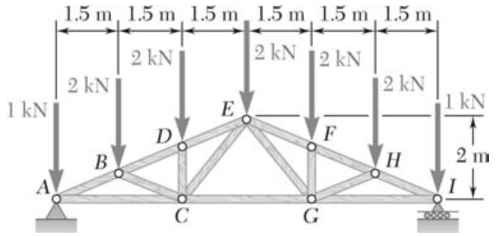
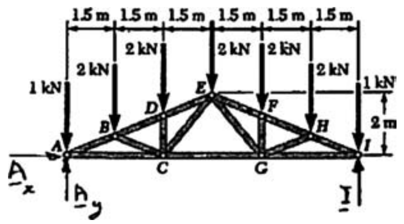


PROBLEM 6.10

Determine the force in each member of the fan roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss



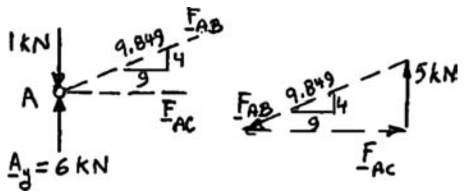
$$\Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

From symmetry of truss and loading:

$$\mathbf{A}_y = \mathbf{I} = \frac{1}{2} \quad \text{Total load}$$

$$\mathbf{A}_y = \mathbf{I} = 6 \text{ kN}\uparrow$$

Free body: Joint A:

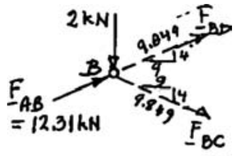


$$\frac{F_{AB}}{9.849} = \frac{F_{AC}}{9} = \frac{5 \text{ kN}}{4}$$

$F_{AB} = 12.31 \text{ kN} \quad C$ ◀

$F_{AC} = 11.25 \text{ kN} \quad T$ ◀

Free body: Joint B:



$$\xrightarrow{+} \Sigma F_x = \frac{9}{9.849}(12.31 \text{ kN} + F_{BD} + F_{DC}) = 0$$

or $\quad F_{BD} + F_{BC} = -12.31 \text{ kN} \quad (1)$

$$+\uparrow \Sigma F_y = \frac{4}{9.849}(12.31 \text{ kN} + F_{BD} - F_{BC}) - 2 \text{ kN} = 0$$

or $\quad F_{BD} - F_{BC} = -7.386 \text{ kN} \quad (2)$

Add (1) and (2): $\quad 2F_{BD} = -19.70 \text{ kN} \qquad F_{BD} = 9.85 \text{ kN} \quad C$ ◀

Subtract (2) from (1): $\quad 2F_{BC} = -4.924 \text{ kN} \qquad F_{BC} = 2.46 \text{ kN} \quad C$ ◀

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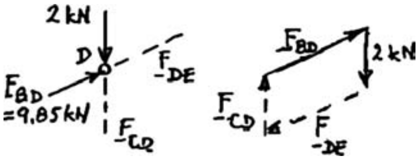
PROBLEM 6.10 (Continued)

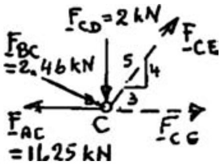
Free body: Joint D:



From force polygon: $F_{CD} = 2.00 \text{ kN} \quad C$ ◀

$F_{DE} = 9.85 \text{ kN} \quad C$ ◀

Free body: Joint C:



$$+\uparrow \Sigma F_y = \frac{4}{5}F_{CE} - \frac{4}{9.849}(2.46 \text{ kN}) - 2 \text{ kN} = 0$$

$F_{CE} = 3.75 \text{ kN} \quad T$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CG} + \frac{3}{5}(3.75 \text{ kN}) + \frac{9}{9.849}(2.46 \text{ kN}) - 11.25 \text{ kN} = 0$$

$$F_{CG} = +6.75 \text{ kN}$$

$F_{CG} = 6.75 \text{ kN} \quad T$ ◀

From the symmetry of the truss and loading:

$F_{EF} = F_{DE}$ $\quad F_{EF} = 9.85 \text{ kN} \quad C$ ◀

$F_{EG} = F_{CE}$ $\quad F_{EG} = 3.75 \text{ kN} \quad T$ ◀

$F_{FG} = F_{CD}$ $\quad F_{FG} = 2.00 \text{ kN} \quad C$ ◀

$F_{FH} = F_{BD}$ $\quad F_{FH} = 9.85 \text{ kN} \quad C$ ◀

$F_{GH} = F_{BC}$ $\quad F_{GH} = 2.46 \text{ kN} \quad C$ ◀

$F_{GI} = F_{AC}$ $\quad F_{GI} = 11.25 \text{ kN} \quad T$ ◀

$F_{HI} = F_{AB}$ $\quad F_{HI} = 12.31 \text{ kN} \quad C$ ◀

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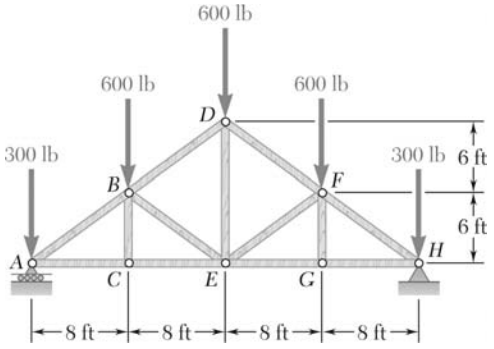
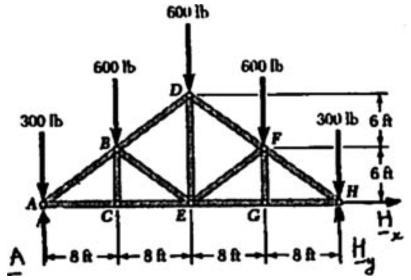


PROBLEM 6.11

Determine the force in each member of the Howe roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss



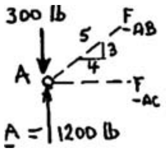
$$\Sigma F_x = 0: \quad \mathbf{H}_x = 0$$

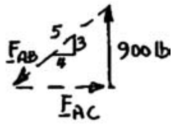
Because of the symmetry of the truss and loading:

$$A = H_y = \frac{1}{2} \text{ Total load}$$

$$\mathbf{A} = \mathbf{H}_y = 1200 \text{ lb}\uparrow$$

Free body: Joint *A*:





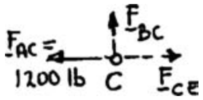
$$\frac{F_{AB}}{5} = \frac{F_{AC}}{4} = \frac{900 \text{ lb}}{3}$$

$\mathbf{F}_{AB} = 1500 \text{ lb} \quad C$ ◀

$\mathbf{F}_{AC} = 1200 \text{ lb} \quad T$ ◀

Free body: Joint *C*:

BC is a zero-force member



$\mathbf{F}_{BC} = 0$

$\mathbf{F}_{CE} = 1200 \text{ lb} \quad T$ ◀

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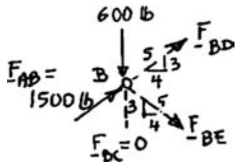
PROBLEM 6.11 (Continued)

Free body: Joint *B*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{BD} + \frac{4}{5}F_{BC} + \frac{4}{5}(1500 \text{ lb}) = 0$$

or $$F_{BD} + F_{BE} = -1500 \text{ lb} \qquad (1)$$

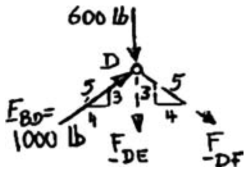
$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BD} - \frac{3}{5}F_{BE} + \frac{3}{5}(1500 \text{ lb}) - 600 \text{ lb} = 0$$

or $$F_{BD} - F_{BE} = -500 \text{ lb} \qquad (2)$$

Add Eqs. (1) and (2): $2F_{BD} = -2000$ lb $\qquad F_{BD} = 1000 \text{ lb} \quad C$ ◀

Subtract (2) from (1): $2F_{BE} = -1000$ lb $\qquad F_{BE} = 500 \text{ lb} \quad C$ ◀

Free Body: Joint *D*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}(1000 \text{ lb}) + \frac{4}{5}F_{DF} = 0$$

$F_{DF} = -1000$ lb $\qquad F_{DF} = 1000 \text{ lb} \quad C$ ◀

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}(1000 \text{ lb}) - \frac{3}{5}(-1000 \text{ lb}) - 600 \text{ lb} - F_{DE} = 0$$

$F_{DE} = +600$ lb $\qquad F_{DE} = 600 \text{ lb} \quad T$ ◀

Because of the symmetry of the truss and loading, we deduce that

$F_{EF} = F_{BE}$ $\qquad F_{EF} = 500 \text{ lb} \quad C$ ◀

$F_{EG} = F_{CE}$ $\qquad F_{EG} = 1200 \text{ lb} \quad T$ ◀

$F_{FG} = F_{BC}$ $\qquad F_{FG} = 0$ ◀

$F_{FH} = F_{AB}$ $\qquad F_{FH} = 1500 \text{ lb} \quad C$ ◀

$F_{GH} = F_{AC}$ $\qquad F_{GH} = 1200 \text{ lb} \quad T$ ◀

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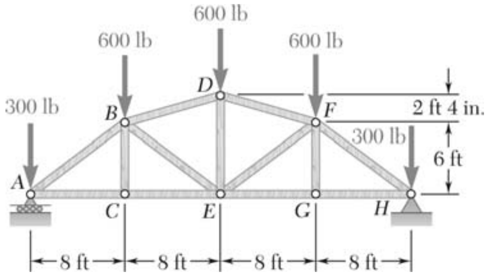
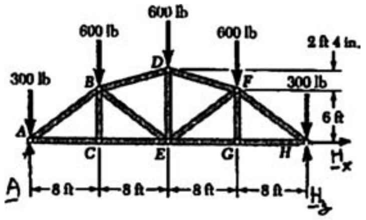


PROBLEM 6.12

Determine the force in each member of the Gambrel roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss



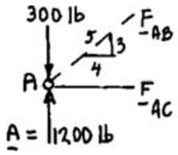
$$\Sigma F_x = 0: \quad \mathbf{H}_x = 0$$

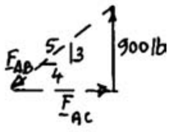
Because of the symmetry of the truss and loading

$$\mathbf{A} = \mathbf{H}_y = \frac{1}{2} \quad \text{Total load}$$

$$\mathbf{A} = \mathbf{H}_y = 1200 \text{ lb} \uparrow$$

Free body: Joint A:



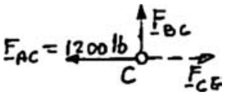


$$\frac{F_{AB}}{5} = \frac{F_{AC}}{4} = \frac{900 \text{ lb}}{3}$$

$\mathbf{F}_{AB} = 1500 \text{ lb}$ C ◀

$\mathbf{F}_{AC} = 1200 \text{ lb}$ T ◀

Free body: Joint C:

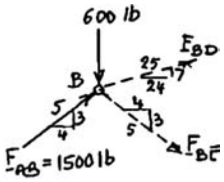


BC is a zero-force member

$$\mathbf{F}_{BC} = 0$$

$F_{CE} = 1200 \text{ lb}$ T ◀

Free body: Joint B:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{24}{25}F_{BD} + \frac{4}{5}F_{BE} + \frac{4}{5}(1500 \text{ lb}) = 0$$

or

$$24F_{BD} + 20F_{BE} = -30{,}000 \text{ lb} \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{7}{25}F_{BD} - \frac{3}{5}F_{BE} + \frac{3}{5}(1500) - 600 = 0$$

or

$$7F_{BD} - 15F_{BE} = -7{,}500 \text{ lb} \tag{2}$$

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PROBLEM 6.12 (Continued)

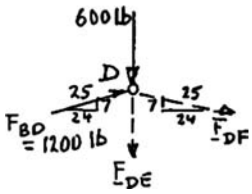
Multiply (1) by (3), (2) by 4, and add:

$$100F_{BD} = -120{,}000 \text{ lb} \qquad F_{BD} = 1200 \text{ lb} \quad C \blacktriangleleft$$

Multiply (1) by 7, (2) by –24, and add:

$$500F_{BE} = -30{,}000 \text{ lb} \qquad F_{BE} = 60.0 \text{ lb} \quad C \blacktriangleleft$$

Free body: Joint *D*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{24}{25}(1200 \text{ lb}) + \frac{24}{25}F_{DF} = 0$$

$$F_{DF} = -1200 \text{ lb} \qquad F_{DF} = 1200 \text{ lb} \quad C \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{7}{25}(1200 \text{ lb}) - \frac{7}{25}(-1200 \text{ lb}) - 600 \text{ lb} - F_{DE} = 0$$

$$F_{DE} = 72.0 \text{ lb} \qquad F_{DE} = 72.0 \text{ lb} \quad T \blacktriangleleft$$

Because of the symmetry of the truss and loading, we deduce that

$$F_{EF} = F_{BE} \qquad F_{EF} = 60.0 \text{ lb} \quad C \blacktriangleleft$$

$$F_{EG} = F_{CE} \qquad F_{EG} = 1200 \text{ lb} \quad T \blacktriangleleft$$

$$F_{FG} = F_{BC} \qquad F_{FG} = 0 \blacktriangleleft$$

$$F_{FH} = F_{AB} \qquad F_{FH} = 1500 \text{ lb} \quad C \blacktriangleleft$$

$$F_{GH} = F_{AC} \qquad F_{GH} = 1200 \text{ lb} \quad T \blacktriangleleft$$

Note: Compare results with those of Problem 6.9.

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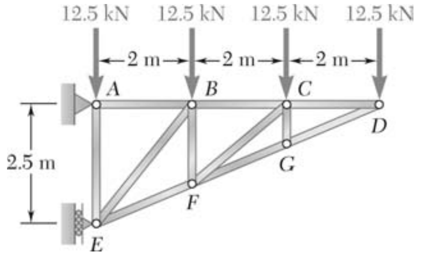


PROBLEM 6.13

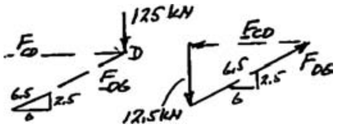
Determine the force in each member of the truss shown.

SOLUTION

Joint D:

$$\frac{12.5\text{ kN}}{2.5} = \frac{F_{CD}}{6} = \frac{F_{DG}}{6.5}$$

$F_{CD} = 30\text{ kN}\quad T$ ◀

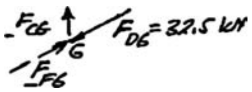


$F_{DG} = 32.5\text{ kN}\quad C$ ◀

Joint G:

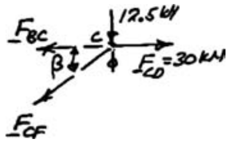
$\nwarrow \Sigma F = 0{:}\quad F_{CG} = 0$ ◀

$\nearrow \Sigma F = 0{:}\quad F_{FG} = 32.5\text{ kN}\quad C$ ◀



Joint C:

$$BF = \frac{2}{3}(2.5\text{ m}) = 1.6667\text{ m}\qquad \beta = \angle BCF = \tan^{-1}\frac{BF}{2} = 39.81°$$



$$+\uparrow \Sigma F_y = 0{:}\quad -12.5\text{ kN} - F_{CF}\sin\beta = 0$$

$$-12.5\text{ kN} - F_{CF}\sin 39.81° = 0$$

$$F_{CF} = -19.526\text{ kN}$$

$F_{CF} = 19.53\text{ kN}\quad C$ ◀

$$\xrightarrow{+} \Sigma F_x = 0{:}\quad 30\text{ kN} - F_{BC} - F_{CF}\cos\beta = 0$$

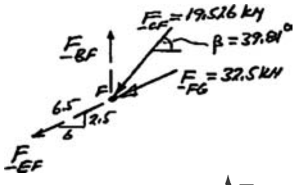
$$30\text{ kN} - F_{BC} - (-19.526\text{ kN})\cos 39.81° = 0$$

$$F_{BC} = +45.0\text{ kN}$$

$F_{BC} = 45.0\text{ kN}\quad T$ ◀

Joint F:

$$\xrightarrow{+} \Sigma F_x = 0{:}\quad -\frac{6}{6.5}F_{EF} - \frac{6}{6.5}(32.5\text{ kN}) - F_{CF}\cos\beta = 0$$



$$F_{EF} = -32.5\text{ kN} - \left(\frac{6.5}{6}\right)(19.526\text{ kN})\cos 39.81°$$

$$F_{EF} = -48.75\text{ kN}$$

$F_{EF} = 48.8\text{ kN}\quad C$ ◀

$$+\uparrow \Sigma F_y = 0{:}\quad F_{BF} - \frac{2.5}{6.5}F_{EF} - \frac{2.5}{6.5}(32.5\text{ kN}) - (19.526\text{ kN})\sin 39.81° = 0$$

$$F_{BF} - \frac{2.5}{6.5}(-48.75\text{ kN}) - 12.5\text{ kN} - 12.5\text{ kN} = 0$$

$$F_{BF} = +6.25\text{ kN}$$

$F_{BF} = 6.25\text{ kN}\quad T$ ◀

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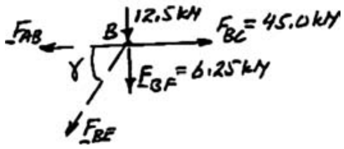
PROBLEM 6.13 (Continued)

<u>Joint *B*</u>:



$$\tan\alpha = \frac{2.5 \text{ m}}{2 \text{ m}}; \quad \gamma = 51.34°$$

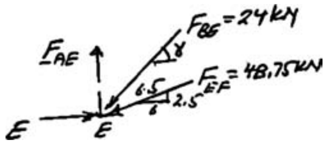
$$+\uparrow \Sigma F_y = 0: \quad -12.5 \text{ kN} - 6.25 \text{ kN} - F_{BE}\sin 51.34° = 0$$

$$F_{BE} = -24.0 \text{ kN} \qquad F_{BE} = 24.0 \text{ kN} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 45.0 \text{ kN} - F_{AB} + (24.0 \text{ kN})\cos 51.34° = 0$$

$$F_{AB} = +60 \text{ kN} \qquad F_{AB} = 60.0 \text{ kN} \quad T \blacktriangleleft$$

<u>Joint *E*</u>:



$$\gamma = 51.34°$$

$$\xrightarrow{+} \Sigma F_y = 0: \quad F_{AE} - (24 \text{ kN})\sin 51.34° - (48.75 \text{ kN})\frac{2.5}{6.5} = 0$$

$$F_{AE} = +37.5 \text{ kN} \qquad F_{AE} = 37.5 \text{ kN} \quad T \blacktriangleleft$$

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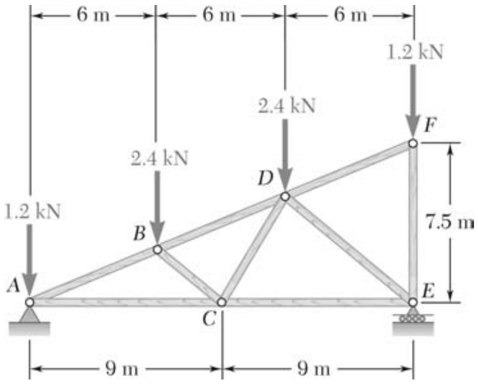
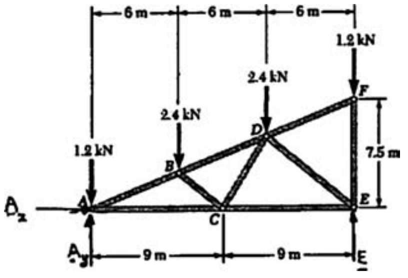


PROBLEM 6.14

Determine the force in each member of the roof truss shown. State whether each member is in tension or compression.

SOLUTION

<u>Free body: Truss</u>



$$\Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

From symmetry of loading:

$$\mathbf{A}_y = \mathbf{E} = \frac{1}{2}\text{ Total load}$$

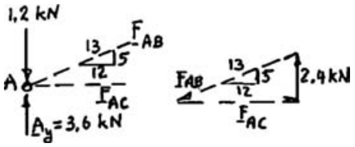
$$\mathbf{A}_y = \mathbf{E} = 3.6\text{ kN}\uparrow$$

We note that *DF* is a zero-force member and that *EF* is aligned with the load. Thus

$$F_{DF} = 0 \blacktriangleleft$$

$$F_{EF} = 1.2\text{ kN} \quad C \blacktriangleleft$$

<u>Free body: Joint *A*:</u>

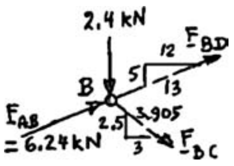


$$\frac{F_{AB}}{13} = \frac{F_{AC}}{12} = \frac{2.4\text{ kN}}{5}$$

$$F_{AB} = 6.24\text{ kN} \quad C \blacktriangleleft$$

$$F_{AC} = 2.76\text{ kN} \quad T \blacktriangleleft$$

<u>Free body: Joint *B*:</u>



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{3}{3.905}F_{BC} + \frac{12}{13}F_{BD} + \frac{12}{13}(6.24\text{ kN}) = 0 \qquad (1)$$

$$+\!\uparrow \Sigma F_y = 0: \quad -\frac{2.5}{3.905}F_{BC} + \frac{5}{13}F_{BD} + \frac{5}{13}(6.24\text{ kN}) - 2.4\text{ kN} = 0 \qquad (2)$$

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PROBLEM 6.14 (Continued)

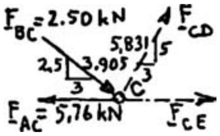
Multiply (1) by 2.5, (2) by 3, and add:

$$\frac{45}{13}F_{BD} + \frac{45}{13}(6.24 \text{ kN}) - 7.2 \text{ kN} = 0, \quad F_{BD} = -4.16 \text{ kN}, \qquad F_{BD} = 4.16 \text{ kN} \quad C \blacktriangleleft$$

Multiply (1) by 5, (2) by –12, and add:

$$\frac{45}{3.905}F_{BC} + 28.8 \text{ kN} = 0, \quad F_{BC} = -2.50 \text{ kN}, \qquad F_{BC} = 2.50 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint C:



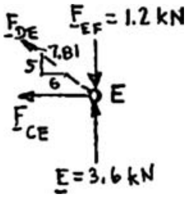
$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{5.831}F_{CD} - \frac{2.5}{3.905}(2.50 \text{ kN}) = 0$$

$$F_{CD} = 1.867 \text{ kN} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} - 5.76 \text{ kN} + \frac{3}{3.905}(2.50 \text{ kN}) + \frac{3}{5.831}(1.867 \text{ kN}) = 0$$

$$F_{CE} = 2.88 \text{ kN} \quad T$$

Free body: Joint E:



$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{7.81}F_{DE} + 3.6 \text{ kN} - 1.2 \text{ kN} = 0$$

$$F_{DE} = -3.75 \text{ kN} \qquad F_{DE} = 3.75 \text{ kN} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{CE} - \frac{6}{7.81}(-3.75 \text{ kN}) = 0$$

$$F_{CE} = +2.88 \text{ kN} \quad F_{CE} = 2.88 \text{ kN} \quad T \qquad \text{(Checks)}$$

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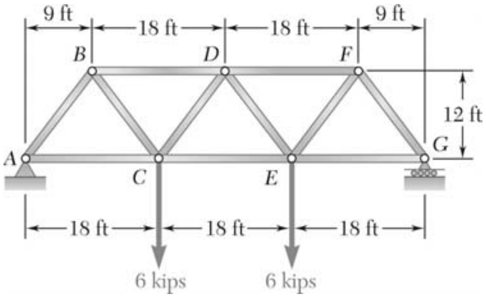
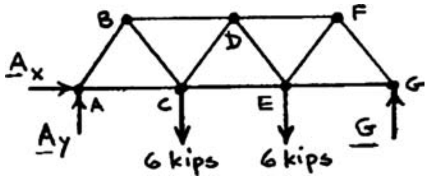


PROBLEM 6.15

Determine the force in each member of the Warren bridge truss shown. State whether each member is in tension or compression.

SOLUTION



Free body: Truss $\qquad \Sigma F_x = 0: \quad A_x = 0$

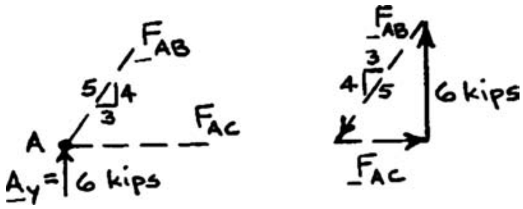
Due to symmetry of truss and loading

$$A_y = G = \frac{1}{2}\text{Total load} = 6 \text{ kips}\uparrow$$

Free body: Joint A: $\qquad \dfrac{F_{AB}}{5} = \dfrac{F_{AC}}{3} = \dfrac{6 \text{ kips}}{4}$

$F_{AB} = 7.50 \text{ kips} \quad C$ ◀

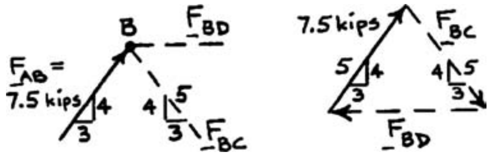
$F_{AC} = 4.50 \text{ kips} \quad T$ ◀



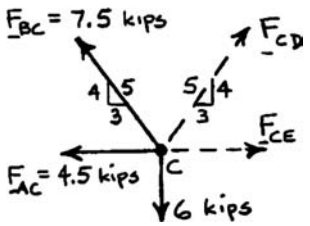
Free body: Joint B: $\qquad \dfrac{F_{BC}}{5} = \dfrac{F_{BD}}{6} = \dfrac{7.5 \text{ kips}}{5}$

$F_{BC} = 7.50 \text{ kips} \quad T$ ◀

$F_{BD} = 9.00 \text{ kips} \quad C$ ◀



Free body: Joint C:



$$+\uparrow \Sigma F_y = 0: \quad \frac{4}{5}(7.5) + \frac{4}{5}F_{CD} - 6 = 0$$

$F_{CD} = 0$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} - 4.5 - \frac{3}{5}(7.5) = 0$$

$+\uparrow\ F_{CE} = +9 \text{ kips}$ $\qquad F_{CE} = 9.00 \text{ kips} \quad T$ ◀

Truss and loading symmetrical about ℄

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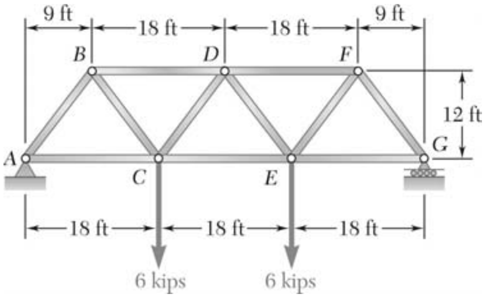


PROBLEM 6.16

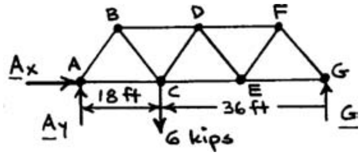
Solve Problem 6.15 assuming that the load applied at E has been removed.

PROBLEM 6.15 Determine the force in each member of the Warren bridge truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss

$$\Sigma F_x = 0: \quad A_x = 0$$



$$+\circlearrowleft \Sigma M_G = 0: \quad 6(36) - A_y(54) = 0 \quad \mathbf{A}_y = 4 \text{ kips} \uparrow$$

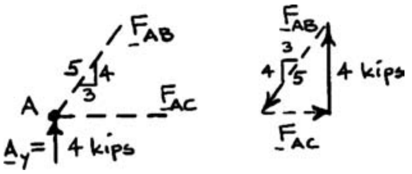
$$+\uparrow \Sigma F_y = 0: \quad 4 - 6 + G = 0 \quad \mathbf{G} = 2 \text{ kips} \uparrow$$

Free body: Joint A:

$$\frac{F_{AB}}{5} = \frac{F_{AC}}{3} = \frac{4 \text{ kips}}{4}$$

$F_{AB} = 5.00 \text{ kips} \quad C$ ◀

$F_{AC} = 3.00 \text{ kips} \quad T$ ◀

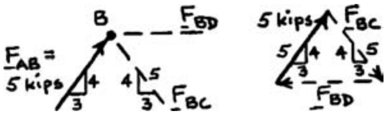


Free body Joint B:

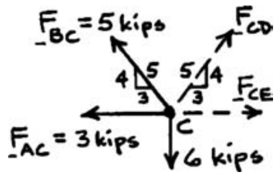
$$\frac{F_{BC}}{5} = \frac{F_{BD}}{6} = \frac{5 \text{ kips}}{5}$$

$F_{BC} = 5.00 \text{ kips} \quad T$ ◀

$F_{BD} = 6.00 \text{ kips} \quad C$ ◀



Free body Joint C:



$$+\uparrow \Sigma M_y = 0: \quad \frac{4}{5}(5) + \frac{4}{5}F_{CD} - 6 = 0$$

$F_{CD} = 2.50 \text{ kips} \quad T$ ◀

$$+\uparrow \Sigma F_x = 0: \quad F_{CE} + \frac{3}{5}(2.5) - \frac{3}{5}(5) - 3 = 0$$

$F_{CE} = 4.50 \text{ kips} \quad T$ ◀

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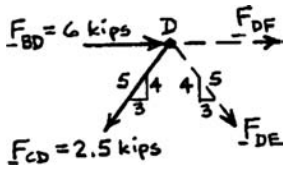
PROBLEM 6.16 (Continued)

Free body: Joint *D*:



$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5}(2.5) - \frac{4}{5}F_{DE} = 0$$

$$F_{DE} = -2.5 \text{ kips} \qquad\qquad F_{DE} = 2.50 \text{ kips} \quad C \blacktriangleleft$$

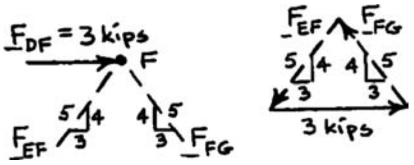
$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{DF} + 6 - \frac{3}{5}(2.5) - \frac{3}{5}(2.5) = 0$$

$$F_{DF} = -3 \text{ kips} \qquad\qquad F_{DF} = 3.00 \text{ kips} \quad C \blacktriangleleft$$

Free body: Joint *F*:

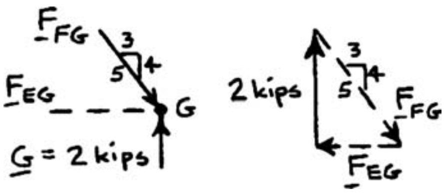
$$\frac{F_{EF}}{5} = \frac{F_{FG}}{5} = \frac{3 \text{ kips}}{6} \qquad\qquad F_{EF} = 2.50 \text{ kips} \quad T \blacktriangleleft$$

$$F_{FG} = 2.50 \text{ kips} \quad C \blacktriangleleft$$



Free body: Joint *G*:

$$\frac{F_{EG}}{3} = \frac{2 \text{ kips}}{4} \qquad\qquad F_{EG} = 1.500 \text{ kips} \quad T \blacktriangleleft$$



Also:

$$\frac{F_{FG}}{5} = \frac{2 \text{ kips}}{4}$$

$$F_{FG} = 2.50 \text{ kips} \quad C \quad \text{(Checks)}$$

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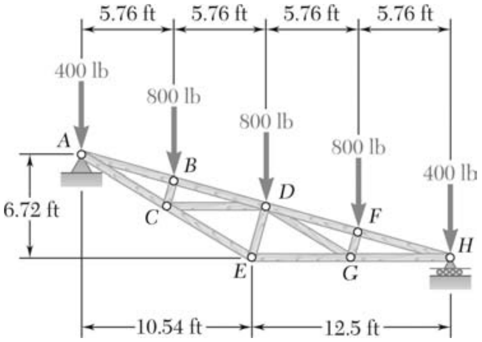
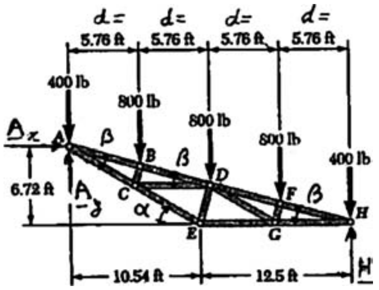


PROBLEM 6.17

Determine the force in member *DE* and in each of the members located to the left of *DE* for the inverted Howe roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:



$$\Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

$$+\circlearrowleft \Sigma M_H = 0: \quad (400\text{ lb})(4d) + (800\text{ lb})(3d) + (800\text{ lb})(2d) + (800\text{ lb})d - A_y(4d) = 0$$

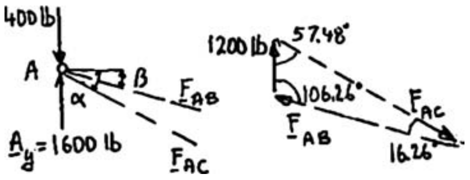
$$A_y = 1600\text{ lb}\uparrow$$

Angles:

$$\tan\alpha = \frac{6.72}{10.54} \quad \alpha = 32.52°$$

$$\tan\beta = \frac{6.72}{23.04} \quad \beta = 16.26°$$

Free body: Joint *A*:



$$\frac{F_{AB}}{\sin 57.48°} = \frac{F_{AC}}{\sin 106.26°} = \frac{1200\text{ lb}}{\sin 16.26°}$$

$$F_{AB} = 3613.8\text{ lb} \quad C$$

$$F_{AC} = 4114.3\text{ lb} \quad T \qquad F_{AB} = 3610\text{ lb} \quad C, \quad F_{AC} = 4110\text{ lb} \quad T \blacktriangleleft$$

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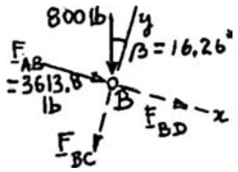
PROBLEM 6.17 (Continued)

Free body: Joint *B*:



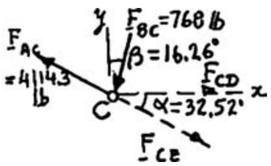
$$+\nearrow \Sigma F_y = 0: \quad -F_{BC} - (800 \text{ lb})\cos 16.26° = 0$$

$$F_{BC} = -768.0 \text{ lb} \qquad F_{BC} = 768 \text{ lb} \quad C \blacktriangleleft$$

$$+\searrow \Sigma F_x = 0: \quad F_{BD} + 3613.8 \text{ lb} + (800 \text{ lb})\sin 16.26° = 0$$

$$F_{BD} = -3837.8 \text{ lb} \qquad F_{BD} = 3840 \text{ lb} \quad C \blacktriangleleft$$

Free body: Joint *C*:



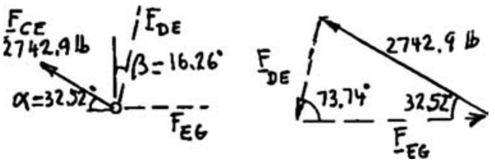
$$+\uparrow \Sigma F_y = 0: \quad -F_{CE}\sin 32.52° + (4114.3 \text{ lb})\sin 32.52° - (768 \text{ lb})\cos 16.26° = 0$$

$$F_{CE} = 2742.9 \text{ lb} \qquad F_{CE} = 2740 \text{ lb} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CD} - (4114.3 \text{ lb})\cos 32.52° + (2742.9 \text{ lb})\cos 32.52° - (768 \text{ lb})\sin 16.26° = 0$$

$$F_{CD} = 1371.4 \text{ lb} \qquad F_{CD} = 1371 \text{ lb} \quad T \blacktriangleleft$$

Free body: Joint *E*:



$$\frac{F_{DE}}{\sin 32.52°} = \frac{2742.9 \text{ lb}}{\sin 73.74°} \qquad F_{DE} = 1536 \text{ lb} \quad C \blacktriangleleft$$

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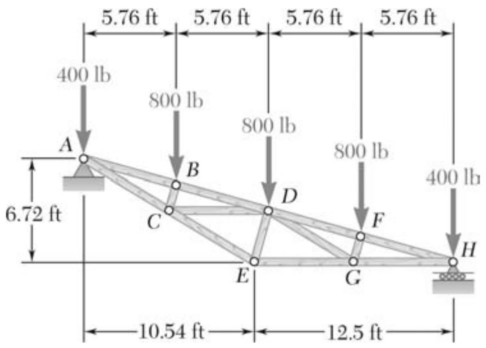
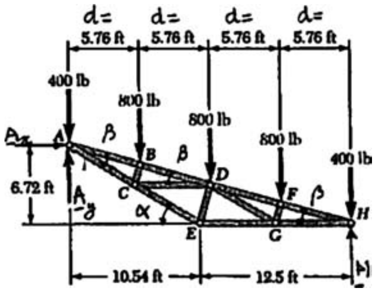


PROBLEM 6.18

Determine the force in each of the members located to the right of *DE* for the inverted Howe roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss



$$+\circlearrowleft \Sigma \mathbf{M}_A = 0: \quad H(4d) - (800 \text{ lb})d - (800 \text{ lb})(2d) - (800 \text{ lb})(3d) - (400 \text{ lb})(4d) = 0$$

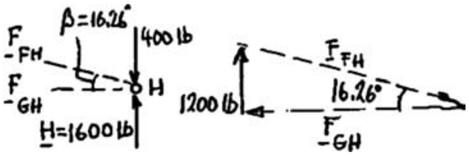
$$\mathbf{H} = 1600 \text{ lb} \uparrow$$

Angles:

$$\tan\alpha = \frac{6.72}{10.54} \quad \alpha = 32.52°$$

$$\tan\beta = \frac{6.72}{23.04} \quad \beta = 16.26°$$

Free body: Joint *H*:



$$F_{GH} = (1200 \text{ lb})\cot 16.26°$$

$$F_{GH} = 4114.3 \text{ lb} \quad T \qquad\qquad F_{GH} = 4110 \text{ lb} \quad T \blacktriangleleft$$

$$F_{FH} = \frac{1200 \text{ lb}}{\sin 16.26°} = 4285.8 \text{ lb} \qquad\qquad F_{FH} = 4290 \text{ lb} \quad C \blacktriangleleft$$

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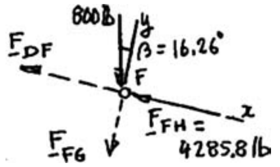
PROBLEM 6.18 (Continued)

Free body Joint F:



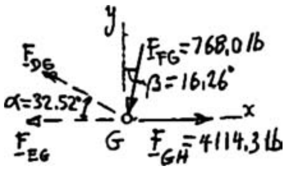
$$+\nearrow \Sigma F_y = 0: \quad -F_{FG} - (800 \text{ lb})\cos 16.26° = 0$$

$$F_{FG} = -768.0 \text{ lb} \qquad F_{FG} = 768 \text{ lb} \quad C \blacktriangleleft$$

$$+\searrow \Sigma F_x = 0: \quad -F_{DF} - 4285.8 \text{ lb} + (800 \text{ lb})\sin 16.26° = 0$$

$$F_{DF} = -4061.8 \text{ lb} \qquad F_{DF} = 4060 \text{ lb} \quad C \blacktriangleleft$$

Free body: Joint G:



$$+\uparrow \Sigma F_y = 0: \quad F_{DG}\sin 32.52° - (768.0 \text{ lb})\cos 16.26° = 0$$

$$F_{DG} = +1371.4 \text{ lb}$$

$$F_{DG} = 1371 \text{ lb} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{EG} + 4114.3 \text{ lb} - (768.0 \text{ lb})\sin 16.26° - (1371.4 \text{ lb})\cos 32.52° = 0$$

$$F_{EG} = +2742.9 \text{ lb} \qquad F_{EG} = 2740 \text{ lb} \quad T \blacktriangleleft$$

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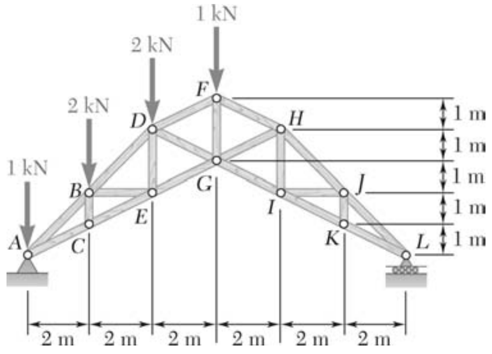
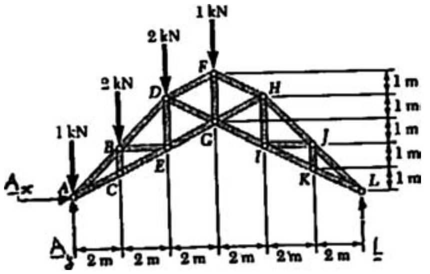


PROBLEM 6.19

Determine the force in each of the members located to the left of *FG* for the scissors roof truss shown. State whether each member is in tension or compression.

SOLUTION

<u>Free Body: Truss</u>



$$\Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

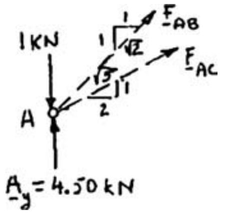
$$+\circlearrowleft \Sigma M_L = 0: \quad (1\text{ kN})(12\text{ m}) + (2\text{ kN})(10\text{ m}) + (2\text{ kN})(8\text{ m}) + (1\text{ kN})(6\text{ m}) - A_y(12\text{ m}) = 0$$

$$\mathbf{A}_y = 4.50\text{ kN}\uparrow$$

We note that *BC* is a zero-force member: $F_{BC} = 0$ ◀

Also: $F_{CE} = F_{AC}$ (1)

<u>Free body: Joint *A*</u>:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{2}}F_{AB} + \frac{2}{\sqrt{5}}F_{AC} = 0 \qquad (2)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{2}}F_{AB} + \frac{1}{\sqrt{5}}F_{AC} + 3.50\text{ kN} = 0 \qquad (3)$$

Multiply (3) by –2 and add (2):

$$-\frac{1}{\sqrt{2}}F_{AB} - 7\text{ kN} = 0$$

$F_{AB} = 9.90$ kN *C* ◀

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PROBLEM 6.19 (Continued)

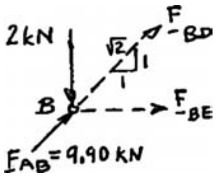
Subtract (3) from (2):

$$\frac{1}{\sqrt{5}}F_{AC} - 3.50\text{ kN} = 0 \quad F_{AC} = 7.826\text{ kN} \qquad F_{AC} = 7.83\text{ kN} \quad T \blacktriangleleft$$

From (1):

$$F_{CE} = F_{AC} = 7.826\text{ kN} \qquad F_{CE} = 7.83\text{ kN} \quad T \blacktriangleleft$$

Free body: Joint B:



$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{2}}F_{BD} + \frac{1}{\sqrt{2}}(9.90\text{ kN}) - 2\text{ kN} = 0$$

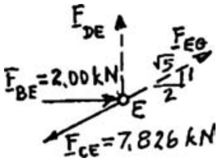
$$F_{BD} = -7.071\text{ kN} \qquad F_{BD} = 7.07\text{ kN} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BE} + \frac{1}{\sqrt{2}}(9.90 - 7.071)\text{kN} = 0$$

$$F_{BE} = -2.000\text{ kN} \qquad F_{BE} = 2.00\text{ kN} \quad C \blacktriangleleft$$

Free body: Joint E:

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}}(F_{EG} - 7.826\text{ kN}) + 2.00\text{ kN} = 0$$

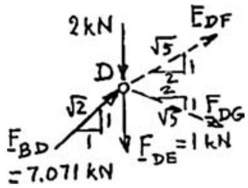


$$F_{EG} = 5.590\text{ kN} \qquad F_{EG} = 5.59\text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad F_{DE} - \frac{1}{\sqrt{5}}(7.826 - 5.590)\text{kN} = 0$$

$$F_{DE} = 1.000\text{ kN} \qquad F_{DE} = 1.000\text{ kN} \quad T \blacktriangleleft$$

Free body: Joint D:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}}(F_{DF} + F_{DG}) + \frac{1}{\sqrt{2}}(7.071\text{ kN})$$

or

$$F_{DF} + F_{DG} = -5.590\text{ kN} \qquad (4)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}(F_{DF} - F_{DG}) + \frac{1}{\sqrt{2}}(7.071\text{ kN}) = 2\text{ kN} - 1\text{ kN} = 0$$

or

$$F_{DE} - F_{DG} = -4.472 \qquad (5)$$

Add (4) and (5):

$$2F_{DF} = -10.062\text{ kN} \qquad F_{DF} = 5.03\text{ kN} \quad C \blacktriangleleft$$

Subtract (5) from (4):

$$2F_{DG} = -1.1180\text{ kN} \qquad F_{DG} = 0.559\text{ kN} \quad C \blacktriangleleft$$

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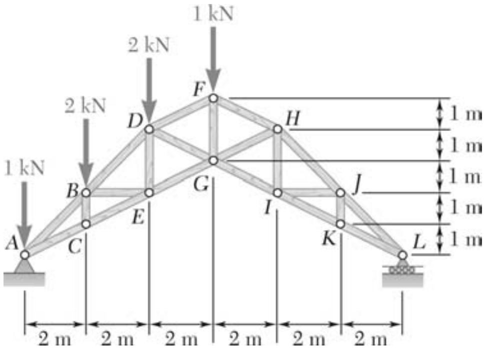
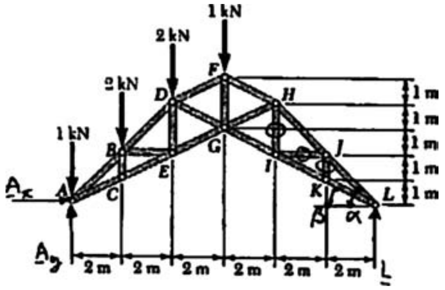


PROBLEM 6.20

Determine the force in member *FG* and in each of the members located to the right of *FG* for the scissors roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss



$$+\circlearrowleft \Sigma M_A = 0: \quad L(12\text{ m}) - (2\text{ kN})(2\text{ m}) - (2\text{ kN})(4\text{ m}) - (1\text{ kN})(6\text{ m}) = 0$$

$$\mathbf{L} = 1.500\text{ kN}\uparrow$$

Angles:

$$\tan\alpha = 1 \qquad \alpha = 45°$$

$$\tan\beta = \frac{1}{2} \qquad \beta = 26.57°$$

Zero-force members:

Examining successively joints *K*, *J*, and *I*, we note that the following members to the right of *FG* are zero-force members: *JK*, *IJ*, and *HI*.

Thus: $F_{HI} = F_{IJ} = F_{JK} = 0$ ◀

We also note that

$$F_{GI} = F_{IK} = F_{KL} \qquad (1)$$

and

$$F_{HJ} = F_{JL} \qquad (2)$$

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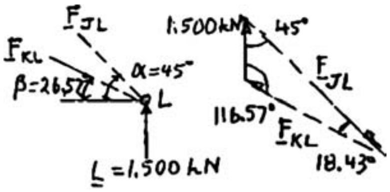
PROBLEM 6.20 (Continued)

Free body: Joint L:



$$\frac{F_{JL}}{\sin 116.57^\circ} = \frac{F_{KL}}{\sin 45^\circ} = \frac{1.500 \text{ kN}}{\sin 18.43^\circ}$$

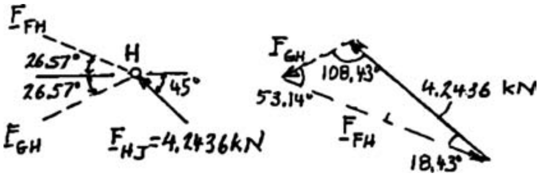
$F_{JL} = 4.2436 \text{ kN}$ $\qquad$ $F_{JL} = 4.24 \text{ kN} \quad C$ ◀

$F_{KL} = 3.35 \text{ kN} \quad T$ ◀

From Eq. (1): $\qquad F_{GI} = F_{IK} = F_{KL}$ $\qquad$ $F_{GI} = F_{IK} = 3.35 \text{ kN} \quad T$ ◀

From Eq. (2): $\qquad F_{HJ} = F_{JL} = 4.2436 \text{ kN}$ $\qquad$ $F_{HJ} = 4.24 \text{ kN} \quad T$ ◀

Free body: Joint H:

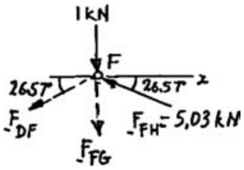


$$\frac{F_{FH}}{\sin 108.43^\circ} = \frac{F_{GH}}{\sin 18.43^\circ} = \frac{4.2436}{\sin 53.14^\circ}$$

$F_{FH} = 5.03 \text{ kN} \quad C$ ◀

$F_{GH} = 1.677 \text{ kN} \quad T$ ◀

Free body: Joint F:



$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{DF}\cos 26.57^\circ - (5.03 \text{ kN})\cos 26.57^\circ = 0$$

$$F_{DF} = -5.03 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{FG} - 1 \text{ kN} + (5.03 \text{ kN})\sin 26.57^\circ - (-5.03 \text{ kN})\sin 26.57^\circ = 0$$

$F_{FG} = 3.500 \text{ kN}$ $\qquad$ $F_{FG} = 3.50 \text{ kN} \quad T$ ◀

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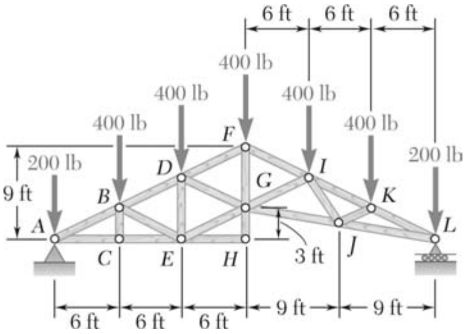
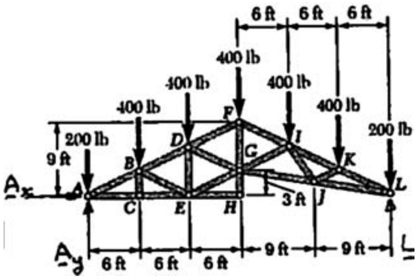


PROBLEM 6.21

Determine the force in each of the members located to the left of line *FGH* for the studio roof truss shown. State whether each member is in tension or compression.

SOLUTION



Free body: Truss $\quad \Sigma F_x = 0: \quad \mathbf{A}_x = 0$

Because of symmetry of loading:

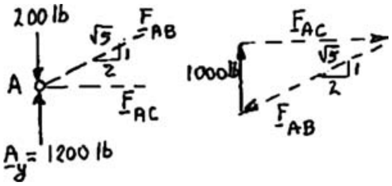
$$A_y = L = \frac{1}{2}\text{ Total load}$$

$$\mathbf{A}_y = \mathbf{L} = 1200 \text{ lb}\uparrow$$

Zero-Force Members. Examining joints *C* and *H*, we conclude that *BC*, *EH*, and *GH* are zero-force members. Thus

$$\mathbf{F}_{BC} = \mathbf{F}_{EH} = 0 \quad ◀$$

Also, $\quad F_{CE} = F_{AC} \quad (1)$



Free body: Joint *A* $\quad \dfrac{F_{AB}}{\sqrt{5}} = \dfrac{F_{AC}}{2} = \dfrac{1000 \text{ lb}}{1}$

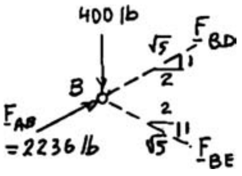
$$F_{AB} = 2236 \text{ lb} \quad C$$

$$F_{AB} = 2240 \text{ lb} \quad C \quad ◀$$

$$F_{AC} = 2000 \text{ lb} \quad T \quad ◀$$

From Eq. (1): $\quad F_{CE} = 2000 \text{ lb} \quad T \quad ◀$

Free body: Joint *B*



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}}F_{BD} + \frac{2}{\sqrt{5}}F_{BE} + \frac{2}{\sqrt{5}}(2236 \text{ lb}) = 0$$

or $\quad F_{BD} + F_{BE} = -2236 \text{ lb} \quad (2)$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}F_{BD} - \frac{1}{\sqrt{5}}F_{BE} + \frac{1}{\sqrt{5}}(2236 \text{ lb}) - 400 \text{ lb} = 0$$

or $\quad F_{BD} - F_{BE} = -1342 \text{ lb} \quad (3)$

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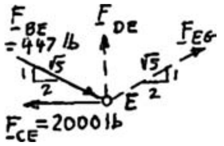
PROBLEM 6.21 (Continued)

Add (2) and (3): $2F_{BD} = -3578 \text{ lb}$ $F_{BD} = 1789 \text{ lb}$ C ◀

Subtract (3) from (1): $2F_{BE} = -894 \text{ lb}$ $F_{BE} = 447 \text{ lb}$ C ◀

<u>Free body: Joint *E*</u>



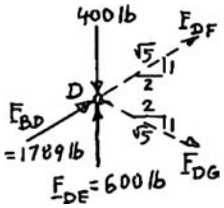
$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{EG} + \frac{2}{\sqrt{5}}(447 \text{ lb}) - 2000 \text{ lb} = 0$$

$F_{EG} = 1789 \text{ lb}$ T ◀

$$+\uparrow \Sigma F_y = 0: \quad F_{DE} + \frac{1}{\sqrt{5}}(1789 \text{ lb}) - \frac{1}{\sqrt{5}}(447 \text{ lb}) = 0$$

$F_{DE} = -600 \text{ lb}$ $F_{DE} = 600 \text{ lb}$ C ◀

<u>Free body: Joint *D*</u>



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{DF} + \frac{2}{\sqrt{5}} F_{DG} + \frac{2}{\sqrt{5}}(1789 \text{ lb}) = 0$$

or $F_{DF} + F_{DG} = -1789 \text{ lb}$ (4)

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{DF} - \frac{1}{\sqrt{5}} F_{DG} + \frac{1}{\sqrt{5}}(1789 \text{ lb}) + 600 \text{ lb} - 400 \text{ lb} = 0$$

or $F_{DF} - F_{DG} = -2236 \text{ lb}$ (5)

Add (4) and (5): $2F_{DF} = -4025 \text{ lb}$ $F_{DF} = 2010 \text{ lb}$ C ◀

Subtract (5) from (4): $2F_{DG} = 447 \text{ lb}$ $F_{DG} = 224 \text{ lb}$ T ◀

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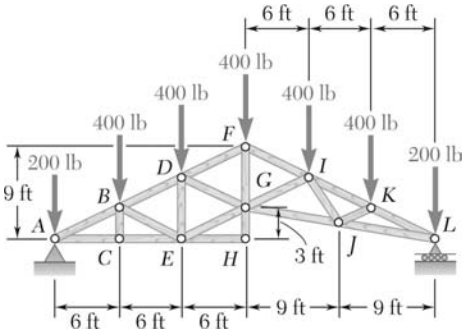


PROBLEM 6.22

Determine the force in member FG and in each of the members located to the right of FG for the studio roof truss shown. State whether each member is in tension or compression.

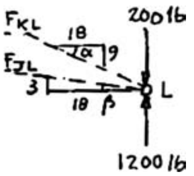
SOLUTION

Reaction at L: Because of the symmetry of the loading,

$$L = \frac{1}{2}\text{Total load}, \quad \mathbf{L} = 1200\text{ lb}\uparrow$$

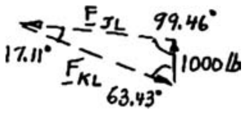
(See F, B, diagram to the left for more details)

Free body: Joint L



$$\alpha = \tan^{-1}\frac{9}{18} = 26.57°$$

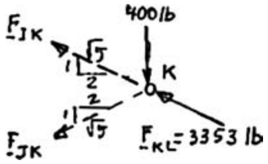
$$\beta = \tan^{-1}\frac{3}{18} = 9.46°$$



$$\frac{F_{JL}}{\sin 63.43°} = \frac{F_{KL}}{\sin 99.46°} = \frac{1000\text{ lb}}{\sin 17.11°} \qquad F_{JL} = 3040\text{ lb} \quad T \blacktriangleleft$$

$$F_{KL} = 3352.7\text{ lb} \quad C \qquad F_{KL} = 3350\text{ lb} \quad C \blacktriangleleft$$

Free body: Joint K



$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{2}{\sqrt{5}}F_{IK} - \frac{2}{\sqrt{5}}F_{JK} - \frac{2}{\sqrt{5}}(3352.7\text{ lb}) = 0$$

or: $$F_{IK} + F_{JK} = -3352.7\text{ lb} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}F_{IK} - \frac{1}{\sqrt{5}}F_{JK} + \frac{1}{\sqrt{5}}(3352.7) - 400 = 0$$

or: $$F_{IK} - F_{JK} = -2458.3\text{ lb} \qquad (2)$$

Add (1) and (2): $$2F_{IK} = -5811.0$$

$$F_{JK} = -2905.5\text{ lb} \qquad F_{IK} = 2910\text{ lb} \quad C \blacktriangleleft$$

Subtract (2) from (1): $$2F_{JK} = -894.4$$

$$F_{JK} = -447.2\text{ lb} \qquad F_{JK} = 447\text{ lb} \quad C \blacktriangleleft$$

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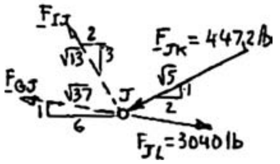
PROBLEM 6.22 (Continued)



Free body: Joint *J*

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{2}{\sqrt{13}}F_{IJ} - \frac{6}{\sqrt{37}}F_{GJ} + \frac{6}{\sqrt{37}}(3040 \text{ lb}) - \frac{2}{\sqrt{5}}(447.2) = 0 \qquad (3)$$

$$+\!\uparrow \Sigma F_y = 0: \quad \frac{3}{\sqrt{13}}F_{IJ} + \frac{1}{\sqrt{37}}F_{GJ} - \frac{1}{\sqrt{37}}(3040 \text{ lb}) - \frac{1}{\sqrt{5}}(447.2) = 0 \qquad (4)$$

Multiply (4) by 6 and add to (3):

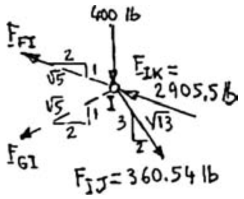
$$\frac{16}{\sqrt{13}}F_{IJ} - \frac{8}{\sqrt{5}}(447.2) = 0$$

$$F_{IJ} = 360.54 \text{ lb} \qquad\qquad F_{IJ} = 361 \text{ lb} \quad T \blacktriangleleft$$

Multiply (3) by 3, (4) by 2, and add:

$$-\frac{16}{\sqrt{37}}(F_{GJ} - 3040) - \frac{8}{\sqrt{5}}(447.2) = 0$$

$$F_{GJ} = 2431.7 \text{ lb} \qquad\qquad F_{GJ} = 2430 \text{ lb} \quad T \blacktriangleleft$$



Free body: Joint *I*

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{2}{\sqrt{5}}F_{FI} - \frac{2}{\sqrt{5}}F_{GI} - \frac{2}{\sqrt{5}}(2905.5) + \frac{2}{\sqrt{13}}(360.54) = 0$$

or $$F_{FI} + F_{GI} = -2681.9 \text{ lb} \qquad (5)$$

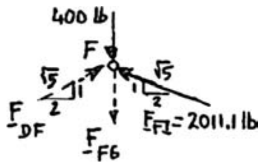
$$+\!\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}F_{FI} - \frac{1}{\sqrt{5}}F_{GI} + \frac{1}{\sqrt{5}}(2905.5) - \frac{3}{\sqrt{13}}(360.54) - 400 = 0$$

or $$F_{FI} - F_{GI} = -1340.3 \text{ lb} \qquad (6)$$

Add (5) and (6): $$2F_{FI} = -4022.2$$

$$F_{FI} = -2011.1 \text{ lb} \qquad\qquad F_{FI} = 2010 \text{ lb} \quad C \blacktriangleleft$$

Subtract (6) from (5): $$2F_{GI} = -1341.6 \text{ lb} \qquad\qquad F_{GI} = 671 \text{ lb} \quad C \blacktriangleleft$$



Free body: Joint *F*

From $$\Sigma F_x = 0: \quad F_{DF} = F_{FI} = 2011.1 \text{ lb} \quad C$$

$$+\!\uparrow \Sigma F_y = 0: \quad F_{FG} - 400 \text{ lb} + 2\left(\frac{1}{\sqrt{5}}2011.1 \text{ lb}\right) = 0$$

$$F_{FG} = +1400 \text{ lb} \qquad\qquad F_{FG} = 1400 \text{ lb} \quad T \blacktriangleleft$$

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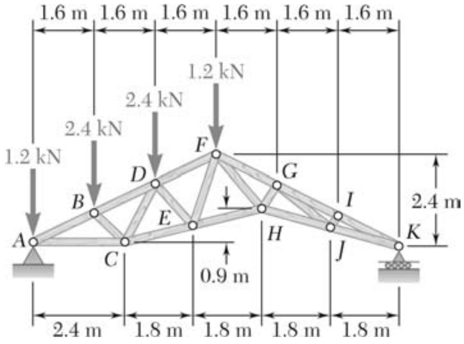
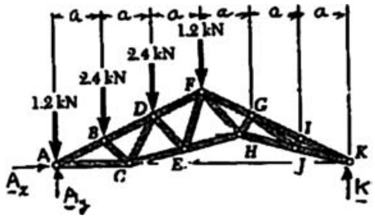


PROBLEM 6.23

Determine the force in each of the members connecting joints *A* through *F* of the vaulted roof truss shown. State whether each member is in tension or compression.

SOLUTION

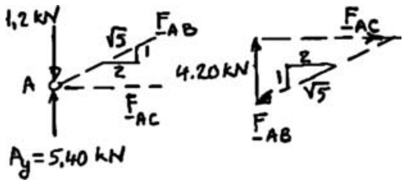


Free body: Truss

$$\Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

$$+\circlearrowleft \Sigma M_K = 0: \quad (1.2\text{ kN})6a + (2.4\text{ kN})5a + (2.4\text{ kN})4a + (1.2\text{ kN})3a$$

$$-A_y(6a) = 0 \quad \mathbf{A}_y = 5.40\text{ kN}\uparrow$$

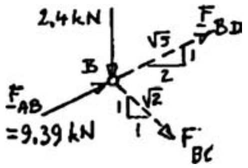


Free body: Joint *A*

$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{AC}}{2} = \frac{4.20\text{ kN}}{1}$$

$$F_{AB} = 9.3915\text{ kN} \qquad F_{AB} = 9.39\text{ kN} \quad C \blacktriangleleft$$

$$F_{AC} = 8.40\text{ kN} \quad T \blacktriangleleft$$



Free body: Joint *B*

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}}F_{BD} + \frac{1}{\sqrt{2}}F_{BC} + \frac{2}{\sqrt{5}}(9.3915) = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}F_{BD} - \frac{1}{\sqrt{2}}F_{BC} + \frac{1}{\sqrt{5}}(9.3915) - 2.4 = 0 \qquad (2)$$

Add (1) and (2):

$$\frac{3}{\sqrt{5}}F_{BD} + \frac{3}{\sqrt{5}}(9.3915\text{ kN}) - 2.4\text{ kN} = 0$$

$$F_{BD} = -7.6026\text{ kN} \qquad F_{BD} = 7.60\text{ kN} \quad C \blacktriangleleft$$

Multiply (2) by –2 and add (1):

$$\frac{3}{\sqrt{2}}F_B + 4.8\text{ kN} = 0$$

$$F_{BC} = -2.2627\text{ kN} \qquad F_{BC} = 2.26\text{ kN} \quad C \blacktriangleleft$$

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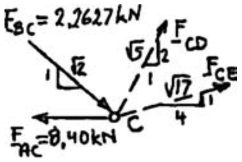
PROBLEM 6.23 (Continued)

<u>Free body: Joint *C*</u>



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{5}} F_{CD} + \frac{4}{\sqrt{17}} F_{CE} + \frac{1}{\sqrt{2}}(2.2627) - 8.40 = 0 \qquad (3)$$

$$+\!\uparrow \Sigma F_y = 0: \quad \frac{2}{\sqrt{5}} F_{CD} + \frac{1}{\sqrt{17}} F_{CE} - \frac{1}{\sqrt{2}}(2.2627) = 0 \qquad (4)$$

Multiply (4) by -4 and add (1):

$$-\frac{7}{\sqrt{5}} F_{CD} + \frac{5}{\sqrt{2}}(2.2627) - 8.40 = 0$$

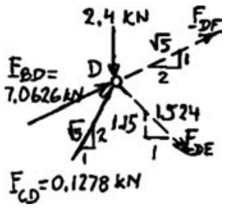
$$F_{CD} = -0.1278 \text{ kN} \qquad F_{CD} = 0.128 \text{ kN} \quad C \blacktriangleleft$$

Multiply (1) by 2 and subtract (2):

$$\frac{7}{\sqrt{17}} F_{CE} + \frac{3}{\sqrt{2}}(2.2627) - 2(8.40) = 0$$

$$F_{CE} = 7.068 \text{ kN} \qquad F_{CE} = 7.07 \text{ kN} \quad T \blacktriangleleft$$

<u>Free body: Joint *D*</u>



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{DF} + \frac{1}{1.524} F_{DE} + \frac{2}{\sqrt{5}}(7.6026)$$

$$+\frac{1}{\sqrt{5}}(0.1278) = 0 \qquad (5)$$

$$+\!\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{DF} - \frac{1.15}{1.524} F_{DE} + \frac{1}{\sqrt{5}}(7.6026)$$

$$+\frac{2}{\sqrt{5}}(0.1278) - 2.4 = 0 \qquad (6)$$

Multiply (5) by 1.15 and add (6):

$$\frac{3.30}{\sqrt{5}} F_{DF} + \frac{3.30}{\sqrt{5}}(7.6026) + \frac{3.15}{\sqrt{5}}(0.1278) - 2.4 = 0$$

$$F_{DF} = -6.098 \text{ kN} \qquad F_{DF} = 6.10 \text{ kN} \quad C \blacktriangleleft$$

Multiply (6) by –2 and add (5):

$$\frac{3.30}{1.524} F_{DE} - \frac{3}{\sqrt{5}}(0.1278) + 4.8 = 0$$

$$F_{DE} = -2.138 \text{ kN} \qquad F_{DE} = 2.14 \text{ kN} \quad C \blacktriangleleft$$

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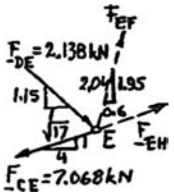
PROBLEM 6.23 (Continued)



Free body: Joint E

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{0.6}{2.04}F_{EF} + \frac{4}{\sqrt{17}}(F_{EH} - F_{CE}) + \frac{1}{1.524}(2.138) = 0 \qquad (7)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1.95}{2.04}F_{EF} + \frac{1}{\sqrt{17}}(F_{EH} - F_{CE}) - \frac{1.15}{1.524}(2.138) = 0 \qquad (8)$$

Multiply (8) by 4 and subtract (7):

$$\frac{7.2}{2.04}F_{EF} - 7.856\text{ kN} = 0 \qquad F_{EF} = 2.23\text{ kN} \quad T \blacktriangleleft$$

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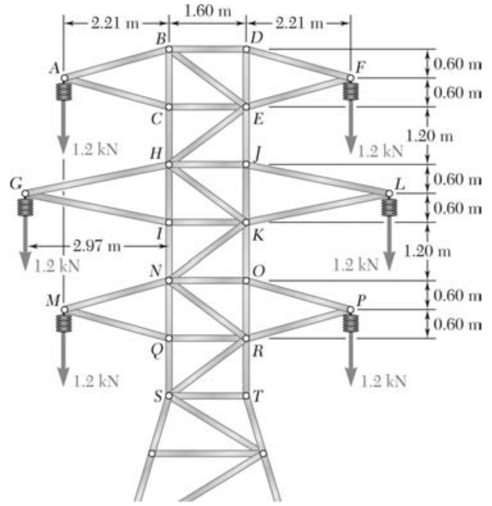
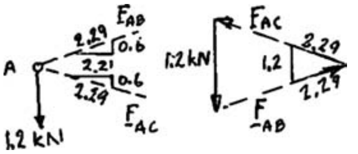


PROBLEM 6.24

The portion of truss shown represents the upper part of a power transmission line tower. For the given loading, determine the force in each of the members located above *HJ*. State whether each member is in tension or compression.

SOLUTION

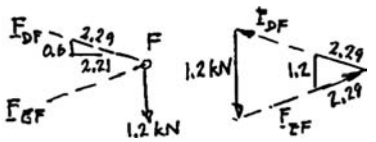


Free body: Joint *A*

$$\frac{F_{AB}}{2.29} = \frac{F_{AC}}{2.29} = \frac{1.2 \text{ kN}}{1.2}$$

$F_{AB} = 2.29 \text{ kN} \quad T$ ◀

$F_{AC} = 2.29 \text{ kN} \quad C$ ◀

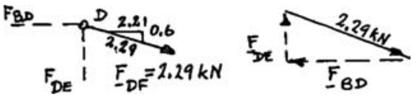


Free body: Joint *F*

$$\frac{F_{DF}}{2.29} = \frac{F_{EF}}{2.29} = \frac{1.2 \text{ kN}}{2.1}$$

$F_{DF} = 2.29 \text{ kN} \quad T$ ◀

$F_{EF} = 2.29 \text{ kN} \quad C$ ◀



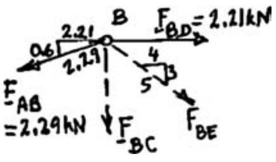
Free body: Joint *D*

$$\frac{F_{BD}}{2.21} = \frac{F_{DE}}{0.6} = \frac{2.29 \text{ kN}}{2.29}$$

$F_{BD} = 2.21 \text{ kN} \quad T$ ◀

$F_{DE} = 0.600 \text{ kN} \quad C$ ◀

Free body: Joint *B*



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{BE} + 2.21 \text{ kN} - \frac{2.21}{2.29}(2.29 \text{ kN}) = 0$$

$F_{BE} = 0$ ◀

$$+\!\uparrow \Sigma F_y = 0: \quad -F_{BC} - \frac{3}{5}(0) - \frac{0.6}{2.29}(2.29 \text{ kN}) = 0$$

$F_{BC} = -0.600 \text{ kN} \qquad F_{BC} = 0.600 \text{ kN} \quad C$ ◀

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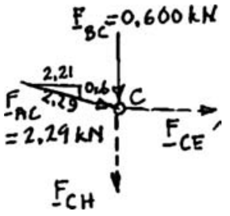
PROBLEM 6.24 (Continued)

Free body: Joint *C*



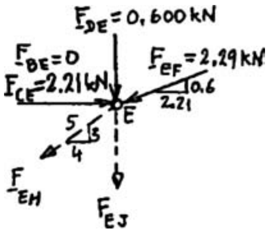
$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} + \frac{2.21}{2.29}(2.29 \text{ kN}) = 0$$

$$F_{CE} = -2.21 \text{ kN} \qquad F_{CE} = 2.21 \text{ kN} \quad C \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad -F_{CH} - 0.600 \text{ kN} - \frac{0.6}{2.29}(2.29 \text{ kN}) = 0$$

$$F_{CH} = -1.200 \text{ kN} \qquad F_{CH} = 1.200 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint *E*



$$\xrightarrow{+} \Sigma F_x = 0: \quad 2.21 \text{ kN} - \frac{2.21}{2.29}(2.29 \text{ kN}) - \frac{4}{5}F_{EH} = 0$$

$$F_{EH} = 0 \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad -F_{EJ} - 0.600 \text{ kN} - \frac{0.6}{2.29}(2.29 \text{ kN}) - 0 = 0$$

$$F_{EJ} = -1.200 \text{ kN} \qquad F_{EJ} = 1.200 \text{ kN} \quad C \blacktriangleleft$$

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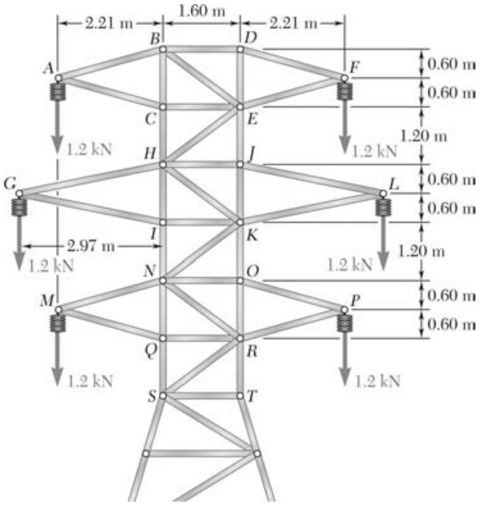
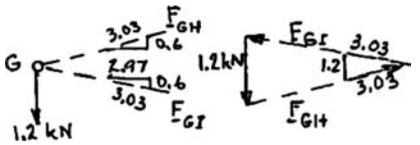


PROBLEM 6.25

For the tower and loading of Problem 6.24 and knowing that $F_{CH} = F_{EJ} = 1.2$ kN C and $F_{EH} = 0$, determine the force in member *HJ* and in each of the members located between *HJ* and *NO*. State whether each member is in tension or compression.

PROBLEM 6.24 The portion of truss shown represents the upper part of a power transmission line tower. For the given loading, determine the force in each of the members located above *HJ*. State whether each member is in tension or compression.

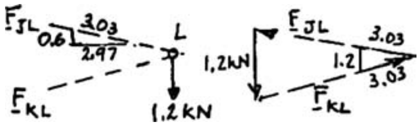
SOLUTION



Free body: Joint *G*

$$\frac{F_{GH}}{3.03} = \frac{F_{GI}}{3.03} = \frac{1.2 \text{ kN}}{1.2} \qquad F_{GH} = 3.03 \text{ kN} \quad T \blacktriangleleft$$

$$F_{GI} = 3.03 \text{ kN} \quad C \blacktriangleleft$$

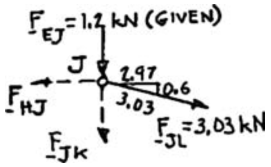


Free body: Joint *L*

$$\frac{F_{JL}}{3.03} = \frac{F_{KL}}{3.03} = \frac{1.2 \text{ kN}}{1.2} \qquad F_{JL} = 3.03 \text{ kN} \quad T \blacktriangleleft$$

$$F_{KL} = 3.03 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint *J*



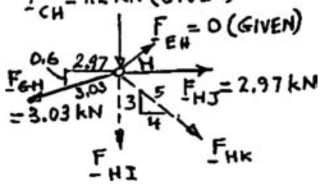
$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{HJ} + \frac{2.97}{3.03}(3.03 \text{ kN}) = 0$$

$$F_{HJ} = 2.97 \text{ kN} \quad T \blacktriangleleft$$

$$+\!\uparrow F_y = 0: \quad -F_{JK} - 1.2 \text{ kN} - \frac{0.6}{3.03}(3.03 \text{ kN}) = 0$$

$$F_{JK} = -1.800 \text{ kN} \qquad F_{JK} = 1.800 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint *H*



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{HK} + 2.97 \text{ kN} - \frac{2.97}{3.03}(3.03 \text{ kN}) = 0$$

$$F_{HK} = 0 \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad -F_{HI} - 1.2 \text{ kN} - \frac{0.6}{3.03}(3.03) \text{ kN} - \frac{3}{5}(0) = 0$$

$$F_{HI} = -1.800 \text{ kN} \qquad F_{HI} = 1.800 \text{ kN} \quad C \blacktriangleleft$$

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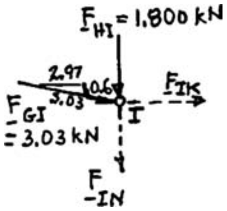
PROBLEM 6.25 (Continued)



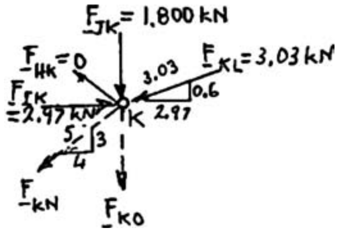
Free body: Joint *I*

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{IK} + \frac{2.97}{3.03}(3.03\text{ kN}) = 0$$

$$F_{IK} = -2.97\text{ kN} \qquad F_{IK} = 2.97\text{ kN} \quad C \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{IN} - 1.800\text{ kN} - \frac{0.6}{3.03}(3.03\text{ kN}) = 0$$

$$F_{IN} = -2.40\text{ kN} \qquad F_{IN} = 2.40\text{ kN} \quad C \blacktriangleleft$$



Free body: Joint *K*

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{4}{5}F_{KN} + 2.97\text{ kN} - \frac{2.97}{3.03}(3.03\text{ kN}) = 0$$

$$F_{KN} = 0 \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{KD} - \frac{0.6}{3.03}(3.03\text{ kN}) - 1.800\text{ kN} - \frac{3}{5}(0) = 0$$

$$F_{KD} = -2.40\text{ kN} \qquad F_{KD} = 2.40\text{ kN} \quad C \blacktriangleleft$$

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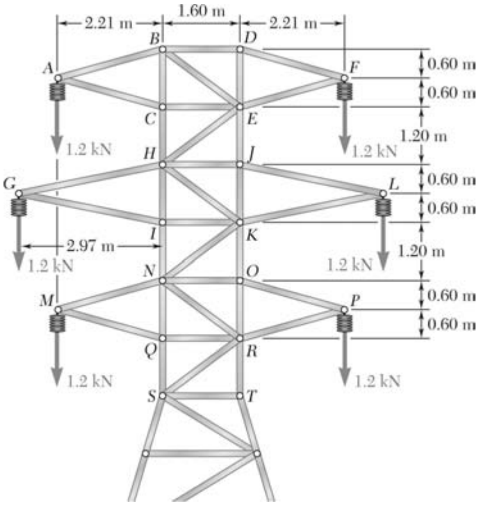
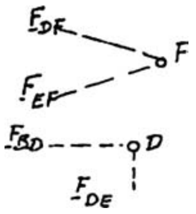


PROBLEM 6.26

Solve Problem 6.24 assuming that the cables hanging from the right side of the tower have fallen to the ground.

PROBLEM 6.24 The portion of truss shown represents the upper part of a power transmission line tower. For the given loading, determine the force in each of the members located above *HJ*. State whether each member is in tension or compression.

SOLUTION



Zero-Force Members.

Considering joint *F*, we note that *DF* and *EF* are zero-force members:

$$F_{DF} = F_{EF} = 0 \blacktriangleleft$$

Considering next joint *D*, we note that *BD* and *DE* are zero-force members:

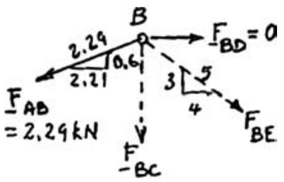
$$F_{BD} = F_{DE} = 0 \blacktriangleleft$$

Free body: Joint *A*

$$\frac{F_{AB}}{2.29} = \frac{F_{AC}}{2.29} = \frac{1.2\text{ kN}}{1.2} \qquad F_{AB} = 2.29\text{ kN} \quad T \blacktriangleleft$$

$$F_{AC} = 2.29\text{ kN} \quad C \blacktriangleleft$$

Free body: Joint *B*



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{BE} - \frac{2.21}{2.29}(2.29\text{ kN}) = 0$$

$$F_{BE} = 2.7625\text{ kN} \qquad F_{BE} = 2.76\text{ kN} \quad T \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad -F_{BC} - \frac{0.6}{2.29}(2.29\text{ kN}) - \frac{3}{5}(2.7625\text{ kN}) = 0$$

$$F_{BC} = -2.2575\text{ kN} \qquad F_{BC} = 2.26\text{ kN} \quad C \blacktriangleleft$$

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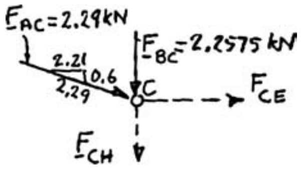
PROBLEM 6.26 (Continued)

Free body: Joint C



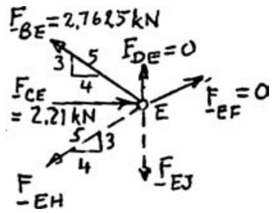
$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} + \frac{2.21}{2.29}(2.29 \text{ kN}) = 0$$

$$F_{CE} = 2.21 \text{ kN} \quad C \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad -F_{CH} - 2.2575 \text{ kN} - \frac{0.6}{2.29}(2.29 \text{ kN}) = 0$$

$$F_{CH} = -2.8575 \text{ kN} \qquad F_{CH} = 2.86 \text{ kN} \quad C \blacktriangleleft$$

Free body: joint E



$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{4}{5}F_{EH} - \frac{4}{5}(2.7625 \text{ kN}) + 2.21 \text{ kN} = 0$$

$$F_{EH} = 0 \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad -F_{EJ} + \frac{3}{5}(2.7625 \text{ kN}) - \frac{3}{5}(0) = 0$$

$$F_{EJ} = +1.6575 \text{ kN} \qquad F_{EJ} = 1.658 \text{ kN} \quad T \blacktriangleleft$$

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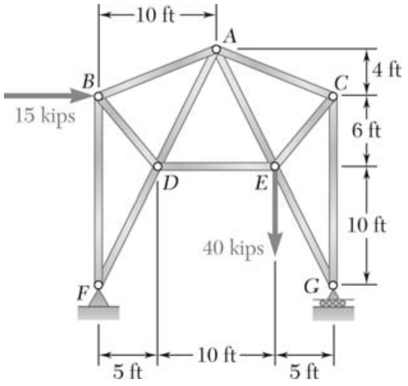
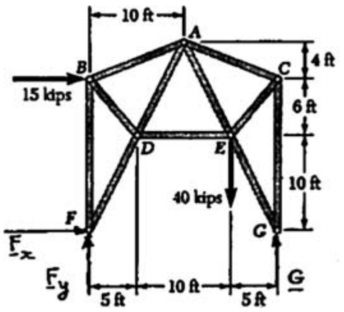


PROBLEM 6.27

Determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION



Free body: Truss

$$+\circlearrowleft \Sigma M_F = 0: \quad G(20\text{ ft}) - (15\text{ kips})(16\text{ ft}) - (40\text{ kips})(15\text{ ft}) = 0$$

$$\mathbf{G} = 42\text{ kips}\uparrow$$

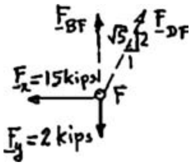
$$\xrightarrow{+} \Sigma F_x = 0: \quad F_x + 15\text{ kips} = 0$$

$$\mathbf{F}_x = 15\text{ kips} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad F_y - 40\text{ kips} + 42\text{ kips} = 0$$

$$\mathbf{F}_y = 2\text{ kips}\downarrow$$

Free body: Joint F



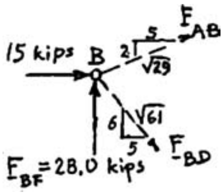
$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{5}} F_{DF} - 15\text{ kips} = 0$$

$$F_{DF} = 33.54\text{ kips} \qquad F_{DF} = 33.5\text{ kips} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad F_{BF} - 2\text{ kips} + \frac{2}{\sqrt{5}}(33.54\text{ kips}) = 0$$

$$F_{BF} = -28.00\text{ kips} \qquad F_{BF} = 28.0\text{ kips} \quad C \blacktriangleleft$$

Free body: Joint B



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{5}{\sqrt{29}} F_{AB} + \frac{5}{\sqrt{61}} F_{BD} + 15\text{ kips} = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{2}{\sqrt{29}} F_{AB} - \frac{6}{\sqrt{61}} F_{BD} + 28\text{ kips} = 0 \qquad (2)$$

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PROBLEM 6.27 (Continued)

Multiply (1) by 6, (2) by 5, and add:

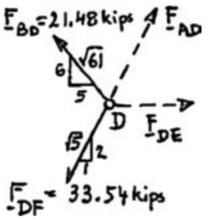
$$\frac{40}{\sqrt{29}}F_{AB} + 230\text{ kips} = 0$$

$F_{AB} = -30.96\text{ kips}$ $\qquad F_{AB} = 31.0\text{ kips}\ \ C$ ◀

Multiply (1) by 2, (2) by –5, and add:

$$\frac{40}{\sqrt{61}}F_{BD} - 110\text{ kips} = 0$$

$F_{BD} = 21.48\text{ kips}$ $\qquad F_{BD} = 21.5\text{ kips}\ \ T$ ◀



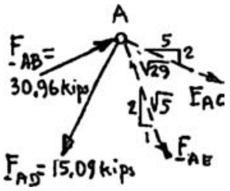
Free body: joint D

$$+\uparrow \Sigma F_y = 0:\quad \frac{2}{\sqrt{5}}F_{AD} - \frac{2}{\sqrt{5}}(33.54) + \frac{6}{\sqrt{61}}(21.48) = 0$$

$F_{AD} = 15.09\text{ kips}\ \ T$ ◀

$$\xrightarrow{+} \Sigma F_x = 0:\quad F_{DE} + \frac{1}{\sqrt{5}}(15.09 - 33.54) - \frac{5}{\sqrt{61}}(21.48) = 0$$

$F_{DE} = 22.0\text{ kips}\ \ T$ ◀



Free body: joint A

$$\xrightarrow{+} \Sigma F_x = 0:\quad \frac{5}{\sqrt{29}}F_{AC} + \frac{1}{\sqrt{5}}F_{AE} + \frac{5}{\sqrt{29}}(30.36) - \frac{1}{\sqrt{5}}(15.09) = 0 \qquad (3)$$

$$+\uparrow \Sigma F_y = 0:\quad -\frac{2}{\sqrt{29}}F_{AC} - \frac{2}{\sqrt{5}}F_{AE} + \frac{2}{\sqrt{29}}(30.96) - \frac{2}{\sqrt{5}}(15.09) = 0 \qquad (4)$$

Multiply (3) by 2 and add (4):

$$\frac{8}{\sqrt{29}}F_{AC} + \frac{12}{\sqrt{29}}(30.96) - \frac{4}{\sqrt{5}}(15.09) = 0$$

$F_{AC} = -28.27\text{ kips,}$ $\qquad F_{AC} = 28.3\text{ kips}\ \ C$ ◀

Multiply (3) by 2 (4) by 5 and add:

$$-\frac{8}{\sqrt{5}}F_{AE} + \frac{20}{\sqrt{29}}(30.96) - \frac{12}{\sqrt{5}}(15.09) = 0$$

$F_{AE} = 9.50\text{ kips}\ \ T$ ◀

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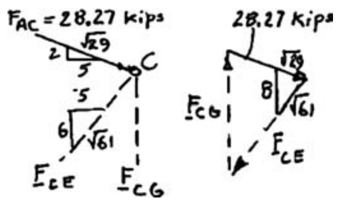
PROBLEM 6.27 (Continued)

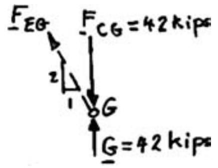


Free body: Joint C

From force triangle

$$\frac{F_{CE}}{\sqrt{61}} = \frac{F_{CG}}{8} = \frac{28.27 \text{ kips}}{\sqrt{29}} \qquad F_{CE} = 41.0 \text{ kips} \quad T \blacktriangleleft$$

$$F_{CG} = 42.0 \text{ kips} \quad C \blacktriangleleft$$



Free body: Joint G

$$\xrightarrow{+} \Sigma F_x = 0: \qquad F_{EG} = 0 \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad 42 \text{ kips} - 42 \text{ kips} = 0 \qquad \text{(Checks)}$$

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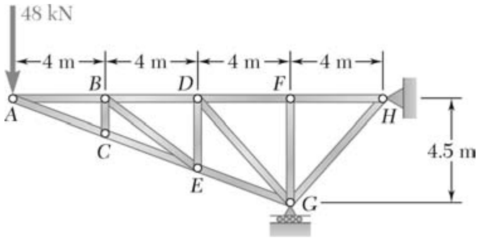
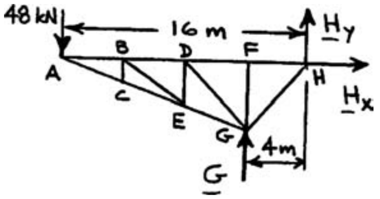


PROBLEM 6.28

Determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss



$$\Sigma F_x = 0: \quad \mathbf{H}_x = 0$$

$$+\circlearrowleft \Sigma M_H = 0: \quad 48(16) - G(4) = 0 \quad \mathbf{G} = 192 \text{ kN} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 192 - 48 + H_y = 0$$

$$H_y = -144 \text{ kN} \quad \mathbf{H}_y = 144 \text{ kN} \downarrow$$

Zero-Force Members:

Examining successively joints *C*, *B*, *E*, *D*, and *F*, we note that the following are zero-force members: *BC*, *BE*, *DE*, *DG*, *FG*

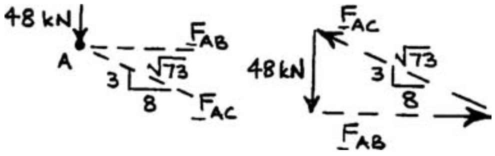
Thus, $F_{BC} = F_{BE} = F_{DE} = F_{DG} = F_{FG} = 0$

Also note: $F_{AB} = F_{BD} = F_{DF} = F_{FH}$ (1)

$F_{AC} = F_{CE} = F_{EG}$ (2)

Free body: Joint *A*:

$$\frac{F_{AB}}{8} = \frac{F_{AC}}{\sqrt{73}} = \frac{48 \text{ kN}}{3}$$



$F_{AB} = 128 \text{ kN}$ $\quad F_{AB} = 128.0 \text{ kN} \quad T$ ◀

$F_{AC} = 136.704 \text{ kN}$ $\quad F_{AC} = 136.7 \text{ kN} \quad C$ ◀

From Eq. (1): $F_{BD} = F_{DF} = F_{FH} = 128.0 \text{ kN} \quad T$ ◀

From Eq. (2): $F_{CE} = F_{EG} = 136.7 \text{ kN} \quad C$ ◀

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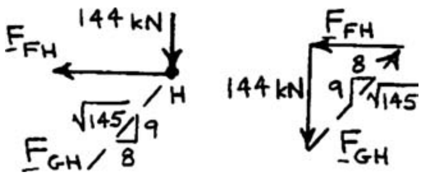
PROBLEM 6.28 (Continued)

<u>Free body: Joint *H*</u>

$$\frac{F_{GH}}{\sqrt{145}} = \frac{144\text{ kN}}{9}$$

$F_{GH} = 192.7\text{ kN}\quad C$ ◀



Also

$$\frac{F_{FH}}{8} = \frac{144\text{ kN}}{9}$$

$F_{FH} = 128.0\text{ kN}\quad T\quad$ (Checks)

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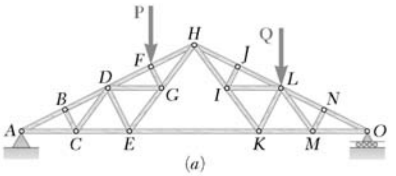
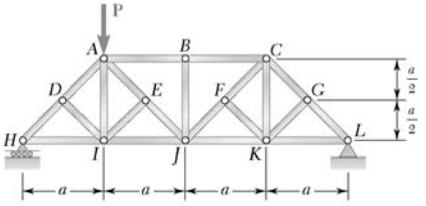
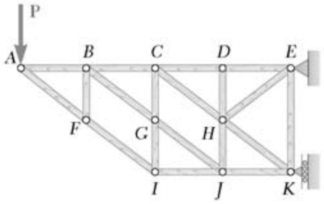


(a)



(a)

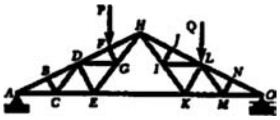


(a)

PROBLEM 6.29

Determine whether the trusses of Problems 6.31*a*, 6.32*a*, and 6.33*a* are simple trusses.

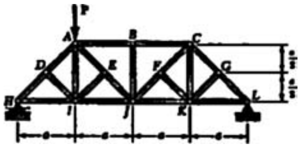
SOLUTION



Truss of Problem 6.31*a*

Starting with triangle *ABC* and adding two members at a time, we obtain joints *D*, *E*, *G*, *F*, and *H*, but cannot go further thus, this truss

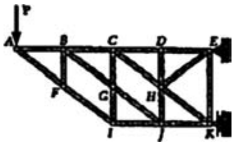
is not a simple truss ◀



Truss of Problem 6.32*a*

Starting with triangle *HDI* and adding two members at a time, we obtain successively joints *A*, *E*, *J*, and *B*, but cannot go further. Thus, this truss

is not a simple truss ◀



Truss of Problem 6.33*a*

Starting with triangle *EHK* and adding two members at a time, we obtain successively joints *D*, *J*, *C*, *G*, *I*, *B*, *F*, and *A*, thus completing the truss. Therefore, this truss

is a simple truss ◀

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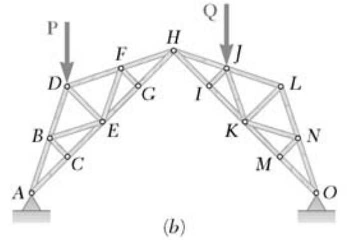
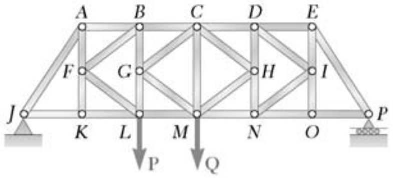


(b)



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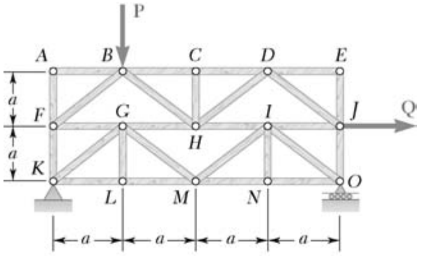
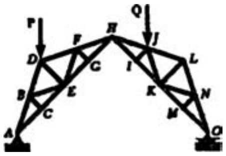


(b)

PROBLEM 6.30

Determine whether the trusses of Problems 6.31*b*, 6.32*b*, and 6.33*b* are simple trusses.

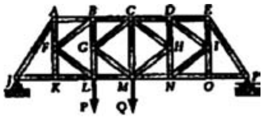
SOLUTION



Truss of Problem 6.31*b*.

Starting with triangle *ABC* and adding two members at a time, we obtain successively joints *E*, *D*, *F*, *G*, and *H*, but cannot go further. Thus, this truss

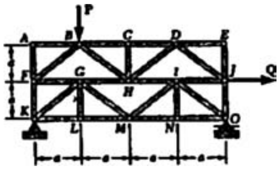
is not a simple truss ◀



Truss of Problem 6.32*b*.

Starting with triangle *CGH* and adding two members at a time, we obtain successively joints *B*, *L*, *F*, *A*, *K*, *J*, then *H*, *D*, *N*, *I*, *E*, *O*, and *P*, thus completing the truss.

Therefore, this truss is a simple truss ◀



Truss of Problem 6.33*b*.

Starting with triangle *GLM* and adding two members at a time, we obtain joints *K* and *I* but cannot continue, starting instead with triangle *BCH*, we obtain joint *D* but cannot continue, thus, this truss

is not a simple truss ◀

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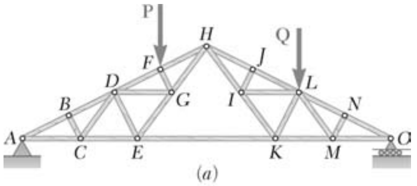
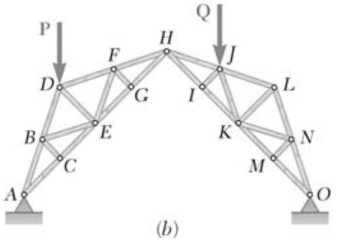


(a)



(b)

PROBLEM 6.31

For the given loading, determine the zero-force members in each of the two trusses shown.

SOLUTION

Truss (*a*)

FB: Joint B: $F_{BC} = 0$

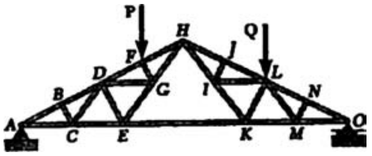
FB: Joint C: $F_{CD} = 0$

FB: Joint J: $F_{IJ} = 0$

FB: Joint I: $F_{IL} = 0$

FB: Joint N: $F_{MN} = 0$

FB: Joint M: $F_{LM} = 0$



(*a*)

The zero-force members, therefore, are BC, CD, IJ, IL, LM, MN ◀

Truss (*b*)

FB: Joint C: $F_{BC} = 0$

FB: Joint B: $F_{BE} = 0$

FB: Joint G: $F_{FG} = 0$

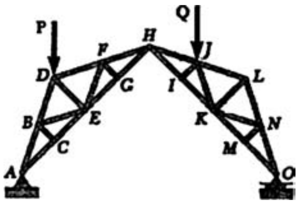
FB: Joint F: $F_{EF} = 0$

FB: Joint E: $F_{DE} = 0$

FB: Joint I: $F_{IJ} = 0$

FB: Joint M: $F_{MN} = 0$

FB: Joint N: $F_{KN} = 0$



(*b*)

The zero-force members, therefore, are $BC, BE, DE, EF, FG, IJ, KN, MN$ ◀

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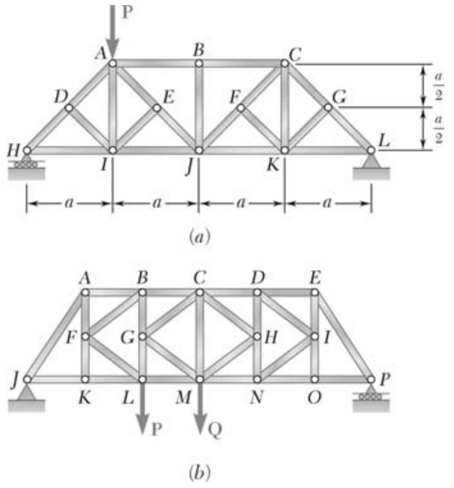
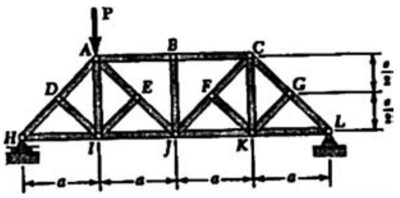


PROBLEM 6.32

For the given loading, determine the zero-force members in each of the two trusses shown.

SOLUTION



(*a*)

<u>Truss (*a*)</u>

FB: Joint *B*: $F_{BJ} = 0$

FB: Joint *D*: $F_{DI} = 0$

FB: Joint *E*: $F_{EI} = 0$

FB: Joint *I*: $F_{AI} = 0$

FB: Joint *F*: $F_{FK} = 0$

FB: Joint *G*: $F_{GK} = 0$

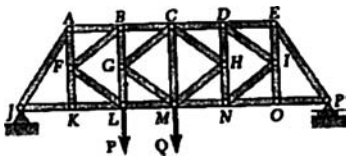
FB: Joint *K*: $F_{CK} = 0$

The zero-force members, therefore, are $AI, BJ, CK, DI, EI, FK, GK$ ◀

<u>Truss (*b*)</u>

FB: Joint *K*: $F_{FK} = 0$

FB: Joint *O*: $F_{IO} = 0$



(*b*)

The zero-force members, therefore, are *FK* and *IO* ◀

All other members are either in tension or compression.

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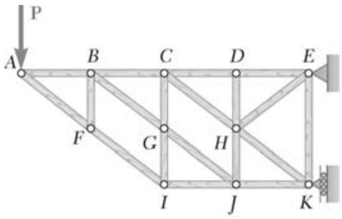
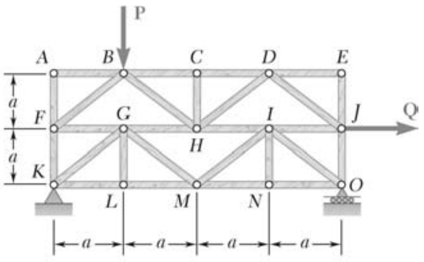
PROBLEM 6.33

For the given loading, determine the zero-force members in each of the two trusses shown.



(a)



(b)

SOLUTION

Truss (a)

FB: Joint F: $F_{BF} = 0$

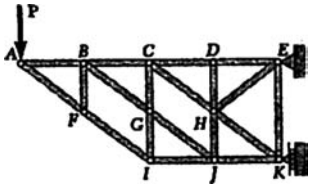
FB: Joint B: $F_{BG} = 0$

FB: Joint G: $F_{GJ} = 0$

FB: Joint D: $F_{DH} = 0$

FB: Joint J: $F_{HJ} = 0$

FB: Joint H: $F_{EH} = 0$



(a)

The zero-force members, therefore, are BF, BG, DH, EH, GJ, HJ ◀

Truss (b)

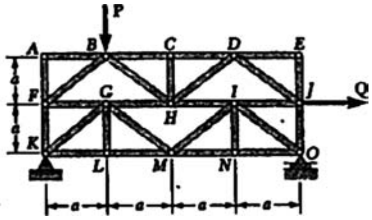
FB: Joint A: $F_{AB} = F_{AF} = 0$

FB: Joint C: $F_{CH} = 0$

FB: Joint E: $F_{DE} = F_{EJ} = 0$

FB: Joint L: $F_{GL} = 0$

FB: Joint N: $F_{IN} = 0$



(b)

The zero-force members, therefore, are $AB, AF, CH, DE, EJ, GL, IN$ ◀

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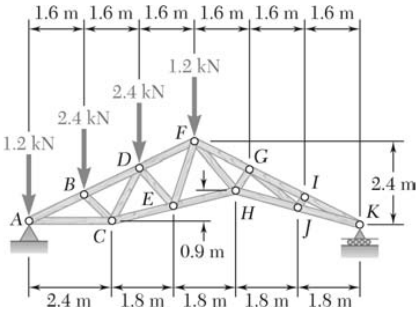
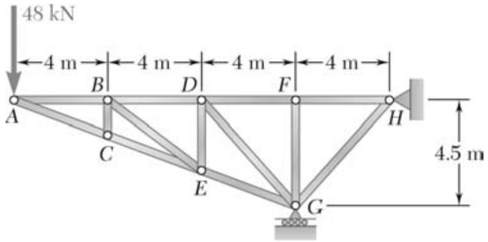
PROBLEM 6.34

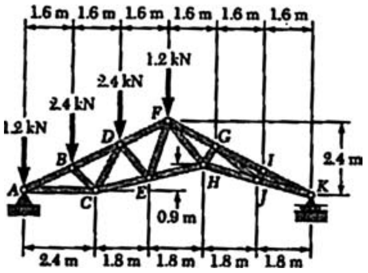
Determine the zero-force members in the truss of (*a*) Problem 6.23, (*b*) Problem 6.28.





SOLUTION

(*a*) Truss of Problem 6.23



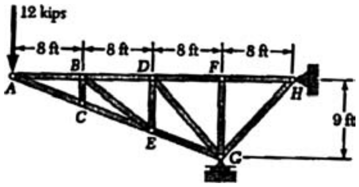
FB: Joint I: $F_{IJ} = 0$

FB: Joint J: $F_{GJ} = 0$

FB: Joint G: $F_{GH} = 0$

The zero-force members, therefore, are GH, GJ, IJ ◀

(*b*) Truss of Problem 6.28



FB: Joint C: $F_{BC} = 0$

FB: Joint B: $F_{BE} = 0$

FB: Joint E: $F_{DE} = 0$

FB: Joint D: $F_{DG} = 0$

FB: Joint F: $F_{FG} = 0$

The zero-force members, therefore, are BC, BE, DE, DG, FG ◀

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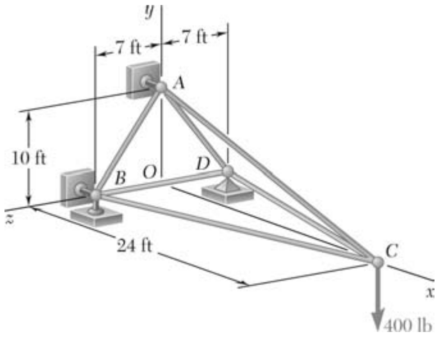
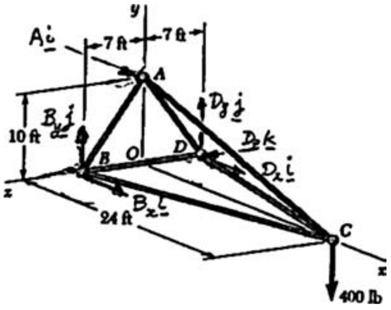


PROBLEM 6.35*

The truss shown consists of six members and is supported by a short link at *A*, two short links at *B*, and a ball and socket at *D*. Determine the force in each of the members for the given loading.

SOLUTION

<u>Free body: Truss</u>



From symmetry:

$$D_x = B_x \quad \text{and} \quad D_y = B_y$$

$$\Sigma M_z = 0: \quad -A(10\text{ ft}) - (400\text{ lb})(24\text{ ft}) = 0$$

$$A = -960\text{ lb}$$

$$\Sigma F_x = 0: \quad B_x + D_x + A = 0$$

$$2B_x - 960\text{ lb} = 0, \quad B_x = 480\text{ lb}$$

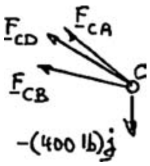
$$\Sigma F_y = 0: \quad B_y + D_y - 400\text{ lb} = 0$$

$$2B_y = 400\text{ lb}$$

$$B_y = +200\text{ lb}$$

Thus $\mathbf{B} = (480\text{ lb})\mathbf{i} + (200\text{ lb})\mathbf{j}$ ◁

<u>Free body: *C*</u>



$$F_{CA} = F_{AC}\frac{\overrightarrow{CA}}{CA} = \frac{F_{AC}}{26}(-24\mathbf{i} + 10\mathbf{j})$$

$$F_{CB} = F_{BC}\frac{\overrightarrow{CB}}{CB} = \frac{F_{BC}}{25}(-24\mathbf{i} + 7\mathbf{k})$$

$$F_{CD} = F_{CD}\frac{\overrightarrow{CD}}{CD} = \frac{F_{CD}}{25}(-24\mathbf{i} - 7\mathbf{k})$$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{CA} + \mathbf{F}_{CB} + \mathbf{F}_{CD} - (400\text{ lb})\mathbf{j} = 0$$

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PROBLEM 6.35* (Continued)

Substituting for $\mathbf{F}_{CA}, \mathbf{F}_{CB}, \mathbf{F}_{CD},$ and equating to zero the coefficients of $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$\mathbf{i}: \quad -\frac{24}{26}F_{AC} - \frac{24}{25}(F_{BC} + F_{CD}) = 0 \qquad (1)$$

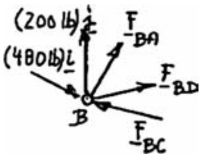
$$\mathbf{j}: \quad \frac{10}{26}F_{AC} - 400 \text{ lb} = 0 \qquad F_{AC} = 1040 \text{ lb} \quad T \blacktriangleleft$$

$$\mathbf{k}: \quad \frac{7}{25}(F_{BC} - F_{CD}) = 0 \quad F_{CD} = F_{BC}$$

Substitute for F_{AC} and F_{CD} in Eq. (1):

$$-\frac{24}{26}(10.40 \text{ lb}) - \frac{24}{25}(2\, F_{BC}) = 0 \quad F_{BC} = -500 \text{ lb} \qquad F_{BC} = F_{CD} = 500 \text{ lb} \quad C \blacktriangleleft$$

Free body: *B*



$$F_{BC} = (500 \text{ lb})\frac{\overrightarrow{CB}}{CB} = -(480 \text{ lb})\mathbf{i} + (140 \text{ lb})\mathbf{k}$$

$$F_{BA} = F_{AB}\frac{\overrightarrow{BA}}{BA} = \frac{F_{AB}}{12.21}(10\mathbf{j} - 7\mathbf{k})$$

$$F_{BD} = -F_{BD}\mathbf{k}$$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{BA} + \mathbf{F}_{BD} + \mathbf{F}_{BC} + (480 \text{ lb})\mathbf{i} + (200 \text{ lb})\mathbf{j} = 0$$

Substituting for $\mathbf{F}_{BA}, \mathbf{F}_{BD}, \mathbf{F}_{BC}$ and equating to zero the coefficients of $\mathbf{j}$ and $\mathbf{k}$:

$$\mathbf{j}: \quad \frac{10}{12.21}F_{AB} + 200 \text{ lb} = 0 \quad F_{AB} = -244.2 \text{ lb} \qquad F_{AB} = 244 \text{ lb} \quad C \blacktriangleleft$$

$$\mathbf{k}: \quad -\frac{7}{12.21}F_{AB} - F_{BD} + 140 \text{ lb} = 0$$

$$F_{BD} = -\frac{7}{12.21}(-244.2 \text{ lb}) + 140 \text{ lb} = +280 \text{ lb} \qquad F_{BD} = 280 \text{ lb} \quad T \blacktriangleleft$$

From symmetry: $\quad F_{AD} = F_{AB} \qquad F_{AD} = 244 \text{ lb} \quad C \blacktriangleleft$

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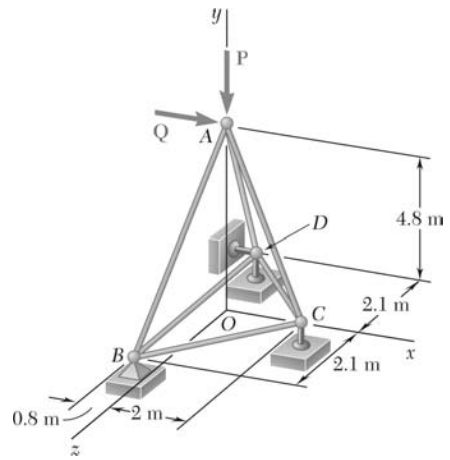
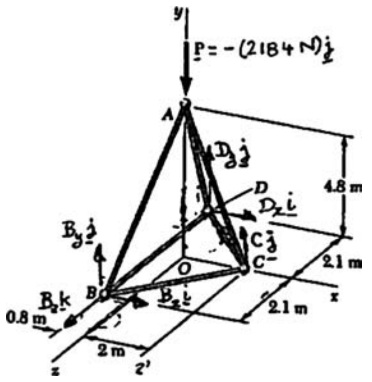


PROBLEM 6.36*

The truss shown consists of six members and is supported by a ball and socket at B, a short link at C, and two short links at D. Determine the force in each of the members for $\mathbf{P} = (-2184\text{ N})\mathbf{j}$ and $\mathbf{Q} = 0$.

SOLUTION



Free body: Truss

From symmetry:

$$D_x = B_x \text{ and } D_y = B_y$$

$$\Sigma F_x = 0: \quad 2B_x = 0$$

$$B_x = D_x = 0$$

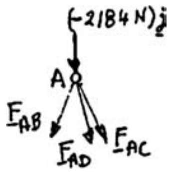
$$\Sigma F_z = 0: \quad B_z = 0$$

$$\Sigma M_{cz} = 0: \quad -2B_y(2.8\text{ m}) + (2184\text{ N})(2\text{ m}) = 0$$

$$B_y = 780\text{ N}$$

Thus $\qquad \mathbf{B} = (780\text{ N})\,j \ \triangleleft$

Free body: A



$$F_{AB} = F_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{F_{AB}}{5.30}(-0.8\mathbf{i} - 4.8\mathbf{j} + 2.1\mathbf{k})$$

$$F_{AC} = F_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{F_{AC}}{5.20}(2\mathbf{i} - 4.8\mathbf{j})$$

$$F_{AD} = F_{AD}\frac{\overrightarrow{AD}}{AD} = \frac{F_{AD}}{5.30}(+0.8\mathbf{i} - 4.8\mathbf{j} - 2.1\mathbf{k})$$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} - (2184\text{ N})\mathbf{j} = 0$$

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PROBLEM 6.36* (Continued)

Substituting for $\mathbf{F}_{AB}, \mathbf{F}_{AC}, \mathbf{F}_{AD},$ and equating to zero the coefficients of $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$\mathbf{i}: \quad -\frac{0.8}{5.30}(F_{AB} + F_{AD}) + \frac{2}{5.20}F_{AC} = 0 \qquad (1)$$

$$\mathbf{j}: \quad -\frac{4.8}{5.30}(F_{AB} + F_{AD}) - \frac{4.8}{5.20}F_{AC} - 2184 \text{ N} = 0 \qquad (2)$$

$$\mathbf{k}: \quad \frac{2.1}{5.30}(F_{AB} - F_{AD}) = 0 \qquad F_{AD} = F_{AB}$$

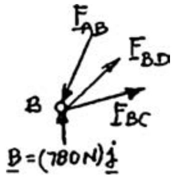
Multiply (1) by –6 and add (2):

$$-\left(\frac{16.8}{5.20}\right)F_{AC} - 2184 \text{ N} = 0, \quad F_{AC} = -676 \text{ N} \qquad F_{AC} = 676 \text{ N} \quad C \blacktriangleleft$$

Substitute for F_{AC} and F_{AD} in (1):

$$-\left(\frac{0.8}{5.30}\right)2F_{AB} + \left(\frac{2}{5.20}\right)(-676 \text{ N}) = 0, \quad F_{AB} = -861.25 \text{ N} \qquad F_{AB} = F_{AD} = 861 \text{ N} \quad C \blacktriangleleft$$

Free body: *B*



$$\mathbf{F}_{AB} = (861.25 \text{ N})\frac{\overrightarrow{AB}}{AB} = -(130 \text{ N})\mathbf{i} - (780 \text{ N})\mathbf{j} + (341.25 \text{ N})\mathbf{k}$$

$$\mathbf{F}_{BC} = F_{BC}\left(\frac{2.8i - 2.1k}{3.5}\right) = F_{BC}(0.8\mathbf{i} - 0.6\mathbf{k})$$

$$\mathbf{F}_{BD} = -F_{BD}\mathbf{k}$$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{AB} + \mathbf{F}_{BC} + \mathbf{F}_{BD} + (780 \text{ N})\mathbf{j} = 0$$

Substituting for $\mathbf{F}_{AB}, \mathbf{F}_{BC}, \mathbf{F}_{BD}$ and equating to zero the coefficients of $\mathbf{i}$ and $\mathbf{k}$:

$$\mathbf{i}: \quad -130 \text{ N} + 0.8\,F_{BC} = 0 \quad F_{BC} = +162.5 \text{ N} \qquad F_{BC} = 162.5 \text{ N} \quad T \blacktriangleleft$$

$$\mathbf{k}: \quad 341.25 \text{ N} - 0.6\,F_{BC} - F_{BD} = 0$$

$$F_{BD} = 341.25 - 0.6(162.5) = +243.75 \text{ N} \qquad F_{BD} = 244 \text{ N} \quad T \blacktriangleleft$$

From symmetry: $\qquad F_{CD} = F_{BC} \qquad F_{CD} = 162.5 \text{ N} \quad T \blacktriangleleft$

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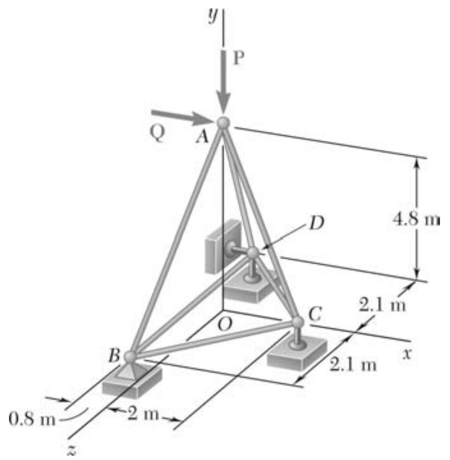
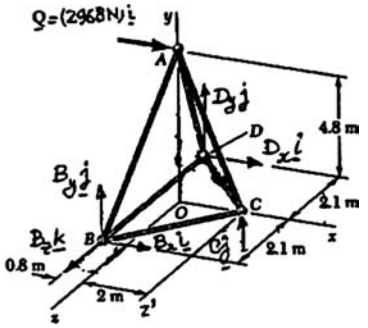


PROBLEM 6.37*

The truss shown consists of six members and is supported by a ball and socket at B, a short link at C, and two short links at D. Determine the force in each of the members for $\mathbf{P} = 0$ and $\mathbf{Q} = (2968\text{ N})\mathbf{i}$.

SOLUTION



Free body: Truss

From symmetry:

$$D_x = B_x \text{ and } D_y = B_y$$

$$\Sigma F_x = 0: \quad 2B_x + 2968\text{ N} = 0$$

$$B_x = D_x = -1484\text{ N}$$

$$\Sigma M_{cz'} = 0: \quad -2B_y(2.8\text{ m}) - (2968\text{ N})(4.8\text{ m}) = 0$$

$$B_y = -2544\text{ N}$$

Thus $\qquad \mathbf{B} = -(1484\text{ N})\mathbf{i} - (2544\text{ N})\mathbf{j}$ ◁

Free body: A

$$F_{AB} = F_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{F_{AB}}{5.30}(-0.8\mathbf{i} - 4.8\mathbf{j} + 2.1\mathbf{k})$$

$$F_{AC} = F_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{F_{AC}}{5.20}(2\mathbf{i} - 4.8\mathbf{j})$$

$$F_{AD} = F_{AD}\frac{\overrightarrow{AD}}{AD} = \frac{F_{AD}}{5.30}(-0.8\mathbf{i} - 4.8\mathbf{j} - 2.1\mathbf{k})$$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + (2968\text{ N})\mathbf{i} = 0$$

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PROBLEM 6.37* (Continued)

Substituting for $\mathbf{F}_{AB}, \mathbf{F}_{AC}, \mathbf{F}_{AD},$ and equating to zero the coefficients of $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$\mathbf{i}: \quad -\frac{0.8}{5.30}(F_{AB}+F_{AD})+\frac{2}{5.20}F_{AC}+2968\text{ N}=0 \qquad (1)$$

$$\mathbf{j}: \quad -\frac{4.8}{5.30}(F_{AB}+F_{AD})-\frac{4.8}{5.20}F_{AC}=0 \qquad (2)$$

$$\mathbf{k}: \quad \frac{2.1}{5.30}(F_{AB}-F_{AD})=0 \qquad F_{AD}=F_{AB}$$

Multiply (1) by –6 and add (2):

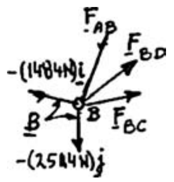
$$-\left(\frac{16.8}{5.20}\right)F_{AC}-6(2968\text{ N})=0, \quad F_{AC}=-5512\text{ N} \qquad F_{AC}=5510\text{ N} \quad C \blacktriangleleft$$

Substitute for F_{AC} and F_{AD} in (2):

$$-\left(\frac{4.8}{5.30}\right)2F_{AB}-\left(\frac{4.8}{5.20}\right)(-5512\text{ N})=0, \quad F_{AB}=+2809\text{ N}$$

$$F_{AB}=F_{AD}=2810\text{ N} \quad T \blacktriangleleft$$

<u>Free body: *B*</u>



$$\mathbf{F}_{AB}=(2809\text{ N})\frac{\overrightarrow{BA}}{BA}=(424\text{ N})\mathbf{i}+(2544\text{ N})\mathbf{j}-(1113\text{ N})\mathbf{k}$$

$$\mathbf{F}_{BC}=F_{BC}\left(\frac{2.8\mathbf{i}-2.1\mathbf{k}}{3.5}\right)=F_{BC}(0.8\mathbf{i}-0.6\mathbf{k})$$

$$\mathbf{F}_{BD}=-F_{BD}\mathbf{k}$$

$$\Sigma\mathbf{F}=0: \quad \mathbf{F}_{AB}+\mathbf{F}_{BC}+\mathbf{F}_{BD}-(1484\text{ N})\mathbf{i}-(2544\text{ N})\mathbf{j}=0$$

Substituting for $\mathbf{F}_{AB}, \mathbf{F}_{BC}, \mathbf{F}_{BD}$ and equating to zero the coefficients of $\mathbf{i}$ and $\mathbf{k}$:

$$\mathbf{i}: \quad +24\text{ N}+0.8F_{BC}-1484\text{ N}=0, \quad F_{BC}=+1325\text{ N} \qquad F_{BC}=1325\text{ N} \quad T \blacktriangleleft$$

$$\mathbf{k}: \quad -1113\text{ N}-0.6F_{BC}-F_{BD}=0$$

$$F_{BD}=-1113\text{ N}-0.6(1325\text{ N})=-1908\text{ N}, \qquad F_{BD}=1908\text{ N} \quad C \blacktriangleleft$$

From symmetry: $F_{CD}=F_{BC}$ $\qquad F_{CD}=1325\text{ N} \quad T \blacktriangleleft$

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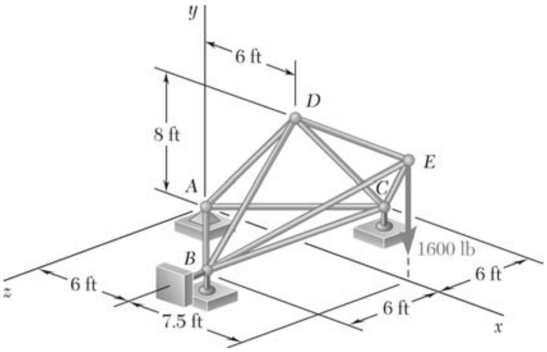
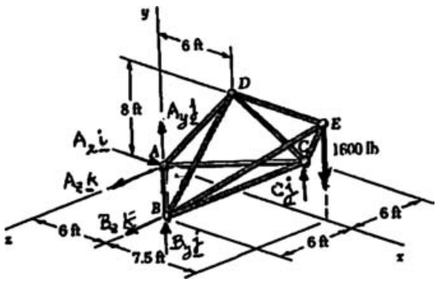


PROBLEM 6.38*

The truss shown consists of nine members and is supported by a ball and socket at *A*, two short links at *B*, and a short link at *C*. Determine the force in each of the members for the given loading.

SOLUTION



Free body: Truss

From symmetry:

$$A_z = B_z = 0$$

$$\Sigma F_x = 0: \quad A_x = 0$$

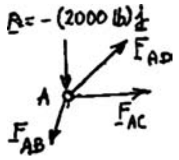
$$\Sigma M_{BC} = 0: \quad A_y(6\text{ ft}) + (1600\text{ lb})(7.5\text{ ft}) = 0$$

$$A_y = -2000\text{ lb} \qquad \mathbf{A} = -(2000\text{ lb})\mathbf{j} \ \triangleleft$$

From symmetry: $B_y = C$

$$\Sigma F_y = 0: \quad 2B_y - 2000\text{ lb} - 1600\text{ lb} = 0$$

$$B_y = 1800\text{ lb} \qquad \mathbf{B} = (1800\text{ lb})\mathbf{j} \ \triangleleft$$



Free body: *A*

$$\Sigma \mathbf{F} = 0: \quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} - (2000\text{ lb})\mathbf{j} = 0$$

$$F_{AB}\frac{\mathbf{i}+\mathbf{k}}{\sqrt{2}} + F_{AC}\frac{\mathbf{i}-\mathbf{k}}{\sqrt{2}} + F_{AD}(0.6\mathbf{i} + 0.8\mathbf{j}) - (2000\text{ lb})\mathbf{j} = 0$$

Factoring **i**, **j**, **k** and equating their coefficient to zero:

$$\frac{1}{\sqrt{2}}F_{AB} + \frac{1}{\sqrt{2}}F_{AC} + 0.6F_{AD} = 0 \qquad (1)$$

$$0.8F_{AD} - 2000\text{ lb} = 0 \qquad F_{AD} = 2500\text{ lb} \quad T \ \blacktriangleleft$$

$$\frac{1}{\sqrt{2}}F_{AB} - \frac{1}{\sqrt{2}}F_{AC} = 0 \qquad F_{AC} = F_{AB}$$

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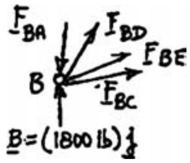
PROBLEM 6.38* (Continued)

Substitute for F_{AD} and F_{AC} into (1):

$$\frac{2}{\sqrt{2}}F_{AB} + 0.6(2500\text{ lb}) = 0, \quad \mathbf{F}_{AB} = -1060.7\text{ lb}, \qquad F_{AB} = F_{AC} = 1061\text{ lb} \quad C \blacktriangleleft$$

Free body: *B*



$$\mathbf{F}_{BA} = F_{AB}\frac{\overrightarrow{BA}}{BA} = +(1060.7\text{ lb})\frac{\mathbf{i}+\mathbf{k}}{\sqrt{2}} = (750\text{ lb})(\mathbf{i}+\mathbf{k})$$

$$\mathbf{F}_{BC} = -F_{BC}\mathbf{k}$$

$$\mathbf{F}_{BD} = F_{BD}(0.8\mathbf{j} - 0.6\mathbf{k})$$

$$\mathbf{F}_{BE} = F_{BE}\frac{\overrightarrow{BE}}{BE} = \frac{F_{BE}}{12.5}(7.5\mathbf{i} + 8\mathbf{j} - 6\mathbf{k})$$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{BA} + \mathbf{F}_{BC} + \mathbf{F}_{BD} + \mathbf{F}_{BE} + (1800\text{ lb})\mathbf{j} = 0$$

Substituting for $\mathbf{F}_{BA}, \mathbf{F}_{BC}, \mathbf{F}_{BD} + \mathbf{F}_{BE}$ and equate to zero the coefficients of $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$\mathbf{i}: \quad 750\text{ lb} + \left(\frac{7.5}{12.5}\right)F_{BE} = 0, \quad F_{BE} = -1250\text{ lb} \qquad F_{BE} = 1250\text{ lb} \quad C \blacktriangleleft$$

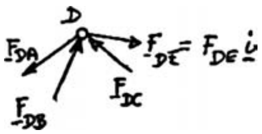
$$\mathbf{j}: \quad 0.8\,F_{BD} + \left(\frac{8}{12.5}\right)(-1250\text{ lb}) + 1800\text{ lb} = 0 \qquad F_{BD} = 1250\text{ lb} \quad C \blacktriangleleft$$

$$\mathbf{k}: \quad 750\text{ lb} - F_{BC} - 0.6(-1250\text{ lb}) - \frac{6}{12.5}(-1250\text{ lb}) = 0$$

$$F_{BC} = 2100\text{ lb} \quad T \blacktriangleleft$$

From symmetry: $\quad F_{BD} = F_{CD} = 1250\text{ lb} \quad C \blacktriangleleft$

Free body: *D*



$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DC} + \mathbf{F}_{DE}\mathbf{i} = 0$$

We now substitute for $\mathbf{F}_{DA}, \mathbf{F}_{DB}, \mathbf{F}_{DC}$ and equate to zero the coefficient of $\mathbf{i}$. Only $\mathbf{F}_{DA}$ contains $\mathbf{i}$ and its coefficient is

$$-0.6\,F_{AD} = -0.6(2500\text{ lb}) = -1500\text{ lb}$$

$$\mathbf{i}: \quad -1500\text{ lb} + F_{DE} = 0 \qquad F_{DE} = 1500\text{ lb} \quad T \blacktriangleleft$$

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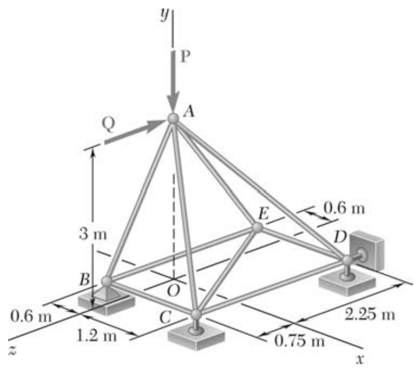
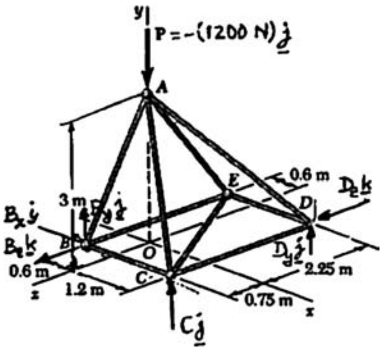


PROBLEM 6.39*

The truss shown consists of nine members and is supported by a ball and socket at B, a short link at C, and two short links at D. (a) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (b) Determine the force in each member for $\mathbf{P} = (-1200\text{ N})\mathbf{j}$ and $\mathbf{Q} = 0$.

SOLUTION



Free body: Truss

$$\Sigma\mathbf{M}_B = 0:\quad 1.8\mathbf{i}\times C\mathbf{j} + (1.8\mathbf{i} - 3\mathbf{k})\times(D_y\mathbf{j} + D\mathbf{k})$$

$$+(0.6\mathbf{i} - 0.75\mathbf{k})\times(-1200\mathbf{j}) = 0$$

$$-1.8\,C\mathbf{k} + 1.8\,D_y\mathbf{k} - 1.8\,D_z\mathbf{j}$$

$$+3D_y\mathbf{i} - 720\mathbf{k} - 900\mathbf{i} = 0$$

Equate to zero the coefficients of $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$:

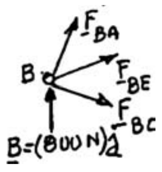
$\mathbf{i}$: $\quad 3D_y - 900 = 0, \quad D_y = 300\text{ N}$

$\mathbf{j}$: $\quad D_z = 0, \qquad \mathbf{D} = (300\text{ N})\mathbf{j}$ ◁

$\mathbf{k}$: $\quad 1.8C + 1.8(300) - 720 = 0 \qquad \mathbf{C} = (100\text{ N})\mathbf{j}$ ◁

$\Sigma\mathbf{F} = 0:\quad \mathbf{B} + 300\mathbf{j} + 100\mathbf{j} - 1200\mathbf{j} = 0 \qquad \mathbf{B} = (800\text{ N})\mathbf{j}$ ◁

Free body: B



$$\Sigma\mathbf{F} = 0:\quad \mathbf{F}_{BA} + \mathbf{F}_{BC} + \mathbf{F}_{BE} + (800\text{ N})\mathbf{j} = 0, \text{ with}$$

$$\mathbf{F}_{BA} = \mathbf{F}_{AB}\frac{\overrightarrow{BA}}{BA} = \frac{F_{AB}}{3.15}(0.6\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k})$$

$$\mathbf{F}_{BC} = F_{BC}\mathbf{i} \qquad \mathbf{F}_{BE} = -F_{BE}\mathbf{k}$$

Substitute and equate to zero the coefficient of $\mathbf{j}$, $\mathbf{i}$, $\mathbf{k}$:

$\mathbf{j}$: $\quad \left(\dfrac{3}{3.315}\right)F_{AB} + 800\text{ N} = 0, \quad F_{AB} = -840\text{ N}, \qquad F_{AB} = 840\text{ N} \quad C$ ◀

$\mathbf{i}$: $\quad \left(\dfrac{0.6}{3.15}\right)(-840\text{ N}) + F_{BC} = 0 \qquad F_{BC} = 160.0\text{ N} \quad T$ ◀

$\mathbf{k}$: $\quad \left(-\dfrac{0.75}{3.15}\right)(-840\text{ N}) - F_{BE} = 0 \qquad F_{BE} = 200\text{ N} \quad T$ ◀

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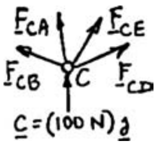
PROBLEM 6.39* (Continued)

Free body C:



$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{CA} + \mathbf{F}_{CB} + \mathbf{F}_{CD} + \mathbf{F}_{CE} + (100\text{ N})\mathbf{j} = 0, \text{ with}$$

$$\mathbf{F}_{CA} = F_{AC}\frac{\overrightarrow{CA}}{CA} = \frac{F_{AC}}{3.317}(-1.2\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k})$$

$$F_{CB} = -(160\text{ N})\,\mathbf{i}$$

$$\mathbf{F}_{CD} = -F_{CD}\mathbf{k} \quad \mathbf{F}_{CE} = \mathbf{F}_{CE}\frac{\overrightarrow{CE}}{CE} = \frac{F_{CE}}{3,499}(-1.8\mathbf{i} - 3\mathbf{k})$$

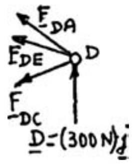
Substitute and equate to zero the coefficient of $\mathbf{j}, \mathbf{i}, \mathbf{k}$:

$\mathbf{j}$: $\left(\frac{3}{3.317}\right)F_{AC} + 100\text{ N} = 0, \quad F_{AC} = -110.57\text{ N}$ $\qquad F_{AC} = 110.6\text{ N} \quad C$ ◀

$\mathbf{i}$: $-\frac{1.2}{3.317}(-110.57) - 160 - \frac{1.8}{3.499}F_{CE} = 0, \quad F_{CE} = -233.3$ $\qquad F_{CE} = 233\text{ N} \quad C$ ◀

$\mathbf{k}$: $-\frac{0.75}{3.317}(-110.57) - F_{CD} - \frac{3}{3.499}(-233.3) = 0$ $\qquad F_{CD} = 225\text{ N} \quad T$ ◀

Free body: D



$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{DA} + \mathbf{F}_{DC} + \mathbf{F}_{DE} + (300\text{ N})\mathbf{j} = 0, \text{ with}$$

$$\mathbf{F}_{DA} = F_{AD}\frac{\overrightarrow{DA}}{DA} = \frac{F_{AD}}{3.937}(-1.2\mathbf{i} + 3\mathbf{j} + 2.25\mathbf{k})$$

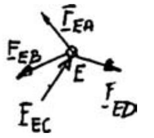
$$\mathbf{F}_{DC} = F_{CD}\mathbf{k} = (225\text{ N})\mathbf{k} \qquad F_{DE} = -F_{DE}\mathbf{i}$$

Substitute and equate to zero the coefficient of $\mathbf{j}, \mathbf{i}, \mathbf{k}$:

$\mathbf{j}$: $\left(\frac{3}{3.937}\right)F_{AD} + 300\text{ N} = 0, \qquad F_{AD} = -393.7\text{ N},$ $\qquad F_{AD} = 394\text{ N} \quad C$ ◀

$\mathbf{i}$: $\left(-\frac{1.2}{3.937}\right)(-393.7\text{ N}) - F_{DE} = 0$ $\qquad F_{DE} = 1200\text{ N} \quad T$ ◀

$\mathbf{k}$: $\left(\frac{2.25}{3.937}\right)(-393.7\text{ N}) + 225\text{ N} = 0 \quad$ (Checks)



Free body: E

Member AE is the only member at E which does not lie in the xz plane.
Therefore, it is a zero-force member.

$F_{AE} = 0$ ◀

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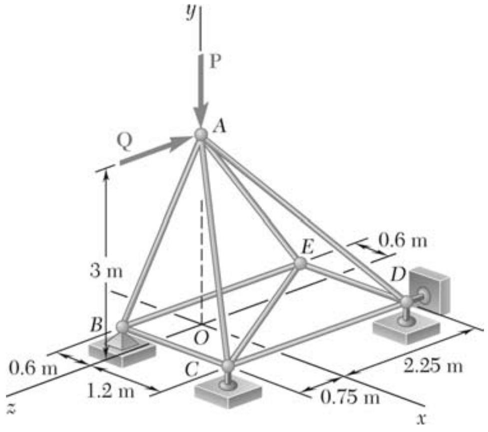


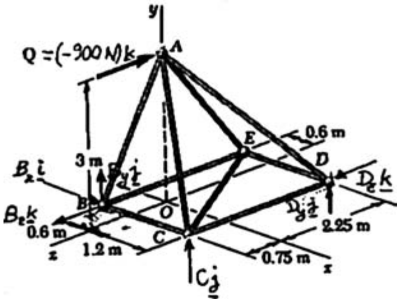
PROBLEM 6.40*

Solve Problem 6.39 for **P** = 0 and **Q** = (–900 N)**k**.

PROBLEM 6.39* The truss shown consists of nine members and is supported by a ball and socket at *B*, a short link at *C*, and two short links at *D*. (*a*) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (*b*) Determine the force in each member for **P** = (–1200 N)**j** and **Q** = 0.

SOLUTION

Free body: Truss



$$\Sigma \mathbf{M}_B = 0: \quad 1.8\mathbf{i} \times C\mathbf{j} + (1.8\mathbf{i} - 3\mathbf{k}) \times (D_y\mathbf{j} + D_z\mathbf{k})$$

$$+ (0.6\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k}) \times (-900\text{N})\mathbf{k} = 0$$

$$1.8C\mathbf{k} + 1.8D_y\mathbf{k} - 1.8D_z\mathbf{j}$$

$$+ 3D_y\mathbf{i} + 540\mathbf{j} - 2700\mathbf{i} = 0$$

Equate to zero the coefficient of $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$3D_y - 2700 = 0 \quad D_y = 900 \text{ N}$$

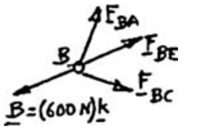
$$-1.8D_z + 540 = 0 \quad D_z = 300 \text{ N}$$

$$1.8C + 1.8D_y = 0 \quad C = -D_y = -900 \text{ N}$$

Thus: $\mathbf{C} = -(900 \text{ N})\mathbf{j} \quad \mathbf{D} = (900 \text{ N})\mathbf{j} + (300 \text{ N})\mathbf{k}$ ◁

$\Sigma \mathbf{F} = 0: \quad \mathbf{B} - 900\mathbf{j} + 900\mathbf{j} + 300\mathbf{k} - 900\mathbf{k} = 0$ $\qquad \mathbf{B} = (600 \text{ N})\mathbf{k}$ ◁

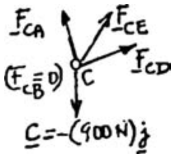
Free body: *B*



Since **B** is aligned with member *BE*:

$F_{AB} = F_{BC} = 0, \quad F_{BE} = 600 \text{ N} \quad T$ ◀

Free body: *C*



$\Sigma \mathbf{F} = 0$: $\mathbf{F}_{CA} + \mathbf{F}_{CD} + \mathbf{F}_{CE} - (900 \text{ N})\mathbf{j} = 0,$ with

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PROBLEM 6.40* (Continued)

$$\mathbf{F}_{CA} = F_{AC}\frac{\overrightarrow{CA}}{CA} = \frac{F_{AC}}{3.317}(-1.2\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k})$$

$$\mathbf{F}_{CD} = -F_{CD}\mathbf{k} \quad \mathbf{F}_{CE} = F_{CE}\frac{\overrightarrow{CE}}{CE} = \frac{F_{CE}}{3.499}(-1.8\mathbf{i} - 3\mathbf{k})$$

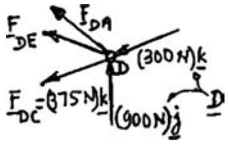
Substitute and equate to zero the coefficient of **j**, **i**, **k**:

j: $\left(\frac{3}{3.317}\right)F_{AC} - 900\text{ N} = 0, \quad F_{AC} = 995.1\text{ N}$ $\qquad F_{AC} = 995\text{ N}\quad T$ ◀

i: $-\frac{1.2}{3.317}(995.1) - \frac{1.8}{3.499}F_{CE} = 0, \quad F_{CE} = -699.8\text{ N}$ $\qquad F_{CE} = 700\text{ N}\quad C$ ◀

k: $-\frac{0.75}{3.317}(995.1) - F_{CD} - \frac{3}{3.499}(-699.8) = 0$ $\qquad F_{CD} = 375\text{ N}\quad T$ ◀

Free body: *D*



$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{DA} + \mathbf{F}_{DE} + (375\text{ N})\mathbf{k} + (900\text{ N})\mathbf{j} + (300\text{ N})\mathbf{k} = 0$$

with $\qquad \mathbf{F}_{DA} = \mathbf{F}_{AD}\frac{\overrightarrow{DA}}{DA} = \frac{F_{AD}}{3.937}(-1.2\mathbf{i} + 3\mathbf{j} + 2.25\mathbf{k})$

and $\qquad F_{DE} = -F_{DE}\,\mathbf{i}$

Substitute and equate to zero the coefficient **j**, **i**, **k**:

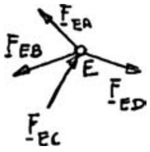
j: $\left(\frac{3}{3.937}\right)F_{AD} + 900\text{ N} = 0, \quad F_{AD} = -1181.1\text{ N}$ $\qquad F_{AD} = 1181\text{N}\quad C$ ◀

i: $-\left(\frac{1.2}{3.937}\right)(-1181.1\text{ N}) - F_{DE} = 0$ $\qquad F_{DE} = 360\text{ N}\quad T$ ◀

k: $\left(\frac{2.25}{3.937}\right)(-1181.1\text{ N} + 375\text{ N} + 300\text{ N} = 0)$ $\qquad$ (Checks)

Free body: *E*

Member *AE* is the only member at *E* which does not lie in the *XZ* plane. Therefore, it is a zero-force member.



$F_{AE} = 0$ ◀

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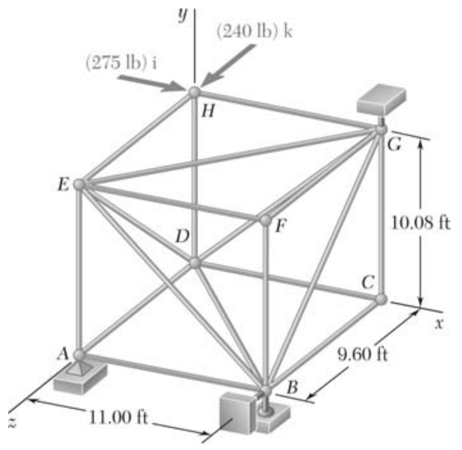


PROBLEM 6.41*

The truss shown consists of 18 members and is supported by a ball and socket at A, two short links at B, and one short link at G. (a) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (b) For the given loading, determine the force in each of the six members joined at E.

SOLUTION

(a) Check simple truss.

(1) start with tetrahedron $BEFG$

(2) Add members BD, ED, GD joining at D.

(3) Add members BA, DA, EA joining at A.

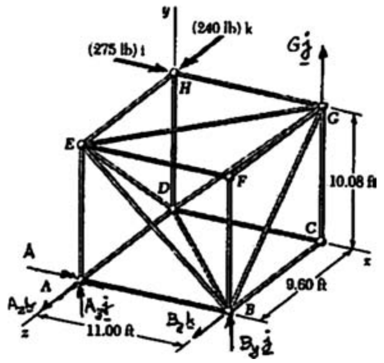
(4) Add members DH, EH, GH joining at H.

(5) Add members BC, DC, GC joining at C.

Truss has been completed: It is a simple truss

Free body: Truss

Check constraints and reactions:



Six unknown reactions-ok-however supports at A and B constrain truss to rotate about AB and support at G prevents such a rotation. Thus,

Truss is completely constrained and reactions are statically determinate

Determination of Reactions:

$$\Sigma \mathbf{M}_A = 0: \quad 11\mathbf{i} \times (B_y\mathbf{j} + B_z\mathbf{k}) + (11\mathbf{i} - 9.6\mathbf{k}) \times G\mathbf{j}$$
$$+ (10.08\mathbf{j} - 9.6\mathbf{k}) \times (275\mathbf{i} + 240\mathbf{k}) = 0$$
$$11B_y\mathbf{k} - 11B_y\mathbf{j} + 11G\mathbf{k} + 9.6G\mathbf{i} - (10.08)(275)\mathbf{k}$$
$$+ (10.08)(240)\mathbf{i} - (9.6)(275)\mathbf{j} = 0$$

Equate to zero the coefficient of **i**, **j**, **k**:

$$\mathbf{i}: \quad 9.6G + (10.08)(240) = 0 \quad G = -252 \text{ lb} \qquad \mathbf{G} = (-252 \text{ lb})\mathbf{j} \ \triangleleft$$

$$\mathbf{j}: \quad -11B_z - (9.6)(275) = 0 \quad B_z = -240 \text{ lb}$$

$$\mathbf{k}: \quad 11B_y + 11(-252) - (10.08)(275) = 0, \quad B_y = 504 \text{ lb} \qquad \mathbf{B} = (504 \text{ lb})\mathbf{j} - (240 \text{ lb})\mathbf{k} \ \triangleleft$$

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PROBLEM 6.41* (Continued)

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + (504 \text{ lb})\mathbf{j} - (240 \text{ lb})\mathbf{k} - (252 \text{ lb})\mathbf{j} + (275 \text{ lb})\mathbf{i} + (240 \text{ lb})\mathbf{k} = 0$$

$\mathbf{A} = -(275 \text{ lb})\mathbf{i} - (252 \text{ lb})\mathbf{j}$ ◁

Zero-force members

The determination of these members will facilitate our solution.

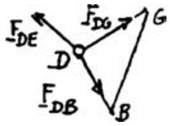
FB: C. Writing $\Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma F_z = 0$ Yields $F_{BC} = F_{CD} = F_{CG} = 0$ ◁

FB: F. Writing $\Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma F_z = 0$ Yields $F_{BF} = F_{EF} = F_{FG} = 0$ ◁

FB: A: Since $A_z = 0$, writing $\Sigma F_z = 0$ Yields $F_{AD} = 0$ ◁

FB: H: Writing $\Sigma F_y = 0$ Yields $F_{DH} = 0$ ◁

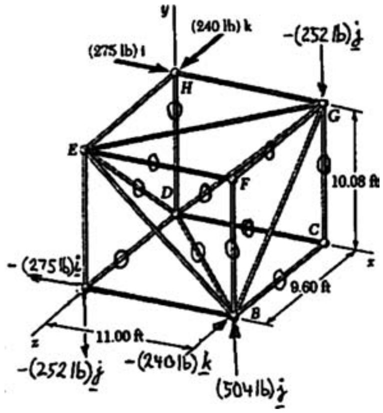
FB: D: Since $F_{AD} = F_{CD} = F_{DH} = 0$, we need consider only members DB, DE, and DG.



Since F_{DE} is the only force not contained in plane BDG, it must be zero. Simple reasonings show that the other two forces are also zero.

$F_{BD} = F_{DE} = F_{DG} = 0$ ◁

The results obtained for the reactions at the supports and for the zero-force members are shown on the figure below. Zero-force members are indicated by a zero ("0").



(*b*) Force in each of the members joined at *E*

We already found that $F_{DE} = F_{EF} = 0$ ◀

Free body: *A* $\Sigma F_y = 0$ Yields $F_{AE} = 252 \text{ lb} \quad T$ ◀

Free body: *H* $\Sigma F_z = 0$ Yields $F_{EH} = 240 \text{ lb} \quad C$ ◀

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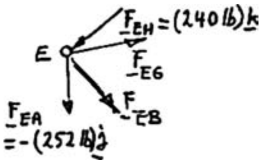
PROBLEM 6.41* (Continued)

Free body: *E* $\Sigma \mathbf{F} = 0\text{:} \quad \mathbf{F}_{EB} + \mathbf{F}_{EG} + (240\text{ lb})\mathbf{k} - (252\text{ lb})\mathbf{j} = 0$



$$\frac{F_{BE}}{14.92}(11\mathbf{i} - 10.08\mathbf{j}) + \frac{F_{EG}}{14.6}(11\mathbf{i} - 9.6\mathbf{k}) + 240\mathbf{k} - 252\mathbf{j} = 0$$

Equate to zero the coefficient of **y** and **k**:

$$\mathbf{j}\text{:} \quad -\left(\frac{10.08}{14.92}\right)F_{BE} - 252 = 0 \qquad F_{BE} = 373\text{ lb} \quad C \blacktriangleleft$$

$$\mathbf{k}\text{:} \quad -\left(\frac{9.6}{14.6}\right)F_{EG} + 240 = 0 \qquad F_{EG} = 365\text{ lb} \quad T \blacktriangleleft$$

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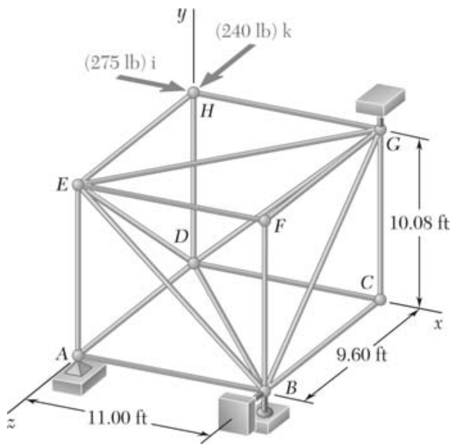


PROBLEM 6.42*

The truss shown consists of 18 members and is supported by a ball and socket at A, two short links at B, and one short link at G. (a) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (b) For the given loading, determine the force in each of the six members joined at G.

SOLUTION

See solution to Problem 6.41 for Part (a) and for reactions and zero-force members.

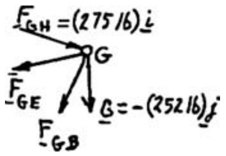
(b) Force in each of the members joined at G.

We already know that

$$F_{CG} = F_{DG} = F_{FG} = 0 \blacktriangleleft$$

Free body: H $\qquad \Sigma F_x = 0$ $\qquad$ Yields $F_{GH} = 275 \text{ lb} \quad C \blacktriangleleft$

Free body: G $\qquad \Sigma \mathbf{F} = 0\text{:} \quad \mathbf{F}_{GB} + \mathbf{F}_{GE} + (275 \text{ lb})\mathbf{i} - (252 \text{ lb})\mathbf{j} = 0$



$$\frac{F_{BG}}{13.92}(-10.08\mathbf{j} + 9.6\mathbf{k}) + \frac{F_{EG}}{14.6}(-11\mathbf{i} + 9.6\mathbf{k}) + 275\mathbf{i} - 252\mathbf{j} = 0$$

Equate to zero the coefficient of **i**, **j**, **k**:

$$\mathbf{i}\text{:} \quad -\left(\frac{11}{14.6}\right)F_{EG} + 275 = 0 \qquad F_{EG} = 365 \text{ lb} \quad T \blacktriangleleft$$

$$\mathbf{j}\text{:} \quad -\left(\frac{10.08}{13.92}\right)F_{BG} - 252 = 0 \qquad F_{BG} = 348 \text{ lb} \quad C \blacktriangleleft$$

$$\mathbf{k}\text{:} \quad \left(\frac{9.6}{13.92}\right)(-348) + \left(\frac{9.6}{14.6}\right)(365) = 0 \qquad \text{(Checks)}$$

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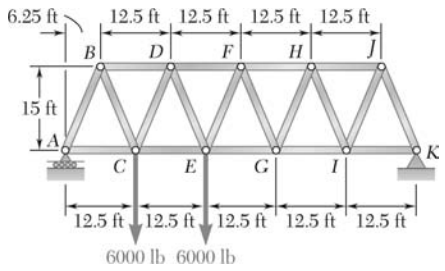
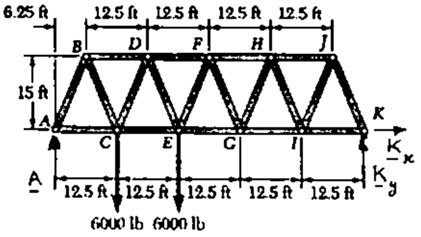


PROBLEM 6.43

A Warren bridge truss is loaded as shown. Determine the force in members *CE*, *DE*, and *DF*.

SOLUTION



Free body: Truss

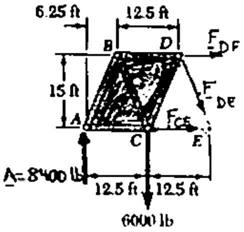
$$\xrightarrow{+} \Sigma F_x = 0: \quad \mathbf{k}_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad k_y(62.5 \text{ ft}) - (6000 \text{ lb})(12.5 \text{ ft}) - (6000 \text{ lb})(25 \text{ ft}) = 0$$

$$\mathbf{k} = \mathbf{k}_y = 3600 \text{ lb} \uparrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A + 3600 \text{ lb} - 6000 \text{ lb} - 6000 \text{ lb} = 0$$

$$\mathbf{A} = 8400 \text{ lb} \uparrow \triangleleft$$



We pass a section through members *CE*, *DE*, and *DF* and use the free body shown.

$$+\circlearrowleft \Sigma M_D = 0: \quad F_{CE}(15 \text{ ft}) - (8400 \text{ lb})(18.75 \text{ ft}) + (6000 \text{ lb})(6.25 \text{ ft}) = 0$$

$$F_{CE} = +8000 \text{ lb} \qquad F_{CE} = 8000 \text{ lb} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 8400 \text{ lb} - 6000 \text{ lb} - \frac{15}{16.25} F_{DE} = 0$$

$$F_{DE} = +2600 \text{ lb} \qquad F_{DE} = 2600 \text{ lb} \quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_E = 0: \quad 6000 \text{ lb}(12.5 \text{ ft}) - (8400 \text{ lb})(25 \text{ ft}) - F_{DF}(15 \text{ ft}) = 0$$

$$F_{DF} = -9000 \text{ lb} \qquad F_{DF} = 9000 \text{ lb} \quad C \blacktriangleleft$$

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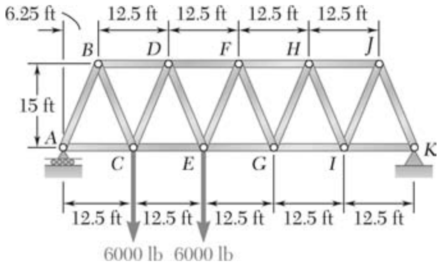


PROBLEM 6.44

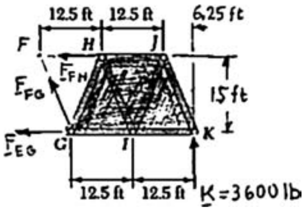
A Warren bridge truss is loaded as shown. Determine the force in members *EG*, *FG*, and *FH*.

SOLUTION

See solution of Problem 6.43 for free-body diagram of truss and determination of reactions:

$$\mathbf{A} = 8400 \text{ lb} \qquad \mathbf{k} = 3600 \text{ lb} \ \triangleleft$$

We pass a section through members *EG*, *FG*, and *FH*, and use the free body shown.



$$+\circlearrowleft \Sigma M_F = 0: \quad (3600 \text{ lb})(31.25 \text{ ft}) - F_{EG}(15 \text{ ft}) = 0$$

$$F_{EG} = +7500 \text{ lb} \qquad F_{EG} = 7500 \text{ lb} \quad T \ \blacktriangleleft$$

$$+\Sigma F_y = 0: \quad \frac{15}{16.25} F_{FG} + 3600 \text{ lb} = 0$$

$$F_{FG} = -3900 \text{ lb} \qquad F_{FG} = 3900 \text{ lb} \quad C \ \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_G = 0: \quad F_{FH}(15 \text{ ft}) + (3600 \text{ lb})(25 \text{ ft}) = 0$$

$$F_{FH} = -6000 \text{ lb} \qquad F_{FH} = 6000 \text{ lb} \quad C \ \blacktriangleleft$$

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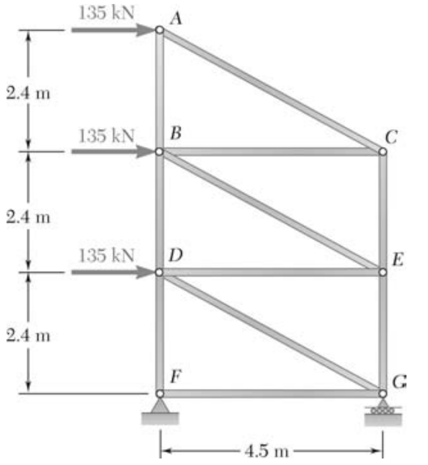
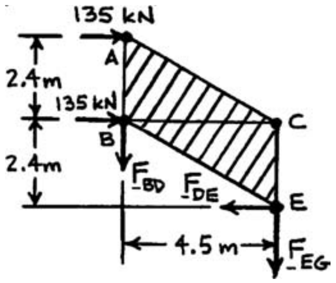


PROBLEM 6.45

Determine the force in members *BD* and *DE* of the truss shown.

SOLUTION



Member *BD*:

$$+\circlearrowleft \Sigma M_E = 0: \quad F_{BD}(4.5\text{ m}) - (135\text{ kN})(4.8\text{ m}) - (135\text{ kN})(2.4\text{ m}) = 0$$

$$F_{BD} = +216\text{ kN} \qquad F_{BD} = 216\text{ kN} \quad T \blacktriangleleft$$

Member *DE*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad 135\text{ kN} + 135\text{ kN} - F_{DE} = 0$$

$$F_{DE} = +270\text{ kN} \qquad F_{DE} = 270\text{ kN} \quad T \blacktriangleleft$$

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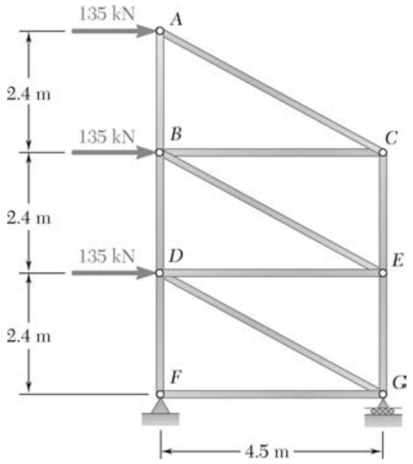
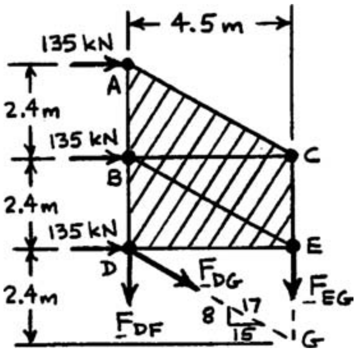


PROBLEM 6.46

Determine the force in members *DG* and *EG* of the truss shown.

SOLUTION



<u>Member *DG*:</u>

$$\xrightarrow{+} \Sigma F_x = 0: \quad 135 \text{ kN} + 135 \text{ kN} + 135 \text{ kN} + \frac{15}{17} F_{DG} = 0$$

$$F_{DG} = -459 \text{ kN} \qquad F_{DG} = 459 \text{ kN} \quad C \blacktriangleleft$$

<u>Member *EG*:</u>

$$+\circlearrowleft \Sigma M_D = 0: \quad (135 \text{ kN})(4.8 \text{ m}) + (135 \text{ kN})(2.4 \text{ m}) + F_{EG}(4.5 \text{ m}) = 0$$

$$F_{EG} = -216 \text{ kN} \qquad F_{EG} = 216 \text{ kN} \quad C \blacktriangleleft$$

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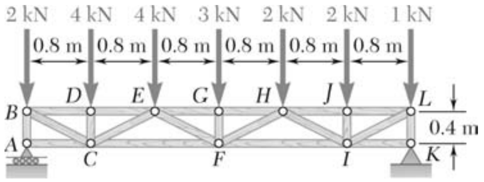
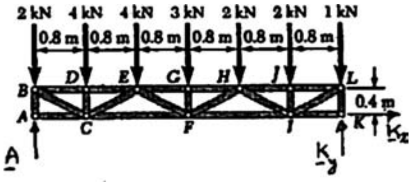


PROBLEM 6.47

A floor truss is loaded as shown. Determine the force in members *CF*, *EF*, and *EG*.

SOLUTION



Free body: Truss

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad k_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad k_y(4.8 \text{ m}) - (4 \text{ kN})(0.8 \text{ m}) - (4 \text{ kN})(1.6 \text{ m}) - (3 \text{ kN})(2.4 \text{ m}) - (2 \text{ kN})(3.2 \text{ m}) - (2 \text{ kN})(4 \text{ m}) - (1 \text{ kN})(4.8 \text{ m}) = 0$$

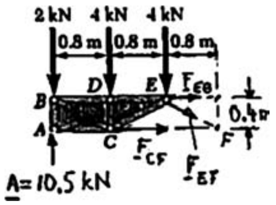
$$k_y = 7.5 \text{ kN}$$

Thus: $\mathbf{k} = 7.5 \text{ kN}\uparrow \triangleleft$

$$+\uparrow \Sigma F_y = 0: \quad A + 7.5 \text{ kN} - 18 \text{ kN} = 0 \quad A = 10.5 \text{ kN}$$

$\mathbf{A} = 10.5 \text{ kN}\uparrow \triangleleft$

We pass a section through members *CF*, *EF*, and *EG* and use the free body shown.



$$+\circlearrowleft \Sigma M_E = 0: \quad F_{CF}(0.4 \text{ m}) - (10.5 \text{ kN})(1.6 \text{ m}) + (2 \text{ kN})(1.6 \text{ m}) + (4 \text{ kN})(0.8 \text{ m}) = 0$$

$$F_{CF} = +26.0 \text{ kN}$$

$F_{CF} = 26.0 \text{ kN} \quad T$ ◀

$$+\uparrow \Sigma F_y = 0: \quad 10.5 \text{ kN} - 2 \text{ kN} - 4 \text{ kN} - 4 \text{ kN} - \frac{1}{\sqrt{5}} F_{EF} = 0$$

$$F_{EF} = +1.118 \text{ kN}$$

$F_{EF} = 1.118 \text{ kN} \quad T$ ◀

$$+\circlearrowleft \Sigma M_F = 0: \quad (2 \text{ kN})(2.4 \text{ m}) + (4 \text{ kN})(1.6 \text{ m}) + (4 \text{ kN})(0.8 \text{ m}) - (10.5 \text{ kN})(2.4 \text{ m}) - F_{EG}(0.4 \text{ m}) = 0$$

$$F_{EG} = -27.0 \text{ kN}$$

$F_{EG} = 27.0 \text{ kN} \quad C$ ◀

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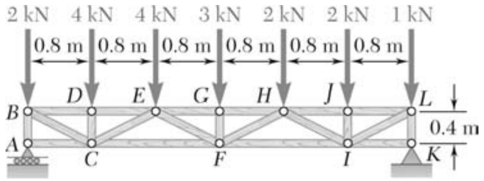


PROBLEM 6.48

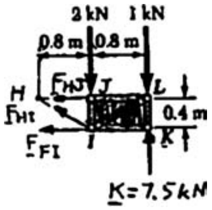
A floor truss is loaded as shown. Determine the force in members *FI*, *HI*, and *HJ*.

SOLUTION

See solution of Problem 6.47 for free-body diagram of truss and determination of reactions:

$$\mathbf{A} = 10.5\text{ kN}\uparrow, \quad \mathbf{k} = 7.5\text{ kN}\uparrow \triangleleft$$

We pass a section through members *FI*, *HI*, and *HJ*, and use the free body shown.



$$+\circlearrowleft \Sigma M_H = 0: \quad (7.5\text{ kN})(1.6\text{ m}) - (2\text{ kN})(0.8\text{ m}) - (1\text{ kN})(1.6\text{ m}) - F_{FI}(0.4\text{ m}) = 0$$

$$F_{FI} = +22.0\text{ kN} \qquad F_{FI} = 22.0\text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{HI} - 2\text{ kN} - 1\text{ kN} + 7.5\text{ kN} = 0$$

$$F_{HI} = -10.06\text{ kN} \qquad F_{HI} = 10.06\text{ kN} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_I = 0: \quad F_{HJ}(0.4\text{ m}) + (7.5\text{ kN})(0.8\text{ m}) - (1\text{ kN})(0.8\text{ m}) = 0$$

$$F_{HJ} = -13.00\text{ kN} \qquad F_{HJ} = 13.00\text{ kN} \quad C \blacktriangleleft$$

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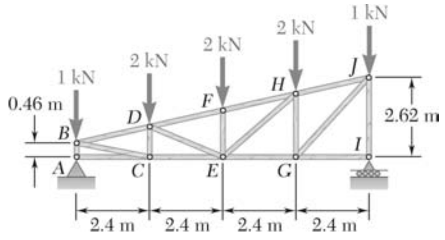


PROBLEM 6.49

A pitched flat roof truss is loaded as shown. Determine the force in members *CE*, *DE*, and *DF*.

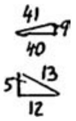
SOLUTION

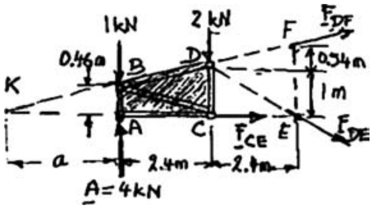
Reactions at supports: Because of the symmetry of the loading,

$$A_x = 0$$

$$A_y = I = \frac{1}{2}\text{(Total load)} = \frac{1}{2}(8\text{ kN}) \qquad \mathbf{A} = \mathbf{I} = 4\text{ kN}\uparrow \triangleleft$$

We pass a section through members *CD*, *DE*, and *DF*, and use the free body shown.
(We moved F_{DE} to E and F_{DF} to F)





$$\text{Slope } BJ = \frac{2.16\text{ m}}{9.6\text{ m}} = \frac{9}{40}$$

$$\text{Slope } DE = \frac{-1\text{ m}}{2.4\text{ m}} = \frac{-5}{12}$$

$$a = \frac{0.46\text{ m}}{\text{Slope } BJ} = \frac{0.46\text{ m}}{\frac{9}{40}} = 2.0444\text{ m}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad F_{CE}(1\text{ m}) + (1\text{ kN})(2.4\text{ m}) - (4\text{ kN})(2.4\text{ m}) = 0$$

$$F_{CE} = +7.20\text{ kN} \qquad F_{CE} = 7.20\text{ kN} \quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_K = 0: \quad (4\text{ kN})(2.0444\text{ m}) - (1\text{ kN})(2.0444\text{ m})$$

$$-(2\text{ kN})(4.4444\text{ m}) - \left(\frac{5}{13}F_{DE}\right)(6.8444\text{ m}) = 0$$

$$F_{DE} = -1.047\text{ kN} \qquad F_{DE} = 1.047\text{ kN} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_E = 0: \quad (1\text{ kN})(4.8\text{ m}) + (2\text{kN})(2.4\text{ m}) - (4\text{ kN})(4.8\text{ m})$$

$$-\left(\frac{40}{41}F_{DF}\right)(1.54\text{ m}) = 0$$

$$F_{DF} = -6.39\text{ kN} \qquad F_{DF} = 6.39\text{ kN} \quad C \blacktriangleleft$$

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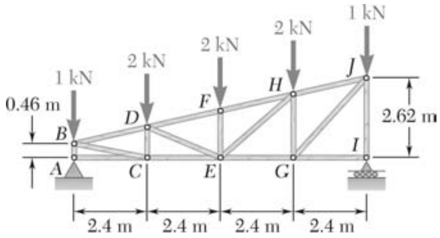


PROBLEM 6.50

A pitched flat roof truss is loaded as shown. Determine the force in members *EG*, *GH*, and *HJ*.

SOLUTION

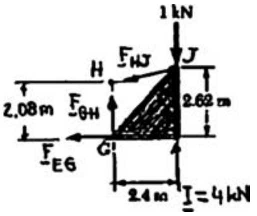
Reactions at supports: Because of the symmetry of the loading,

$$A_x = 0$$

$$A_y = I = \frac{1}{2}\text{(Total load)} = \frac{1}{2}(8\text{ kN})$$

$\mathbf{A} = \mathbf{I} = 4\text{ kN}\uparrow$ ◁

We pass a section through members *EG*, *GH*, and *HJ*, and use the free body shown.



$$+\circlearrowleft \Sigma M_H = 0: \quad (4\text{ kN})(2.4\text{ m}) - (1\text{ kN})(2.4\text{ m}) - F_{EG}(2.08\text{ m}) = 0$$

$$F_{EG} = +3.4615\text{ kN}$$

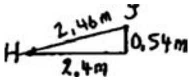
$F_{EG} = 3.46\text{ kN}\quad T$ ◀

$$+\circlearrowleft \Sigma M_J = 0: \quad -F_{GH}(2.4\text{ m}) - F_{EG}(2.62\text{ m}) = 0$$

$$F_{GH} = -\frac{2.62}{2.4}(3.4615\text{ kN})$$

$$F_{GH} = -3.7788\text{ kN}$$

$F_{GH} = 3.78\text{ kN}\quad C$ ◀



$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{EG} - \frac{2.4}{2.46}F_{HJ} = 0$$

$$F_{HJ} = -\frac{2.46}{2.4}F_{EG} = -\frac{2.46}{2.4}(3.4615\text{ kN})$$

$$F_{HJ} = -3.548\text{ kN}$$

$F_{HJ} = 3.55\text{ kN}\quad C$ ◀

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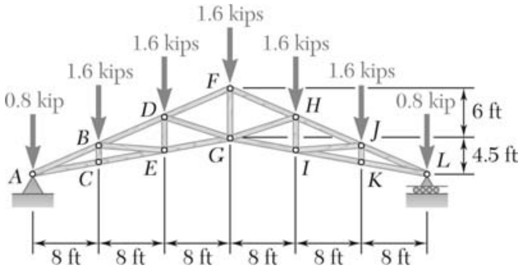


PROBLEM 6.51

A Howe scissors roof truss is loaded as shown. Determine the force in members *DF*, *DG*, and *EG*.

SOLUTION

Reactions at supports.

Because of symmetry of loading.

$$A_x = 0, \quad A_y = L = \frac{1}{2}(\text{Total load}) = \frac{1}{2}(9.60 \text{ kips}) = 4.80 \text{ kips}$$

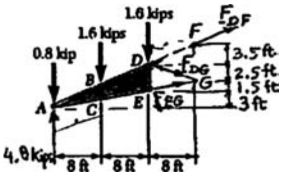
$$\mathbf{A} = \mathbf{L} = 4.80 \text{ kips} \uparrow \blacktriangleleft$$

We pass a section through members *DF*, *DG*, and *EG*, and use the free body shown.

We slide $\mathbf{F}_{DF}$ to apply it at *F*:

$$+\circlearrowleft \Sigma M_G = 0: \quad (0.8 \text{ kip})(24 \text{ ft}) + (1.6 \text{ kips})(16 \text{ ft}) + (1.6 \text{ kips})(8 \text{ ft})$$

$$-(4.8 \text{ kips})(24 \text{ ft}) - \frac{8F_{DF}}{\sqrt{8^2 + 3.5^2}}(6 \text{ ft}) = 0$$



$$F_{DF} = -10.48 \text{ kips}, \quad F_{DF} = 10.48 \text{ kips} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(1.6 \text{ kips})(8 \text{ ft}) - (1.6 \text{ kips})(16 \text{ ft})$$

$$-\frac{2.5F_{DG}}{\sqrt{8^2 + 2.5^2}}(16 \text{ ft}) - \frac{8F_{DG}}{\sqrt{8^2 + 2.5^2}}(7 \text{ ft}) = 0$$

$$F_{DG} = -3.35 \text{ kips}, \quad F_{DG} = 3.35 \text{ kips} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (0.8 \text{ kips})(16 \text{ ft}) + (1.6 \text{ kips})(8 \text{ ft}) - (4.8 \text{ kips})(16 \text{ ft}) - \frac{8F_{EG}}{\sqrt{8^2 + 1.5^2}}(4 \text{ ft}) = 0$$

$$F_{EG} = +13.02 \text{ kips}, \quad F_{EG} = 13.02 \text{ kips} \quad T \blacktriangleleft$$

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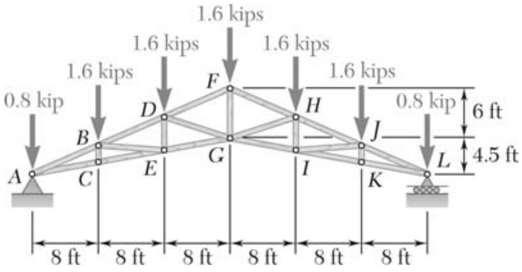


PROBLEM 6.52

A Howe scissors roof truss is loaded as shown. Determine the force in members *GI*, *HI*, and *HJ*.

SOLUTION

<u>Reactions at supports</u>. Because of symmetry of loading:

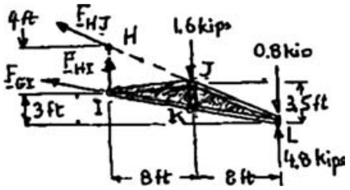
$$A_x = 0, \qquad A_y = L = \frac{1}{2}(\text{Total load})$$

$$= \frac{1}{2}(9.60 \text{ kips})$$

$$= 4.80 \text{ kips}$$

$\mathbf{A} = \mathbf{L} = 4.80 \text{ kips} \uparrow$ ◀

We pass a section through members *GI*, *HI*, and *HJ*, and use the <u>free body shown</u>.



$$+\circlearrowleft \Sigma M_H = 0: \quad -\frac{16F_{GI}}{\sqrt{16^2 + 3^2}}(4 \text{ ft}) + (4.8 \text{ kips})(16 \text{ ft}) - (0.8 \text{ kip})(16 \text{ ft}) - (1.6 \text{ kips})(8 \text{ ft}) = 0$$

$$F_{GI} = +13.02 \text{ kips}$$

$F_{GI} = 13.02 \text{ kips} \quad T$ ◀

$$+\circlearrowleft \Sigma M_L = 0: \quad (1.6 \text{ kips})(8 \text{ ft}) - F_{HI}(16 \text{ ft}) = 0$$

$$F_{HI} = +0.800 \text{ kips}$$

$F_{HI} = 0.800 \text{ kips} \quad T$ ◀

We slide $\mathbf{F}_{HG}$ to apply it at *H*.

$$+\circlearrowleft \Sigma M_I = 0: \quad \frac{8F_{HJ}}{\sqrt{8^2 + 3.5^2}}(4 \text{ ft}) + (4.8 \text{ kips})(16 \text{ ft}) - (1.6 \text{ kips})(8 \text{ ft}) - (0.8 \text{ kip})(16 \text{ ft}) = 0$$

$$F_{HJ} = -13.97 \text{ kips}$$

$F_{HJ} = 13.97 \text{ kips} \quad C$ ◀

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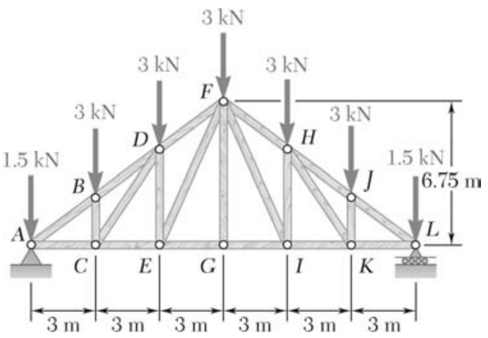


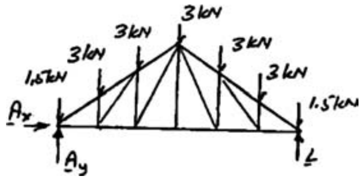
PROBLEM 6.53

A Pratt roof truss is loaded as shown. Determine the force in members *CE*, *DE*, and *DF*.

SOLUTION

Free body: Entire truss

$$\Sigma F_x = 0: \quad A_x = 0$$



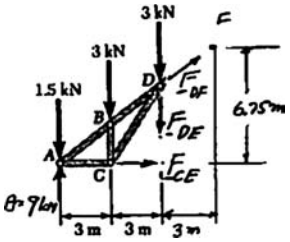
$$\text{Total load } = 5(3 \text{ kN}) + 2(1.5 \text{ kN})$$
$$= 18 \text{ kN}$$

By symmetry:

$$A_y = L = \frac{1}{2}(18 \text{ kN})$$

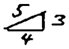
$\mathbf{A} = \mathbf{L} = 9 \text{ kN}$ ◀

Free body: Portion *ACD*



Note: Slope of *ABDF* is

$$\frac{6.75}{9.00} = \frac{3}{4}$$



Force in *CE*:

$$+\circlearrowleft \Sigma M_D = 0: \quad F_{CE}\left(\frac{2}{3} \times 6.75 \text{ m}\right) - (9 \text{ kN})(6 \text{ m}) + (1.5 \text{ kN})(6 \text{ m}) + (3 \text{ kN})(3 \text{ m}) = 0$$

$$F_{CE}(4.5 \text{ m}) - 36 \text{ kN}\cdot\text{m} = 0$$

$$F_{CE} = +8 \text{ kN}$$

$F_{CE} = 8 \text{ kN} \quad T$ ◀

Force in *DE*:

$$+\circlearrowleft \Sigma M_A = 0: \quad F_{DE}(6 \text{ m}) + (3 \text{ kN})(6 \text{ m}) + (3 \text{ kN})(3 \text{ m}) = 0$$

$$F_{DE} = -4.5 \text{ kN}$$

$F_{DE} = 4.5 \text{ kN} \quad C$ ◀

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PROBLEM 6.53 (Continued)

<u>Force in *DF*</u>:

Sum moments about *E* where F_{CE} and F_{DE} intersect.

$$+\circlearrowleft \Sigma M_E = 0: \quad (1.5 \text{ kN})(6 \text{ m}) - (9 \text{ kN})(6 \text{ m}) + (3 \text{ kN})(3 \text{ m}) + \frac{4}{5}F_{CE}\left(\frac{2}{3}\times 6.75 \text{ m}\right) = 0$$

$$\frac{4}{5}F_{CE}(4.5 \text{ m}) - 36 \text{ kN}\cdot\text{m}$$

$$F_{CE} = -10.00 \text{ kN}$$

$F_{CE} = 10.00 \text{ kN} \quad C$ ◀

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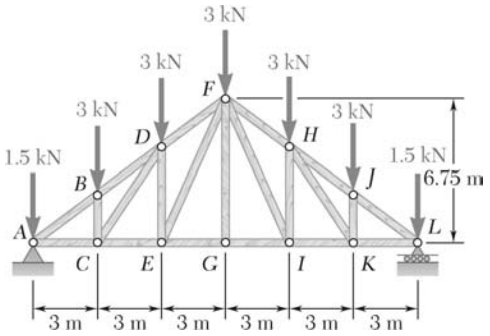
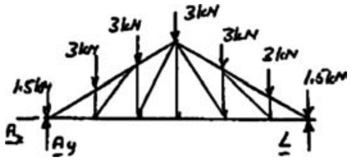


PROBLEM 6.54

A Pratt roof truss is loaded as shown. Determine the force in members *FH*, *FI*, and *GI*.

SOLUTION

<u>Free body: Entire truss</u>



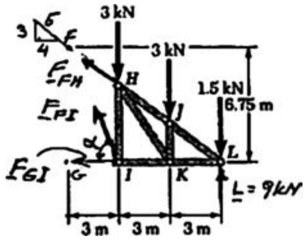
$$\Sigma F_x = 0: \quad A_x = 0$$

$$\text{Total load } = 5(3\text{ kN}) + 2(1.5\text{ kN}) = 18\text{ kN}$$

By symmetry:

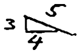
$$\mathbf{A}_y = \mathbf{L} = \frac{1}{2}(18) = 9\text{ kN}\uparrow$$

<u>Free body: Portion *HIL*</u>



Slope of *FHJL*

$$\frac{6.75}{9.00} = \frac{3}{4}$$



$$\tan\alpha = \frac{FG}{GI} = \frac{6.75\text{ m}}{3\text{ m}} \qquad \alpha = 66.04°$$

<u>Force in *FH*</u>:

$$+\circlearrowleft \Sigma M_I = 0: \quad \frac{4}{5}F_{FH}\left(\frac{2}{3}\times 6.75\text{ m}\right) + (9\text{ kN})(6\text{ m}) - (1.5\text{ kN})(6\text{ m}) - (3\text{ kN})(3\text{ m}) = 0$$

$$\frac{4}{5}F_{FH}(4.5\text{ m}) + 36\text{ kN}\cdot\text{m}$$

$$F_{FH} = -10.00\text{ kN} \qquad F_{FH} = 10.00\text{ kN} \quad C \blacktriangleleft$$

<u>Force in *FI*</u>:

$$+\circlearrowleft \Sigma M_L = 0: \quad F_{FI}\sin\alpha(6\text{ m}) - (3\text{ kN})(6\text{ m}) - (3\text{ kN})(3\text{ m}) = 0$$

$$F_{FI}\sin 66.04°(6\text{ m}) = 27\text{ kN}\cdot\text{m}$$

$$F_{FI} = +4.92\text{ kN} \qquad F_{FI} = 4.92\text{ kN} \quad T \blacktriangleleft$$

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PROBLEM 6.54 (Continued)

Force in *GI*:

$$+\circlearrowleft \Sigma M_H = 0: \quad F_{GI}(6.75 \text{ m}) + (3 \text{ kN})(3 \text{ m}) + (3 \text{ kN})(6 \text{ m}) + (1.5 \text{ kN})(9 \text{ m}) - (9 \text{ kN})(9 \text{ m}) = 0$$

$$F_{GI}(6.75 \text{ m}) = +40.5 \text{ kN} \cdot \text{m}$$

$$F_{GI} = +6.00 \text{ kN} \qquad F_{GI} = 6.00 \text{ kN} \quad T \blacktriangleleft$$

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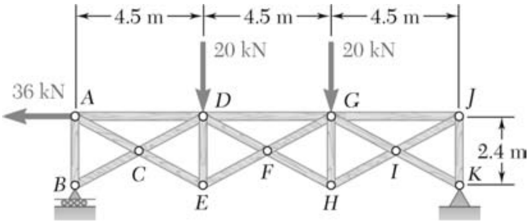
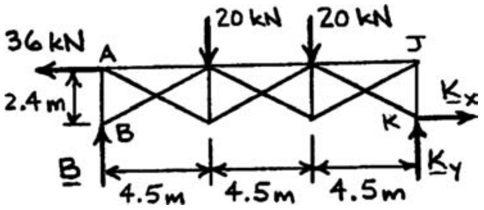


PROBLEM 6.55

Determine the force in members *AD*, *CD*, and *CE* of the truss shown.

SOLUTION

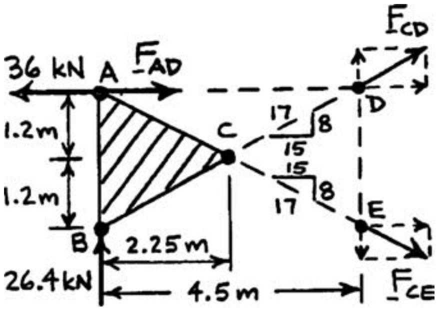
Reactions:



$$+\circlearrowleft \Sigma M_k = 0: \quad 36(2.4) - B(13.5) + 20(9) + 20(4.5) = 0 \qquad \mathbf{B} = 26.4 \text{ kN} \uparrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -36 + K_x = 0 \qquad \mathbf{K}_x = 36 \text{ kN} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad 26.4 - 20 - 20 + K_y = 0 \quad \mathbf{K}_y = 13.6 \text{ kN} \uparrow$$



$$+\circlearrowleft \Sigma M_C = 0: \quad 36(1.2) - 26.4(2.25) - F_{AD}(1.2) = 0$$

$$F_{AD} = -13.5 \text{ kN} \qquad F_{AD} = 13.5 \text{ kN} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad \left(\frac{8}{17} F_{CD}\right)(4.5) = 0 \qquad F_{CD} = 0 \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad \left(\frac{15}{17} F_{CE}\right)(2.4) - 26.4(4.5) = 0$$

$$F_{CE} = +56.1 \text{ kN} \qquad F_{CE} = 56.1 \text{ kN} \quad T \blacktriangleleft$$

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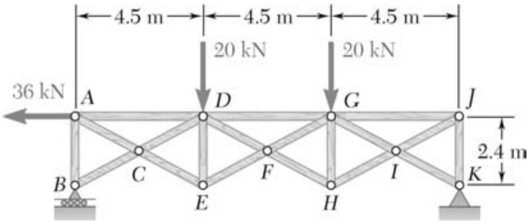


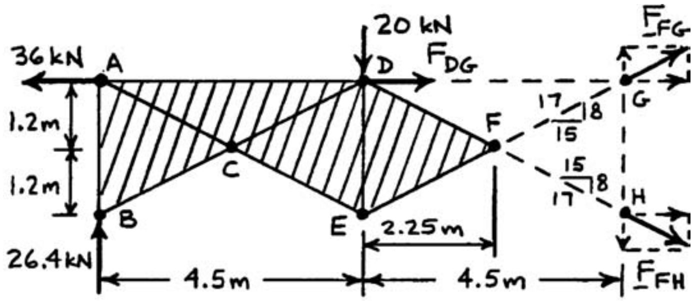
PROBLEM 6.56

Determine the force in members *DG*, *FG*, and *FH* of the truss shown.

SOLUTION

See the solution to Problem 6.55 for free-body diagram and analysis to determine the reactions at the supports *B* and *K*.

$$\mathbf{B} = 26.4 \text{ kN}\uparrow; \quad \mathbf{K}_x = 36.0 \text{ kN}\longrightarrow; \quad \mathbf{K}_y = 13.60 \text{ kN}\uparrow$$



$$+\circlearrowleft \Sigma M_F = 0: \quad 36(1.2) - 26.4(6.75) + 20(2.25) - F_{DG}(1.2) = 0$$

$F_{DG} = -75$ kN $F_{DG} = 75.0$ kN *C* ◀

$$+\circlearrowleft \Sigma M_D = 0: \quad -26.4(4.5) + \left(\frac{8}{17}F_{FG}\right)(4.5) = 0$$

$F_{FG} = +56.1$ kN $F_{FG} = 56.1$ kN *T* ◀

$$+\circlearrowleft \Sigma M_G = 0: \quad 20(4.5) - 26.4(9) + \left(\frac{15}{17}F_{FH}\right)(2.4) = 0$$

$F_{FH} = +69.7$ kN $F_{FH} = 69.7$ kN *T* ◀

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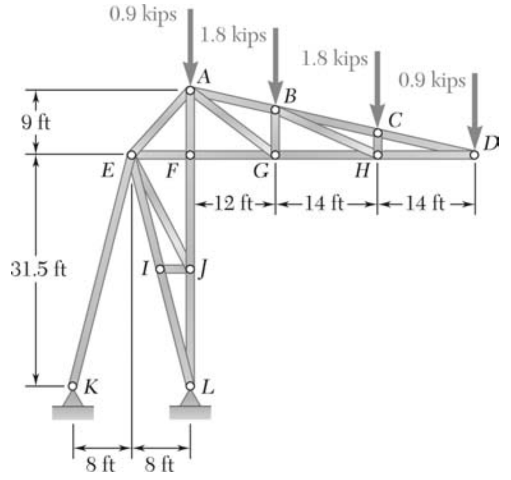
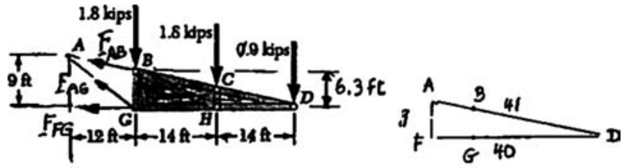


PROBLEM 6.57

A stadium roof truss is loaded as shown. Determine the force in members *AB*, *AG*, and *FG*.

SOLUTION

We pass a section through members *AB*, *AG*, and *FG*, and use the free body shown.



$$+\circlearrowleft \Sigma M_G = 0: \quad \left(\frac{40}{41}F_{AB}\right)(6.3\text{ ft}) - (1.8\text{ kips})(14\text{ ft}) - (0.9\text{ kips})(28\text{ ft}) = 0$$

$F_{AB} = +8.20\text{ kips}$ $\qquad F_{AB} = 8.20\text{ kips}\quad T$ ◀

$$+\circlearrowleft \Sigma M_D = 0: \quad -\left(\frac{3}{5}F_{AG}\right)(28\text{ ft}) + (1.8\text{ kips})(28\text{ ft}) + (1.8\text{ kips})(14\text{ ft}) = 0$$

$F_{AG} = +4.50\text{ kips}$ $\qquad F_{AG} = 4.50\text{ kips}\quad T$ ◀

$$+\circlearrowleft M_A = 0: \quad -F_{FG}(9\text{ ft}) - (1.8\text{ kips})(12\text{ ft}) - (1.8\text{ kips})(26\text{ ft}) - (0.9\text{ kips})(40\text{ ft}) = 0$$

$F_{FG} = -11.60\text{ kips}$ $\qquad F_{FG} = 11.60\text{ kips}\quad C$ ◀

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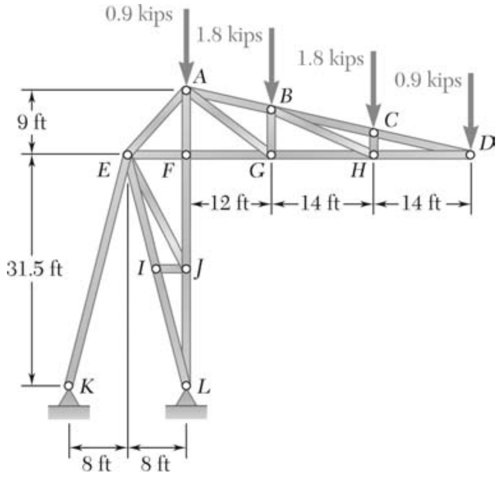
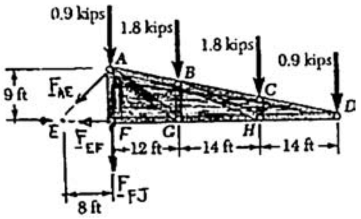


PROBLEM 6.58

A stadium roof truss is loaded as shown. Determine the force in members *AE*, *EF*, and *FJ*.

SOLUTION

We pass a section through members *AE*, *EF*, and *FJ*, and use the free body shown.



$$+\circlearrowleft \Sigma M_F = 0: \quad \left(\frac{8}{\sqrt{8^2+9^2}} F_{AE}\right)(9\text{ ft}) - (1.8\text{ kips})(12\text{ ft}) - (1.8\text{ kips})(26\text{ ft}) - (0.9\text{ kips})(40\text{ ft}) = 0$$

$$F_{AE} = +17.46\text{ kips} \qquad\qquad F_{AE} = 17.46\text{ kips}\quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -F_{EF}(9\text{ ft}) - (1.8\text{ kips})(12\text{ ft}) - (1.8\text{ kips})(26\text{ ft}) - (0.9\text{ kips})(40\text{ ft}) = 0$$

$$F_{EF} = -11.60\text{ kips} \qquad\qquad F_{EF} = 11.60\text{ kips}\quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_E = 0: \quad -F_{FJ}(8\text{ ft}) - (0.9\text{ kips})(8\text{ ft}) - (1.8\text{ kips})(20\text{ ft}) - (1.8\text{ kips})(34\text{ ft}) - (0.9\text{ kips})(48\text{ ft}) = 0$$

$$F_{FJ} = -18.45\text{ kips} \qquad\qquad F_{FJ} = 18.45\text{ kips}\quad C \blacktriangleleft$$

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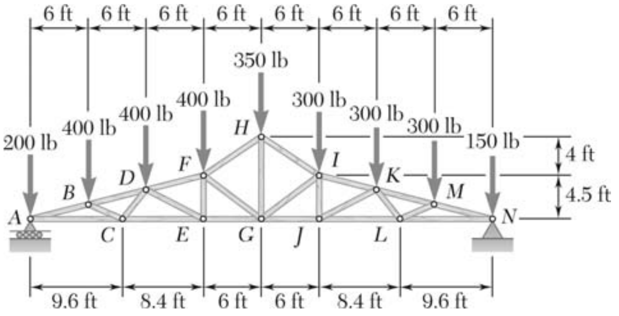
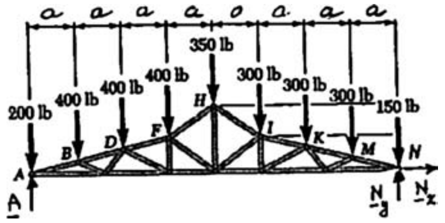


PROBLEM 6.59

A Polynesian, or duopitch, roof truss is loaded as shown. Determine the force in members *DF*, *EF*, and *EG*.

SOLUTION

<u>Free body: Truss</u>



$$\Sigma F_x = 0: \quad N_x = 0$$

$$+\circlearrowleft \Sigma M_N = 0: \quad (200 \text{ lb})(8a) + (400 \text{ lb})(7a + 6a + 5a) + (350 \text{ lb})(4a) + (300 \text{ lb})(3a + 2a + a) - A(8a) = 0$$

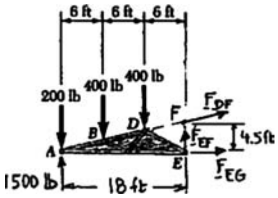
$$\mathbf{A} = 1500 \text{ lb} \uparrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 1500 \text{ lb} - 200 \text{ lb} - 3(400 \text{ lb}) - 350 \text{ lb} - 3(300 \text{ lb}) - 150 \text{ lb} + N_y = 0$$

$$N_y = 1300 \text{ lb} \quad \mathbf{N} = 1300 \text{ lb} \uparrow \triangleleft$$

We pass a section through *DF*, *EF*, and *EG*, and use the <u>free body shown</u>.

(We apply $\mathbf{F}_{DF}$ at *F*)



$$+\circlearrowleft \Sigma M_E = 0: \quad (200 \text{ lb})(18 \text{ ft}) + (400 \text{ lb})(12 \text{ ft}) + (400 \text{ lb})(6 \text{ ft}) - (1500 \text{ lb})(18 \text{ ft})$$

$$- \left(\frac{18}{\sqrt{18^2 + 4.5^2}} F_{DF} \right)(4.5 \text{ ft}) = 0$$

$$F_{DF} = -3711 \text{ lb} \quad F_{DF} = 3710 \text{ lb} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad F_{EF}(18 \text{ ft}) - (400 \text{ lb})(6 \text{ ft}) - (400 \text{ lb})(12 \text{ ft}) = 0$$

$$F_{EF} = +400 \text{ lb} \qquad F_{EF} = 400 \text{ lb} \quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_F = 0: \quad F_{EG}(4.5 \text{ ft}) - (1500 \text{ lb})(18 \text{ ft}) + (200 \text{ lb})(18 \text{ ft}) + (400 \text{ lb})(12 \text{ ft}) + (400 \text{ lb})(6 \text{ ft}) = 0$$

$$F_{EG} = +3600 \text{ lb} \quad F_{EG} = 3600 \text{ lb} \quad T \blacktriangleleft$$

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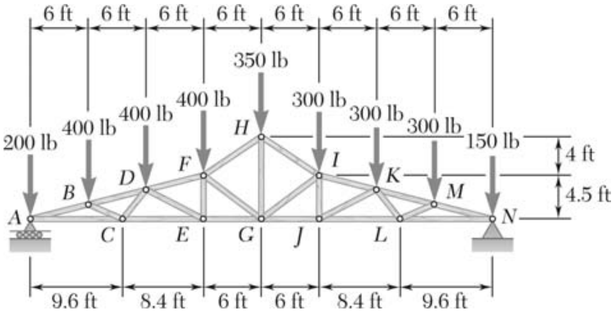


PROBLEM 6.60

A Polynesian, or duopitch, roof truss is loaded as shown. Determine the force in members *HI*, *GI*, and *GJ*.

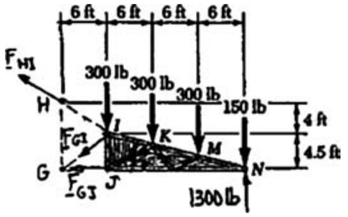
SOLUTION

See solution of Problem 6.59 for reactions:

$$\mathbf{A} = 1500 \text{ lb}\uparrow, \quad \mathbf{N} = 1300 \text{ lb}\uparrow \triangleleft$$

We pass a section through *HI*, *GI*, and *GJ*, and use the free body shown.

(We apply F_{HI} at *H*.)



$$+\circlearrowleft \Sigma M_G = 0: \quad \left(\frac{6}{\sqrt{6^2+4^2}} F_{HI}\right)(8.5 \text{ ft}) + (1300 \text{ lb})(24 \text{ ft}) - (300 \text{ lb})(6 \text{ ft})$$
$$- (300 \text{ lb})(12 \text{ ft}) - (300 \text{ lb})(18 \text{ ft}) - (150 \text{ lb})(24 \text{ ft}) = 0$$

$$F_{HI} = -2375.4 \text{ lb} \quad F_{HI} = 2375 \text{ lb} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_I = 0: \quad (1300 \text{ lb})(18 \text{ ft}) - (300 \text{ lb})(6 \text{ ft}) - (300 \text{ lb})(12 \text{ ft})$$
$$- (150 \text{ lb})(18 \text{ ft}) - F_{GJ}(4.5 \text{ ft}) = 0$$

$$F_{GJ} = +3400 \text{ lb} \quad F_{GJ} = 3400 \text{ lb} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{4}{5} F_{GI} - \frac{6}{\sqrt{6^2+4^2}}(-2375.4 \text{ lb}) - 3400 \text{ lb} = 0$$

$$F_{GI} = -1779.4 \text{ lb} \quad F_{GI} = 1779 \text{ lb} \quad C \blacktriangleleft$$

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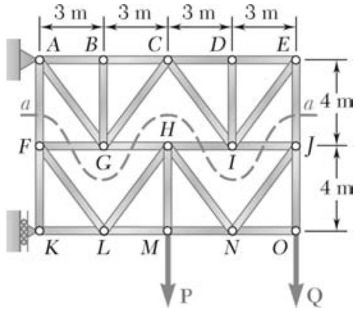
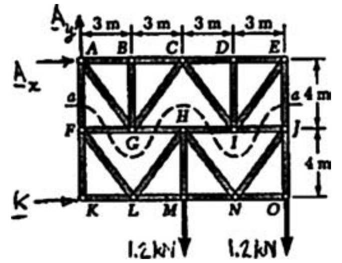


PROBLEM 6.61

Determine the force in members *AF* and *EJ* of the truss shown when $P = Q = 1.2$ kN. (*Hint:* Use section *aa*.)

SOLUTION

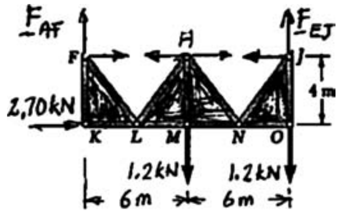
Free body: Entire truss



$$+\circlearrowleft \Sigma M_A = 0: \quad K(8 \text{ m}) - (1.2 \text{ kN})(6 \text{ m}) - (1.2 \text{ kN})(12 \text{ m}) = 0$$

$$K = +2.70 \text{ kN} \qquad \mathbf{K} = 2.70 \text{ kN} \longrightarrow \triangleleft$$

Free body: Lower portion



$$+\circlearrowleft \Sigma M_F = 0: \quad F_{EJ}(12 \text{ m}) + (2.70 \text{ kN})(4 \text{ m}) - (1.2 \text{ kN})(6 \text{ m}) - (1.2 \text{ kN})(12 \text{ m}) = 0$$

$$F_{EJ} = +0.900 \text{ kN} \qquad F_{EJ} = 0.900 \text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad F_{AF} + 0.9 \text{ kN} - 1.2 \text{ kN} - 1.2 \text{ kN} = 0$$

$$F_{AF} = +1.500 \text{ kN} \qquad F_{AF} = 1.500 \text{ kN} \quad T \blacktriangleleft$$

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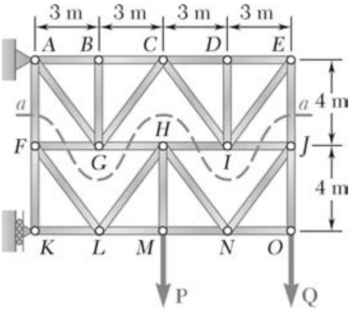
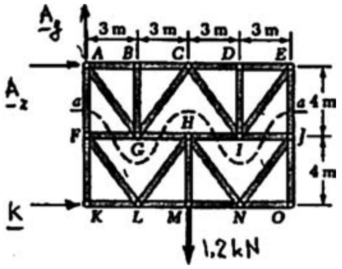


PROBLEM 6.62

Determine the force in members *AF* and *EJ* of the truss shown when $P = 1.2$ kN and $Q = 0$. (*Hint:* Use section *aa*.)

SOLUTION

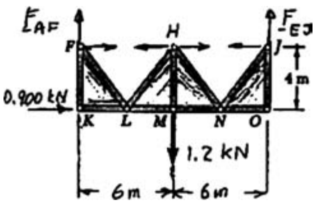
Free body: Entire truss



$$+\circlearrowleft \Sigma M_A = 0: \quad K(8 \text{ m}) - (1.2 \text{ kN})(6 \text{ m}) = 0$$

$$K = +0.900 \text{ kN} \qquad \mathbf{K} = 0.900 \text{ kN} \longrightarrow \triangleleft$$

Free body: Lower portion



$$+\circlearrowleft \Sigma M_F = 0: \quad F_{EJ}(12 \text{ m}) + (0.900 \text{ kN})(4 \text{ m}) - (1.2 \text{ kN})(6 \text{ m}) = 0$$

$$F_{EJ} = +0.300 \text{ kN} \qquad F_{EJ} = 0.300 \text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad F_{AF} + 0.300 \text{ kN} - 1.2 \text{ kN} = 0$$

$$F_{AF} = +0.900 \text{ kN} \qquad F_{AF} = 1.900 \text{ kN} \quad T \blacktriangleleft$$

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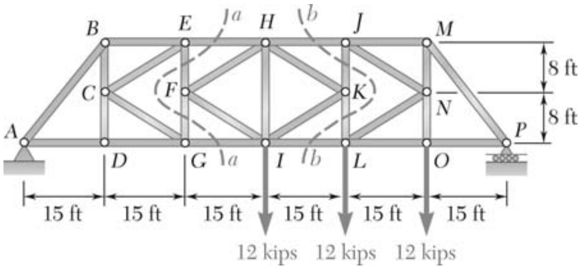
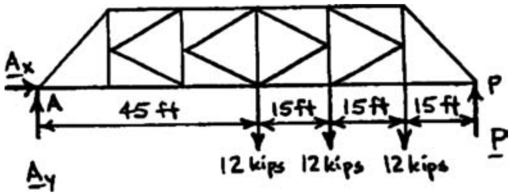


PROBLEM 6.63

Determine the force in members *EH* and *GI* of the truss shown. (*Hint:* Use section *aa*.)

SOLUTION

<u>Reactions</u>:

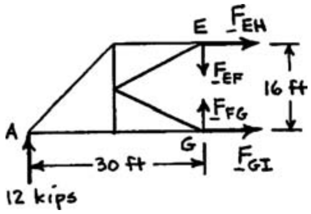


$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_P = 0: \quad 12(45) + 12(30) + 12(15) - A_y(90) = 0$$

$$\mathbf{A}_y = 12 \text{ kips} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 12 - 12 - 12 - 12 + P = 0 \qquad \mathbf{P} = 24 \text{ kips} \uparrow$$



$$+\circlearrowleft \Sigma M_G = 0: \quad -(12 \text{ kips})(30 \text{ ft}) - F_{EH}(16 \text{ ft}) = 0$$

$$F_{EH} = -22.5 \text{ kips} \qquad F_{EH} = 22.5 \text{ kips} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{GI} - 22.5 \text{ kips} = 0 \qquad F_{GI} = 22.5 \text{ kips} \quad T \blacktriangleleft$$

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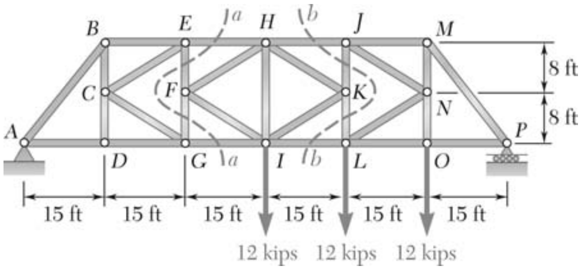
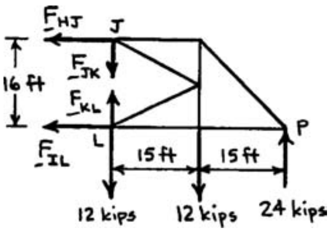


PROBLEM 6.64

Determine the force in members *HJ* and *IL* of the truss shown. (*Hint:* Use section *bb*.)

SOLUTION

See the solution to Problem 6.63 for free body diagram and analysis to determine the reactions at supports *A* and *P*.



$$\mathbf{A}_x = 0; \quad \mathbf{A}_y = 12.00 \text{ kips} \uparrow; \quad \mathbf{P} = 24.0 \text{ kips} \uparrow$$

$$+\circlearrowleft \Sigma M_L = 0: \quad F_{HJ}(16 \text{ ft}) - (12 \text{ kips})(15 \text{ ft}) + (24 \text{ kips})(30 \text{ ft}) = 0$$

$$F_{HJ} = -33.75 \text{ kips} \qquad F_{HJ} = 33.8 \text{ kips} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 33.75 \text{ kips} - F_{IL} = 0$$

$$F_{IL} = +33.75 \text{ kips} \qquad F_{IL} = 33.8 \text{ kips} \quad T \blacktriangleleft$$

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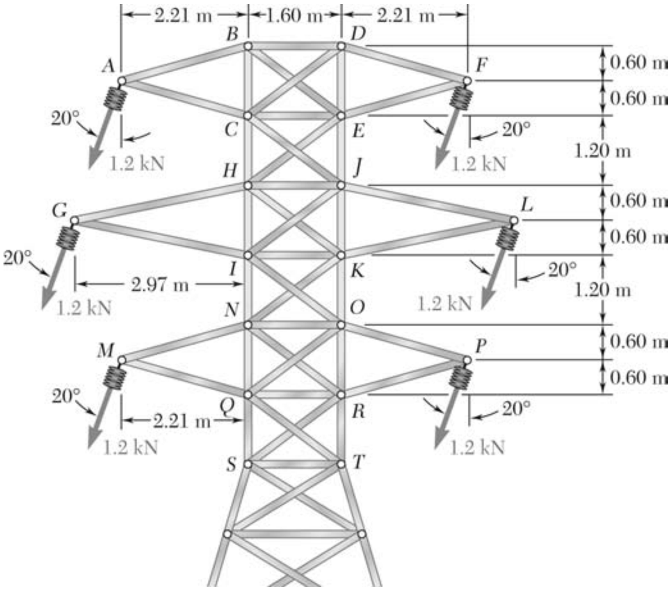


PROBLEM 6.65

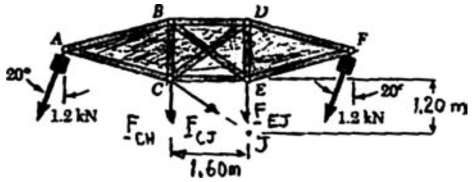
The diagonal members in the center panels of the power transmission line tower shown are very slender and can act only in tension; such members are known as *counters*. For the given loading, determine (*a*) which of the two counters listed below is acting, (*b*) the force in that counter.

Counters *CJ* and *HE*.

SOLUTION

Free body: Portion *ABDFEC* of tower

We assume that counter *CJ* is acting and show the forces exerted by that counter and by members *CH* and *EJ*.



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{CJ} - 2(1.2\text{ kN})\sin 20° = 0 \quad F_{CJ} = +1.026\text{ kN}$$

Since *CJ* is found to be in tension, our assumption was correct. Thus, the answers are

(*a*) *CJ* ◀

(*b*) 1.026 kN *T* ◀

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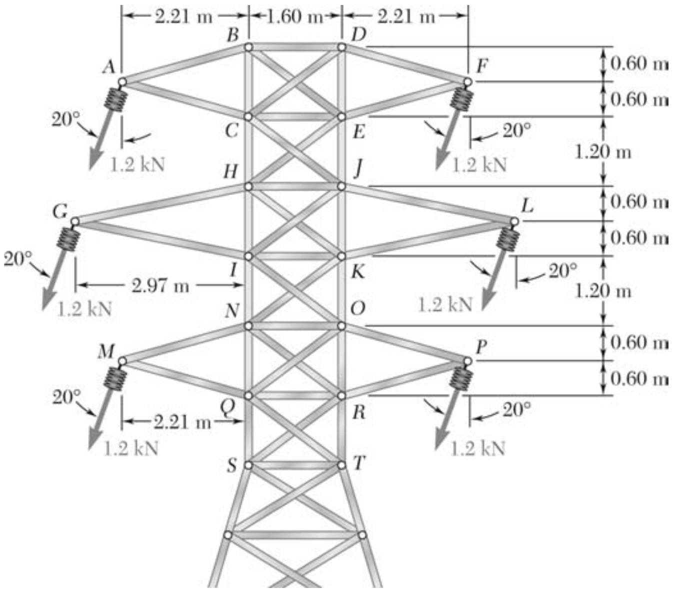


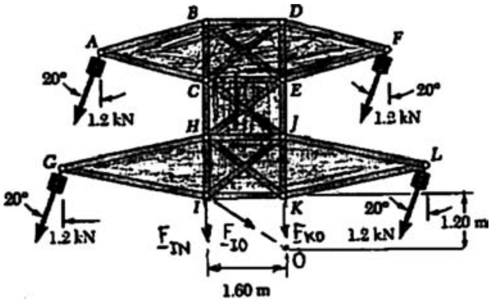
PROBLEM 6.66

The diagonal members in the center panels of the power transmission line tower shown are very slender and can act only in tension; such members are known as *counters*. For the given loading, determine (*a*) which of the two counters listed below is acting, (*b*) the force in that counter.

Counters *IO* and *KN*.

SOLUTION

Free body: Portion of tower shown



We assume that counter *IO* is acting and show the forces exerted by that counter and by members *IN* and *KO*.

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5} F_{IO} - 4(1.2 \text{ kN}) \sin 20° = 0 \qquad F_{IO} = +2.05 \text{ kN}$$

Since *IO* is found to be in tension, our assumption was correct. Thus, the answers are

(*a*) *IO* ◀

(*b*) 2.05 kN *T* ◀

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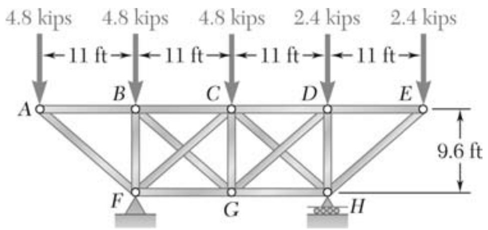
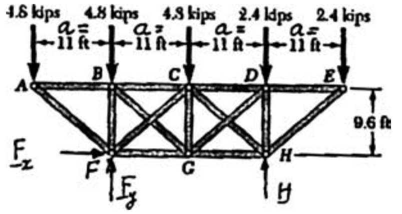


PROBLEM 6.67

The diagonal members in the center panels of the truss shown are very slender and can act only in tension; such members are known as *counters*. Determine the forces in the counters that are acting under the given loading.

SOLUTION

Free body: Truss



$$\Sigma F_x = 0: \quad F_x = 0$$

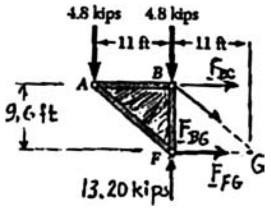
$$+\circlearrowleft \Sigma M_H = 0: \quad 4.8(3a) + 4.8(2a) + 4.8a - 2.4a - F_y(2a) = 0$$

$F_y = +13.20$ kips $\qquad\qquad$ $\mathbf{F} = 13.20 \text{ kips} \uparrow$ ◁

$$+\uparrow \Sigma F_y = 0: \quad H + 13.20 \text{ kips} - 3(4.8 \text{ kips}) - 2(2.4 \text{ kips}) = 0$$

$H = +6.00$ kips $\qquad\qquad$ $\mathbf{H} = 6.00 \text{ kips} \uparrow$ ◁

Free body: *ABF*



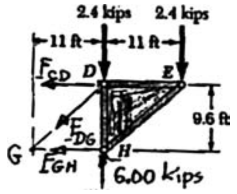
We assume that counter *BG* is acting.

$$+\uparrow \Sigma F_y = 0: \quad -\frac{9.6}{14.6} F_{BG} + 13.20 - 2(4.8) = 0$$

$F_{BG} = +5.475$ $\qquad\qquad$ $F_{BG} = 5.48 \text{ kips} \quad T$ ◀

Since *BG* is in tension, our assumption was correct

Free body: *DEH*



We assume that counter *DG* is acting.

$$+\Sigma F_y = 0: \quad -\frac{9.6}{14.6} F_{DG} + 6.00 - 2(2.4) = 0$$

$F_{DG} = +1.825$ $\qquad\qquad$ $F_{DG} = 1.825 \text{ kips} \quad T$ ◀

Since *DG* is in tension, O.K.

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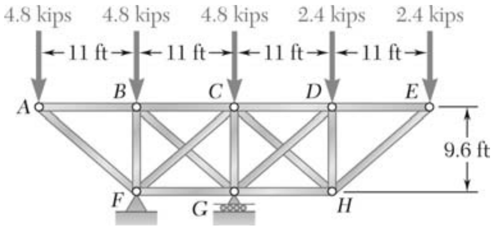
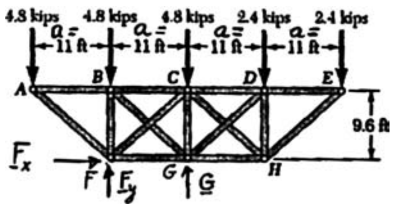


PROBLEM 6.68

The diagonal members in the center panels of the truss shown are very slender and can act only in tension; such members are known as *counters*. Determine the forces in the counters that are acting under the given loading.

SOLUTION

Free body: Truss

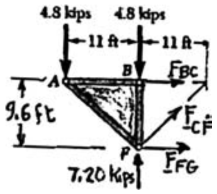


$$\Sigma F_x = 0: \quad F_x = 0$$

$$+\circlearrowleft \Sigma M_G = 0: \quad -F_y a + 4.8(2a) + 4.8a - 2.4a - 2.4(2a) = 0$$

$$F_y = 7.20 \qquad\qquad F = 7.20 \text{ kips} \uparrow \triangleleft$$

Free body: *ABF*



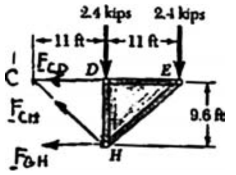
We assume that counter *CF* is acting.

$$+\uparrow \Sigma F_y = 0: \frac{9.6}{14.6} F_{CF} + 7.20 - 2(4.8) = 0$$

$$F_{CF} = +3.65 \qquad\qquad F_{CF} = 3.65 \text{ kips} \quad T \blacktriangleleft$$

Since *CF* is in tension, O.K.

Free body: *DEH*



We assume that counter *CH* is acting.

$$+\Sigma F_y = 0: \quad \frac{9.6}{14.6} F_{CH} - 2(2.4 \text{ kips}) = 0$$

$$F_{CH} = +7.30 \qquad\qquad F_{CH} = 7.30 \text{ kips} \quad T \blacktriangleleft$$

Since *CH* is in tension, O.K.

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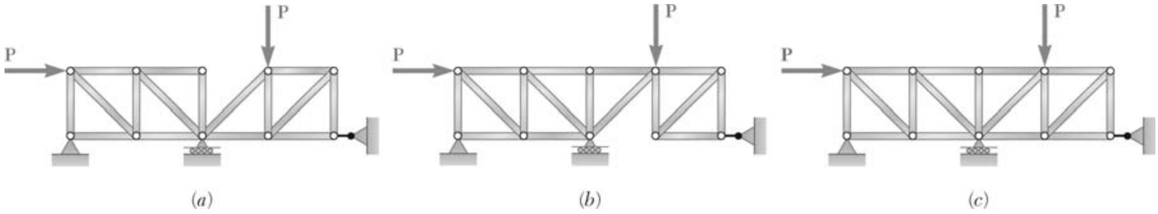
PROBLEM 6.69

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



SOLUTION

Structure (a)

Number of members:

$$m = 16$$

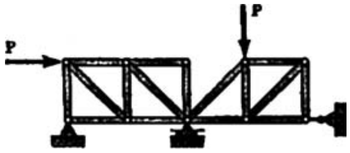
Number of joints:

$$n = 10$$

Reaction components:

$$r = 4$$

$$m + r = 20, \quad 2n = 20$$

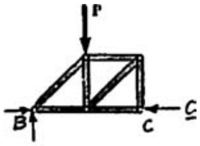


(a)

Thus: $m + r = 2n$ ◁

To determine whether the structure is actually completely constrained and determinate, we must try to find the reactions at the supports. We divide the structure into two simple trusses and draw the free-body diagram of each truss.

This is a properly supported simple truss – O.K.



This is an improperly supported simple truss. (Reaction at C passes through B. Thus, Eq. $\Sigma M_B = 0$ cannot be satisfied.)

Structure is improperly constrained ◀

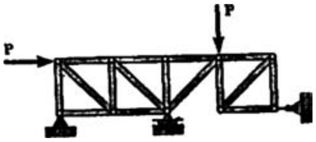
Structure (b)

$$m = 16$$

$$n = 10$$

$$r = 4$$

$$m + r = 20, \quad 2n = 20$$



(b)

Thus: $m + r = 2n$ ◁

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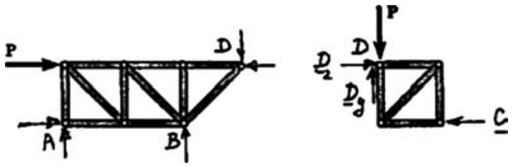
PROBLEM 6.69 (Continued)

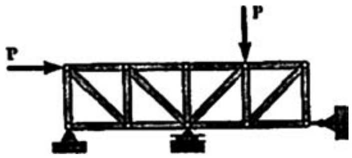
We must again try to find the reactions at the supports dividing the structure as shown.



Both portions are simply supported simple trusses.

Structure is completely constrained and determinate ◀

Structure (c)


(c)

$$m = 17$$
$$n = 10$$
$$r = 4$$
$$m + r = 21, \quad 2n = 20$$

Thus: $m + r > 2n$ ◁

This is a simple truss with an extra support which causes reactions (and forces in members) to be indeterminate.

Structure is completely constrained and indeterminate ◀

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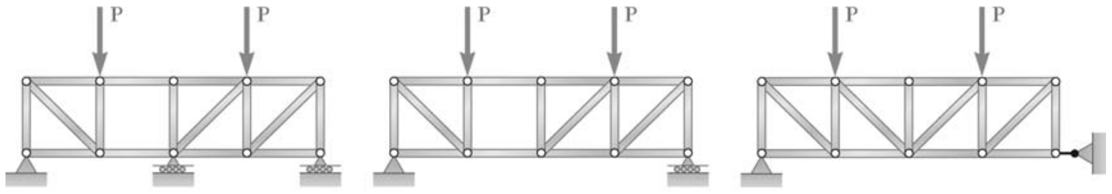
PROBLEM 6.70

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



(a) (b) (c)

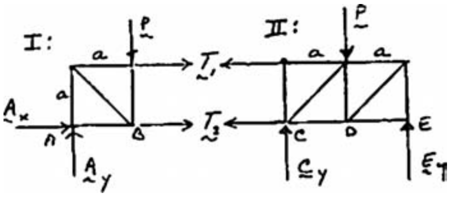
SOLUTION

<u>Structure (a)</u>:

Non-simple truss with $r = 4,\ \ m = 16,\ \ n = 10$

so $m + r = 20 = 2n,$ but must examine further.

FBD Sections:



FBD I: $\Sigma M_A = 0 \ \Rightarrow\ T_1$

II: $\Sigma F_x = 0 \ \Rightarrow\ T_2$

I: $\Sigma F_x = 0 \ \Rightarrow\ A_x$

I: $\Sigma F_y = 0 \ \Rightarrow\ A_y$

II: $\Sigma M_E = 0 \ \Rightarrow\ C_y$

II: $\Sigma F_y = 0 \ \Rightarrow\ E_y$

Since each section is a simple truss with reactions determined,

structure is completely constrained and determinate. ◀

Non-simple truss with $r = 3,\ \ m = 16,\ \ n = 10$

<u>Structure (b)</u>:

so $m + r = 19 < 2n = 20$ structure is partially constrained ◀

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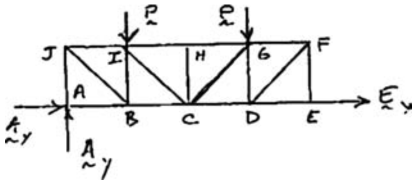
PROBLEM 6.70 (Continued)

Structure (c):



Simple truss with $r = 3, \;\; m = 17, \;\; n = 10 \;\; m + r = 20 = 2n,$ but the horizontal reaction forces A_x and E_x are collinear and no equilibrium equation will resolve them, so the structure is improperly constrained and indeterminate ◀

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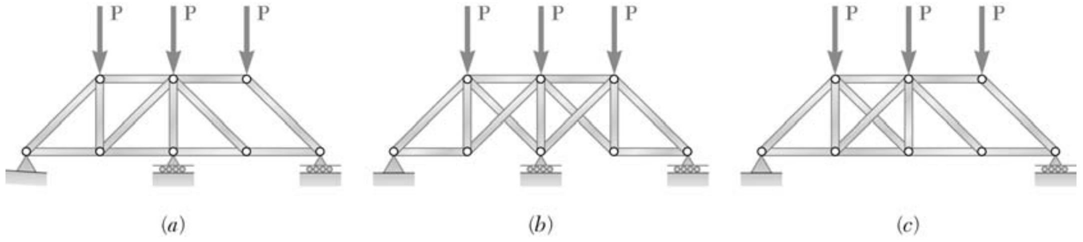
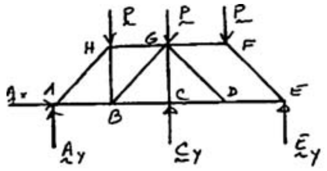
PROBLEM 6.71

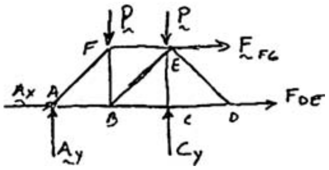
Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



SOLUTION

Structure (a):





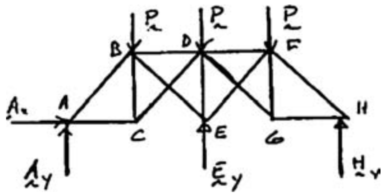
Non-simple truss with $r = 4,\ m = 12,\ n = 8$ so $r + m = 16 = 2n,$
check for determinacy:

One can solve joint *F* for forces in *EF*, *FG* and then solve joint *E* for $\mathbf{E}_y$ and force in *DE*.

This leaves a simple truss *ABCDGH* with

$r = 3,\ m = 9,\ n = 6$ so $r + m = 12 = 2n$ Structure is completely constrained and determinate ◀

Structure (b):



Simple truss (start with *ABC* and add joints alphabetically to complete truss) with $r = 4,\ m = 13,\ n = 8$

so $r + m = 17 > 2n = 16$ Constrained but indeterminate ◀

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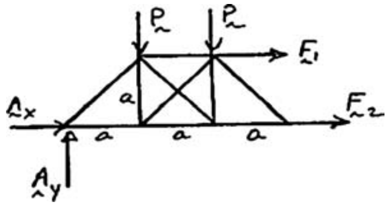
PROBLEM 6.71 (Continued)

Structure (c):



Non-simple truss with $r = 3,\ m = 13,\ n = 8$ so $r + m = 16 = 2n.$ To further examine, follow procedure in Part (a) above to get truss at left.

Since $\mathbf{F}_1 \neq 0$ (from solution of joint F),

$\Sigma M_A = aF_1 \curvearrowright \neq 0$ and there is no equilibrium.

Structure is improperly constrained ◀

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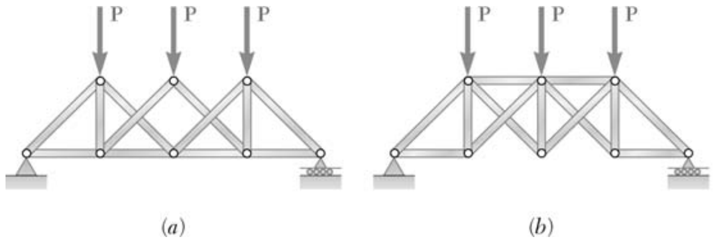
PROBLEM 6.72

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



SOLUTION

Structure (a)

Number of members:

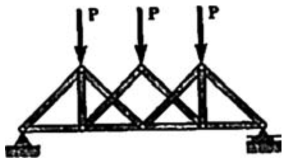
$$m = 12$$

Number of joints:

$$n = 8$$

Reaction components:

$$r = 3$$



$$m + r = 15, \quad 2n = 16$$

Thus: $m + r < 2n$ ◁

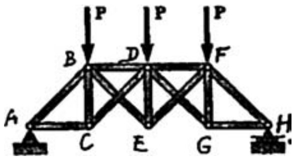
Structure is partially constrained ◀

Structure (b)

$$m = 13, \; n = 8$$

$$r = 3$$

$$m + r = 16, \quad 2n = 16$$



Thus: $m + r = 2n$ ◁

To verify that the structure is actually completely constrained and determinate, we observe that it is a simple truss (follow lettering to check this) and that it is simply supported by a pin-and-bracket and a roller. Thus:

Structure is completely constrained and determinate ◀

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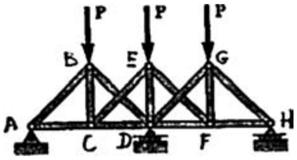
PROBLEM 6.72 (Continued)

Structure (c)



$$m = 13, \quad n = 8$$
$$r = 4$$
$$m + r = 17, \quad 2n = 16$$

Thus: $m + r > 2n$ ◁

Structure is completely constrained and indeterminate ◀

This result can be verified by observing that the structure is a simple truss (follow lettering to check thus), therefore rigid, and that its supports involve 4 unknowns.

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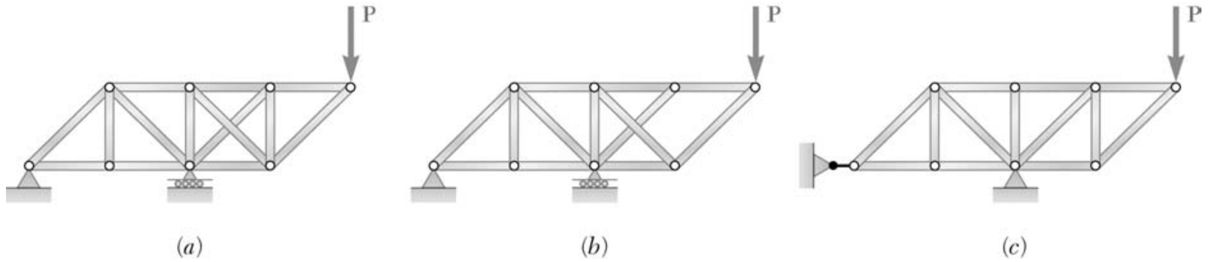
PROBLEM 6.73

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



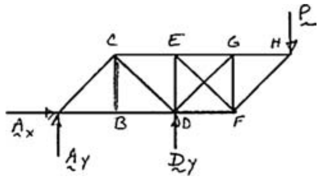
SOLUTION

Structure (a): Rigid truss with $r = 3, \quad m = 14, \quad n = 8$

so $r + m = 17 > 2n = 16$

so completely constrained but indeterminate ◀

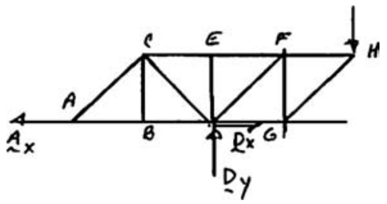
Structure (b): Simple truss (start with *ABC* and add joints alphabetically), with



$r = 3, m = 13, n = 8 \quad \text{so} \quad r + m = 16 = 2n$

so completely constrained and determinate ◀

Structure (c):



Simple truss with $r = 3, \quad m = 13, \quad n = 8$ so $r + m = 16 = 2n,$ but horizontal reactions $(A_x$ and $D_x)$ are collinear so cannot be resolved by any equilibrium equation.

structure is improperly constrained ◀

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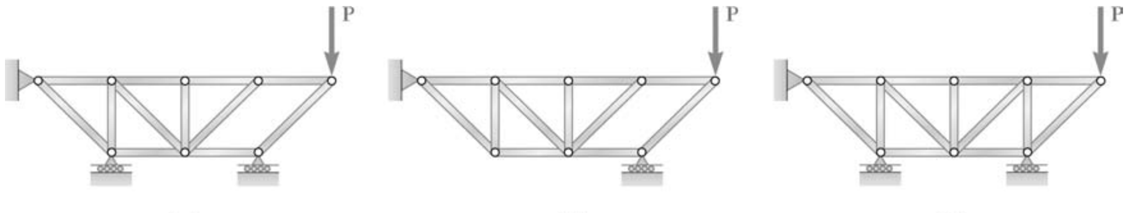
PROBLEM 6.74

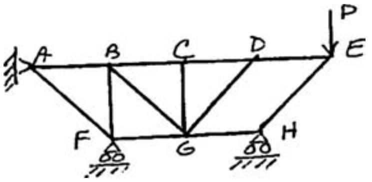
Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



(a) (b) (c)

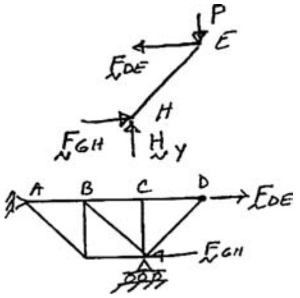
SOLUTION

Structure (a):



No. of members	$m = 12$
No. of joints	$n = 8 \quad m + r = 16 = 2n$
No. of react. comps.	$r = 4 \quad$ unks = eqns

FBD of EH:

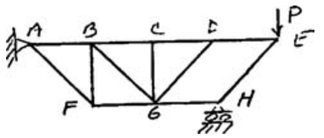


$$\Sigma M_H = 0 \longrightarrow F_{DE}; \ \Sigma F_x = 0 \longrightarrow F_{GH}; \ \Sigma F_y = 0 \longrightarrow H_y$$

Then *ABCDGF* is a simple truss and all forces can be determined.

This example is completely constrained and determinate. ◀

Structure (b):



No. of members	$m = 12$
No. of joints	$n = 8 \quad m + r = 15 < 2n = 16$
No. of react. comps.	$r = 3 \quad$ unks < eqns

partially constrained ◀

Note: Quadrilateral *DEHG* can collapse with joint *D* moving downward: in (a) the roller at *F* prevents this action.

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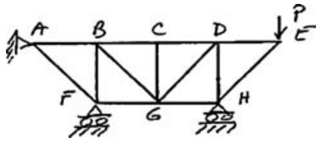
PROBLEM 6.74 (Continued)

Structure (c):



No. of members $m = 13$

No. of joints $n = 8 \quad m + r = 17 > 2n = 16$

No. of react. comps. $r = 4 \quad$ unks $>$ eqns

completely constrained but indeterminate ◀

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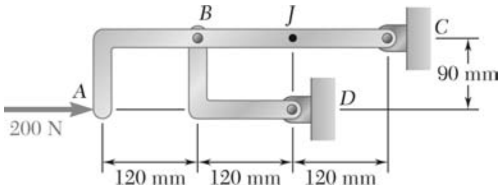
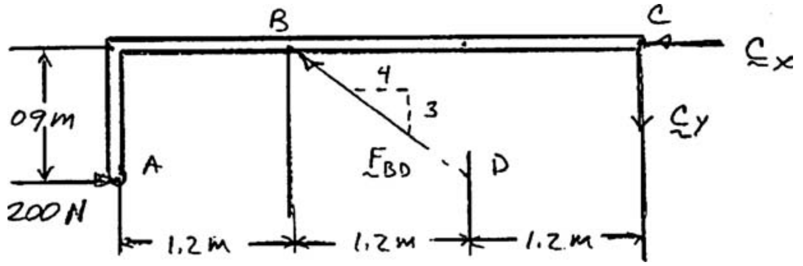


PROBLEM 6.75

For the frame and loading shown, determine the force acting on member *ABC* (*a*) at *B*, (*b*) at *C*.

SOLUTION

FBD ABC:



Note: *BD* is two-force member

(*a*) $\circlearrowleft \Sigma M_C = 0\text{:} \quad (0.09\text{ m})(200\text{ N}) - (2.4\text{ m})\left(\frac{3}{5}F_{BD}\right) = 0$

$\mathbf{F}_{BD} = 125.0\text{ N} \searrow 36.9°$ ◀

(*b*) $\xrightarrow{} \Sigma F_x = 0\text{:} \quad 200\text{ N} - \frac{4}{5}(125\text{ N}) - C_x = 0 \qquad \mathbf{C}_x = 100\text{ N} \longleftarrow$

$\uparrow \Sigma F_y = 0\text{:} \quad \frac{3}{5}F_{BD} - C_y = 0 \qquad \mathbf{C}_y = \frac{3}{5}(125\text{ N}) = 75\text{ N} \downarrow$

$\mathbf{C} = 125.0\text{ N} \nearrow 36.9°$ ◀

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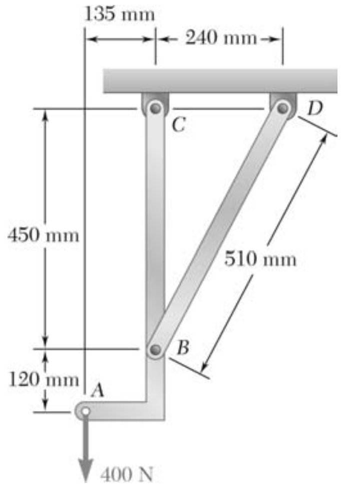


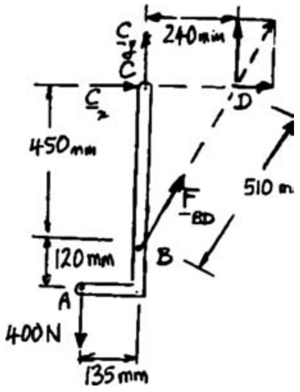
PROBLEM 6.76

Determine the force in member *BD* and the components of the reaction at *C*.

SOLUTION

We note that *BD* is a two-force member. The force it exerts on *ABC*, therefore, is directed along the *BD*.

Free body: *ABC*



Attaching $\mathbf{F}_{BD}$ at *D* and resolving it into components, we write

$$+\circlearrowleft \Sigma M_C = 0: \quad (400 \text{ N})(135 \text{ mm}) + \left(\frac{450}{510} F_{BD}\right)(240 \text{ mm}) = 0$$

$$F_{BD} = -255 \text{ N} \qquad F_{BD} = 255 \text{ N} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + \frac{240}{510}(-255 \text{ N}) = 0$$

$$C_x = +120.0 \text{ N} \qquad \mathbf{C}_x = 120.0 \text{ N} \longrightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 400 \text{ N} + \frac{450}{510}(-255 \text{ N}) = 0$$

$$C_y = +625 \text{ N} \qquad \mathbf{C}_y = 625 \text{ N} \uparrow \blacktriangleleft$$

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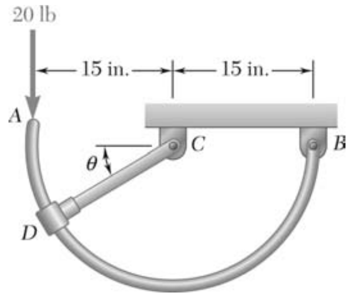
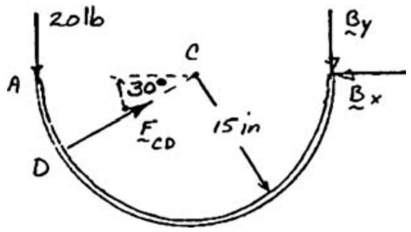


PROBLEM 6.77

Rod *CD* is fitted with a collar at *D* that can be moved along rod *AB*, which is bent in the shape of an arc of circle. For the position when $\theta = 30°$, determine (*a*) the force in rod *CD*, (*b*) the reaction at *B*.

SOLUTION

FBD:



(*a*) $\circlearrowleft \Sigma M_C = 0: \quad (15 \text{ in.})(20 \text{ lb} - B_y) = 0$ $\qquad \mathbf{B}_y = 20 \text{ lb} \downarrow$

$\uparrow \Sigma F_y = 0: \; -20 \text{ lb} + F_{CD} \sin 30° - 20 \text{ lb} = 0$ $\qquad F_{CD} = 80.0 \text{ lb} \quad T$ ◀

(*b*) $\longrightarrow \Sigma F_x = 0: \quad (80 \text{ lb})\cos 30° - B_x = 0$

$\mathbf{B}_x = 69.282 \text{ lb} \longleftarrow$

so $\mathbf{B} = 72.1 \text{ lb} \measuredangle 16.10°$ ◀

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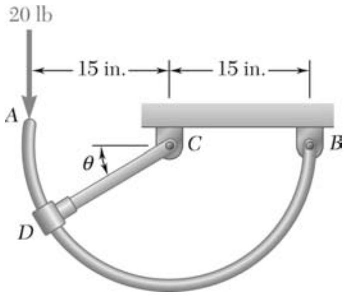


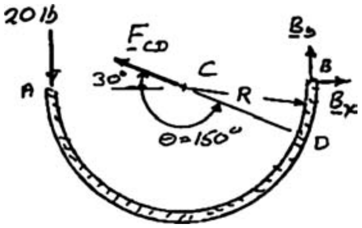
PROBLEM 6.78

Solve Problem 6.77 when $\theta = 150°$.

PROBLEM 6.77 Rod *CD* is fitted with a collar at *D* that can be moved along rod *AB*, which is bent in the shape of an arc of circle. For the position when $\theta = 30°$, determine (*a*) the force in rod *CD*, (*b*) the reaction at *B*.

SOLUTION

Note that *CD* is a two-force member, $\mathbf{F}_{CD}$ must be directed along *DC*.



(*a*) $+\circlearrowleft \Sigma M_B = 0: \quad (20\text{ lb})(2R) - (F_{CD}\sin 30°)R = 0$

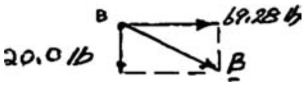
$F_{CD} = 80\text{ lb}$ $\qquad F_{CD} = 80.0\text{ lb} \quad T$ ◀

(*b*) $+\circlearrowleft \Sigma M_C = 0: \quad (20\text{ lb})R + (B_y)R = 0$

$B_y = -20\text{ lb}$ $\qquad \mathbf{B}_y = 20.0\text{ lb}\downarrow$

$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{CD}\cos 30° + B_x = 0$

$-(20\text{ lb})\cos 30° + B_x = 0$



$B_x = 69.28\text{ lb} \quad \mathbf{B}_x = 69.28\text{ lb}\longrightarrow$

$\mathbf{B} = 72.1\text{ lb}$ ↘ $16.10°$ ◀

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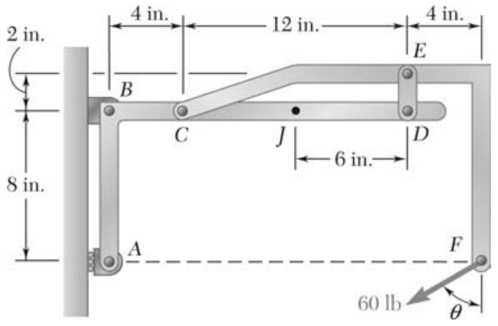
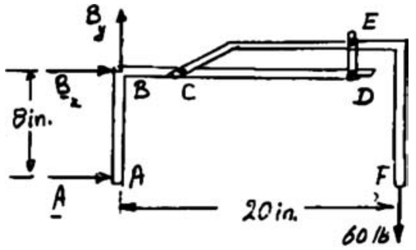


PROBLEM 6.79

Determine the components of all forces acting on member *ABCD* when $\theta = 0$.

SOLUTION

Free body: Entire assembly



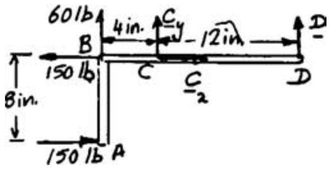
$$+\circlearrowleft \Sigma M_B = 0: \quad A(8 \text{ in.}) - (60 \text{ lb})(20 \text{ in.}) = 0$$

$A = 150$ lb $\qquad \mathbf{A} = 150.0 \text{ lb} \longrightarrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 150.0 \text{ lb} = 0 \quad B_x = -150 \text{ lb} \qquad \mathbf{B}_x = 150.0 \text{ lb} \longleftarrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad B_y - 60.0 \text{ lb} = 0 \quad B_y = +60.0 \text{ lb} \qquad \mathbf{B}_y = 60.0 \text{ lb} \uparrow$ ◀

Free body: Member *ABCD* We note that **D** is directed along *DE*, since *DE* is a two-force member.



$$+\circlearrowleft \Sigma M_C = 0: \quad D(12) - (60 \text{ lb})(4) + (150 \text{ lb})(8) = 0$$

$D = -80$ lb $\qquad \mathbf{D} = 80.0 \text{ lb} \downarrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + 150.0 - 150.0 = 0 \quad C_x = 0$

$+\uparrow \Sigma F_y = 0: \quad C_y + 60.0 - 80.0 = 0 \quad C_y = +20.0 \text{ lb} \qquad \mathbf{C} = 20.0 \text{ lb} \uparrow$ ◀

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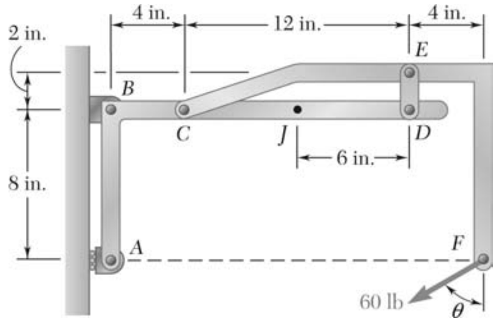
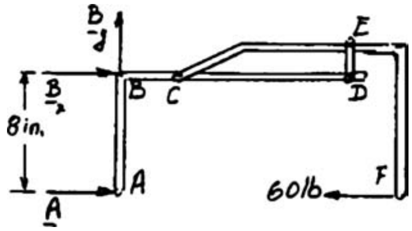


PROBLEM 6.80

Determine the components of all forces acting on member *ABCD* when $\theta = 90°$.

SOLUTION

Free body: Entire assembly



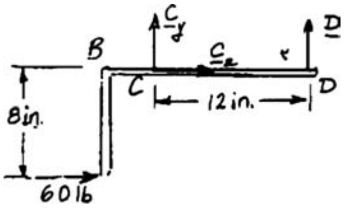
$$+\circlearrowleft \Sigma M_B = 0: \quad A(8\text{ in.}) - (60\text{ lb})(8\text{ in.}) = 0$$

$$A = +60.0\text{ lb} \qquad \mathbf{A} = 60.0\text{ lb} \longrightarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 60\text{ lb} - 60\text{ lb} = 0 \quad B_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad B_y = 0 \qquad \mathbf{B} = 0 \blacktriangleleft$$

Free body: Member *ABCD* — We note that **D** is directed along *DE*, since *DE* is a two-force member.



$$+\circlearrowleft \Sigma M_C = 0: \quad D(12\text{ in.}) + (60\text{ lb})(8\text{ in.}) = 0$$

$$D = -40.0\text{ lb} \qquad \mathbf{D} = 40.0\text{ lb} \downarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + 60\text{ lb} = 0$$

$$C_x = -60\text{ lb} \qquad \mathbf{C}_x = 60.0\text{ lb} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 40\text{ lb} = 0$$

$$C_y = +40\text{ lb} \qquad \mathbf{C}_y = 40.0\text{ lb} \uparrow \blacktriangleleft$$

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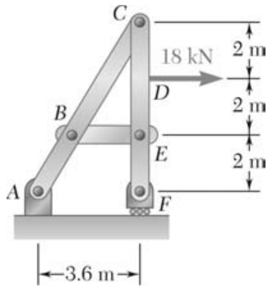
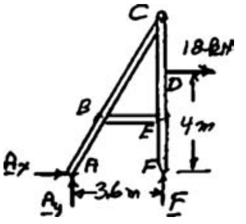


PROBLEM 6.81

For the frame and loading shown, determine the components of all forces acting on member *ABC*.

SOLUTION

Free body: Entire frame



$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 18 \text{ kN} = 0$

$A_x = -18 \text{ kN} \qquad \mathbf{A}_x = 18.00 \text{ kN} \longleftarrow$ ◀

$+\circlearrowleft \Sigma M_E = 0: \quad -(18 \text{ kN})(4 \text{ m}) - A_y(3.6 \text{ m}) = 0$

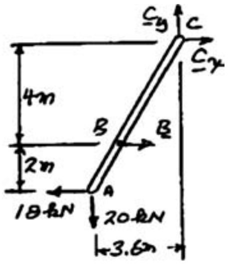
$A_y = -20 \text{ kN} \qquad \mathbf{A}_y = 20.0 \text{ kN} \downarrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad -20 \text{ kN} + F = 0$

$F = +20 \text{ kN} \quad \mathbf{F} = 20 \text{ kN} \uparrow$

Free body: Member *ABC*

Note: BE is a two-force member. Thus **B** is directed along line *BE*.



$+\circlearrowleft \Sigma M_C = 0: \quad B(4 \text{ m}) - (18 \text{ kN})(6 \text{ m}) + (20 \text{ kN})(3.6 \text{ m}) = 0$

$B = 9 \text{ kN} \qquad \mathbf{B} = 9.00 \text{ kN} \longrightarrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 18 \text{ kN} + 9 \text{ kN} = 0$

$C_x = 9 \text{ kN} \qquad \mathbf{C}_x = 9.00 \text{ kN} \longrightarrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad C_y - 20 \text{ kN} = 0$

$C_y = 20 \text{ kN} \qquad \mathbf{C}_y = 20.0 \text{ kN} \uparrow$ ◀

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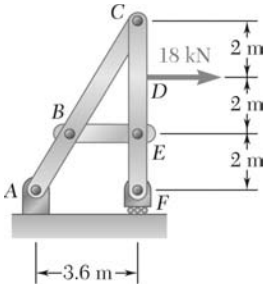


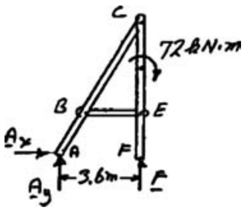
PROBLEM 6.82

Solve Problem 6.81 assuming that the 18-kN load is replaced by a clockwise couple of magnitude 72 kN · m applied to member *CDEF* at Point *D*.

PROBLEM 6.81 For the frame and loading shown, determine the components of all forces acting on member *ABC*.

SOLUTION

Free body: Entire frame



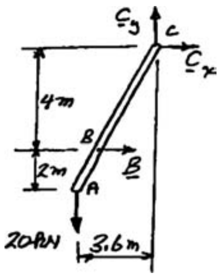
$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$

$+\circlearrowright \Sigma M_F = 0: \quad -72\text{ kN}\cdot\text{m} - A_y(3.6\text{ m}) = 0$

$A_y = -20\text{ kN} \quad \mathbf{A}_y = 20\text{ kN}\downarrow$ $\qquad \mathbf{A} = 20.0\text{ kN}\downarrow$ ◀

Free body: Member *ABC*

Note: BE is a two-force member. Thus **B** is directed along line *BE*.



$+\circlearrowright \Sigma M_C = 0: \quad B(4\text{ m}) + (20\text{ kN})(3.6\text{ m}) = 0$

$B = -18\text{ kN}$ $\qquad \mathbf{B} = 18.00\text{ kN}\longleftarrow$ ◀

$\xrightarrow{+} F_x = 0: \quad -18\text{ kN} + C_x = 0$

$C_x = 18\text{ kN}$ $\qquad \mathbf{C}_x = 18.00\text{ kN}\longrightarrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad C_y - 20\text{ kN} = 0$

$C_y = 20\text{ kN}$ $\qquad \mathbf{C}_y = 20.0\text{ kN}\uparrow$ ◀

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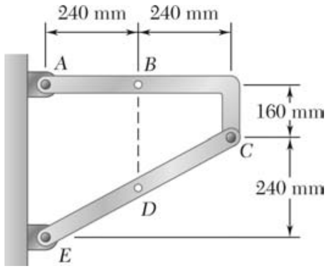


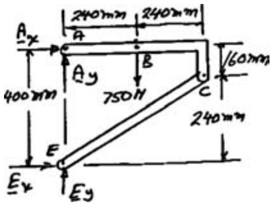
PROBLEM 6.83

Determine the components of the reactions at A and E if a 750-N force directed vertically downward is applied (a) at B, (b) at D.

SOLUTION

<u>Free-body: Entire Frame</u>

The following analysis is valid for both (a) and (b) since position of load on its line of action is immaterial.



$$+\circlearrowleft \Sigma M_E = 0: \quad -(750 \text{ N})(240 \text{ mm}) - A_x(400 \text{ mm}) = 0$$

$$A_x = -450 \text{ N} \quad \mathbf{A}_x = 450 \text{ N} \longleftarrow$$

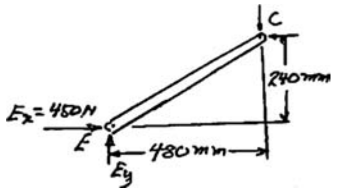
$$\xrightarrow{+} \Sigma F_x = 0: \quad E_x - 450 \text{ N} = 0 \quad E_x = 450 \text{ N} \quad \mathbf{E}_x = 450 \text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + E_y - 750 \text{ N} = 0 \qquad (1)$$

(a) <u>Load applied at B</u>.

<u>Free body: Member CE</u>

CE is a two-force member. Thus, the reaction at E must be directed along CE.



$$\frac{E_y}{450 \text{ N}} = \frac{240 \text{ mm}}{480 \text{ mm}}; \quad \mathbf{E}_y = 225 \text{ N} \uparrow$$

From Eq. (1): $\quad A_y + 225 - 750 = 0; \quad \mathbf{A}_y = 525 \text{ lb} \uparrow$

Thus, reactions are:

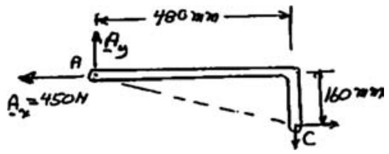
$$\mathbf{A}_x = 450 \text{ N} \longleftarrow, \qquad \mathbf{A}_y = 525 \text{ lb} \uparrow \blacktriangleleft$$

$$\mathbf{E}_x = 450\text{N} \longrightarrow, \qquad \mathbf{E}_y = 225 \text{ lb} \uparrow \blacktriangleleft$$

(b) <u>Load applied at D</u>.

<u>Free body: Member AC</u>

AC is a two-force member. Thus, the reaction at A must be directed along AC.



$$\frac{A_y}{450 \text{ N}} = \frac{160 \text{ mm}}{480 \text{ mm}} \quad \mathbf{A}_y = 150.0 \text{ N} \uparrow$$

From Eq. (1):

$$A_y + E_y - 750 \text{ N} = 0$$

$$150 \text{ N} + E_y - 750 \text{ N} = 0$$

$$E_y = 600 \text{ N} \quad \mathbf{E}_y = 600 \text{ N} \uparrow$$

Thus, reactions are:

$$\mathbf{A}_x = 450 \text{ N} \longleftarrow, \qquad \mathbf{A}_y = 150.0 \text{ N} \uparrow \blacktriangleleft$$

$$\mathbf{E}_x = 450 \text{ N} \longrightarrow, \qquad \mathbf{E}_y = 600 \text{ N} \uparrow \blacktriangleleft$$

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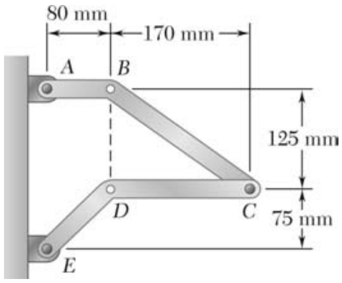


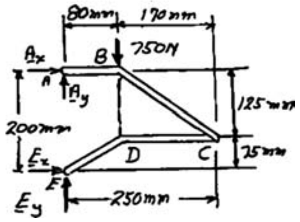
PROBLEM 6.84

Determine the components of the reactions at A and E if a 750-N force directed vertically downward is applied (a) at B, (b) at D.

SOLUTION

Free body: Entire frame

The following analysis is valid for both (a) and (b) since position of load on its line of action is immaterial.



$$+\circlearrowleft \Sigma M_E = 0: \quad -(750 \text{ N})(80 \text{ mm}) - A_x(200 \text{ mm}) = 0$$

$$A_x = -300 \text{ N} \quad \mathbf{A}_x = 300 \text{ N} \longleftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad E_x - 300 \text{ N} = 0 \quad E_x = 300 \text{ N} \quad \mathbf{E}_x = 300 \text{ N} \longrightarrow$$

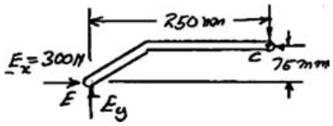
$$+\uparrow \Sigma F_y = 0: \quad A_y + E_y - 750 \text{ N} = 0 \qquad (1)$$

(a) Load applied at B.

Free body: Member CE

CE is a two-force member. Thus, the reaction at E must be directed along CE.

$$\frac{E_y}{300 \text{ N}} = \frac{75 \text{ mm}}{250 \text{ mm}} \quad E_y = 90 \text{ N} \uparrow$$



From Eq. (1): $A_y + 90 \text{ N} - 750 \text{ N} = 0 \qquad A_y = 660 \text{ N} \uparrow$

Thus reactions are:

$$\mathbf{A}_x = 300 \text{ N} \longleftarrow, \qquad \mathbf{A}_y = 660 \text{ N} \uparrow \blacktriangleleft$$

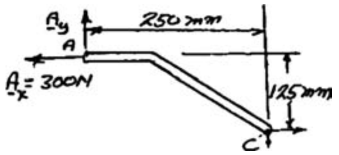
$$\mathbf{E}_x = 300 \text{ N} \longrightarrow, \qquad \mathbf{E}_y = 90.0 \text{ N} \uparrow \blacktriangleleft$$

(b) Load applied at D.

Free body: Member AC

AC is a two-force member. Thus, the reaction at A must be directed along AC.

$$\frac{A_y}{300 \text{ N}} = \frac{125 \text{ mm}}{250 \text{ mm}} \quad A_y = 150 \text{ N} \uparrow$$



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PROBLEM 6.84 (Continued)

From Eq. (1): $A_y + E_y - 750 \text{ N} = 0$

$$150 \text{ N} + E_y - 750 \text{ N} = 0$$

$$E_y = 600 \text{ N} \quad \mathbf{E}_y = 600 \text{ N} \uparrow$$

Thus, reactions are: $\mathbf{A}_x = 300 \text{ N} \longleftarrow, \quad \mathbf{A}_y = 150.0 \text{ N} \uparrow$ ◀

$\mathbf{E}_x = 300 \text{ N} \longrightarrow, \quad \mathbf{E}_y = 600 \text{ N} \uparrow$ ◀

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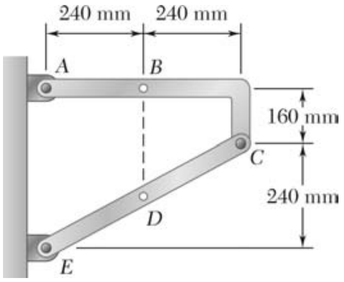


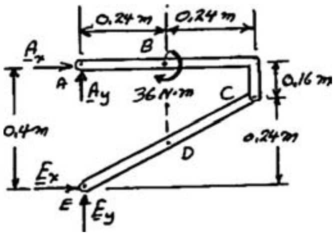
PROBLEM 6.85

Determine the components of the reactions at A and E if the frame is loaded by a clockwise couple of magnitude 36 N · m applied (a) at B, (b) at D.

SOLUTION

Free body: Entire frame

The following analysis is valid for both (a) and (b) since the point of application of the couple is immaterial.



$$+\circlearrowleft \Sigma M_E = 0: \quad -36\ \text{N}\cdot\text{m} - A_x(0.4\ \text{m}) = 0$$

$$A_x = -90\ \text{N} \qquad \mathbf{A}_x = 90.0\ \text{N} \longleftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -90 + E_x = 0$$

$$E_x = 90\ \text{N} \qquad \mathbf{E}_x = 90.0\ \text{N} \longrightarrow$$

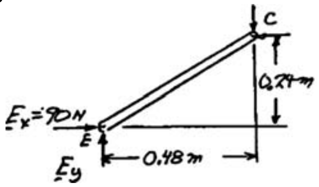
$$+\uparrow \Sigma F_y = 0: \quad A_y + E_y = 0 \qquad (1)$$

(a) Couple applied at B.

Free body: Member CE

AC is a two-force member. Thus, the reaction at E must be directed along EC.

$$\frac{E_y}{90\ \text{N}} = \frac{0.24\ \text{m}}{0.48\ \text{m}}; \quad \mathbf{E}_y = 45.0\ \text{N}\uparrow$$



From Eq. (1): $\quad A_y + 45\ \text{N} = 0$

$$A_y = -45\ \text{N} \qquad \mathbf{A}_y = 45.0\ \text{N}\downarrow$$

Thus, reactions are

$$\mathbf{A}_x = 90.0\ \text{N} \longleftarrow, \qquad \mathbf{A}_y = 45.0\ \text{N}\downarrow \blacktriangleleft$$

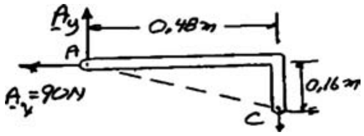
$$\mathbf{E}_x = 90.0\ \text{N} \longrightarrow, \qquad \mathbf{E}_y = 45.0\ \text{N}\uparrow \blacktriangleleft$$

(b) Couple applied at D.

Free body: Member AC

AC is a two-force member. Thus, the reaction at A must be directed along AC.

$$\frac{A_y}{90\ \text{N}} = \frac{0.16\ \text{m}}{0.48\ \text{m}}; \quad \mathbf{A}_y = 30\ \text{N}\uparrow$$



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PROBLEM 6.85 (Continued)

From Eq. (1):

$$A_y + E_y = 0$$

$$30\text{ N} + E_y = 0$$

$$E_y = -30\text{ N} \quad \mathbf{E}_y = 30\text{ N}\downarrow$$

Thus, reactions are:

$$\mathbf{A}_x = 90.0\text{ N}\longleftarrow, \quad \mathbf{A}_y = 30.0\text{ N}\uparrow \blacktriangleleft$$

$$\mathbf{E}_x = -90.0\text{ N}\longrightarrow, \quad \mathbf{E}_y = 30.0\text{ N}\downarrow \blacktriangleleft$$

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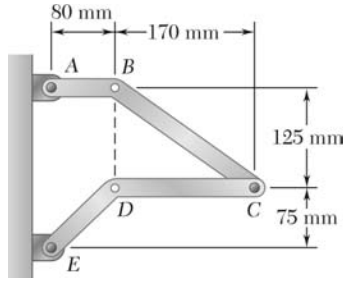


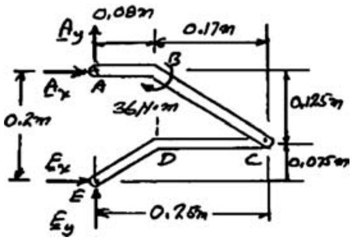
PROBLEM 6.86

Determine the components of the reactions at A and E if the frame is loaded by a clockwise couple of magnitude 36 N · m applied (a) at B, (b) at D.

SOLUTION

Free body: Entire frame

The following analysis is valid for both (a) and (b) since the point of application of the couple is immaterial.



$$+\circlearrowleft \Sigma M_E = 0: \quad -36\ \text{N}\cdot\text{m} - A_x(0.2\ \text{m}) = 0$$

$$A_x = -180\ \text{N} \qquad \mathbf{A}_x = 180\ \text{N} \longleftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -180\ \text{N} + E_x = 0$$

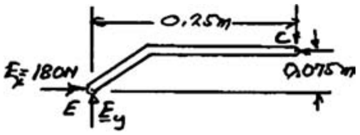
$$E_x = 180\ \text{N} \qquad \mathbf{E}_x = 180\ \text{N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + E_y = 0 \qquad (1)$$

(a) Couple applied at B.

Free body: Member CE

AC is a two-force member. Thus, the reaction at E must be directed along EC.



$$\frac{E_y}{180\ \text{N}} = \frac{0.075\ \text{m}}{0.25\ \text{m}} \qquad \mathbf{E}_y = 54\ \text{N}\uparrow$$

From Eq. (1): $\quad A_y + 54\ \text{N} = 0$

$$A_y = -54\ \text{N} \qquad \mathbf{A}_y = 54.0\ \text{N}\downarrow$$

Thus reactions are

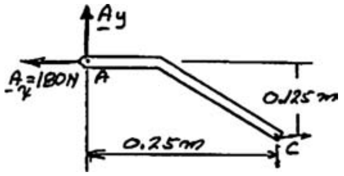
$$\mathbf{A}_x = 180.0\ \text{N} \longleftarrow, \qquad \mathbf{A}_y = 54.0\ \text{N}\downarrow \blacktriangleleft$$

$$\mathbf{E}_x = 180.0\ \text{N} \longrightarrow, \qquad \mathbf{E}_y = 54.0\ \text{N}\uparrow \blacktriangleleft$$

(b) Couple applied at D.

Free body: Member AC

AC is a two-force member. Thus, the reaction at A must be directed along EC.



$$\frac{A_y}{180\ \text{N}} = \frac{0.125\ \text{m}}{0.25\ \text{m}} \qquad A_y = 90\ \text{N}\uparrow$$

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PROBLEM 6.86 (Continued)

From Eq. (1): $A_y + E_y = 0$

$$90 \text{ N} + E_y = 0$$

$$E_y = -90 \text{ N} \quad \mathbf{E}_y = 90 \text{ N} \downarrow$$

Thus, reactions are

$$\mathbf{A}_x = 180.0 \text{ N} \longleftarrow, \quad \mathbf{A}_y = 90.0 \text{ N} \uparrow \blacktriangleleft$$

$$\mathbf{E}_x = -180.0 \text{ N} \longrightarrow, \quad \mathbf{E}_y = 90.0 \text{ N} \downarrow \blacktriangleleft$$

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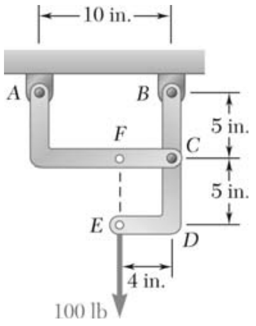
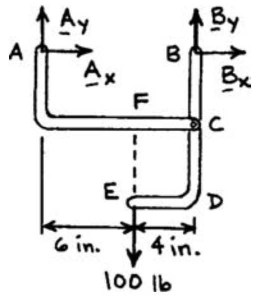


PROBLEM 6.87

Determine the components of the reactions at A and B, (a) if the 500-N load is applied as shown, (b) if the 500-N load is moved along its line of action and is applied at Point F.

SOLUTION

Free body: Entire frame



Analysis is valid for either (a) or (b), since position of 100-lb load on its line of action is immaterial.

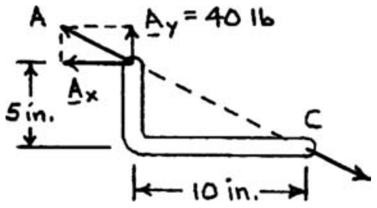
$$+\circlearrowleft \Sigma M_A = 0: \quad B_y(10) - (100 \text{ lb})(6) = 0 \quad B_y = +60 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 60 - 100 = 0 \qquad A_y = +40 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + B_x = 0 \qquad (1)$$

(a) Load applied at E.

Free body: Member AC



Since AC is a two-force member, the reaction at A must be directed along CA. We have

$$\frac{A_x}{10 \text{ in.}} = \frac{40 \text{ lb}}{5 \text{ in.}} \qquad \mathbf{A}_x = 80.0 \text{ lb} \longleftarrow, \quad \mathbf{A}_y = 40.0 \text{ lb} \uparrow \blacktriangleleft$$

From Eq. (1): $\quad -80 + B_x = 0 \quad B_x = +80 \text{ lb}$

Thus, $\qquad \mathbf{B}_x = 80.0 \text{ lb} \longrightarrow, \quad \mathbf{B}_y = 60.0 \text{ lb} \uparrow \blacktriangleleft$

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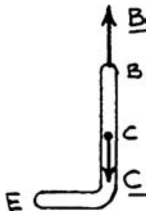
PROBLEM 6.87 (Continued)



(*b*) Load applied at *F*.

Free body: Member *BCD*

Since *BCD* is a two-force member (with forces applied at *B* and *C* only), the reaction at *B* must be directed along *CB*. We have therefore

$$B_x = 0$$

The reaction at *B* is $\mathbf{B}_x = 0$ $\qquad \mathbf{B}_y = 60.0 \text{ lb} \uparrow$ ◀

From Eq. (1): $A_x + 0 = 0 \quad A_x = 0$

The reaction at *A* is $\mathbf{A}_x = 0$ $\qquad \mathbf{A}_y = 40.0 \text{ lb} \uparrow$ ◀

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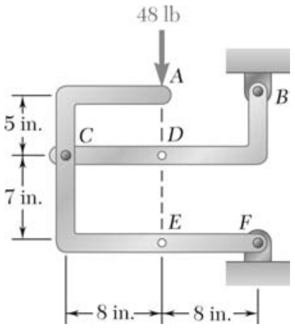
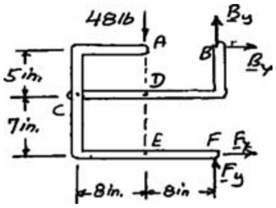


PROBLEM 6.88

The 48-lb load can be moved along the line of action shown and applied at A, D, or E. Determine the components of the reactions at B and F if the 48-lb load is applied (a) at A, (b) at D, (c) at E.

SOLUTION

Free body: Entire frame



The following analysis is valid for (a), (b) and (c) since the position of the load along its line of action is immaterial.

$$+\circlearrowleft \Sigma M_F = 0: \quad (48\,\text{lb})(8\text{ in.}) - B_x(12\text{ in.}) = 0$$

$$B_x = 32\text{ lb} \quad \mathbf{B}_x = 32\text{ lb} \longrightarrow$$

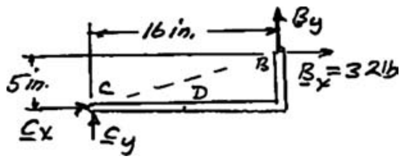
$$\xrightarrow{+} \Sigma F_x = 0: \quad 32\text{ lb} + F_x = 0$$

$$F_x = -32\text{ lb} \quad \mathbf{F}_x = 32\text{ lb} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + F_y - 48\text{ lb} = 0 \qquad (1)$$

(a) Load applied at A.

Free body: Member CDB



CDB is a two-force member. Thus, the reaction at B must be directed along BC.

$$\frac{B_y}{32\text{ lb}} = \frac{5\text{ in.}}{16\text{ in.}} \qquad \mathbf{B}_y = 10\text{ lb}\uparrow$$

From Eq. (1): $10\text{ lb} + F_y - 48\text{ lb} = 0$

$$F_y = 38\text{ lb} \quad \mathbf{F}_y = 38\text{ lb}\uparrow$$

Thus reactions are:

$$\mathbf{B}_x = 32.0\text{ lb} \longrightarrow, \qquad \mathbf{B}_y = 10.00\text{ lb}\uparrow \blacktriangleleft$$

$$\mathbf{F}_x = 32.0\text{ lb} \longleftarrow, \qquad \mathbf{F}_y = 38.0\text{ lb}\uparrow \blacktriangleleft$$

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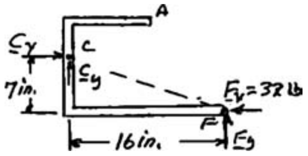
PROBLEM 6.88 (Continued)

(*b*) Load applied at *D*.

Free body: Member *ACF*.



ACF is a two-force member. Thus, the reaction at *F* must be directed along *CF*.

$$\frac{F_y}{32\text{ lb}} = \frac{7\text{ in.}}{16\text{ in.}} \qquad \mathbf{F}_y = 14\text{ lb}\uparrow$$

From Eq. (1): $B_y + 14\text{ lb} - 48\text{ lb} = 0$

$$B_y = 34\text{ lb} \quad B_y = 34\text{ lb}\uparrow$$

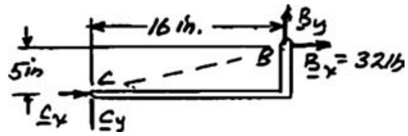
Thus, reactions are:

$\mathbf{B}_x = 32.0\text{ lb}\leftarrow$, $\mathbf{B}_y = 34.0\text{ lb}\uparrow$ ◀

$\mathbf{F}_x = 32.0\text{ lb}\rightarrow$, $\mathbf{F}_y = 14.00\text{ lb}\uparrow$ ◀

(*c*) Load applied at *E*.

Free body: Member *CDB*



This is the same free body as in Part (*a*).

Reactions are same as (*a*) ◀

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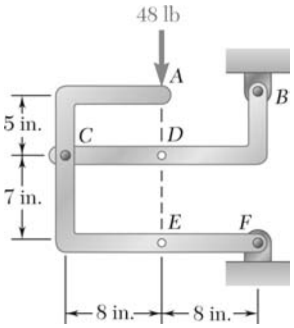


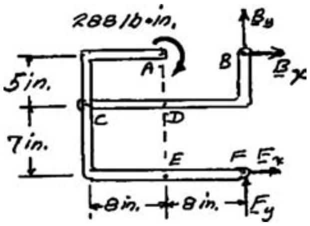
PROBLEM 6.89

The 48-lb load is removed and a 288-lb · in. clockwise couple is applied successively at A, D, and E. Determine the components of the reactions at B and F if the couple is applied (a) at A, (b) at D, (c) at E.

SOLUTION

Free body: Entire frame

The following analysis is valid for (a), (b), and (c), since the point of application of the couple is immaterial.



$$+\circlearrowleft \Sigma M_F = 0: \quad -288 \text{ lb}\cdot\text{in.} - B_x(12 \text{ in.}) = 0$$

$$B_x = -24 \text{ lb} \quad \mathbf{B}_x = 24 \text{ lb} \longleftarrow$$

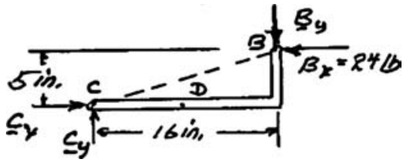
$$\xrightarrow{+} \Sigma F_x = 0: \quad -24 \text{ lb} + F_x = 0$$

$$F_x = 24 \text{ lb} \quad \mathbf{F}_x = 24 \text{ lb} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + F_y = 0 \qquad (1)$$

(a) Couple applied at A.

Free body: Member CDB



CDB is a two-force member. Thus, reaction at B must be directed along BC.

$$\frac{B_y}{24 \text{ lb}} = \frac{5 \text{ in.}}{16 \text{ in.}} \qquad \mathbf{B}_y = 7.5 \text{ lb} \downarrow$$

From Eq. (1): $\quad -7.5 \text{ lb} + F_y = 0$

$$F_y = 7.5 \text{ lb} \quad \mathbf{F}_y = 7.5 \text{ lb} \uparrow$$

Thus, reactions are:

$$\mathbf{B}_x = 24.0 \text{ lb} \longleftarrow, \qquad \mathbf{B}_y = 7.50 \text{ lb} \downarrow \blacktriangleleft$$

$$\mathbf{F}_x = 24.0 \text{ lb} \longrightarrow, \qquad \mathbf{F}_y = 7.50 \text{ lb} \uparrow \blacktriangleleft$$

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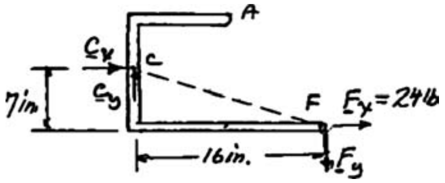
PROBLEM 6.89 (Continued)

(*b*) Couple applied at *D*.

Free body: Member *ACF*.



ACF is a two-force member. Thus, the reaction at *F* must be directed along *CF*.

$$\frac{F_y}{24\text{ lb}} = \frac{7\text{ in.}}{16\text{ in.}} \qquad \mathbf{F}_y = 10.5\text{ lb}\downarrow$$

From Eq. (1): $B_y - 10.5\text{ lb}$:

$$B_y = +10.5\text{ lb} \quad \mathbf{B}_y = 10.5\text{ lb}\uparrow$$

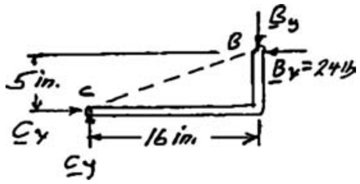
Thus, reactions are:

$\mathbf{B}_x = 24.0\text{ lb}\longleftarrow, \qquad \mathbf{B}_y = 10.50\text{ lb}\uparrow$ ◀

$\mathbf{F}_x = 24.0\text{ lb}\longrightarrow, \qquad \mathbf{F}_y = 10.50\text{ lb}\downarrow$ ◀

(*c*) Couple applied at *E*.

Free body: Member *CDB*



This is the same free body as in Part (*a*).

Reactions are same as in (*a*) ◀

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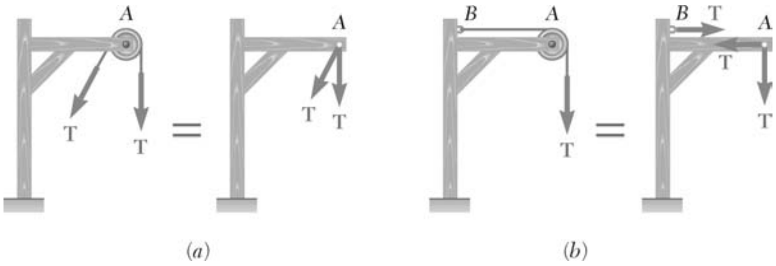
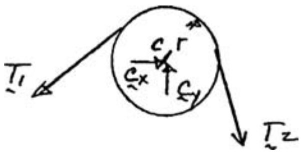
PROBLEM 6.90

(*a*) Show that when a frame supports a pulley at *A*, an equivalent loading of the frame and of each of its component parts can be obtained by removing the pulley and applying at *A* two forces equal and parallel to the forces that the cable exerted on the pulley. (*b*) Show that if one end of the cable is attached to the frame at a point *B*, a force of magnitude equal to the tension in the cable should also be applied at *B*.



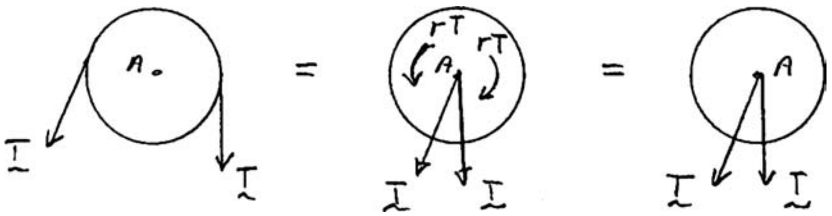
SOLUTION

First note that, when a cable or cord passes over a *frictionless, motionless* pulley, the tension is unchanged.

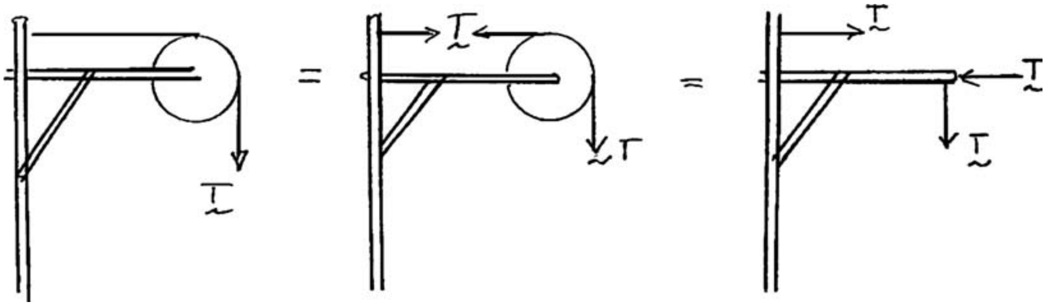


$$\circlearrowleft \Sigma M_C = 0: \quad rT_1 - rT_2 = 0 \quad T_1 = T_2$$

(*a*) Replace each force with an equivalent force-couple.



(*b*) Cut cable and replace forces on pulley with equivalent pair of forces at *A* as above.



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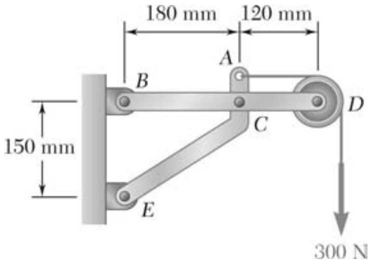
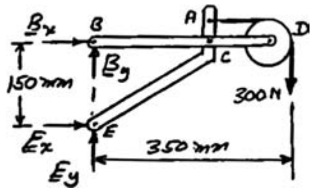


PROBLEM 6.91

Knowing that the pulley has a radius of 50 mm, determine the components of the reactions at B and E.

SOLUTION

Free body: Entire assembly



$$+\circlearrowleft \Sigma M_E = 0: \quad -(300\text{ N})(350\text{ mm}) - B_x(150\text{ mm}) = 0$$

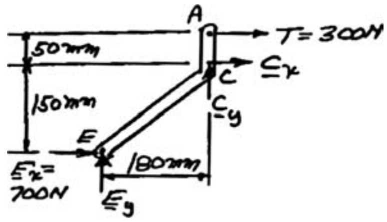
$$B_x = -700\text{ N} \qquad \mathbf{B}_x = 700\text{ N} \longleftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -700N + E_x = 0$$

$$E_x = 700\text{ N} \qquad \mathbf{E}_x = 700\text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + E_y - 300\text{ N} = 0 \qquad (1)$$

Free body: Member *ACE*



$$+\circlearrowleft \Sigma M_C = 0: \quad (700\text{ N})(150\text{ mm}) - (300\text{ N})(50\text{ mm}) - E_y(180\text{ mm}) = 0$$

$$E_y = 500\text{ N} \qquad \mathbf{E}_y = 500\text{ N}\uparrow$$

From Eq. (1): $\quad B_y + 500\text{ N} - 300\text{ N} = 0$

$$B_y = -200\text{ N} \qquad \mathbf{B}_y = 200\text{ N}\downarrow$$

Thus, reactions are:

$$\mathbf{B}_x = 700\text{ N} \longleftarrow, \qquad \mathbf{B}_y = 200\text{ N}\downarrow \blacktriangleleft$$

$$\mathbf{E}_x = 700\text{ N} \longrightarrow, \qquad \mathbf{E}_y = 500\text{ N}\uparrow \blacktriangleleft$$

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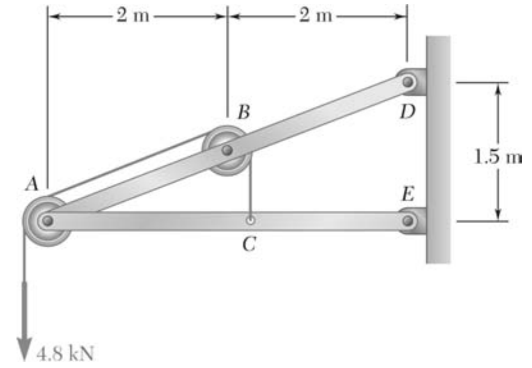
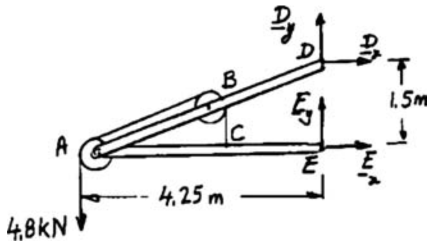


PROBLEM 6.92

Knowing that each pulley has a radius of 250 mm, determine the components of the reactions at D and E.

SOLUTION

Free body: Entire assembly



$$+\circlearrowleft \Sigma M_E = 0: \quad (4.8 \text{ kN})(4.25 \text{ m}) - D_x(1.5 \text{ m}) = 0$$

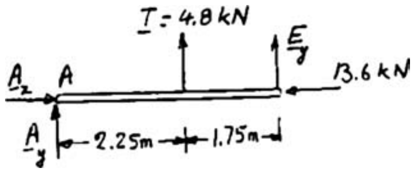
$$D_x = +13.60 \text{ kN} \qquad \mathbf{D}_x = 13.60 \text{ kN} \longrightarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad E_x + 13.60 \text{ kN} = 0$$

$$E_x = -13.60 \text{ kN} \qquad \mathbf{E}_x = 13.60 \text{ kN} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad D_y + E_y - 4.8 \text{ kN} = 0 \qquad (1)$$

Free body: Member ACE



$$+\circlearrowleft \Sigma M_A = 0: \quad (4.8 \text{ kN})(2.25 \text{ m}) + E_y(4 \text{ m}) = 0$$

$$E_y = -2.70 \text{ kN} \qquad \mathbf{E}_y = 2.70 \text{ kN} \downarrow \blacktriangleleft$$

From Eq. (1):

$$D_y - 2.70 \text{ kN} - 4.80 \text{ kN} = 0$$

$$D_y = +7.50 \text{ kN} \qquad \mathbf{D}_y = 7.50 \text{ kN} \uparrow \blacktriangleleft$$

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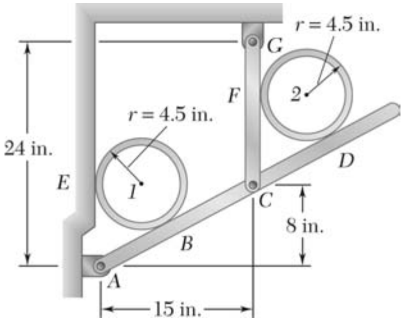
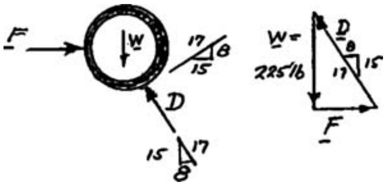


PROBLEM 6.93

Two 9-in.-diameter pipes (pipe 1 and pipe 2) are supported every 7.5 ft by a small frame like that shown. Knowing that the combined weight of each pipe and its contents is 30 lb/ft and assuming frictionless surfaces, determine the components of the reactions at *A* and *G*.

SOLUTION

Free-body: Pipe 2



$$W = (30 \text{ lb/ft})(7.5 \text{ ft}) = 225 \text{ lb}$$

$$\frac{F}{8} = \frac{D}{17} = \frac{225 \text{ lb}}{15}$$

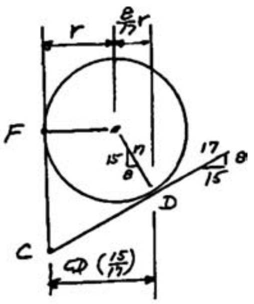
$$\mathbf{F} = 120 \text{ lb} \longrightarrow$$

$$\mathbf{D} = 255 \text{ lb} \nwarrow$$

Geometry of pipe 2 $\qquad r = 4.5 \text{ in.}$

By symmetry: $\qquad CF = CD \qquad (1)$

Equate horizontal distance:



$$r + \frac{8}{17} r = CD\left(\frac{15}{17}\right)$$

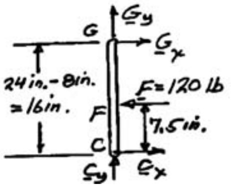
$$\frac{25}{17} r = CD\left(\frac{15}{17}\right)$$

$$CD = \frac{25}{15} r = \frac{5}{3} r$$

From Eq. (1): $\qquad CF = \frac{5}{3} r = \frac{5}{3}(4.5 \text{ in.})$

$$CF = 7.5 \text{ in.}$$

Free-body: Member *CFG*



$$+\circlearrowleft \Sigma M_C = 0: \quad (120 \text{ lb})(7.5 \text{ in.}) - G_x(16 \text{ in.}) = 0$$

$$G_x = 56.25 \text{ lb} \qquad \mathbf{G}_x = 56.3 \text{ lb} \longrightarrow \blacktriangleleft$$

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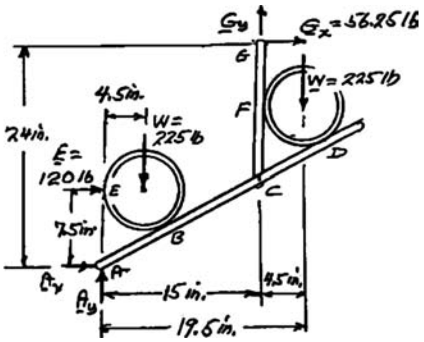
PROBLEM 6.93 (Continued)

Free body: Frame and pipes



Note: Pipe 2 is similar to pipe 1.

$$AE = CF = 7.5 \text{ in.}$$

$$\mathbf{E} = \mathbf{F} = 120 \text{ lb}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad G_y(15 \text{ in.}) - (56.25 \text{ lb})(24 \text{ in.}) - (225 \text{ lb})(4.5 \text{ in.})$$

$$-(225 \text{ lb})(19.5 \text{ in.}) - (120 \text{ lb})(7.5 \text{ in.}) = 0$$

$$G_y = 510 \text{ lb} \qquad \mathbf{G}_y = 510 \text{ lb} \uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 120 \text{ lb} + 56.25 \text{ lb} = 0$$

$$A_x = 176.25 \text{ lb} \qquad \mathbf{A}_x = 176.3 \text{ lb} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 510 \text{ lb} - 225 \text{ lb} - 225 \text{ lb} = 0$$

$$A_y = -60 \text{ lb} \qquad \mathbf{A}_y = 60.0 \text{ lb} \downarrow \blacktriangleleft$$

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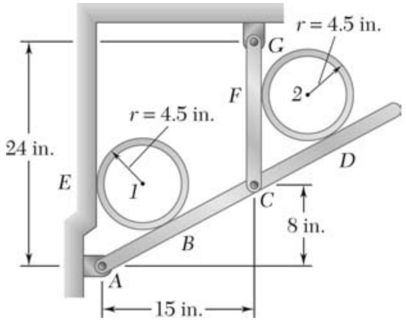


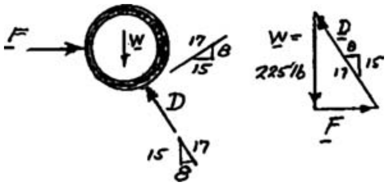
PROBLEM 6.94

Solve Problem 6.93 assuming that pipe 1 is removed and that only pipe 2 is supported by the frames.

PROBLEM 6.93 Two 9-in.-diameter pipes (pipe 1 and pipe 2) are supported every 7.5 ft by a small frame like that shown. Knowing that the combined weight of each pipe and its contents is 30 lb/ft and assuming frictionless surfaces, determine the components of the reactions at *A* and *G*.

SOLUTION

Free-body: Pipe 2 $\quad W = (30 \text{ lb/ft})(7.5 \text{ ft}) = 225 \text{ lb}$



$$\frac{F}{8} = \frac{D}{17} = \frac{225 \text{ lb}}{15}$$

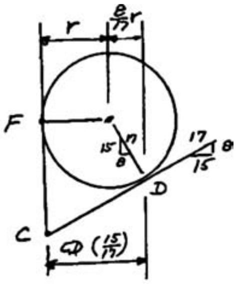
$$\mathbf{F} = 120 \text{ lb} \longrightarrow$$

$$\mathbf{D} = 255 \text{ lb} \nwarrow$$

Geometry of pipe 2 $\quad r = 4.5 \text{ in.}$

By symmetry: $\quad CF = CD \quad (1)$

Equate horizontal distance:



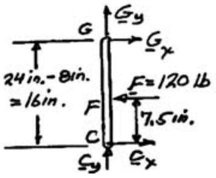
$$r + \frac{8}{17}r = CD\left(\frac{15}{17}\right)$$

$$\frac{25}{17}r = CD\left(\frac{15}{17}\right)$$

$$CD = \frac{25}{15}r = \frac{5}{3}r$$

From Eq. (1): $\quad CF = \frac{5}{3}r = \frac{5}{3}(4.5 \text{ in.})$

$$CF = 7.5 \text{ in.}$$



Free body: Member *CFG*

$$+\circlearrowleft \Sigma M_C = 0: \quad (120 \text{ lb})(7.5 \text{ in.}) - G_x(16 \text{ in.}) = 0$$

$$G_x = 56.25 \text{ lb} \qquad \mathbf{G}_x = 56.3 \text{ lb} \longrightarrow \blacktriangleleft$$

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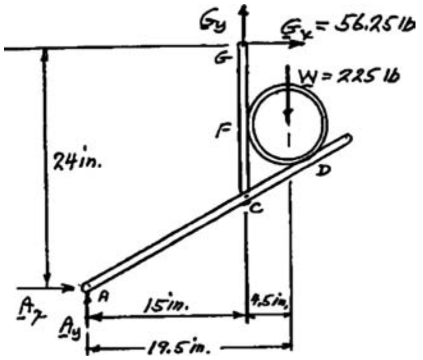
PROBLEM 6.94 (Continued)

Free body: Frame and pipe 2

$\mathbf{G}_x = 56.3 \text{ lb} \longrightarrow$ ◀



$+\circlearrowleft \Sigma M_A = 0: \quad G_y(15 \text{ in.}) - (56.25 \text{ lb})(24 \text{ in.}) - (225 \text{ lb})(19.5 \text{ in.}) = 0$

$G_y = 382.5 \text{ lb} \qquad \mathbf{G}_y = 383 \text{ lb} \uparrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 56.25 \text{ lb}$

$A_x = -56.25 \text{ lb} \qquad \mathbf{A}_x = 56.3 \text{ lb} \longleftarrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad A_y + 382.5 \text{ lb} - 225 \text{ lb} = 0$

$A_y = -157.5 \text{ lb} \qquad \mathbf{A}_y = 157.5 \downarrow$ ◀

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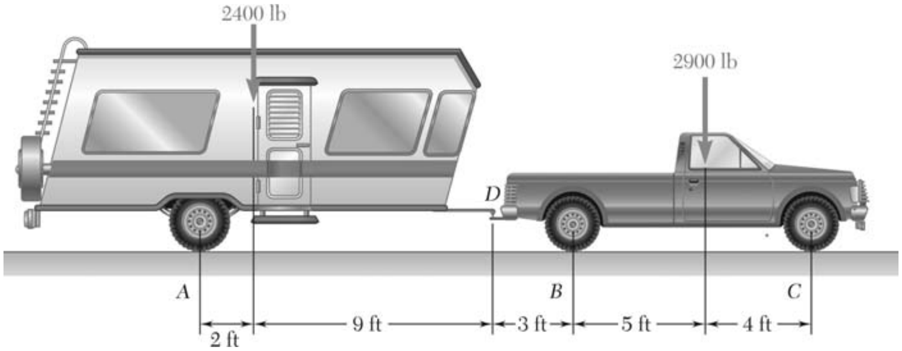
PROBLEM 6.95

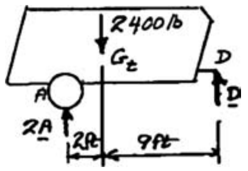
A trailer weighing 2400 lb is attached to a 2900-lb pickup truck by a ball-and-socket truck hitch at *D*. Determine (*a*) the reactions at each of the six wheels when the truck and trailer are at rest, (*b*) the additional load on each of the truck wheels due to the trailer.



SOLUTION

(*a*) Free body: Trailer

(We shall denote by *A*, *B*, *C* the reaction at one wheel)



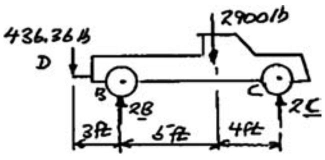
$$+\circlearrowleft \Sigma M_A = 0: \quad -(2400 \text{ lb})(2 \text{ ft}) + D(11 \text{ ft}) = 0$$

$$D = 436.36 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad 2A - 2400 \text{ lb} + 436.36 \text{ lb} = 0$$

$$A = 981.82 \text{ lb} \qquad \mathbf{A} = 982 \text{ lb} \uparrow \blacktriangleleft$$

Free body: Truck



$$+\circlearrowleft \Sigma M_B = 0: \quad (436.36 \text{ lb})(3 \text{ ft}) - (2900 \text{ lb})(5 \text{ ft}) + 2C(9 \text{ ft}) = 0$$

$$C = 732.83 \text{ lb} \qquad \mathbf{C} = 733 \text{ lb} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2B - 436.36 \text{ lb} - 2900 \text{ lb} + 2(732.83 \text{ lb}) = 0$$

$$B = 935.35 \text{ lb} \qquad \mathbf{B} = 935 \text{ lb} \uparrow \blacktriangleleft$$

(*b*) Additional load on truck wheels

Use free body diagram of truck without 2900 lb.

$$+\circlearrowleft \Sigma M_B = 0: \quad (436.36 \text{ lb})(3 \text{ ft}) + 2C(9 \text{ ft}) = 0$$

$$C = -72.73 \text{ lb} \qquad \Delta C = -72.7 \text{ lb} \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2B - 436.36 \text{ lb} - 2(72.73 \text{ lb}) = 0$$

$$B = 290.9 \text{ lb} \qquad \Delta B = +291 \text{ lb} \blacktriangleleft$$

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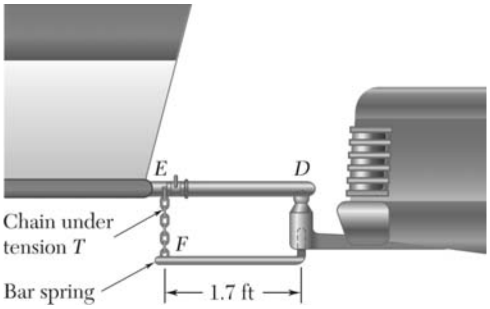


PROBLEM 6.96

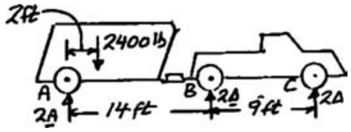
In order to obtain a better weight distribution over the four wheels of the pickup truck of Problem 6.95, a compensating hitch of the type shown is used to attach the trailer to the truck. The hitch consists of two bar springs (only one is shown in the figure) that fit into bearings inside a support rigidly attached to the truck. The springs are also connected by chains to the trailer frame, and specially designed hooks make it possible to place both chains in tension. (*a*) Determine the tension *T* required in each of the two chains if the additional load due to the trailer is to be evenly distributed over the four wheels of the truck. (*b*) What are the resulting reactions at each of the six wheels of the trailer-truck combination?

PROBLEM 6.95 A trailer weighing 2400 lb is attached to a 2900-lb pickup truck by a ball-and-socket truck hitch at *D*. Determine (*a*) the reactions at each of the six wheels when the truck and trailer are at rest, (*b*) the additional load on each of the truck wheels due to the trailer.

SOLUTION

(*a*) We small first find the additional reaction Δ at each wheel due the trailer.

Free body diagram (Same Δ at each truck wheel)



$$+\circlearrowleft \Sigma M_A = 0: \quad -(2400 \text{ lb})(2 \text{ ft}) + 2\Delta(14 \text{ ft}) + 2\Delta(23 \text{ ft}) = 0$$

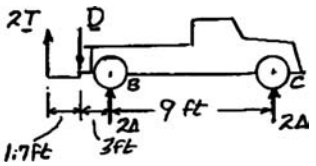
$$\Delta = 64.86 \text{ lb}$$

$$+\uparrow \Sigma F_Y = 0: \quad 2A - 2400 \text{ lb} + 4(64.86 \text{ lb}) = 0;$$

$$A = 1070 \text{ lb}; \qquad \mathbf{A} = 1070 \text{ lb} \uparrow$$

Free body: Truck

(Trailer loading only)



$$+\circlearrowleft \Sigma M_D = 0: \quad 2\Delta(12 \text{ ft}) + 2\Delta(3 \text{ ft}) - 2T(1.7 \text{ ft}) = 0$$

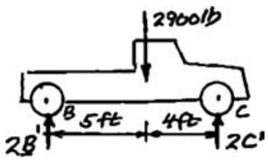
$$T = 8.824\Delta$$

$$= 8.824(64.86 \text{ lb})$$

$$T = 572.3 \text{ lb} \qquad T = 572 \text{ lb} \blacktriangleleft$$

Free body: Truck

(Truck weight only)



$$+\circlearrowleft \Sigma M_B = 0: \quad -(2900 \text{ lb})(5 \text{ ft}) + 2C'(9 \text{ ft}) = 0$$

$$C' = 805.6 \text{ lb} \qquad \mathbf{C}' = 805.6 \text{ lb} \uparrow$$

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PROBLEM 6.96 (Continued)

$$+\uparrow \Sigma F_y = 0: \quad 2B' - 2900 \text{ lb} + 2(805.6 \text{ lb}) = 0$$

$$B' = 644.4 \text{ lb} \qquad \mathbf{B}' = 644.4 \text{ lb} \uparrow$$

Actual reactions

$$B = B' + \Delta = 644.4 \text{ lb} + 64.86 = 709.2 \text{ lb} \qquad \mathbf{B} = 709 \text{ lb} \uparrow \blacktriangleleft$$

$$C = C' + \Delta = 805.6 \text{ lb} + 64.86 = 870.46 \text{ lb} \qquad \mathbf{C} = 870 \text{ lb} \uparrow \blacktriangleleft$$

(From Part *a*): $\mathbf{A} = 1070 \text{ lb} \uparrow \blacktriangleleft$

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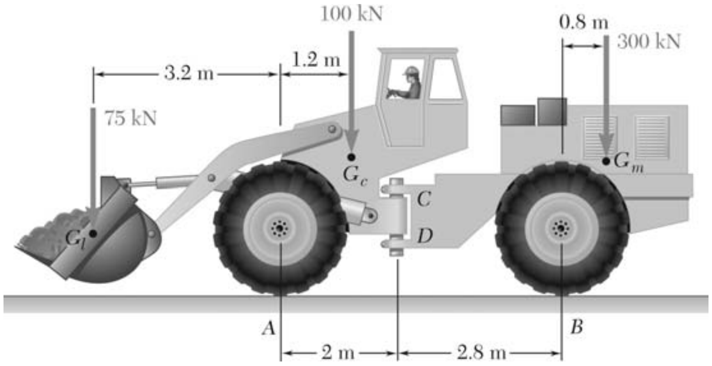
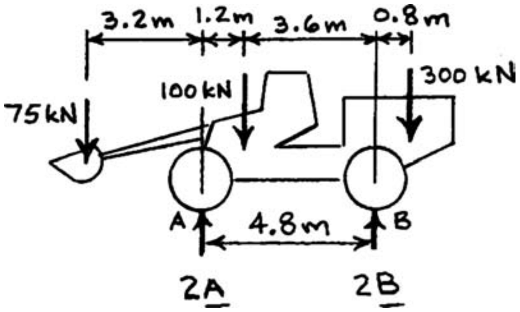
PROBLEM 6.97

The cab and motor units of the front-end loader shown are connected by a vertical pin located 2 m behind the cab wheels. The distance from C to D is 1 m. The center of gravity of the 300-kN motor unit is located at G_m, while the centers of gravity of the 100-kN cab and 75-kN load are located, respectively, at G_c and G_l. Knowing that the machine is at rest with its brakes released, determine (*a*) the reactions at each of the four wheels, (*b*) the forces exerted on the motor unit at C and D.



SOLUTION

(*a*) Free body: Entire machine



$\mathbf{A} =$ Reaction at each front wheel

$\mathbf{B} =$ Reaction at each rear wheel

$$+\circlearrowleft \Sigma M_A = 0: \quad 75(3.2\text{ m}) - 100(1.2\text{ m}) + 2B(4.8\text{ m}) - 300(5.6\text{ m}) = 0$$

$$2B = 325\text{ kN} \qquad \mathbf{B} = 162.5\text{ kN}\uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2A + 325 - 75 - 100 - 300 = 0$$

$$2A = 150\text{ kN} \qquad \mathbf{A} = 75.0\text{ kN}\uparrow \blacktriangleleft$$

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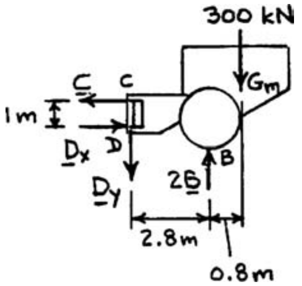
PROBLEM 6.97 (Continued)

(*b*) Free body: Motor unit

$$+\circlearrowright \Sigma M_D = 0: \quad C(1\text{ m}) + 2B(2.8\text{ m}) - 300(3.6\text{ m}) = 0$$

$$C = 1080 - 5.6B \qquad (1)$$



Recalling $\quad B = 162.5\text{ kN}, \quad C = 1080 - 5.6(162.5) = 170\text{ kN}$

$\mathbf{C} = 170.0\text{ kN} \longleftarrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad D_x - 170 = 0$ $\qquad \mathbf{D}_x = 170.0\text{ kN} \longrightarrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad 2(162.5) - D_y - 300 = 0$ $\qquad \mathbf{D}_y = 25.0\text{ kN} \downarrow$ ◀

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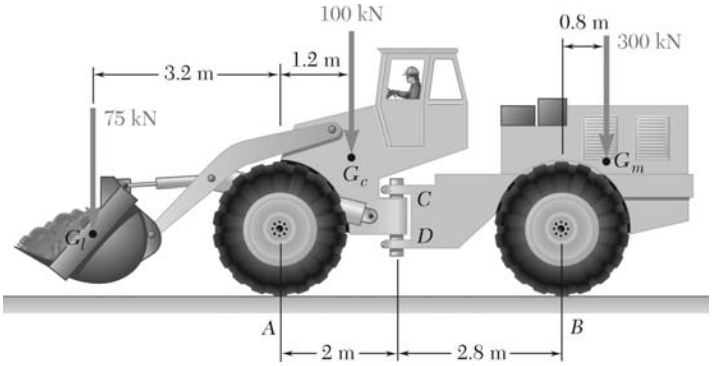
PROBLEM 6.98

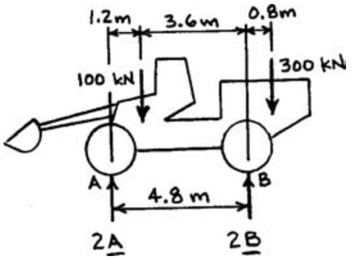
Solve Problem 6.97 assuming that the 75-kN load has been removed.

PROBLEM 6.97 The cab and motor units of the front-end loader shown are connected by a vertical pin located 2 m behind the cab wheels. The distance from C to D is 1 m. The center of gravity of the 300-kN motor unit is located at G_m, while the centers of gravity of the 100-kN cab and 75-kN load are located, respectively, at G_c and G_l. Knowing that the machine is at rest with its brakes released, determine (*a*) the reactions at each of the four wheels, (*b*) the forces exerted on the motor unit at C and D.



SOLUTION

(*a*) Free body: Entire machine



$\mathbf{A}$ = Reaction at each front wheel

$\mathbf{B}$ = Reaction at each rear wheel

$$+\circlearrowleft \Sigma M_A = 0: \quad 2B(4.8 \text{ m}) - 100(1.2 \text{ m}) - 300(5.6 \text{ m}) = 0$$

$$2B = 375 \text{ kN} \qquad \mathbf{B} = 187.5 \text{ kN}\uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2A + 375 - 100 - 300 = 0$$

$$2A = 25 \text{ kN} \qquad \mathbf{A} = 12.50 \text{ kN}\uparrow \blacktriangleleft$$

(*b*) Free body: Motor unit

See solution of Problem 6.97 for free body diagram and derivation of Eq. (1). With $B = 187.5$ kN, we have

$$C = 1080 - 5.6(187.5) = 30 \text{ kN}$$

$$\mathbf{C} = 30.0 \text{ kN} \longleftarrow \blacktriangleleft$$

$$\overset{+}{\longrightarrow} \Sigma F_x = 0: \quad D_x - 30 = 0 \qquad \mathbf{D}_x = 30.0 \text{ kN} \longrightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2(187.5) - D_y - 300 = 0 \qquad \mathbf{D}_y = 75.0 \text{ kN}\downarrow \blacktriangleleft$$

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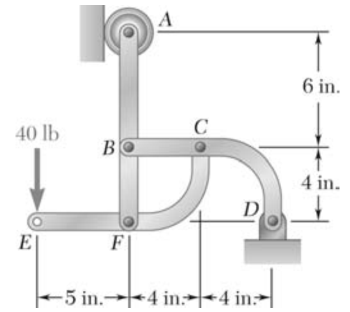
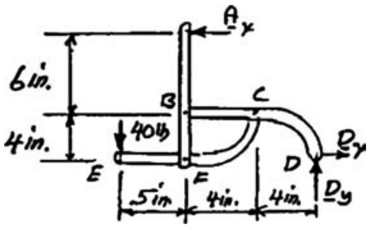


PROBLEM 6.99

For the frame and loading shown, determine the components of the forces acting on member *CFE* at *C* and *F*.

SOLUTION

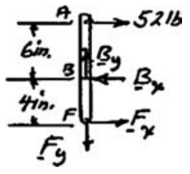
<u>Free body: Entire frame</u>



$$+\circlearrowleft \Sigma M_D = 0: \quad (40\text{ lb})(13\text{ in.}) + A_x(10\text{ in.}) = 0$$

$$A_x = -52\text{ lb}, \qquad \mathbf{A}_x = 52\text{ lb} \longrightarrow$$

<u>Free body: Member *ABF*</u>



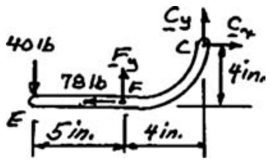
$$+\circlearrowleft \Sigma M_B = 0: \quad -(52\text{ lb})(6\text{ in.}) + F_x(4\text{ in.}) = 0$$

$$F_x = +78\text{ lb}$$

<u>Free body: Member *CFE*</u>

From above:

$$\mathbf{F}_x = 78.0\text{ lb} \longleftarrow \blacktriangleleft$$



$$+\circlearrowleft \Sigma M_C = 0: \quad (40\text{ lb})(9\text{ in.}) - (78\text{ lb})(4\text{ in.}) - F_y(4\text{ in.}) = 0$$

$$F_y = +12\text{ lb} \qquad \mathbf{F}_y = 12.00\text{ lb} \uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 78\text{ lb} = 0$$

$$C_x = +78\text{ lb} \qquad \mathbf{C}_x = 78.0\text{ lb} \longrightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -40\text{ lb} + 12\text{ lb} + C_y = 0; \quad C_y = +28\text{ lb} \qquad \mathbf{C}_y = 28.0\text{ lb} \uparrow \blacktriangleleft$$

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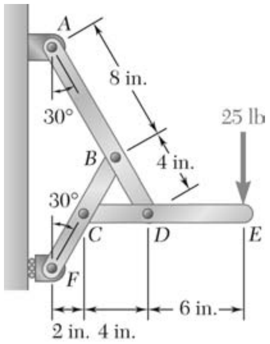
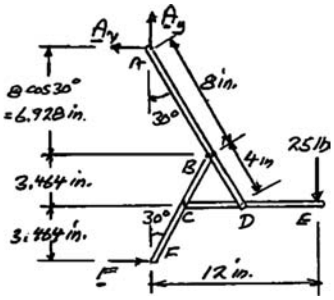


PROBLEM 6.100

For the frame and loading shown, determine the components of the forces acting on member *CDE* at *C* and *D*.

SOLUTION

Free body: Entire frame



$+\uparrow \Sigma M_y = 0: \quad A_y - 25 \text{ lb} = 0$

$A_y = 25 \text{ lb} \qquad \mathbf{A}_y = 25 \text{ lb} \uparrow$

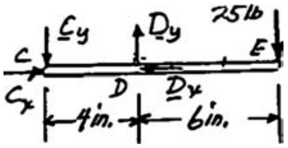
$+\circlearrowleft \Sigma M_F = 0: \quad A_x(6.928 + 2 \times 3.464) - (25 \text{ lb})(12 \text{ in.}) = 0$

$A_x = 21.651 \text{ lb} \qquad \mathbf{A}_y = 21.65 \text{ lb} \longleftarrow$

$\xrightarrow{+} \Sigma F_x = 0: \quad F - 21.651 \text{ lb} = 0$

$F = 21.651 \text{ lb} \qquad \mathbf{F} = 21.65 \text{ lb} \longrightarrow$

Free body: Member *CDE*



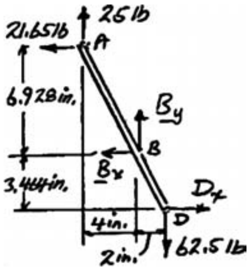
$+\circlearrowleft \Sigma M_C = 0: \quad D_y(4 \text{ in.}) - (25 \text{ lb})(10 \text{ in.}) = 0$

$D_y = +62.5 \text{ lb} \qquad \mathbf{D}_y = 62.5 \text{ lb} \uparrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad -C_y + 62.5 \text{ lb} - 25 \text{ lb} = 0$

$C_y = +37.5 \text{ lb} \qquad \mathbf{C}_y = 37.5 \text{ lb} \downarrow$ ◀

Free body: Member *ABD*



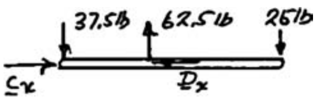
$+\circlearrowleft \Sigma M_B = 0: \quad D_x(3.464 \text{ in.}) + (21.65 \text{ lb})(6.928 \text{ in.})$

$-(25 \text{ lb})(4 \text{ in.}) - (62.5 \text{ lb})(2 \text{ in.})$

$D_x = +21.65 \text{ lb}$

Return to free body: Member *CDE*

From above



$D_x = +21.65 \text{ lb} \qquad \mathbf{D}_x = 21.7 \text{ lb} \longleftarrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 21.65 \text{ lb}$

$C_x = +21.65 \text{ lb} \qquad \mathbf{C}_x = 21.7 \text{ lb} \longrightarrow$ ◀

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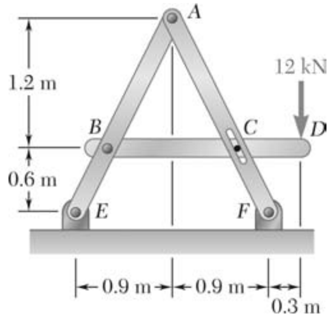
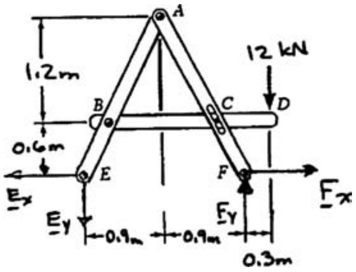


PROBLEM 6.101

For the frame and loading shown, determine the components of all forces acting on member *ABE*.

SOLUTION

FBD Frame:



$$\circlearrowleft \Sigma M_E = 0: \quad (1.8\text{ m})F_y - (2.1\text{ m})(12\text{ kN}) = 0$$

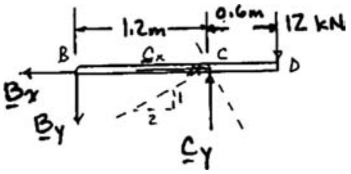
$$\mathbf{F}_y = 14.00\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: \quad -E_y + 14.00\text{ kN} - 12\text{ kN} = 0$$

$$E_y = 2\text{ kN} \downarrow$$

$$\mathbf{E}_y = 2.00\text{ kN} \downarrow \blacktriangleleft$$

FBD member *BCD*:



$$\circlearrowleft \Sigma M_B = 0: \quad (1.2\text{ m})C_y - (12\text{ kN})(1.8\text{ m}) = 0 \quad C_y = 18.00\text{ kN} \uparrow$$

But **C** is $\perp$ *ACF*, so $\quad C_x = 2C_y; \quad C_x = 36.0\text{ kN} \longrightarrow$

$$\longrightarrow \Sigma F_x = 0: \quad -B_x + C_x = 0 \quad B_x = C_x = 36.0\text{ kN}$$

$$B_x = 36.0\text{ kN} \longleftarrow \text{ on } BCD$$

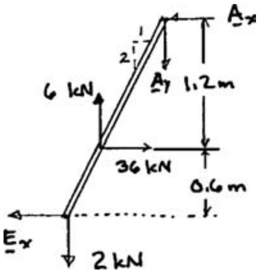
$$\uparrow \Sigma F_y = 0: \quad -B_y + 18.00\text{ kN} - 12\text{ kN} = 0 \quad B_y = 6.00\text{ kN} \downarrow \text{ on } BCD$$

On *ABE*:

$$\mathbf{B}_x = 36.0\text{ kN} \longrightarrow \blacktriangleleft$$

$$\mathbf{B}_y = 6.00\text{ kN} \uparrow \blacktriangleleft$$

FBD member *ABE*:



$$\circlearrowleft \Sigma M_A = 0: \quad (1.2\text{ m})(36.0\text{ kN}) - (0.6\text{ m})(6.00\text{ kN}) + (0.9\text{ m})(2.00\text{ kN}) - (1.8\text{ m})(E_x) = 0$$

$$\mathbf{E}_x = 23.0\text{ kN} \longleftarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: \quad -23.0\text{ kN} + 36.0\text{ kN} - A_x = 0 \qquad \mathbf{A}_x = 13.00\text{ kN} \longleftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad -2.00\text{ kN} + 6.00\text{ kN} - A_y = 0 \qquad \mathbf{A}_y = 4.00\text{ kN} \downarrow \blacktriangleleft$$

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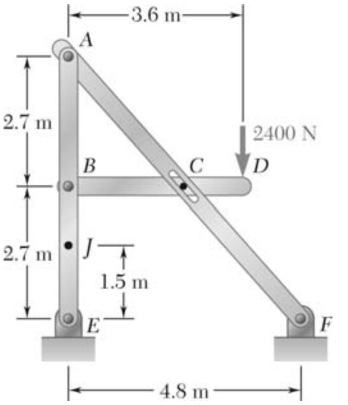


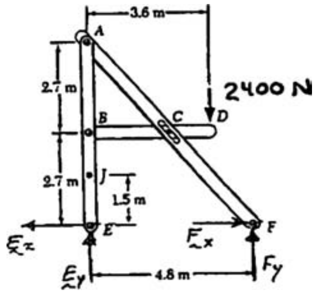
PROBLEM 6.102

For the frame and loading shown, determine the components of all forces acting on member *ABE*.

PROBLEM 6.101 For the frame and loading shown, determine the components of all forces acting on member *ABE*.

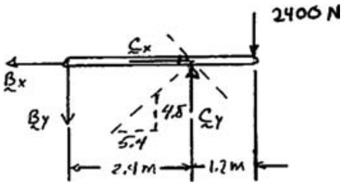
SOLUTION

FBD Frame:



$\circlearrowleft \Sigma M_F = 0: \quad (1.2 \text{ m})(2400 \text{ N}) - (4.8 \text{ m})E_y = 0$ $\qquad \mathbf{E}_y = 600 \text{ N}\uparrow$ ◀

FBD member *BC*:



$$C_y = \frac{4.8}{5.4}C_x = \frac{8}{9}C_x$$

$\circlearrowleft \Sigma M_C = 0: \quad (2.4 \text{ m})B_y - (1.2 \text{ m})(2400 \text{ N}) = 0 \quad B_y = 1200 \text{ N}\downarrow$

on *ABE*: $\qquad \mathbf{B}_y = 1200 \text{ N}\uparrow$ ◀

$\uparrow \Sigma F_y = 0: \quad -1200 \text{ N} + C_y - 2400 \text{ N} = 0 \quad C_y = 3600 \text{ N}\uparrow$

so $\qquad C_x = \frac{9}{8}C_y \quad C_x = 4050 \text{ N} \longrightarrow$

$\longrightarrow \Sigma F_x = 0: \quad -B_x + C_x = 0 \quad B_x = 4050 \text{ N} \longleftarrow$ on *BC*

on *ABE*: $\qquad \mathbf{B}_x = 4050 \text{ N} \longrightarrow$ ◀

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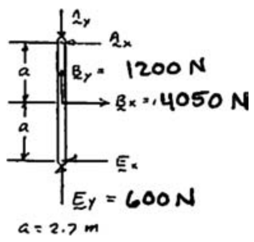
PROBLEM 6.102 (Continued)

FBD member *AB0E*:



$$\circlearrowleft \Sigma M_A = 0: \quad a(4050 \text{ N}) - 2aE_x = 0$$

$$E_x = 2025 \text{ N} \qquad \mathbf{E}_x = 2025 \text{ N} \longleftarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: \quad -A_x + (4050 - 2025) \text{ N} = 0 \qquad \mathbf{A}_x = 2025 \text{ N} \longleftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad 600 \text{ N} + 1200 \text{ N} - A_y = 0 \qquad \mathbf{A}_y = 1800 \text{ N} \downarrow \blacktriangleleft$$

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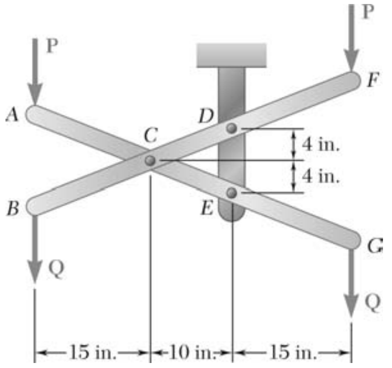
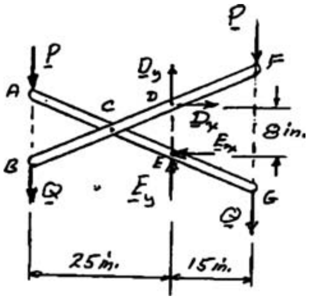


PROBLEM 6.103

Knowing that $P = 15$ lb and $Q = 65$ lb, determine the components of the forces exerted (*a*) on member *BCDF* at *C* and *D*, (*b*) on member *ACEG* at *E*.

SOLUTION

<u>Free body: Entire frame</u>



$$+\circlearrowleft \Sigma M_E = 0: \quad (P+Q)(25\text{ in.}) - (P+Q)(15\text{ in.}) - D_x(8\text{ in.}) = 0$$

$$D_x = (P+Q)\frac{10}{8}$$

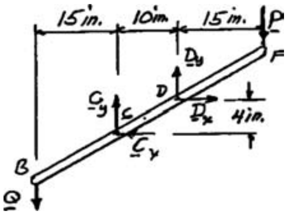
$$\mathbf{D}_x = (P+Q)\frac{10}{8} \longrightarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -E_x + (P+Q)\frac{10}{8} = 0$$

$$E_x = (P+Q)\frac{10}{8}; \quad \mathbf{E}_x = (P+Q)\frac{10}{8} \longleftarrow$$

<u>Free body: Member *BCDF*</u>

From above:



$$\mathbf{D}_x = (P+Q)\frac{10}{8} \longrightarrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -C_x + D_x = 0 \qquad \mathbf{C}_x = (P+Q)\frac{10}{8} \longleftarrow \triangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -(P+Q)\frac{10}{8}(4\text{ in.}) - P(15\text{ in.}) + Q(25\text{ in.}) + C_y(10\text{ in.})$$

$$C_y = \frac{1}{10}(-20P + 20Q)$$

$$C_y = 2Q - 2P \qquad \mathbf{C}_y = 2Q - 2P \uparrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad D_y + (2Q - 2P) = P - Q = 0$$

$$D_y = -Q + 3P \qquad \mathbf{D}_y = -Q + 3P \uparrow \triangleleft$$

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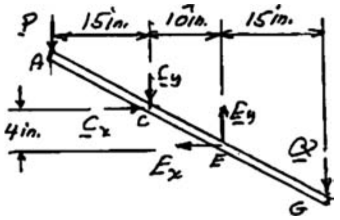
PROBLEM 6.103 (Continued)

Free body: Member *ACEG*



From above

$$\mathbf{E}_x = (P+Q)\frac{10}{8} \longleftarrow \triangleleft$$

and $C_y = 2Q - P$

$$+\uparrow \Sigma F_y = 0: \quad E_y - C_y - P - Q = 0$$

$$E_y - (2Q - 2P) - P - Q = 0$$

$$E_y = 3Q - P \qquad \mathbf{E}_y = 3Q - P \uparrow \triangleleft$$

$$P = 15 \text{ lb} \quad \text{and} \quad Q = 65 \text{ lb}$$

$$C_x = (P+Q)\frac{10}{8} = (15+65)\frac{10}{8} = +100 \text{ lb} \qquad \mathbf{C}_x = 100.0 \text{ lb} \longleftarrow \blacktriangleleft$$

$$C_y = 2Q - 2P = 2(65) - 2(15) = +100 \text{ lb} \qquad \mathbf{C}_y = 100.0 \text{ lb} \uparrow \blacktriangleleft$$

$$D_x = (P+Q)\frac{10}{8} = (15+65)\frac{10}{8} = +100 \text{ lb} \qquad \mathbf{D}_x = 100.0 \text{ lb} \longrightarrow \blacktriangleleft$$

$$D_y = -Q + 3P = -65 + 3(15) = -20 \text{ lb} \qquad \mathbf{D}_y = 20.0 \text{ lb} \downarrow \blacktriangleleft$$

$$E_x = (P+Q)\frac{10}{8} = (15+65)\frac{10}{8} = +100 \text{ lb} \qquad \mathbf{E}_x = 100.0 \text{ lb} \longleftarrow \blacktriangleleft$$

$$E_y = 3Q - P = 3(65) - 15 = +180 \text{ lb} \qquad \mathbf{E}_y = 180.0 \text{ lb} \uparrow \blacktriangleleft$$

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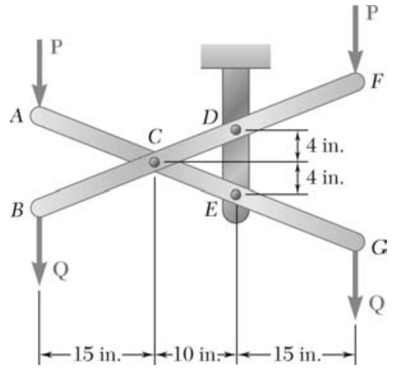


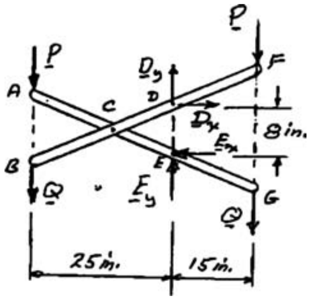
PROBLEM 6.104

Knowing that $P = 25$ lb and $Q = 55$ lb, determine the components of the forces exerted (*a*) on member *BCDF* at *C* and *D*, (*b*) on member *ACEG* at *E*.

SOLUTION

<u>Free body: Entire frame</u>

$$+\circlearrowleft \Sigma M_E = 0: \quad (P+Q)(25 \text{ in.}) - (P+Q)(15 \text{ in.}) - D_x(8 \text{ in.}) = 0$$



$$D_x = (P+Q)\frac{10}{8}$$

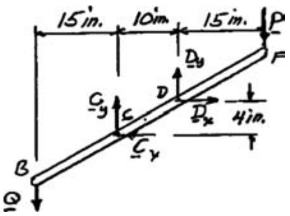
$$\mathbf{D}_x = (P+Q)\frac{10}{8} \longrightarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -E_x + (P+Q)\frac{10}{8} = 0$$

$$E_x = (P+Q)\frac{10}{8}; \quad \mathbf{E}_x = (P+Q)\frac{10}{8} \longleftarrow$$

<u>Free body: Member *BCDF*</u>

From above:



$$\mathbf{D}_x = (P+Q)\frac{10}{8} \longrightarrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -C_x + D_x = 0 \qquad \mathbf{C}_x = (P+Q)\frac{10}{8} \longleftarrow \triangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -(P+Q)\frac{10}{8}(4 \text{ in.}) - P(15 \text{ in.}) + Q(25 \text{ in.}) + C_y(10 \text{ in.})$$

$$C_y = \frac{1}{10}(-20P + 20Q)$$

$$C_y = 2Q - 2P \qquad \mathbf{C}_y = 2Q - 2P \uparrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad D_y + (2Q - 2P) - P - Q = 0$$

$$D_y = -Q + 3P \qquad \mathbf{D}_y = -Q + 3P \uparrow \triangleleft$$

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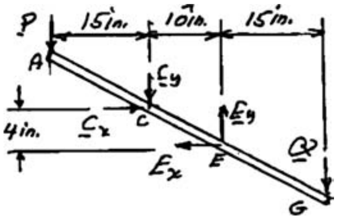
PROBLEM 6.104 (Continued)

Free body: Member *ACEG*



From above:

$$\mathbf{E}_x = (P+Q)\frac{10}{8} \longleftarrow \triangleleft$$

and $$C_y = 2Q - P$$

$$+\uparrow \Sigma F_y = 0: \quad E_y - C_y - P - Q = 0$$

$$E_y - (2Q - 2P) - P - Q = 0$$

$$E_y = 3Q - P \qquad \mathbf{E}_y = 3Q - P \uparrow \triangleleft$$

$$P = 25 \text{ lb} \quad \text{and} \quad Q = 55 \text{ lb}$$

$$C_x = (P+Q)\frac{10}{8} = (25+55)\frac{10}{8} = +100 \text{ lb} \qquad \mathbf{C}_x = 100.0 \longleftarrow \blacktriangleleft$$

$$C_y = 2Q - 2P = 2(55) - 2(25) = +60 \text{ lb} \qquad \mathbf{C}_y = 60.0 \text{ lb} \uparrow \blacktriangleleft$$

$$D_x = (P+Q)\frac{10}{8} = (25+55)\frac{10}{8} = +100 \text{ lb} \qquad \mathbf{D}_x = 100.0 \text{ lb} \longrightarrow \blacktriangleleft$$

$$D_y = -Q + 3P = -55 + 3(25) = +20 \text{ lb} \qquad \mathbf{D}_y = 20.0 \text{ lb} \uparrow \blacktriangleleft$$

$$E_x = (P+Q)\frac{10}{8} = (25+55)\frac{10}{8} = +100 \text{ lb} \qquad \mathbf{E}_x = 100.0 \text{ lb} \longleftarrow \blacktriangleleft$$

$$E_y = 3Q - P = 3(55) - 25 = +140 \text{ lb} \qquad \mathbf{E}_y = 140.0 \text{ lb} \uparrow \blacktriangleleft$$

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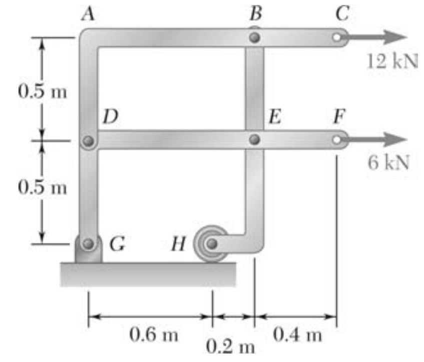
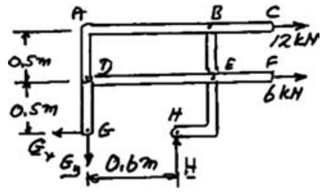


PROBLEM 6.105

For the frame and loading shown, determine the components of the forces acting on member *DABC* at *B* and *D*.

SOLUTION

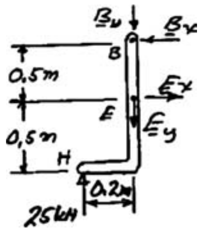
<u>Free body: Entire frame</u>



$$+\circlearrowleft \Sigma M_G = 0: \quad H(0.6 \text{ m}) - (12 \text{ kN})(1 \text{ m}) - (6 \text{ kN})(0.5 \text{ m}) = 0$$

$$H = 25 \text{ kN} \qquad \mathbf{H} = 25 \text{ kN} \uparrow$$

<u>Free body: Member *BEH*</u>

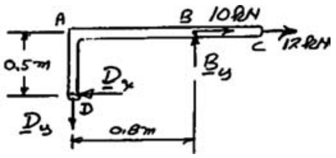


$$+\circlearrowleft \Sigma M_F = 0: \quad B_x(0.5 \text{ m}) - (25 \text{ kN})(0.2 \text{ m}) = 0$$

$$B_x = +10 \text{ kN}$$

<u>Free body: Member *DABC*</u>

From above: $\mathbf{B}_x = 10.00 \text{ kN} \longrightarrow$ ◀



$$+\circlearrowleft \Sigma M_D = 0: \quad -B_y(0.8 \text{ m}) + (10 \text{ kN} + 12 \text{ kN})(0.5 \text{ m}) = 0$$

$$B_y = +13.75 \text{ kN} \qquad \mathbf{B}_y = 13.75 \text{ kN} \uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -D_x + 10 \text{ kN} + 12 \text{ kN} = 0$$

$$D_x = +22 \text{ kN} \qquad \mathbf{D}_x = 22.0 \text{ kN} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -D_y + 13.75 \text{ kN} = 0$$

$$D_y = +13.75 \text{ kN} \qquad \mathbf{D}_y = 13.75 \text{ kN} \downarrow \blacktriangleleft$$

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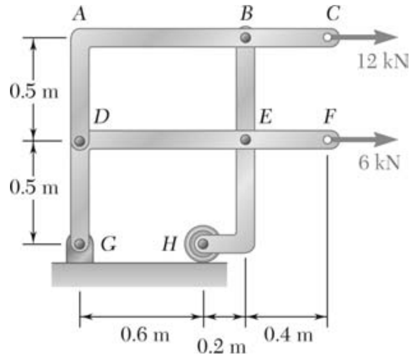


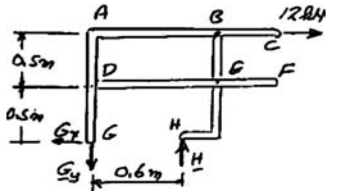
PROBLEM 6.106

Solve Problem 6.105 assuming that the 6-kN load has been removed.

PROBLEM 6.105 For the frame and loading shown, determine the components of the forces acting on member *DABC* at *B* and *D*.

SOLUTION

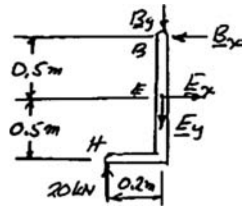
Free body: Entire frame



$$+\circlearrowleft \Sigma M_G = 0: \quad H(0.6\text{ m}) - (12\text{ kN})(1\text{ m}) = 0$$

$$H = 20\text{ kN} \qquad \mathbf{H} = 20\text{ kN}\uparrow$$

Free body: Member *BEH*

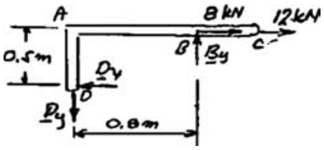


$$+\circlearrowleft \Sigma M_E = 0: \quad B_x(0.5\text{ m}) - (20\text{ kN})(0.2\text{ m}) = 0$$

$$B_x = +8\text{ kN}$$

Free body: Member *DABC*

From above: $\mathbf{B}_x = -8.00\text{ kN} \longrightarrow$ ◀



$$+\circlearrowleft \Sigma M_D = 0: \quad -B_y(0.8\text{ m}) + (8\text{ kN} + 12\text{ kN})(0.5\text{ m}) = 0$$

$$B_y = +12.5\text{ kN} \qquad \mathbf{B}_y = 12.50\text{ kN}\uparrow \text{ ◀}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -D_x + 8\text{ kN} + 12\text{ kN} = 0$$

$$D_x = +20\text{ kN} \qquad \mathbf{D}_x = 20.0\text{ kN} \longleftarrow \text{ ◀}$$

$$+\uparrow \Sigma F_y = 0: \quad -D_y + 12.5\text{ kN} = 0; \quad D_y = +12.5\text{ kN} \qquad \mathbf{D}_y = 12.50\text{ kN}\downarrow \text{ ◀}$$

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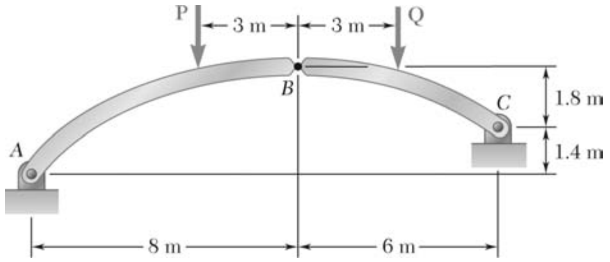
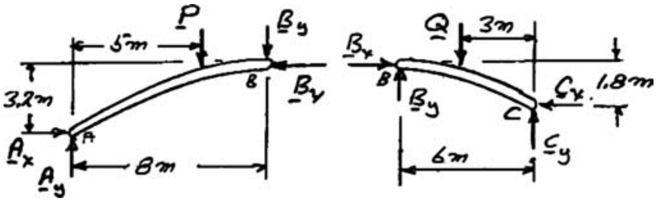


PROBLEM 6.107

The axis of the three-hinge arch ABC is a parabola with vertex at B. Knowing that P = 112 kN and Q = 140 kN, determine (*a*) the components of the reaction at A, (*b*) the components of the force exerted at B on segment AB.

SOLUTION



Free body: Segment AB:

$$+\circlearrowleft \Sigma M_A = 0: \quad B_x(3.2 \text{ m}) - B_y(8 \text{ m}) - P(5 \text{ m}) = 0 \qquad (1)$$

0.75 (Eq. 1)

$$B_x(2.4 \text{ m}) - B_y(6 \text{ m}) - P(3.75 \text{ m}) = 0 \qquad (2)$$

Free body: Segment BC:

$$+\circlearrowright \Sigma M_C = 0: \quad B_x(1.8 \text{ m}) + B_y(6 \text{ m}) - Q(3 \text{ m}) = 0 \qquad (3)$$

Add (2) and (3):

$$4.2B_x - 3.75P - 3Q = 0$$

$$B_x = (3.75P + 3Q)/4.2 \qquad (4)$$

Eq. (1):

$$(3.75P + 3Q)\frac{3.2}{4.2} - 8B_y - 5P = 0$$

$$B_y = (-9P + 9.6Q)/33.6 \qquad (5)$$

Given that $P = 112$ kN and $Q = 140$ kN

(*a*) Reaction at A:

Considering again AB as a free body

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B_x = 0; \quad A_x = B_x = 200 \text{ kN}$$

$\mathbf{A}_x = 200$ kN $\longrightarrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad A_y - P - B_y = 0$$

$$A_y - 112 \text{ kN} - 10 \text{ kN} = 0$$

$$A_y = +122 \text{ kN}$$

$\mathbf{A}_y = 122.0$ kN $\uparrow$ ◀

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PROBLEM 6.107 (Continued)

(*b*) Force exerted at *B* on *AB*

Eq. (4): $B_x = (3.75 \times 112 + 3 \times 140)/4.2 = 200 \text{ kN}$

$\mathbf{B}_x = 200 \text{ kN} \longleftarrow$ ◀

Eq. (5): $B_y = (-9 \times 112 + 9.6 \times 140)/33.6 = +10 \text{ kN}$

$\mathbf{B}_y = 10.00 \text{ kN} \downarrow$ ◀

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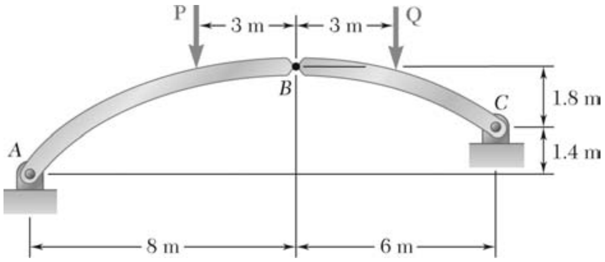
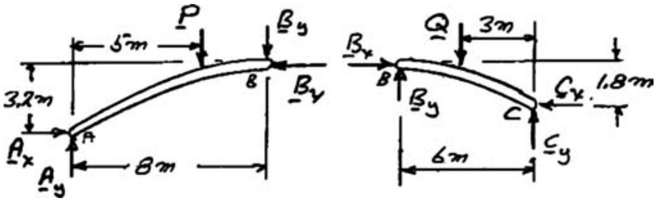


PROBLEM 6.108

The axis of the three-hinge arch ABC is a parabola with vertex at B. Knowing that P = 140 kN and Q = 112 kN, determine (a) the components of the reaction at A, (b) the components of the force exerted at B on segment AB.

SOLUTION



Free body: Segment *AB*:

$$+\circlearrowleft \Sigma M_A = 0: \quad B_x(3.2\text{ m}) - B_y(8\text{ m}) - P(5\text{ m}) = 0 \qquad (1)$$

0.75 (Eq. 1) $\quad B_x(2.4\text{ m}) - B_y(6\text{ m}) - P(3.75\text{ m}) = 0 \qquad (2)$

Free body: Segment *BC*:

$$+\circlearrowleft \Sigma M_C = 0: \quad B_x(1.8\text{ m}) + B_y(6\text{ m}) - Q(3\text{ m}) = 0 \qquad (3)$$

Add (2) and (3): $\quad 4.2B_x - 3.75P - 3Q = 0$

$$B_x = (3.75P + 3Q)/4.2 \qquad (4)$$

Eq. (1): $\quad (3.75P + 3Q)\dfrac{3.2}{4.2} - 8B_y - 5P = 0$

$$B_y = (-9P + 9.6Q)/33.6 \qquad (5)$$

Given that $P = 140\text{ kN}$ and $Q = 112\text{ kN}$

(a) Reaction at A:

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B_x = 0; \quad A_x = B_x = 205\text{ kN} \qquad \mathbf{A}_x = 205\text{ kN} \longrightarrow ◀$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - P - B_y = 0$$

$$A_y - 140\text{ kN} - (-5.5\text{ kN}) = 0$$

$$A_y = 134.5\text{ kN} \qquad \mathbf{A}_y = 134.5\text{ kN} \uparrow ◀$$

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PROBLEM 6.108 (Continued)

(*b*) Force exerted at *B* on *AB*

Eq. (4): $B_x = (3.75 \times 140 + 3 \times 112)/4.2 = 205 \text{ kN}$

$\mathbf{B}_x = 205 \text{ kN} \longleftarrow$ ◀

Eq. (5): $B_y = (-9 \times 140 + 9.6 \times 112)/33.6 = -5.5 \text{ kN}$

$\mathbf{B}_y = 5.5 \text{ kN} \uparrow$ ◀

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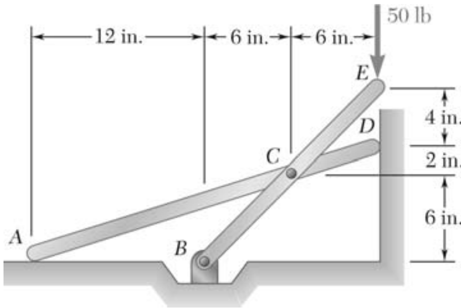
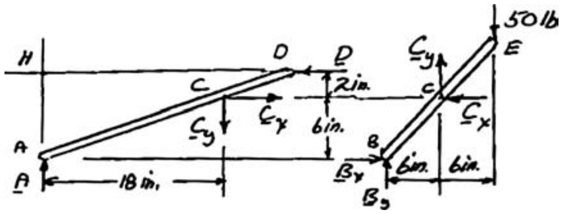


PROBLEM 6.109

Knowing that the surfaces at A and D are frictionless, determine the forces exerted at B and C on member BCE.

SOLUTION



<u>Free body of Member *ACD*</u>

$$+\circlearrowleft \Sigma M_H = 0: \quad C_x(2\text{ in.}) - C_y(18\text{ in.}) = 0 \quad C_x = 9C_y \qquad (1)$$

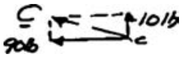
<u>Free body of Member *BCE*</u>

$$+\circlearrowleft \Sigma M_B = 0: \quad C_x(6\text{ in.}) + C_y(6\text{ in.}) - (50\text{ lb})(12\text{ in.}) = 0$$

Substitute from (1):

$$9C_y(6) + C_y(6) - 600 = 0$$

$$C_y = +10\text{ lb}; \quad C_x = 9C_y = 9(10) = +90\text{ lb}$$



$\mathbf{C} = 90.6\text{ lb} \nwarrow 6.34°$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x - 90\text{ lb} = 0 \qquad B_x = 90\text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + 10\text{ lb} - 50\text{ lb} = 0 \qquad B_y = 40\text{ lb}$$

$\mathbf{B} = 98.5\text{ lb} \nearrow 24.0°$ ◀

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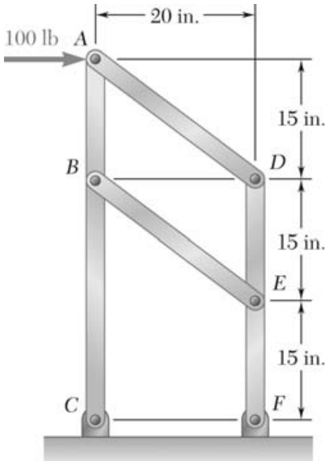
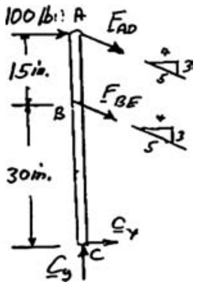


PROBLEM 6.110

For the frame and loading shown, determine (*a*) the reaction at *C*, (*b*) the force in member *AD*.

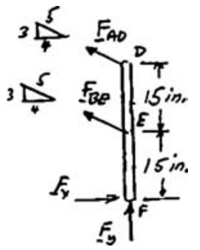
SOLUTION

Free body: Member *ABC*



$$+\circlearrowleft \Sigma M_C = 0: \quad +(100\text{ lb})(45\text{ in.}) + \frac{4}{5}F_{AD}(45\text{ in.}) + \frac{4}{5}F_{BE}(30\text{ in.}) = 0$$

$$3F_{AD} + 2F_{BF} = -375\text{ lb} \qquad (1)$$



Free Body: Member *DEF*

$$+\circlearrowleft \Sigma M_F = 0: \quad \frac{4}{5}F_{AD}(30\text{ in.}) + \frac{4}{5}F_{BF}(15\text{ in.}) = 0$$

$$F_{BE} = -2F_{AD} \qquad (2)$$

(*a*) Substitute from (2) into (1)

$$3F_{AD} + 2(-2F_{AD}) = -375\text{ lb}$$

$$F_{AD} = +375\text{ lb} \qquad F_{AD} = 375\text{ lb ten.} \blacktriangleleft$$

(2)

$$F_{BE} = -2F_{AD} = -2(375\text{ lb})$$

$$F_{BE} = -750\text{ lb} \quad F_{BE} = 750\text{ lb comp.}$$

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PROBLEM 6.110 (Continued)

(*b*) Return to free body of member *ABC*

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + 100 \text{ lb} + \frac{4}{5}F_{AD} + \frac{4}{5}F_{BE} = 0$$

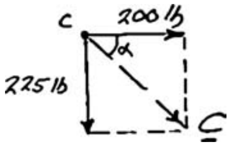
$$C_x + 100 + \frac{4}{5}(375) + \frac{4}{5}(-750) = 0$$

$$C_x = +200 \text{ lb} \quad \mathbf{C}_x = 200 \text{ lb} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - \frac{3}{5}F_{AD} - \frac{3}{5}F_{BF} = 0$$

$$C_y - \frac{3}{5}(375) - \frac{3}{5}(-750) = 0$$

$$C_y = -225 \text{ lb} \quad \mathbf{C}_y = 225 \text{ lb} \downarrow$$



$$\alpha = 48.37^\circ$$

$$C = 301.0 \text{ lb}$$

$$\mathbf{C} = 301 \text{ lb} \searrow 48.4^\circ \blacktriangleleft$$

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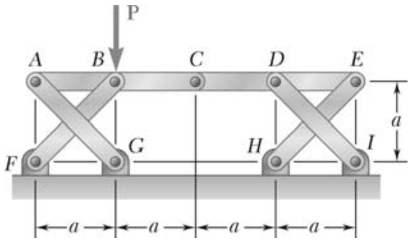
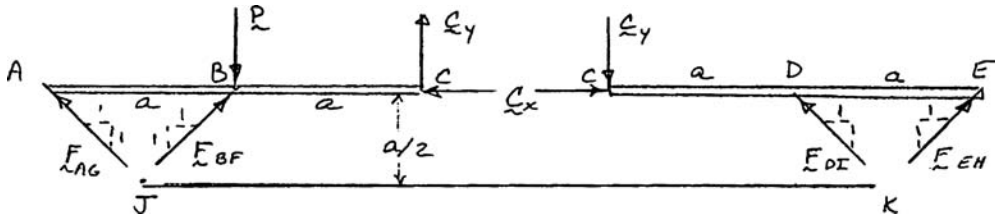


PROBLEM 6.111

Members *ABC* and *CDE* are pin-connected at *C* and supported by four links. For the loading shown, determine the force in each link.

SOLUTION

Member FBDs:



I II

From FBD I: $\circlearrowleft \Sigma M_J = 0: \quad \frac{a}{2}C_x + \frac{3a}{2}C_y - \frac{a}{2}P = 0 \quad C_x + 3C_y = P$

FBD II: $\circlearrowleft \Sigma M_K = 0: \quad \frac{a}{2}C_x - \frac{3a}{2}C_y = 0 \quad C_x - 3C_y = 0$

Solving: $C_x = \frac{P}{2}; \quad C_y = \frac{P}{6}$ as drawn

FBD I: $\circlearrowleft \Sigma M_B = 0: \quad aC_y - a\frac{1}{\sqrt{2}}F_{AG} = 0 \quad F_{AG} = \sqrt{2}C_y = \frac{\sqrt{2}}{6}P$ $\qquad F_{AG} = \frac{\sqrt{2}}{6}P \quad C$ ◀

$\rightarrow \Sigma F_x = 0: \quad -\frac{1}{\sqrt{2}}F_{AG} + \frac{1}{\sqrt{2}}F_{BF} - C_x = 0 \quad F_{BF} = F_{AG} + C_x\sqrt{2} = \frac{\sqrt{2}}{6}P + \frac{\sqrt{2}}{2}P$

$F_{BF} = \frac{2\sqrt{2}}{3}P \quad C$ ◀

FBD II: $\circlearrowleft \Sigma M_D = 0: \quad a\frac{1}{\sqrt{2}}F_{EH} + aC_y = 0 \quad F_{EH} = -\sqrt{2}C_y = -\frac{\sqrt{2}}{6}P$ $\qquad F_{EH} = \frac{\sqrt{2}}{6}P \quad T$ ◀

$\rightarrow \Sigma F_x = 0: \quad C_x - \frac{1}{\sqrt{2}}F_{DI} + \frac{1}{\sqrt{2}}F_{EH} = 0 \quad F_{DI} = F_{EH} + C_x\sqrt{2} = -\frac{\sqrt{2}}{6}P + \frac{\sqrt{2}}{2}P$

$F_{DI} = \frac{\sqrt{2}}{3}P \quad C$ ◀

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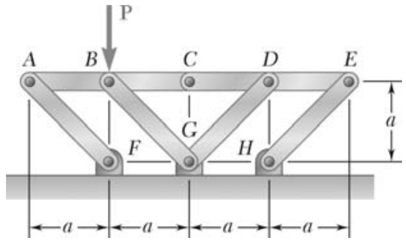
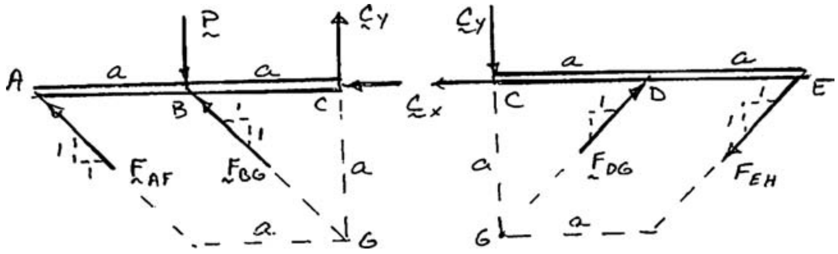


PROBLEM 6.112

Members *ABC* and *CDE* are pin-connected at *C* and supported by four links. For the loading shown, determine the force in each link.

SOLUTION

Member FBDs:



I **II**

FBD I: $\circlearrowleft \Sigma M_B = 0: \quad aC_y - a\dfrac{1}{\sqrt{2}}F_{AF} = 0 \quad F_{AF} = \sqrt{2}C_y$

FBD II: $\circlearrowleft M_D = 0: \quad aC_y - a\dfrac{1}{\sqrt{2}}F_{EH} = 0 \quad F_{EH} = \sqrt{2}C_y$

FBDs combined: $\circlearrowleft \Sigma M_G = 0: \quad aP - a\dfrac{1}{\sqrt{2}}F_{AF} - a\dfrac{1}{\sqrt{2}}F_{EH} = 0 \quad P = \dfrac{1}{\sqrt{2}}\sqrt{2}C_y + \dfrac{1}{\sqrt{2}}\sqrt{2}C_y$

$$C_y = \frac{P}{2} \qquad \text{so } F_{AF} = \frac{\sqrt{2}}{2}P \quad C \blacktriangleleft$$

$$F_{EH} = \frac{\sqrt{2}}{2}P \quad T \blacktriangleleft$$

FBD I: $\uparrow \Sigma F_y = 0: \quad \dfrac{1}{\sqrt{2}}F_{AF} + \dfrac{1}{\sqrt{2}}F_{BG} - P + C_y = 0 \quad \dfrac{P}{2} + \dfrac{1}{\sqrt{2}}F_{BG} - P + \dfrac{P}{2} = 0$

$$F_{BG} = 0 \blacktriangleleft$$

FBD II: $\uparrow \Sigma F_y = 0: \quad -C_y + \dfrac{1}{\sqrt{2}}F_{DG} - \dfrac{1}{\sqrt{2}}F_{EH} = 0 \quad -\dfrac{P}{2} + \dfrac{1}{\sqrt{2}}F_{DG} - \dfrac{P}{2} = 0$

$$F_{DG} = \sqrt{2}\,P \quad C \blacktriangleleft$$

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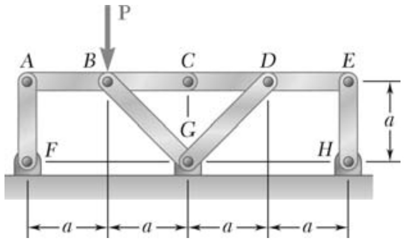
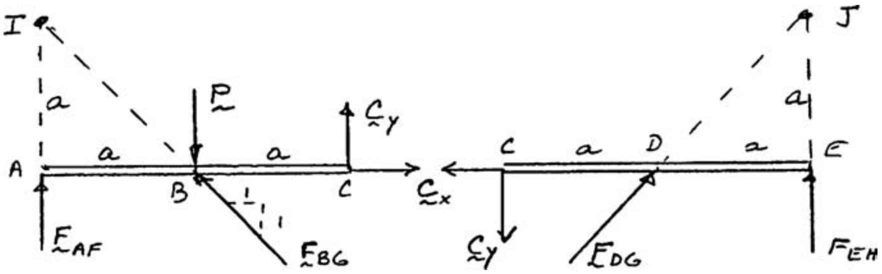


PROBLEM 6.113

Members *ABC* and *CDE* are pin-connected at *C* and supported by four links. For the loading shown, determine the force in each link.

SOLUTION

Member FBDs:



I **II**

FBD I: $\circlearrowleft \Sigma M_I = 0{:}\quad 2aC_y + aC_x - aP = 0 \quad 2C_y + C_x = P$

FBD II: $\circlearrowleft \Sigma M_J = 0{:}\quad 2aC_y - aC_x = 0 \quad 2C_y - C_x = 0$

Solving: $C_x = \dfrac{P}{2};\quad C_y = \dfrac{P}{4}$ as shown

FBD I: $\rightarrow \Sigma F_x = 0{:}\quad -\dfrac{1}{\sqrt{2}}F_{BG} + C_x = 0 \quad F_{BG} = C_x\sqrt{2}$ $\qquad F_{BG} = \dfrac{P}{\sqrt{2}}\ C$ ◀

$\uparrow \Sigma F_y = 0{:}\quad F_{AF} - P + \dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{2}P\right) + \dfrac{P}{4} = 0$ $\qquad F_{AF} = \dfrac{P}{4}\ C$ ◀

FBD II: $\rightarrow \Sigma F_x = 0{:}\quad -C_x + \dfrac{1}{\sqrt{2}}F_{DG} = 0 \quad F_{DG} = C_x\sqrt{2}$ $\qquad F_{DG} = \dfrac{P}{\sqrt{2}}\ C$ ◀

$\uparrow \Sigma F_y = 0{:}\quad -C_y + \dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{2}P\right) + F_{EH} = 0 \quad F_{EH} = \dfrac{P}{4} - \dfrac{P}{2} = -\dfrac{P}{4}$ $\qquad F_{EH} = \dfrac{P}{4}\ T$ ◀

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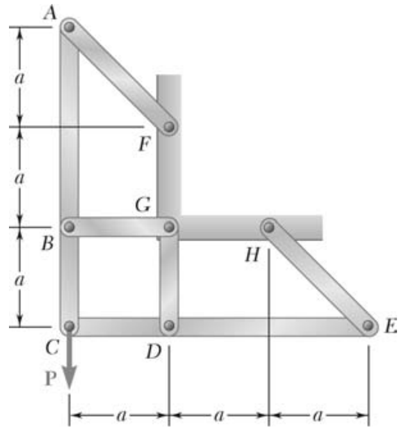
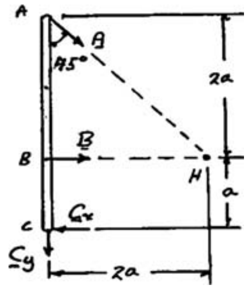


PROBLEM 6.114

Members *ABC* and *CDE* are pin-connected at *C* and supported by the four links *AF*, *BG*, *DG*, and *EH*. For the loading shown, determine the force in each link.

SOLUTION

<u>Free body: Member *ABC*</u>



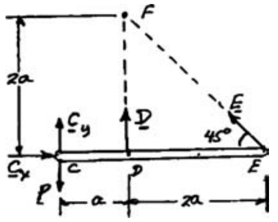
$$+\circlearrowleft \Sigma M_H = 0: \quad C_x(a) - C_y(2a) = 0$$

$$C_x = 2C_y$$

Note: This checks that for 3-force member the forces are concurrent.

<u>Free body: Member *CDE*</u>

$$+\circlearrowleft \Sigma M_F = 0: \quad C_x(2a) - C_y(a) + P(a) = 0$$



$$2C_x - C_y + P = 0$$

$$2(2C_y) - C_y + P = 0 \qquad C_y = -\frac{1}{3}P \ \triangleleft$$

$$C_x = 2C_y: \qquad C_x = -\frac{2}{3}P \ \triangleleft$$

$$\xrightarrow{+} \Sigma F = 0: \quad C_x - \frac{E}{\sqrt{2}} = 0; \quad -\frac{2}{3}P - \frac{E}{\sqrt{2}} = 0$$

$$E = -\frac{2\sqrt{2}}{3}P \qquad F_{EH} = \frac{2\sqrt{2}}{3}P \ \text{ comp.} \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_E = 0: \quad D(2a) + C_y(3a) - P(3a) = 0$$

$$D(2a) - \frac{P}{3}(3a) - P(3a) = 0$$

$$D = +2P \qquad F_{DG} = 2P \quad T \ \blacktriangleleft$$

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PROBLEM 6.114 (Continued)

Return to free body of *ABC*

$$+\downarrow \Sigma F_y = 0: \quad \frac{A}{\sqrt{2}} + C_y = 0$$

$$\frac{A}{\sqrt{2}} - \frac{P}{3} = 0$$

$$A = +\frac{\sqrt{2}}{3}P \qquad F_{AF} = \frac{\sqrt{2}}{3}P \quad T \blacktriangleleft$$

$$+\circlearrowright \Sigma M_A = 0: \quad B(2a) - C_x(3a) = 0$$

$$B(2a) + \frac{2}{3}P(3a) = 0$$

$$B = -P \qquad F_{BG} = P \quad C \blacktriangleleft$$

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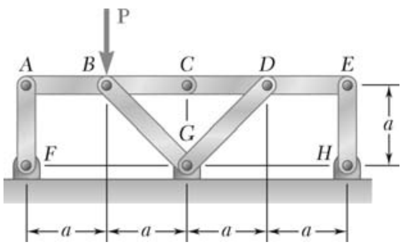


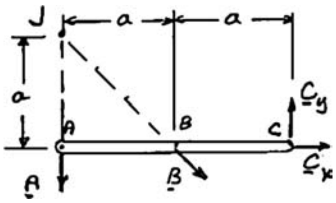
PROBLEM 6.115

Solve Problem 6.113 assuming that the force **P** is replaced by a clockwise couple of moment $\mathbf{M}_0$ applied to member *CDE* at *D*.

PROBLEM 6.113 Members *ABC* and *CDE* are pin-connected at *C* and supported by four links. For the loading shown, determine the force in each link.

SOLUTION

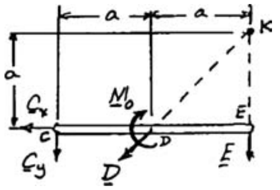
Free body: Member *ABC*



$$+\circlearrowleft \Sigma M_J = 0: \quad C_y(2a) + C_x(a) = 0$$

$$C_x = -2C_y$$

Free body: Member *CDE*



$$+\circlearrowleft \Sigma M_K = 0: \quad C_y(2a) - C_x(a) - M_0 = 0$$

$$C_y(2a) - (-2C_y)(a) - M_0 = 0 \qquad C_y = \frac{M_0}{4a} \triangleleft$$

$$C_x = -2C_y: \qquad C_x = -\frac{M_0}{2a} \triangleleft$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0: \quad \frac{D}{\sqrt{2}} + C_x = 0; \quad \frac{D}{\sqrt{2}} - \frac{M_0}{2a} = 0$$

$$D = \frac{M_0}{\sqrt{2}a} \qquad F_{DG} = \frac{M_0}{\sqrt{2}a} \quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad E(a) - C_y(a) + M_0 = 0$$

$$E(a) - \left(\frac{M_0}{4a}\right)(a) + M_0 = 0$$

$$E = -\frac{3}{4}\frac{M_0}{a} \qquad F_{EH} = \frac{3}{4}\frac{M_0}{a} \quad C \blacktriangleleft$$

Return to Free Body of *ABC*

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad \frac{B}{\sqrt{2}} + C_x = 0; \quad \frac{B}{\sqrt{2}} - \frac{M_0}{2a} = 0$$

$$B = \frac{M_0}{\sqrt{2}a} \qquad F_{BG} = \frac{M_0}{\sqrt{2}a} \quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_B = 0: \quad A(a) + C_y(a); \quad A(a) + \frac{M_0}{4a}(a) = 0$$

$$A = -\frac{M_0}{4a} \qquad F_{AF} = \frac{M_0}{4a} \quad C \blacktriangleleft$$

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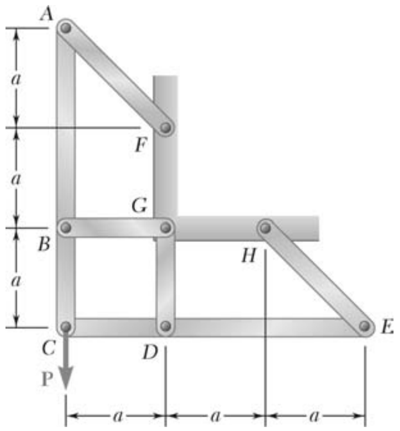
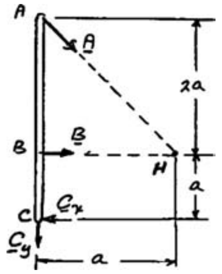


PROBLEM 6.116

Solve Problem 6.114 assuming that the force **P** is replaced by a clockwise couple of moment $\mathbf{M}_0$ applied to member *CDE* at *D*.

PROBLEM 6.114 Members *ABC* and *CDE* are pin-connected at *C* and supported by the four links *AF*, *BG*, *DG*, and *EH*. For the loading shown, determine the force in each link.

SOLUTION

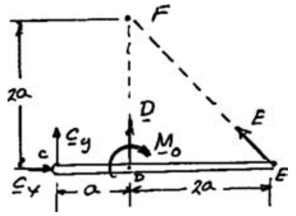


Free body: Member *ABC*

$$+\circlearrowright \Sigma M_H = 0: \quad C_x(a) - C_y(2a) = 0$$

$$C_x = 2C_y$$

Free body: Member *CDE*



$$+\circlearrowright \Sigma M_F = 0: \quad C_x(2a) - C_y(a) - M_0 = 0$$

$$(2C_y)(2a) - C_y(a) - M_0 = 0 \qquad C_y = \frac{M_0}{3a} \triangleleft$$

$$C_x = 2C_y: \qquad C_x = \frac{2M_0}{3a} \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - \frac{E}{\sqrt{2}} = 0; \quad \frac{2M_0}{3a} - \frac{E}{\sqrt{2}} = 0$$

$$E = \frac{2\sqrt{2}}{3}\frac{M_0}{a} \qquad F_{EH} = \frac{2\sqrt{2}}{3}\frac{M_0}{a} \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad D + \frac{E}{\sqrt{2}} + C_y = 0$$

$$D + \frac{2\sqrt{2}}{3}\frac{M_0}{a}\frac{1}{\sqrt{2}} + \frac{M_0}{3a} = 0$$

$$D = -\frac{M_0}{a} \qquad F_{DG} = \frac{M_0}{a} \quad C \blacktriangleleft$$

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PROBLEM 6.116 (Continued)

Return to free body of *ABC*

$$+\downarrow \Sigma F_y = 0: \quad \frac{A}{\sqrt{2}} + C_y = 0; \quad \frac{A}{\sqrt{2}} + \frac{M_0}{3a} = 0$$

$$A = -\frac{\sqrt{2}}{3}\frac{M_0}{a} \qquad\qquad F_{AF} = \frac{\sqrt{2}}{3}\frac{M_0}{a} \quad C \blacktriangleleft$$

$$+\circlearrowright \Sigma M_A = 0: \quad B(2a) - C_x(3a) = 0$$

$$B(2a) - \left(\frac{2}{3}\frac{M_0}{a}\right)(3a) = 0$$

$$B = +\frac{M_0}{a} \qquad\qquad F_{BG} = \frac{M_0}{a} \quad T \blacktriangleleft$$

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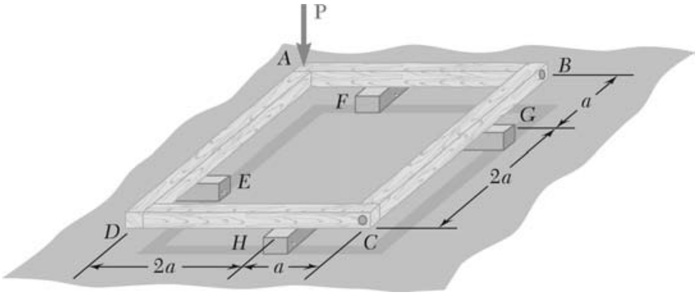
PROBLEM 6.117

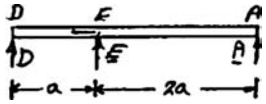
Four beams, each of length $3a$, are held together by single nails at A, B, C, and D. Each beam is attached to a support located at a distance a from an end of the beam as shown. Assuming that only vertical forces are exerted at the connections, determine the vertical reactions at E, F, G, and H.



SOLUTION

We shall draw the free body of each member. Force **P** will be applied to member *AFB*. Starting with member *AED*, we shall express all forces in terms of reaction E.

Member *AFB*:



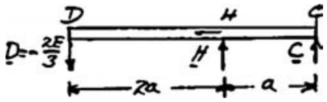
$$+\circlearrowleft \Sigma M_D = 0: \quad A(3a) + E(a) = 0$$

$$A = -\frac{E}{3}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -D(3a) - E(2a) = 0$$

$$D = -\frac{2E}{3}$$

Member *DHC*:



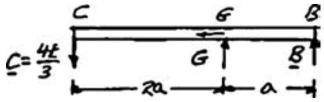
$$+\circlearrowleft \Sigma M_C = 0: \quad \left(-\frac{2E}{3}\right)(3a) - H(a) = 0$$

$$H = -2E \qquad (1)$$

$$+\circlearrowleft \Sigma M_H = 0: \quad \left(-\frac{2E}{3}\right)(2a) + C(a) = 0$$

$$C = +\frac{4E}{3}$$

Member *CGB*:



$$+\circlearrowleft \Sigma M_B = 0: \quad +\left(\frac{4E}{3}\right)(3a) - G(a) = 0$$

$$G = +4E \qquad (2)$$

$$+\circlearrowleft \Sigma M_G = 0: \quad +\left(\frac{4E}{3}\right)(2a) + B(a) = 0$$

$$B = -\frac{8E}{3}$$

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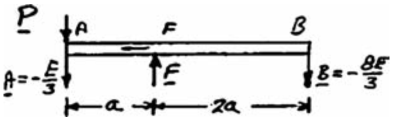
PROBLEM 6.117 (Continued)

Member *AFB*:



$$+\uparrow \Sigma F_y = 0: \quad F - A - B - P = 0$$

$$F - \left(-\frac{E}{3}\right) - \left(-\frac{8E}{3}\right) - P = 0$$

$$F = P - 3E \qquad (3)$$

$$+\circlearrowleft \Sigma M_A = 0: \quad F(a) - B(3a) = 0$$

$$(P - 3E)(a) - \left(-\frac{8E}{3}\right)(3a) = 0$$

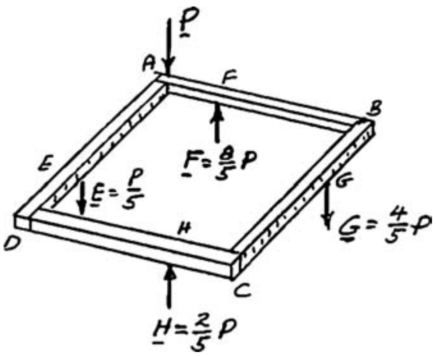
$$P - 3E + 8E = 0; \quad E = -\frac{P}{5} \qquad \mathbf{E} = \frac{P}{5}\downarrow \blacktriangleleft$$

Substitute $E = -\frac{P}{5}$ into Eqs. (1), (2), and (3).

$$H = -2E = -2\left(-\frac{P}{5}\right) \qquad H = +\frac{2P}{5} \qquad \mathbf{H} = \frac{2P}{5}\uparrow \blacktriangleleft$$

$$G = +4E = 4\left(-\frac{P}{5}\right) \qquad G = -\frac{4P}{5} \qquad \mathbf{G} = \frac{4P}{5}\downarrow \blacktriangleleft$$

$$F = P - 3E = P - 3\left(-\frac{P}{5}\right) \qquad F = +\frac{8P}{5} \qquad \mathbf{F} = \frac{8P}{5}\uparrow \blacktriangleleft$$



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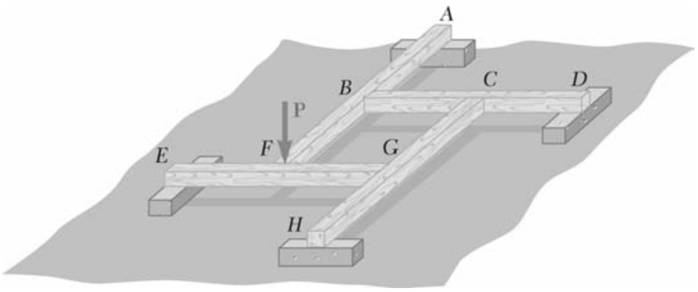
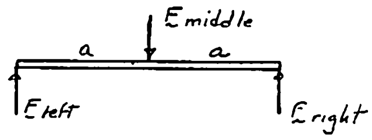
PROBLEM 6.118

Four beams, each of length $2a$, are nailed together at their midpoints to form the support system shown. Assuming that only vertical forces are exerted at the connections, determine the vertical reactions at A, D, E, and H.



SOLUTION

Note that, if we assume P is applied to EG, each individual member FBD looks like



so $$2F_{\text{left}} = 2F_{\text{right}} = F_{\text{middle}}$$

Labeling each interaction force with the letter corresponding to the joint of its application, we see that

$$B = 2A = 2F$$

$$C = 2B = 2D$$

$$G = 2C = 2H$$

$$P + F = 2G(= 4C = 8B = 16F) = 2E$$

From $$P + F = 16F, \quad F = \frac{P}{15}$$ so $\mathbf{A} = \frac{P}{15}\uparrow$ ◀

$\mathbf{D} = \frac{2P}{15}\uparrow$ ◀

$\mathbf{H} = \frac{4P}{15}\uparrow$ ◀

$\mathbf{E} = \frac{8P}{15}\uparrow$ ◀

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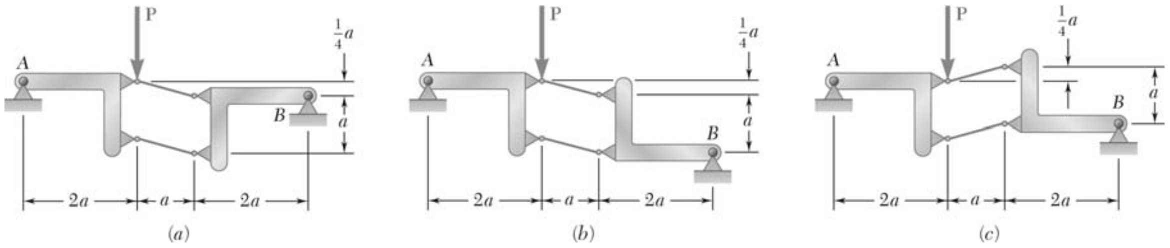
PROBLEM 6.119

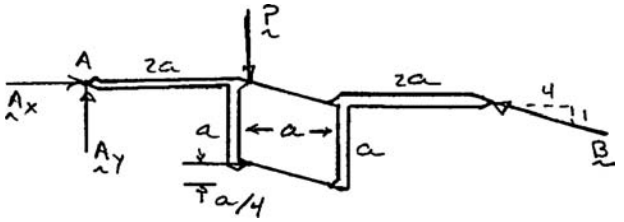
Each of the frames shown consists of two L-shaped members connected by two rigid links. For each frame, determine the reactions at the supports and indicate whether the frame is rigid.



SOLUTION

Note: In all three cases, the right member has only three forces acting, two of which are parallel. Thus the third force, at B, must be parallel to the link forces.

(*a*) **FBD whole:**



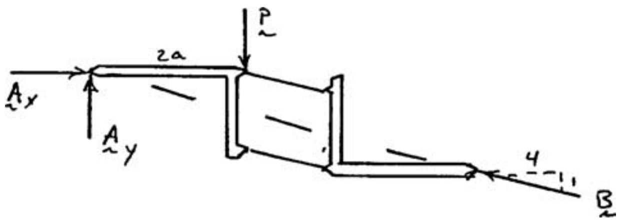
$\circlearrowleft \Sigma M_A = 0: \quad -2aP - \frac{a}{4}\frac{4}{\sqrt{17}}B + 5a\frac{1}{\sqrt{17}}B = 0 \quad B = 2.06P$ **B** $= 2.06P$ ⦣ $14.04°$ ◀

$\rightarrow \Sigma F_x = 0: \quad A_x - \frac{4}{\sqrt{17}}B = 0 \qquad \mathbf{A}_x = 2P \leftarrow$

$\uparrow \Sigma F_y = 0: \quad A_y - P + \frac{1}{\sqrt{17}}B = 0 \quad \mathbf{A}_y = \frac{P}{2}\uparrow$ **A** $= 2.06P$ ⦣ $14.04°$ ◀

rigid ◀

(*b*) **FBD whole:**



Since **B** passes through A, $\circlearrowleft \Sigma M_A = 2aP = 0$ only if $P = 0$

no equilibrium if $P \neq 0$ not rigid ◀

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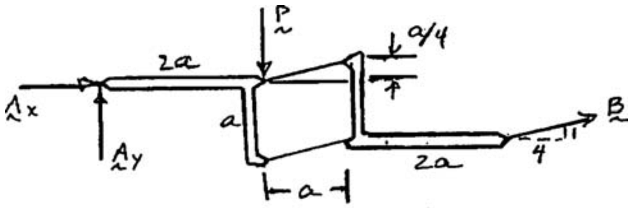
PROBLEM 6.119 (Continued)

(*c*) **FBD whole:**



$$\circlearrowleft \Sigma M_A = 0: \quad 5a\frac{1}{\sqrt{17}}B + \frac{3a}{4}\frac{4}{\sqrt{17}}B - 2aP = 0 \quad B = \frac{\sqrt{17}}{4}P$$

B $= 1.031P \angle 14.04°$ ◀

$$\xrightarrow{} \Sigma F_x = 0: \quad A_x + \frac{4}{\sqrt{17}}B = 0 \quad A_x = -P$$

$$\uparrow \Sigma F_y = 0: \quad A_y - P + \frac{1}{\sqrt{17}}B = 0 \quad A_y = P - \frac{P}{4} = \frac{3P}{4}$$

A $= 1.250P \searrow 36.9°$ ◀

System is rigid ◀

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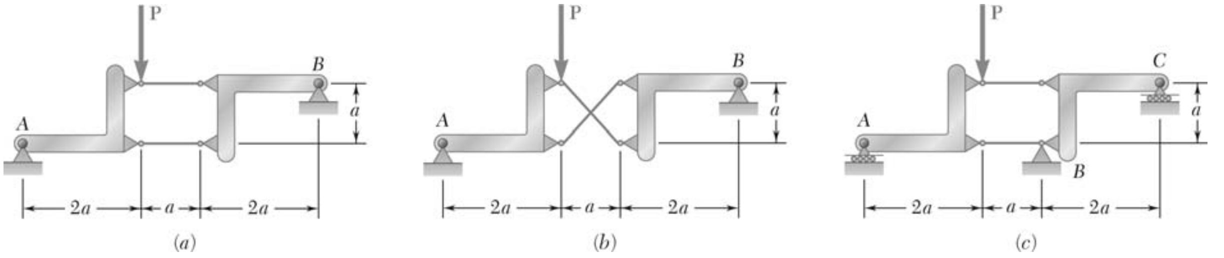
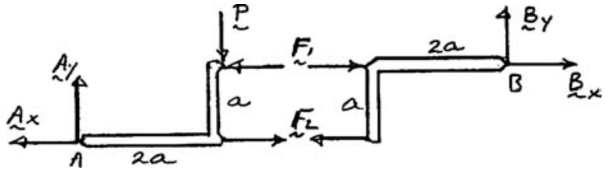
PROBLEM 6.120

Each of the frames shown consists of two L-shaped members connected by two rigid links. For each frame, determine the reactions at the supports and indicate whether the frame is rigid.



SOLUTION

(*a*) **Member FBDs:**



I II

FBD I: $\circlearrowleft \Sigma M_A = 0: \quad aF_1 - 2aP = 0 \quad F_1 = 2P; \uparrow \Sigma F_y = 0: \quad A_y - P = 0 \quad \mathbf{A}_y = P \uparrow$

FBD II: $\circlearrowleft \Sigma M_B = 0: \quad -aF_2 = 0 \quad F_2 = 0$

$\longrightarrow \Sigma F_x = 0: \quad B_x + F_1 = 0, \quad B_x = -F_1 = -2P \quad \mathbf{B}_x = 2P \longrightarrow$

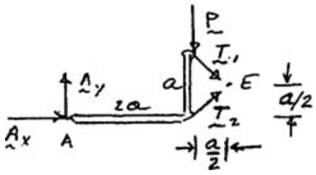
$\uparrow \Sigma F_y = 0: \quad B_y = 0$ so $\mathbf{B} = 2P \longrightarrow$ ◀

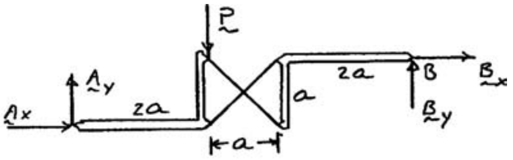
FBD I: $\longrightarrow \Sigma F_x = 0: \quad -A_x - F_1 + F_2 = 0 \quad A_x = F_2 - F_1 = 0 - 2P \quad \mathbf{A}_x = 2P \longrightarrow$

so $\mathbf{A} = 2.24P \measuredangle 26.6°$ ◀

frame is rigid ◀

(*b*) **FBD left:** **FBD whole:**





I II

FBD I: $\circlearrowleft \Sigma M_E = 0: \quad \frac{a}{2}P + \frac{a}{2}A_x - \frac{5a}{2}A_y = 0 \quad A_x - 5A_y = -P$

FBD II: $\circlearrowleft \Sigma M_B = 0: \quad 3aP + aA_x - 5aA_y = 0 \quad A_x - 5A_y = -3P$

This is impossible unless $P = 0$ not rigid ◀

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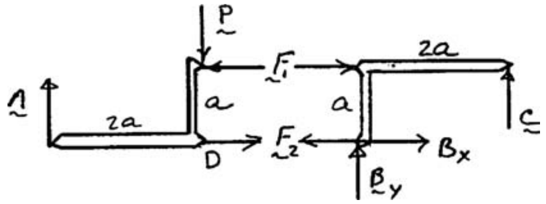
PROBLEM 6.120 (Continued)

(*c*) **Member FBDs:**



I II

FBD I: $\Sigma F_y = 0$: $A - P = 0$ **A** $= P \uparrow$ ◀

$\circlearrowleft \Sigma M_D = 0$: $aF_1 - 2aA = 0$ $F_1 = 2P$

$\rightarrow \Sigma F_x = 0$: $F_2 - F_1 = 0$ $F_2 = 2P$

FBD II: $\circlearrowleft \Sigma M_B = 0$: $2aC - aF_1 = 0$ $C = \frac{F_1}{2} = P$ **C** $= P \uparrow$ ◀

$\rightarrow \Sigma F_x = 0$: $F_1 - F_2 + B_x = 0$ $B_x = P - P = 0$

$\uparrow \Sigma F_x = 0$: $B_y + C = 0$ $B_y = -C = -P$ **B** $= P$ ◀

Frame is rigid ◀

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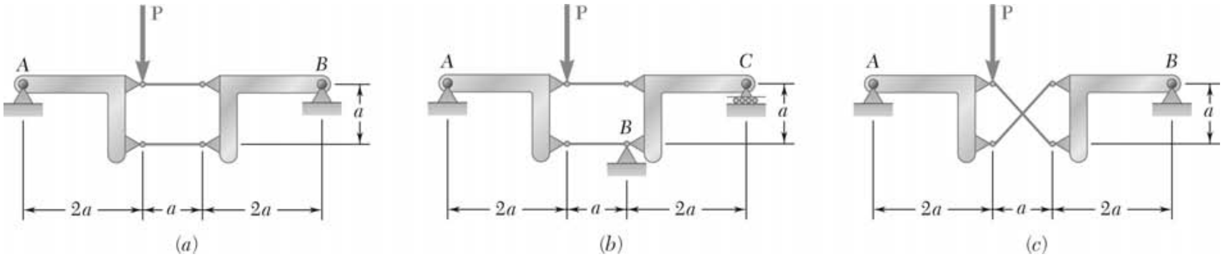
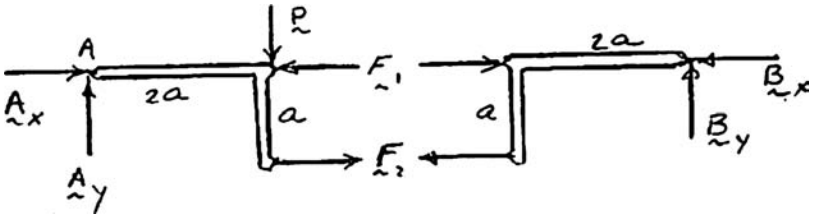
PROBLEM 6.121

Each of the frames shown consists of two L-shaped members connected by two rigid links. For each frame, determine the reactions at the supports and indicate whether the frame is rigid.



SOLUTION

(*a*) **Member FBDs:**

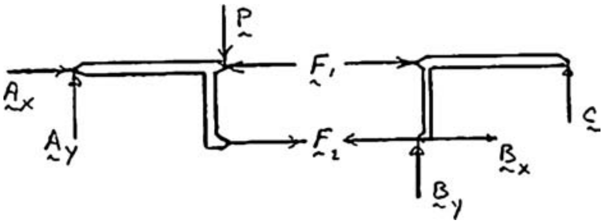


FBD II: $\uparrow \Sigma F_y = 0$: $B_y = 0$ $\quad \circlearrowleft \Sigma M_B = 0$: $aF_2 = 0$ $\quad F_2 = 0$

FBD I: $\circlearrowleft \Sigma M_A = 0$: $aF_2 - 2aP = 0$ but $F_2 = 0$

so $P = 0$ $\quad$ not rigid for $P \neq 0$ ◀

(*b*) **Member FBDs:**



Note: 7 Unknowns $(A_x, A_y, B_x, B_y, F_1, F_2, C)$ but only 6 independent equations.

System is statically indeterminate ◀

System is, however, rigid ◀

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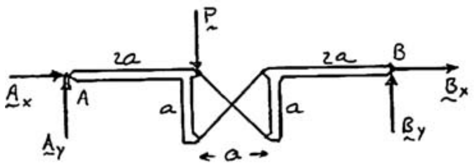
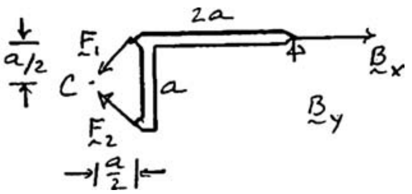
PROBLEM 6.121 (Continued)

(*c*) **FBD whole:** **FBD right:**



I



II

FBD I: $\circlearrowleft \Sigma M_A = 0: \quad 5aB_y - 2aP = 0$ $\qquad \mathbf{B}_y = \frac{2}{5}P \uparrow$

$\uparrow \Sigma F_y = 0: \quad A_y - P + \frac{2}{5}P = 0$ $\qquad \mathbf{A}_y = \frac{3}{5}P \uparrow$

FBD II: $\circlearrowleft \Sigma M_c = 0: \quad \frac{a}{2}B_x - \frac{5a}{2}B_y = 0 \quad B_x = 5B_y$ $\qquad \mathbf{B}_x = 2P \longrightarrow$

FBD I: $\longrightarrow \Sigma F_x = 0: \quad A_x + B_x = 0 \quad A_x = -B_x$ $\qquad \mathbf{A}_x = 2P \longleftarrow$

$\mathbf{A} = 2.09P \searrow 16.70°$ ◀

$\mathbf{B} = 2.04P \nearrow 11.31°$ ◀

System is rigid ◀

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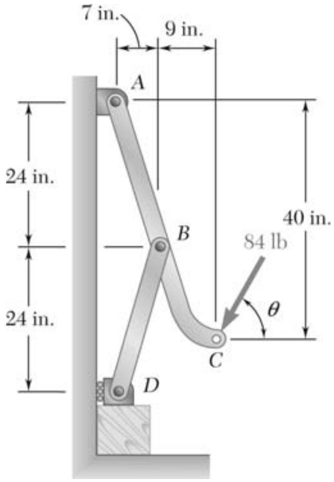


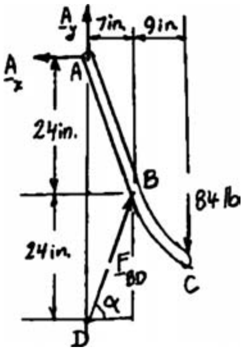
PROBLEM 6.122

An 84-lb force is applied to the toggle vise at *C*. Knowing that $\theta = 90°$, determine (*a*) the vertical force exerted on the block at *D*, (*b*) the force exerted on member *ABC* at *B*.

SOLUTION

We note that *BD* is a two-force member.

Free body: Member *ABC*



We have $$BD = \sqrt{(7)^2 + (24)^2} = 25 \text{ in.}$$

$$(F_{BD})_x = \frac{7}{25}F_{BD}, \quad (F_{BD})_y = \frac{24}{25}F_{BD}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (F_{BD})_x(24) + (F_{BD})_y(7) - 84(16) = 0$$

$$\left(\frac{7}{25}F_{BD}\right)(24) + \left(\frac{24}{25}F_{BD}\right)(7) = 84(16)$$

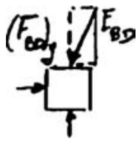
$$\frac{336}{25}F_{BD} = 1344$$

$$F_{BD} = 100 \text{ lb}$$

$$\tan\alpha = \frac{24}{7} \quad \alpha = 73.7°$$

(*b*) Force exerted at *B*: $\mathbf{F}_{BD} = 100.0 \text{ lb} \measuredangle 73.7°$ ◀

(*a*) Vertical force exerted on block



$$(F_{BD})_y = \frac{24}{25}F_{BD} = \frac{24}{25}(100 \text{ lb}) = 96 \text{ lb}$$

$(\mathbf{F}_{BD})_y = 96.0 \text{ lb} \downarrow$ ◀

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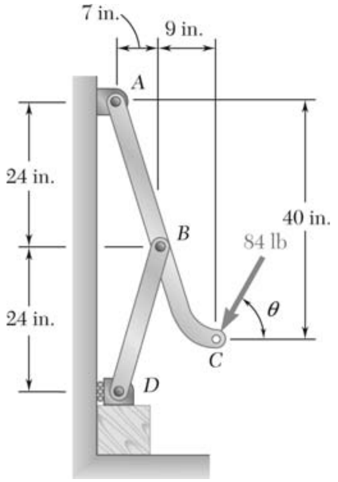


PROBLEM 6.123

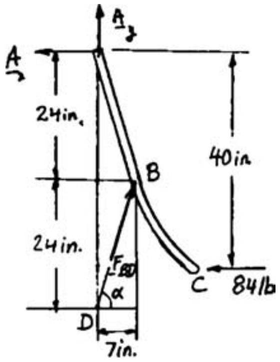
Solve Problem 6.122 when $\theta = 0$.

PROBLEM 6.122 An 84-lb force is applied to the toggle vise at *C*. Knowing that $\theta = 90°$, determine (*a*) the vertical force exerted on the block at *D*, (*b*) the force exerted on member *ABC* at *B*.

SOLUTION

We note that *BD* is a two-force member.

Free body: Member *ABC*



We have $$BD = \sqrt{(7)^2 + (24)^2} = 25\text{ in.}$$

$$(F_{BD})_x = \frac{7}{25}F_{BD}, \quad (F_{BD})_y = \frac{24}{25}F_{BD}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (F_{BD})_x(24) + (F_{BD})_y(7) - 84(40) = 0$$

$$\left(\frac{7}{25}F_{BD}\right)(24) + \left(\frac{24}{25}F_{BD}\right)(7) = 84(40)$$

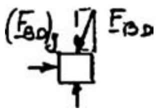
$$\frac{336}{25}F_{BD} = 3360$$

$$F_{BD} = 250\text{ lb}$$

$$\tan\alpha = \frac{24}{7} \quad \alpha = 73.7°$$

(*b*) Force exerted at *B*: $\mathbf{F}_{BD} = 250.0\text{ lb} \measuredangle 73.7°$ ◀

(*a*) Vertical force exerted on block



$$(F_{BD})_y = \frac{24}{25}F_{BD} = \frac{24}{25}(250\text{ lb}) = 240\text{ lb}$$

$(\mathbf{F}_{BD})_y = 240\text{ lb}\downarrow$ ◀

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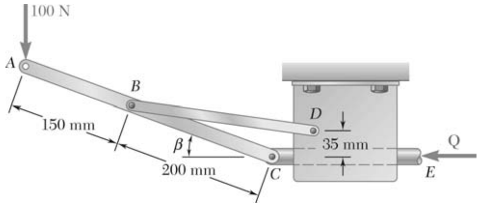


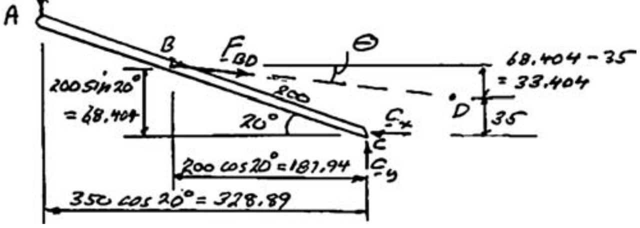
PROBLEM 6.124

The control rod *CE* passes through a horizontal hole in the body of the toggle system shown. Knowing that link *BD* is 250 mm long, determine the force **Q** required to hold the system in equilibrium when $\beta = 20°$.

SOLUTION

We note that *BD* is a two-force member.

Free body: Member *ABC*



Dimensions in mm

Since $$BD = 250, \quad \theta = \sin^{-1}\frac{33.404}{250}; \quad \theta = 7.679°$$

$$+\circlearrowleft \Sigma M_C = 0: \quad (F_{BD}\sin\theta)187.94 - (F_{BD}\cos\theta)68.404 + (100\text{ N})328.89 = 0$$

$$F_{BD}[187.94\sin 7.679° - 68.404\cos 7.679°] = 32889$$

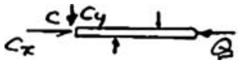
$$F_{BD} = 770.6\text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad (770.6\text{ N})\cos 7.679° = C_x = 0$$

$$C_x = +763.7\text{ N}$$

Member *CQ*:

$$\Sigma F_x = 0: \quad Q = C_x = 763.7\text{ N}$$



$\mathbf{Q} = 764\text{ N} \longleftarrow$ ◀

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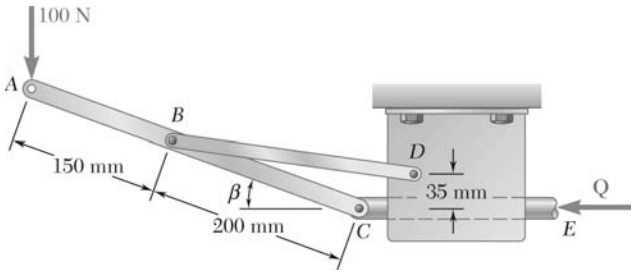


PROBLEM 6.125

Solve Problem 6.124 when (*a*) $\beta = 0$, (*b*) $\beta = 6°$.

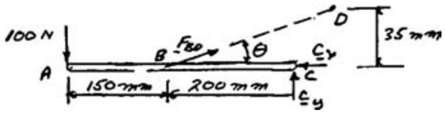
PROBLEM 6.124 The control rod *CE* passes through a horizontal hole in the body of the toggle system shown. Knowing that link *BD* is 250 mm long, determine the force **Q** required to hold the system in equilibrium when $\beta = 20°$.

SOLUTION

We note that *BD* is a two-force member.

(*a*) $\underline{\beta = 0}$:

Free body: Member *ABC*



Since $BD = 250 \text{ mm}, \quad \sin\theta = \dfrac{35 \text{ mm}}{250 \text{ mm}}; \quad \theta = 8.048°$

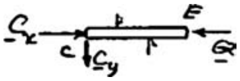
$$+\circlearrowleft \Sigma M_C = 0: \quad (100 \text{ N})(350 \text{ mm}) - F_{BD}\sin\theta(200 \text{ mm}) = 0$$

$$F_{BD} = 1250 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BD}\cos\theta - C_x = 0$$

$$(1250 \text{ N})(\cos 8.048°) - C_x = 0 \qquad \mathbf{C}_x = 1237.7 \text{ N}$$

Member *CE*:



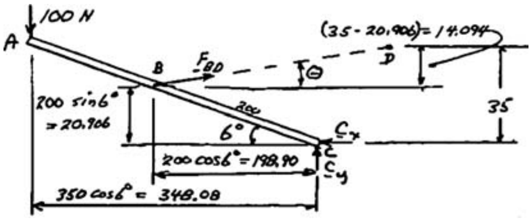
$$\xrightarrow{+} \Sigma F_x = 0: \quad (1237.7 \text{ N}) - Q = 0$$

$$Q = 1237.7 \text{ N}$$

$\mathbf{Q} = 1238 \text{ N} \longleftarrow$ ◀

(*b*) $\underline{\beta = 6°}$

Free body: Member *ABC*



Dimensions in mm

Since $BD = 250 \text{ mm}, \quad \theta = \sin^{-1}\dfrac{14.094 \text{ mm}}{250 \text{ mm}}$

$$\theta = 3.232°$$

$$+\circlearrowleft \Sigma M_C = 0: \quad (F_{BD}\sin\theta)198.90 + (F_{BD}\cos\theta)20.906 - (100 \text{ N})348.08 = 0$$

$$F_{BD}[198.90\sin 3.232° + 20.906\cos 3.232°] = 34808$$

$$F_{BD} = 1084.8 \text{ N}$$

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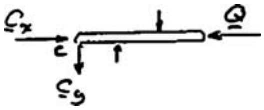
PROBLEM 6.125 (Continued)

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BD} \cos\theta - C_x = 0$$

$$(1084.8 \text{ N}) \cos 3.232° - C_x = 0$$

$$C_x = +1083.1 \text{ N}$$

Member *DE*: $\Sigma F_x = 0: \quad Q = C_x$



$$Q = 1083.1 \text{ N}$$

$\mathbf{Q} = 1083 \text{ N} \longleftarrow$ ◀

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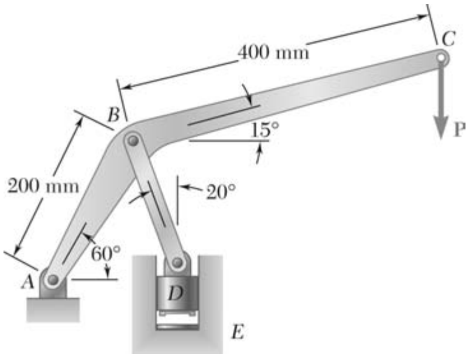
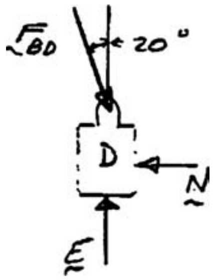


PROBLEM 6.126

The press shown is used to emboss a small seal at E. Knowing that $P = 250$ N, determine (a) the vertical component of the force exerted on the seal, (b) the reaction at A.

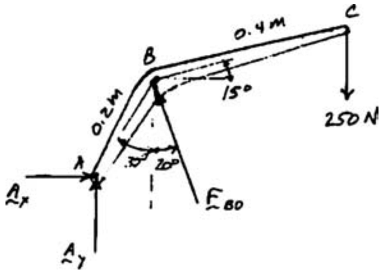
SOLUTION

FBD Stamp *D*:



$$\uparrow \Sigma F_y = 0: \quad E - F_{BD}\cos 20° = 0, \quad E = F_{BD}\cos 20°$$

FBD *ABC*:



$$\circlearrowleft \Sigma M_A = 0: \quad (0.2\text{ m})(\sin 30°)(F_{BD}\cos 20°) + (0.2\text{ m})(\cos 30°)(F_{BD}\sin 20°)$$

$$-[(0.2\text{ m})\sin 30° + (0.4\text{ m})\cos 15°](250\text{ N}) = 0$$

$$F_{BD} = 793.64\text{ N} \quad C$$

and, from above,

$$E = (793.64\text{ N})\cos 20°$$

(a) $\quad \mathbf{E} = 746\text{ N}\downarrow$ ◀

$$\rightarrow \Sigma F_x = 0: \quad A_x - (793.64\text{ N})\sin 20° = 0$$

$$\mathbf{A}_x = 271.44\text{ N} \rightarrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y + (793.64\text{ N})\cos 20° - 250\text{ N} = 0$$

$$\mathbf{A}_y = 495.78\text{ N}\downarrow$$

so (b) $\quad \mathbf{A} = 565\text{ N} \nwarrow 61.3°$ ◀

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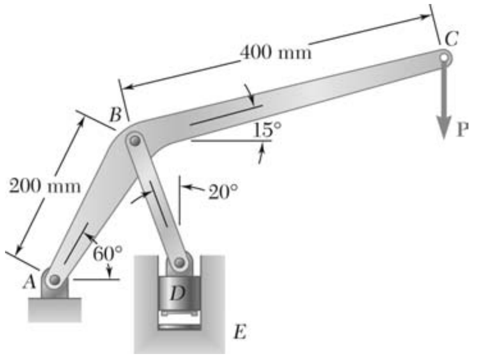
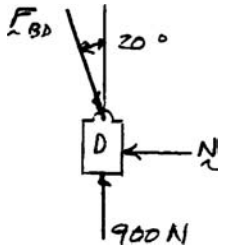


PROBLEM 6.127

The press shown is used to emboss a small seal at E. Knowing that the vertical component of the force exerted on the seal must be 900 N, determine (*a*) the required vertical force $\mathbf{P}$, (*b*) the corresponding reaction at A.

SOLUTION

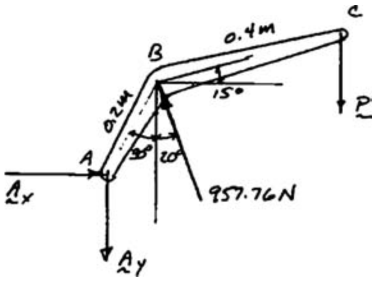
FBD Stamp *D*:



$$+\uparrow \Sigma F_y = 0: \quad 900\text{ N} - F_{BD}\cos 20° = 0, \quad F_{BD} = 957.76\text{ N} \quad C$$

(*a*)

FBD *ABC*:



$$\circlearrowleft \Sigma M_A = 0: \quad [(0.2\text{ m})(\sin 30°)](957.76\text{ N})\cos 20° + [(0.2\text{ m})(\cos 30°)](957.76\text{ N})\sin 20°$$

$$-[(0.2\text{ m})\sin 30° + (0.4\text{ m})\cos 15°]P = 0$$

$$P = 301.70\text{ N}, \qquad \mathbf{P} = 302\text{ N}\downarrow \blacktriangleleft$$

(*b*) $\quad \xrightarrow{+} \Sigma F_x = 0: \quad A_x - (957.76\text{ N})\sin 20° = 0$

$$\mathbf{A}_x = 327.57\text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad -A_y + (957.76\text{ N})\cos 20° - 301.70\text{ N} = 0$$

$$\mathbf{A}_y = 598.30\text{ N}\downarrow$$

so $\qquad \mathbf{A} = 682\text{ N} \measuredangle 61.3° \blacktriangleleft$

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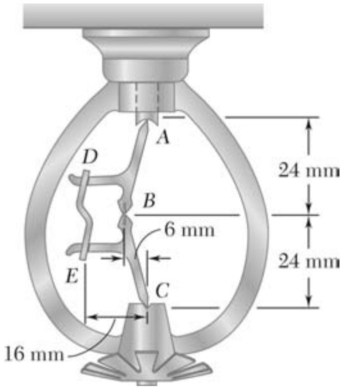
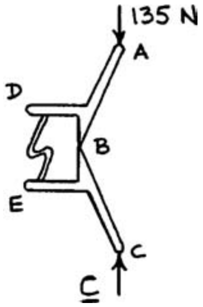


PROBLEM 6.128

Water pressure in the supply system exerts a downward force of 135 N on the vertical plug at *A*. Determine the tension in the fusible link *DE* and the force exerted on member *BCE* at *B*.

SOLUTION

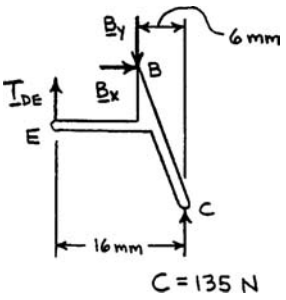


Free body: Entire linkage

$$+\Sigma F_y = 0: \quad C - 135 = 0$$

$$C = +135 \text{ N}$$

Free body: Member *BCE*



$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (135 \text{ N})(6 \text{ mm}) - T_{DE}(10 \text{ mm}) = 0$$

$$T_{DE} = 81.0 \text{ N} \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 135 + 81 - B_y = 0$$

$$B_y = +216 \text{ N} \qquad \mathbf{B} = 216 \text{ N} \downarrow \blacktriangleleft$$

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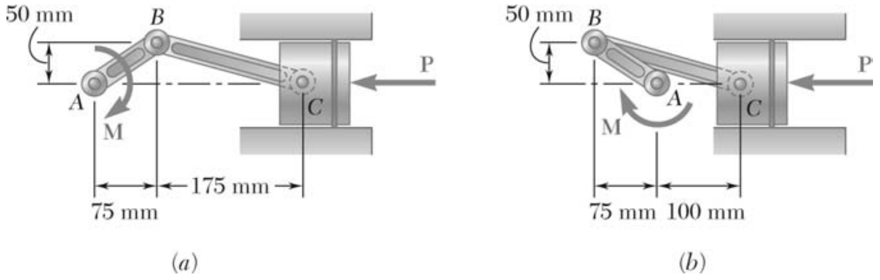
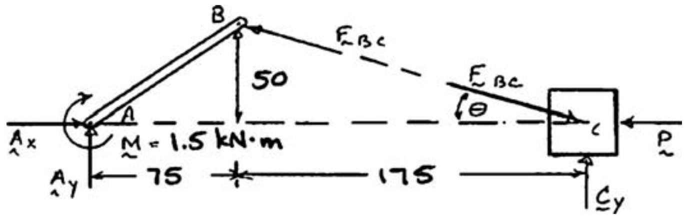
PROBLEM 6.129

A couple **M** of magnitude 1.5 kN · m is applied to the crank of the engine system shown. For each of the two positions shown, determine the force **P** required to hold the system in equilibrium.



SOLUTION

(*a*) **FBDs:**



Dimensions in mm

Note: $\tan\theta = \dfrac{50 \text{ mm}}{175 \text{ mm}}$

$= \dfrac{2}{7}$

FBD whole: $\circlearrowleft \Sigma M_A = 0{:} \quad (0.250 \text{ m})C_y - 1.5 \text{ kN}\cdot\text{m} = 0 \quad C_y = 6.00 \text{ kN}$

FBD piston: $\uparrow \Sigma F_y = 0{:} \quad C_y - F_{BC}\sin\theta = 0 \quad F_{BC} = \dfrac{C_y}{\sin\theta} = \dfrac{6.00 \text{ kN}}{\sin\theta}$

$\longrightarrow \Sigma F_x = 0{:} \quad F_{BC}\cos\theta - P = 0$

$$P = F_{BC}\cos\theta = \frac{6.00 \text{ kN}}{\tan\theta} = 7 \text{ kips}$$

P = 21.0 kN ⟵ ◀

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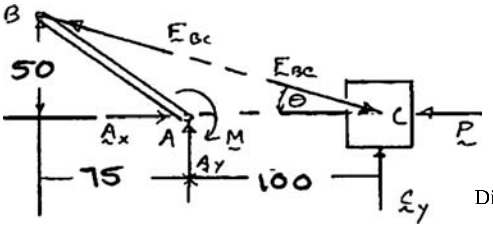
PROBLEM 6.129 (Continued)

(*b*) **FBDs:**



Dimensions in mm

Note: $\tan\theta = \dfrac{2}{7}$ as above

FBD whole: $\Sigma M_A = 0{:}\quad (0.100\text{ m})C_y - 1.5\text{ kN}\cdot\text{m} = 0 \quad C_y = 15\text{ kN}$

$$\Sigma F_y = 0{:}\quad C_y - F_{BC}\sin\theta = 0 \quad F_{BC} = \frac{C_y}{\sin\theta}$$

$$\rightarrow \Sigma F_x = 0{:}\quad F_{BC}\cos\theta - P = 0$$

$$P = F_{BC}\cos\theta = \frac{C_y}{\tan\theta} = \frac{15\text{ kN}}{2/7} \qquad \mathbf{P} = 52.5\text{ kN} \leftarrow ◀$$

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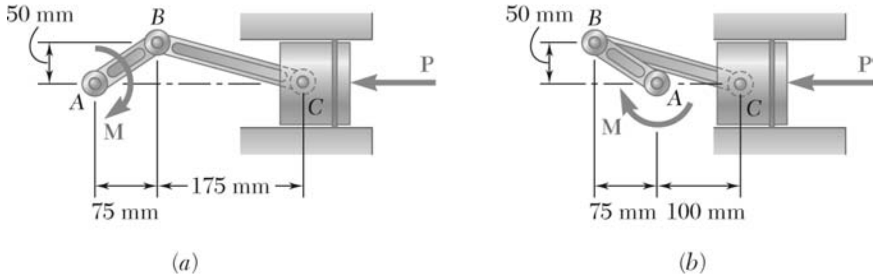
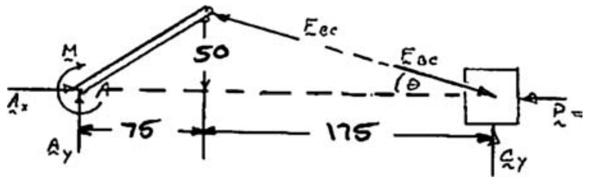
PROBLEM 6.130

A force **P** of magnitude 16 kN is applied to the piston of the engine system shown. For each of the two positions shown, determine the couple **M** required to hold the system in equilibrium.



SOLUTION

(*a*) **FBDs:**



Dimensions in mm

Note: $\tan\theta = \dfrac{50 \text{ mm}}{175 \text{ mm}}$

$= \dfrac{2}{7}$

FBD piston: $\xrightarrow{} \Sigma F_x = 0\!: \quad F_{BC}\cos\theta - P = 0 \quad F_{BC} = \dfrac{P}{\cos\theta}$

$\uparrow \Sigma F_y = 0\!: \quad C_y - F_{BC}\sin\theta = 0 \quad C_y = F_{BC}\sin\theta = P\tan\theta = \dfrac{2}{7}P$

FBD whole: $\circlearrowleft \Sigma M_A = 0\!: \quad (0.250 \text{ m})C_y - M = 0$

$$M = (0.250 \text{ m})\left(\frac{2}{7}\right)(16 \text{ kN})$$

$$= 1.14286 \text{ kN}\cdot\text{m}$$

$\mathbf{M} = 1143 \text{ N}\cdot\text{m} \circlearrowright$ ◀

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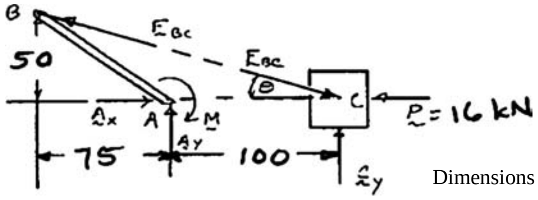
PROBLEM 6.130 (Continued)

(*b*) **FBDs:**



Note: $\tan\theta = \dfrac{2}{7}$ as above

FBD piston: as above $C_y = P\tan\theta = \dfrac{2}{7}P$

FBD whole: $\circlearrowleft \Sigma M_A = 0\text{:} \quad (0.100\text{ m})C_y - M = 0 \quad M = (0.100\text{ m})\dfrac{2}{7}(16\text{ kN})$

$M = 0.45714\text{ kN}\cdot\text{m}$ $\mathbf{M} = 457\text{ N}\cdot\text{m} \circlearrowright$ ◀

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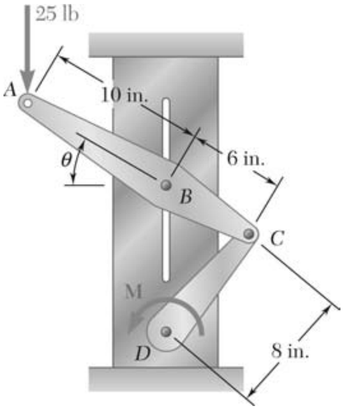
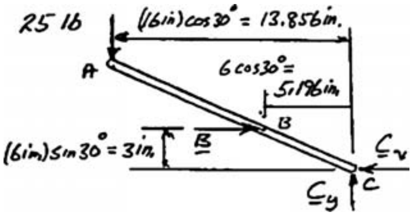


PROBLEM 6.131

The pin at B is attached to member ABC and can slide freely along the slot cut in the fixed plate. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when $\theta = 30°$.

SOLUTION

Free body: Member ABC



$$+\circlearrowleft \Sigma M_C = 0: \quad (25\text{ lb})(13.856\text{ in.}) - B(3\text{ in.}) = 0$$

$$B = +115.47\text{ lb}$$

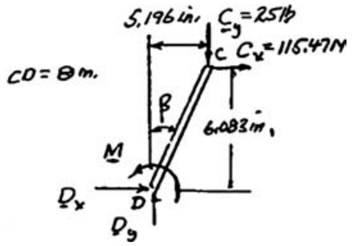
$$+\uparrow \Sigma F_y = 0: \quad -25\text{ lb} + C_y = 0$$

$$C_y = +25\text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 115.47\text{ lb} - C_x = 0$$

$$C_x = +115.47\text{ lb}$$

Free body: Member CD



$$\beta = \sin^{-1}\frac{5.196}{8}; \quad \beta = 40.505°$$

$$CD\cos\beta = (8\text{ in.})\cos 40.505° = 6.083\text{ in.}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad M - (25\text{ lb})(5.196\text{ in.}) - (115.47\text{ lb})(6.083\text{ in.}) = 0$$

$$M = +832.3\text{ lb}\cdot\text{in.}$$

$$\mathbf{M} = 832\text{ lb}\cdot\text{in.} \blacktriangleleft$$

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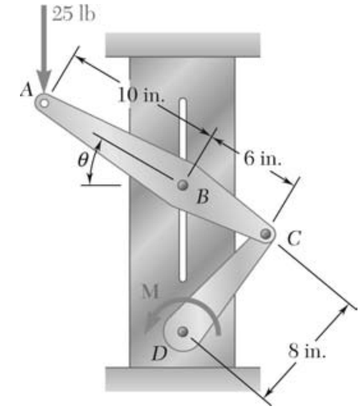
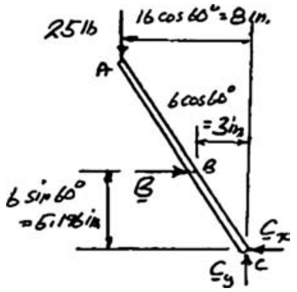


PROBLEM 6.132

The pin at B is attached to member ABC and can slide freely along the slot cut in the fixed plate. Neglecting the effect of friction, determine the couple $\mathbf{M}$ required to hold the system in equilibrium when $\theta = 60°$.

SOLUTION

<u>Free body: Member *ABC*</u>



$$+\circlearrowleft \Sigma M_C = 0: \quad (25 \text{ lb})(8 \text{ in.}) - B(5.196 \text{ in.}) = 0$$

$$B = +38.49 \text{ lb}$$

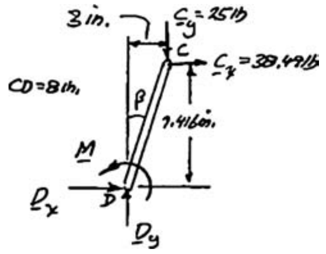
$$\xrightarrow{+} \Sigma F_x = 0: \quad 38.49 \text{ lb} - C_x = 0$$

$$C_x = +38.49 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad -25 \text{ lb} + C_y = 0$$

$$C_y = +25 \text{ lb}$$

<u>Free body: Member *CD*</u>



$$\beta = \sin^{-1}\frac{3}{8}; \quad \beta = 22.024°$$

$$CD\cos\beta = (8 \text{ in.})\cos 22.024° = 7.416 \text{ in.}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad M - (25 \text{ lb})(3 \text{ in.}) - (38.49 \text{ lb})(7.416 \text{ in.}) = 0$$

$$M = +360.4 \text{ lb}\cdot\text{in.} \qquad \mathbf{M} = 360 \text{ lb}\cdot\text{in.} \circlearrowleft \blacktriangleleft$$

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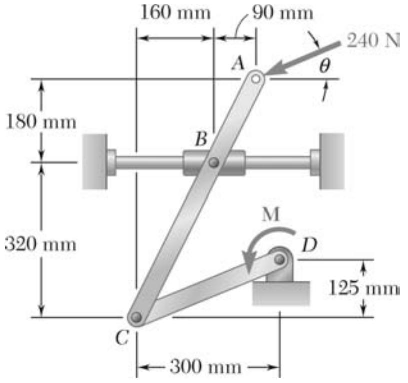
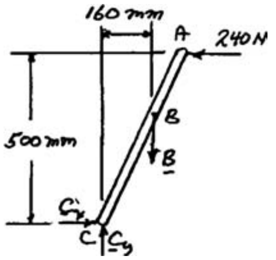


PROBLEM 6.133

Arm *ABC* is connected by pins to a collar at *B* and to crank *CD* at *C*. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when $\theta = 0$.

SOLUTION

Free body: Member *ABC*



$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 240 \text{ N} = 0$$

$$C_x = +240 \text{ N}$$

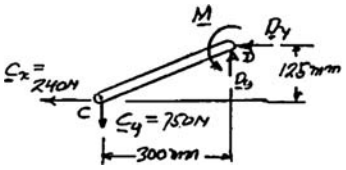
$$+\circlearrowleft \Sigma M_C = 0: \quad (240 \text{ N})(500 \text{ mm}) - B(160 \text{ mm}) = 0$$

$$B = +750 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 750 \text{ N} = 0$$

$$C_y = +750 \text{ N}$$

Free body: Member *CD*



$$+\circlearrowleft \Sigma M_D = 0: \quad M + (750 \text{ N})(300 \text{ mm}) - (240 \text{ N})(125 \text{ mm}) = 0$$

$$M = -195 \times 10^3 \text{ N} \cdot \text{mm}$$

$$\mathbf{M} = 195.0 \text{ kN} \cdot \text{m} \circlearrowright \blacktriangleleft$$

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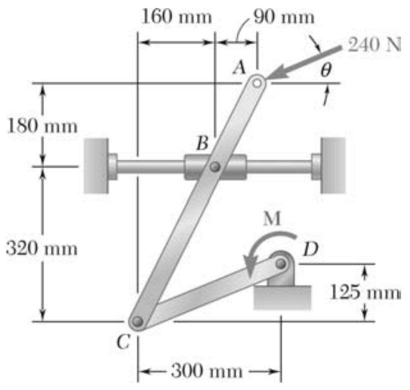
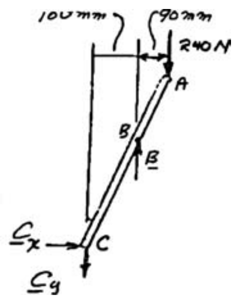


PROBLEM 6.134

Arm *ABC* is connected by pins to a collar at *B* and to crank *CD* at *C*. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when $\theta = 90°$.

SOLUTION

Free body: Member *ABC*

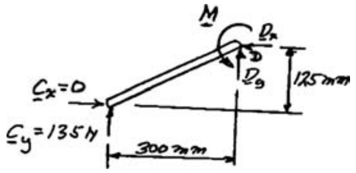


$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad C_y(160 \text{ mm}) - (240 \text{ N})(90 \text{ mm}) = 0$$

$$C_y = +135 \text{ N}$$

Free body: Member *CD*



$$+\circlearrowleft \Sigma M_D = 0: \quad M - (135 \text{ N})(300 \text{ mm}) = 0$$

$$M = +40.5 \times 10^3 \text{ N} \cdot \text{mm}$$

$$\mathbf{M} = 40.5 \text{ kN} \cdot \text{m} \circlearrowleft \blacktriangleleft$$

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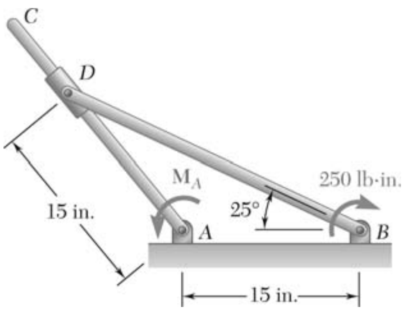


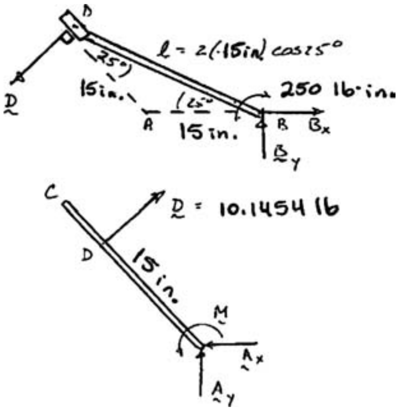
PROBLEM 6.135

Two rods are connected by a slider block as shown. Neglecting the effect of friction, determine the couple $\mathbf{M}_A$ required to hold the system in equilibrium.

SOLUTION

Note: $\mathbf{D} \perp AC$

Member FBDs:



$\circlearrowleft \Sigma M_B = 0$: $\quad lD\sin 115° - 250 \text{ lb} \cdot \text{in.} = 0$

$$D = \frac{250 \text{ lb} \cdot \text{in.}}{2(15 \text{ in.})\cos 25° \sin 115°}$$

$$= 10.1454 \text{ lb}$$

$\circlearrowleft \Sigma M_A = 0$: $\quad M_A - (15 \text{ in.})(10.1454 \text{ lb}) = 0$

$$\mathbf{M}_A = 152.2 \text{ lb} \cdot \text{in.} \circlearrowright \blacktriangleleft$$

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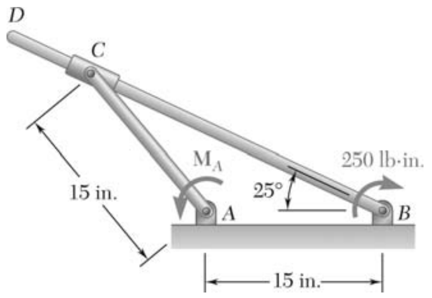


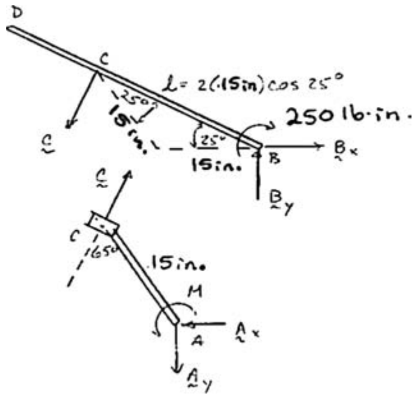
PROBLEM 6.136

Two rods are connected by a slider block as shown. Neglecting the effect of friction, determine the couple $\mathbf{M}_A$ required to hold the system in equilibrium.

SOLUTION

Note: $\mathbf{C} \perp BD$

Member FBD's:



$$\circlearrowleft \Sigma M_B = 0: \quad lC - 250 \text{ lb}\cdot\text{in.} = 0$$

$$C = \frac{250 \text{ lb}\cdot\text{in.}}{2(15 \text{ in.})\cos 25°}$$

$$C = 9.1948 \text{ lb}$$

$$\circlearrowleft \Sigma M_A = 0: \quad M_A - (15 \text{ in.})C \sin 65° = 0$$

$$M_A = (15 \text{ in.})(9.1948 \text{ lb})\sin 65°$$

$$\mathbf{M}_A = 125.0 \text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

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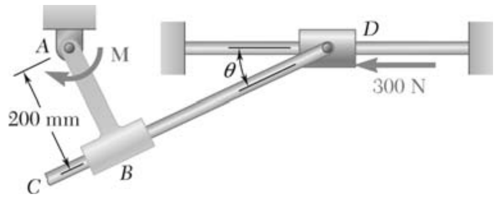


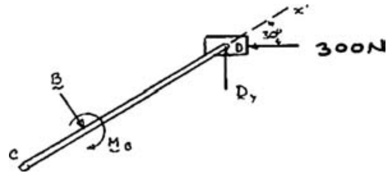
PROBLEM 6.137

Rod CD is attached to the collar D and passes through a collar welded to end B of lever AB. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when $\theta = 30°$.

SOLUTION

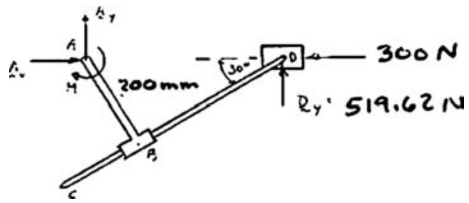
Note: $\mathbf{B} \perp CD$

FBD *DC*: $\nearrow \Sigma F_{x'} = 0: \quad D_y \sin 30° - (300\text{ N})\cos 30° = 0$



$$D_y = \frac{300\text{ N}}{\tan 30°} = 519.62\text{ N}$$

FBD machine:



$$\circlearrowleft \Sigma M_A = 0: \quad \frac{0.200\text{ m}}{\sin 30°} 519.62\text{ N} - M = 0$$

$M = 207.85\text{ N}\cdot\text{m}$ $\qquad$ $\mathbf{M} = 208\text{ N}\cdot\text{m}$ ↻ ◀

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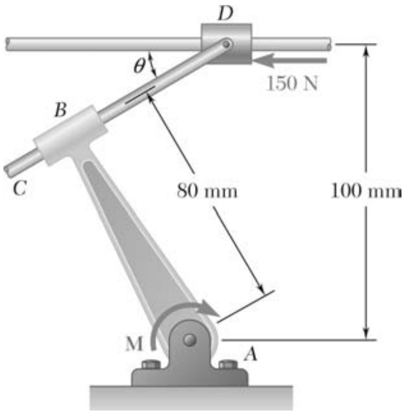
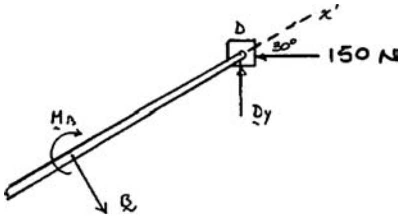


PROBLEM 6.138

Rod *CD* is attached to the collar *D* and passes through a collar welded to end *B* of lever *AB*. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when $\theta = 30°$.

SOLUTION

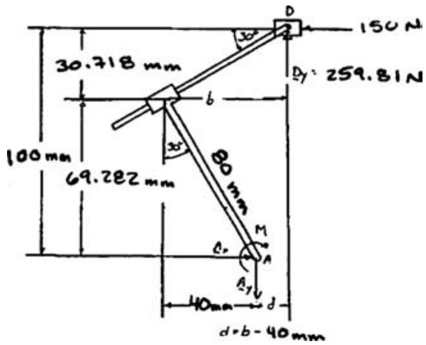
FBD *DC*:



$$\nearrow \Sigma F_{x'} = 0: \quad D_y \sin 30° - (150 \text{ N}) \cos 30° = 0$$

$$D_y = (150 \text{ N}) \text{ ctn } 30° = 259.81 \text{ N}$$

FBD machine:



$$\circlearrowleft \Sigma M_A = 0: \quad (0.100 \text{ m})(150 \text{ N}) + d(259.81 \text{ N}) - M = 0$$

$$d = b - 0.040 \text{ m}$$

$$b = \frac{0.030718 \text{ m}}{\tan 30}$$

so

$$b = 0.053210 \text{ m}$$

$$d = 0.0132100 \text{ m}$$

$$M = 18.4321 \text{ N} \cdot \text{m}$$

$$\mathbf{M} = 18.43 \text{ N} \cdot \text{m.} \circlearrowright \blacktriangleleft$$

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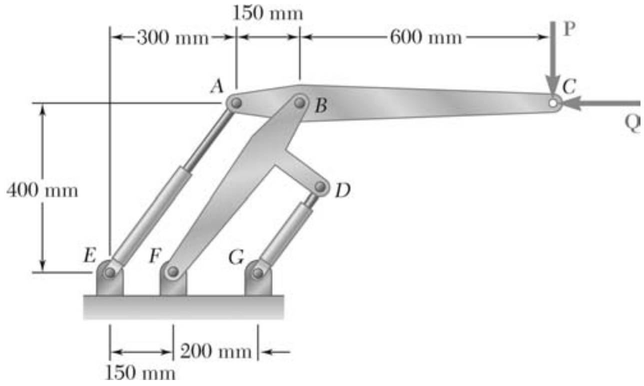
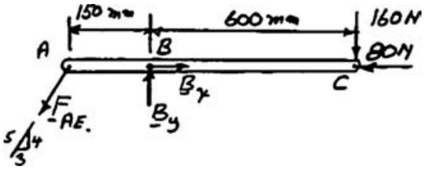


PROBLEM 6.139

Two hydraulic cylinders control the position of the robotic arm *ABC*. Knowing that in the position shown the cylinders are parallel, determine the force exerted by each cylinder when $P = 160$ N and $Q = 80$ N.

SOLUTION

Free body: Member *ABC*



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{4}{5}F_{AE}(150 \text{ mm}) - (160 \text{ N})(600 \text{ mm}) = 0$$

$$F_{AE} = +800 \text{ N} \qquad\qquad F_{AE} = 800 \text{ N} \quad T \blacktriangleleft$$

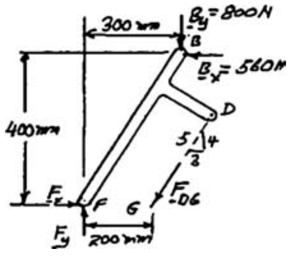
$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{3}{5}(800 \text{ N}) + B_x - 80 \text{ N} = 0$$

$$B_x = +560 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5}(800 \text{ N}) + B_y - 160 \text{ N} = 0$$

$$B_y = +800 \text{ N}$$

Free body: Member *BDF*



$$+\circlearrowleft \Sigma M_F = 0: \quad (560 \text{ N})(400 \text{ mm}) - (800 \text{ N})(300 \text{ mm}) - \frac{4}{5}F_{DG}(200 \text{ mm}) = 0$$

$$F_{DG} = -100 \text{ N} \qquad\qquad F_{DG} = 100.0 \text{ N} \quad C \blacktriangleleft$$

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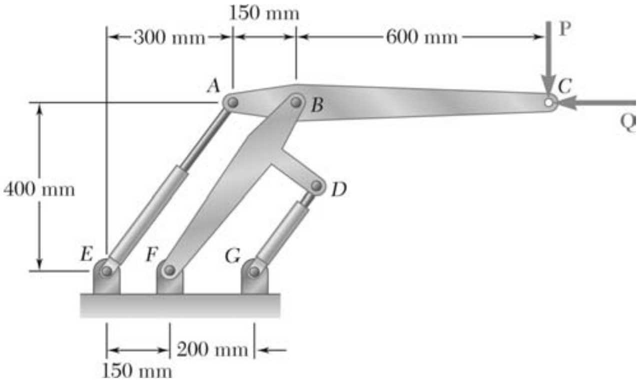
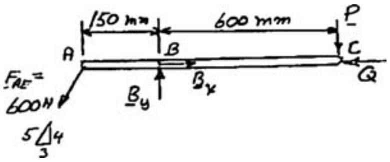
PROBLEM 6.140

Two hydraulic cylinders control the position of the robotic arm *ABC*. In the position shown, the cylinders are parallel and both are in tension. Knowing the $F_{AE} = 600$ N and $F_{DG} = 50$ N, determine the forces **P** and **Q** applied at *C* to arm *ABC*.



SOLUTION



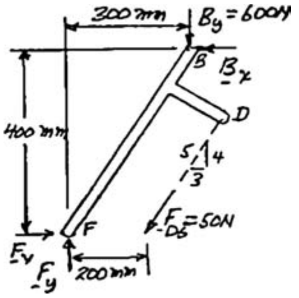
Free body: Member *ABC*

$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{4}{5}(600\text{ N})(150\text{ mm}) - P(600\text{ mm}) = 0$$

$P = +120$ N $\qquad$ $\mathbf{P} = 120.0\text{ N}\downarrow$ ◀

$$+\circlearrowleft \Sigma M_C = 0: \quad \frac{4}{5}(600)(750\text{ mm}) - B_y(600\text{ mm}) = 0$$

$$B_y = +600\text{ N}$$



Free body: Member *BDF*

$$+\circlearrowleft \Sigma M_F = 0: \quad B_x(400\text{ mm}) - (600\text{ N})(300\text{ mm})$$

$$-\frac{4}{5}(50\text{ N})(200\text{ mm}) = 0$$

$$B_x = +470\text{ N}$$

Return to free body: Member *ABC*

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{3}{5}(600\text{ N}) + 470\text{ N} - Q = 0$$

$Q = +110$ N $\qquad$ $\mathbf{Q} = 110.0\text{ N} \longleftarrow$ ◀

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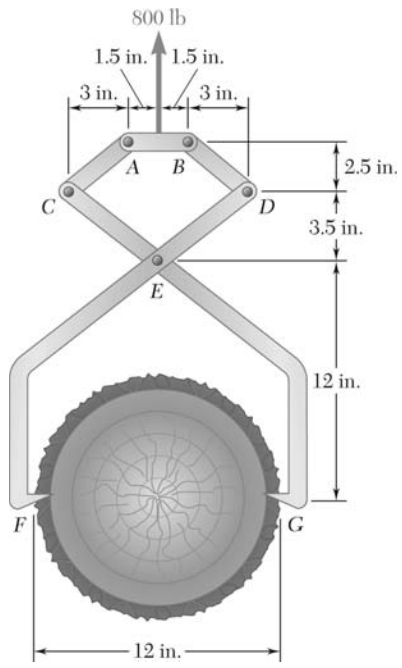
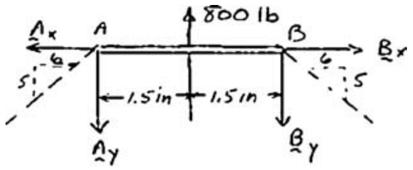


PROBLEM 6.141

A log weighing 800 lb is lifted by a pair of tongs as shown. Determine the forces exerted at *E* and *F* on tong *DEF*.

SOLUTION

FBD *AB*:



By symmetry: $A_y = B_y = 400 \text{ lb}$

and $A_x = B_x$

$$= \frac{6}{5}(400 \text{ lb})$$

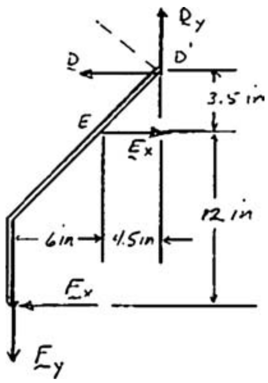
$$= 480 \text{ lb}$$

Note: $\mathbf{D} = -\mathbf{B}$

so $D_x = 480 \text{ lb}$

$D_y = 400 \text{ lb}$

FBD *DEF*:



$$\circlearrowleft \Sigma M_F = (10.5 \text{ in.})(400 \text{ lb}) + (15.5 \text{ in.})(480 \text{ lb}) - (12 \text{ in.})E_x = 0$$

$E_x = 970 \text{ lb}$ $\mathbf{E} = 970 \text{ lb} \longrightarrow$ ◀

$$\longrightarrow \Sigma F_x = 0: \quad -480 \text{ lb} + 970 \text{ lb} - F_x = 0$$

$$F_x = 490 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad 400 \text{ lb} - F_y = 0$$

$$F_y = 400 \text{ lb}$$

$\mathbf{F} = 633 \text{ lb} \measuredangle\ 39.2°$ ◀

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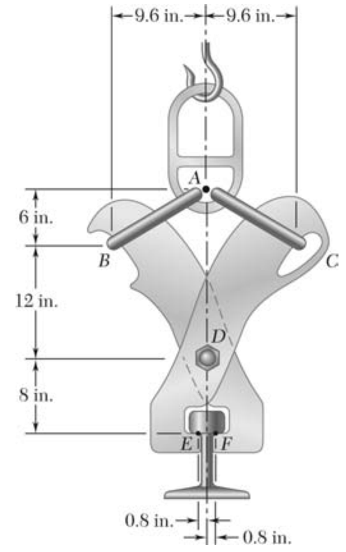
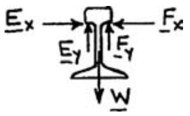
PROBLEM 6.142



A 39-ft length of railroad rail of weight 44 lb/ft is lifted by the tongs shown. Determine the forces exerted at D and F on tong BDF.

SOLUTION

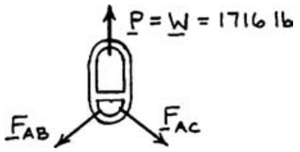
Free body: Rail



$$W = (39\text{ ft})(44\text{ lb/ft}) = 1716\text{ lb}$$

By symmetry $\quad E_y = F_y = \dfrac{1}{2}W = 858\text{ lb}$

Free body: Upper link

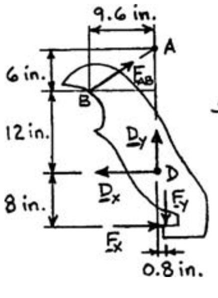


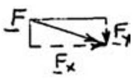
By symmetry, $\quad (F_{AB})_y = (F_{AC})_y = \dfrac{1}{2}W = 858\text{ lb}$

Since AB is a two-force member,

$$\frac{(F_{AB})_x}{9.6} = \frac{(F_{AB})_y}{6} \qquad (F_{AB})_x = \frac{9.6}{6}(858) = 1372.8\text{ lb}$$

Free Body: Tong BDF





$+\circlearrowleft \Sigma M_D = 0$: $\quad$ (Attach F_{AB} at A)

$$F_x(8) - (F_{AB})_x(18) - F_y(0.8) = 0$$

$$F_x(8) - (1372.8\text{ lb})(18) - (858\text{ lb})(0.8) = 0$$

$$F_x = +3174.6\text{ lb}$$

$\mathbf{F} = 3290\text{ lb} \searrow 15.12°$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $\quad -D_x + (F_{AB})_x + F_x = 0$

$$D_x = (F_{AB})_x + F_x = 1372.8 + 3174.6 = 4547.4\text{ lb}$$

$+\uparrow \Sigma F_y = 0$: $\quad D_y + (F_{AB})_y - F_y = 0$

$$D_y = 0$$

$\mathbf{D} = 4550\text{ lb} \longleftarrow$ ◀

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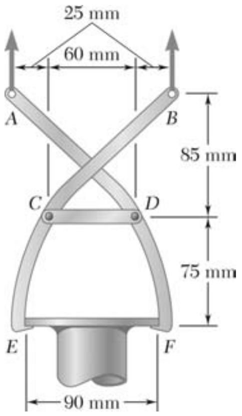
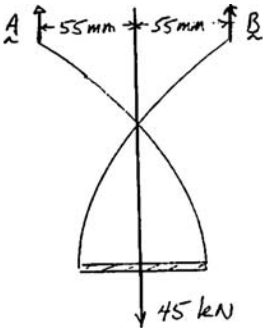


PROBLEM 6.143

The tongs shown are used to apply a total upward force of 45 kN on a pipe cap. Determine the forces exerted at *D* and *F* on tong *ADF*.

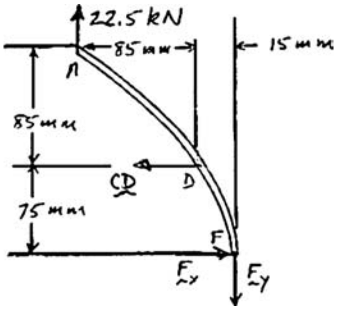
SOLUTION

FBD whole:



By symmetry $\mathbf{A} = \mathbf{B} = 22.5 \text{ kN} \uparrow$

FBD *ADF*:



$$\circlearrowleft \Sigma M_F = 0: \quad (75 \text{ mm})CD - (100 \text{ mm})(22.5 \text{ kN}) = 0$$

$$\mathbf{CD} = 30.0 \text{ kN} \longleftarrow \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: \quad F_x - CD = 0$$

$$F_x = CD = 30 \text{ kN}$$

$$\uparrow \Sigma F_y = 0: \quad 22.5 \text{ kN} - F_y = 0$$

$$F_y = 22.5 \text{ kN}$$

so $\mathbf{F} = 37.5 \text{ kN} \searrow 36.9° \blacktriangleleft$

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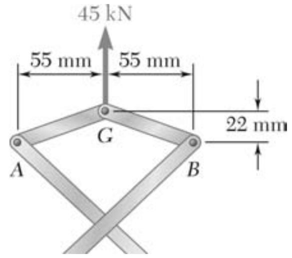
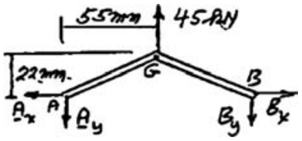


PROBLEM 6.144

If the toggle shown is added to the tongs of Problem 6.143 and a single vertical force is applied at G, determine the forces exerted at D and F on tong ADF.

SOLUTION



Free body: Toggle

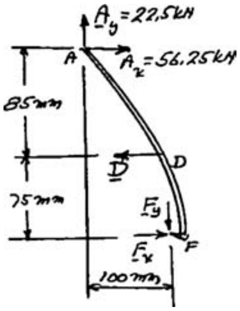
By symmetry $$A_y = \frac{1}{2}(45 \text{ kN}) = 22.5 \text{ kN}$$

AG is a two-force member

$$\frac{22.5 \text{ kN}}{22 \text{ mm}} = \frac{A_x}{55 \text{ mm}}$$

$$A_x = 56.25 \text{ kN}$$

Free body: Tong ADF



$$+\uparrow \Sigma F_y = 0: \quad 22.5 \text{ kN} - F_y = 0$$

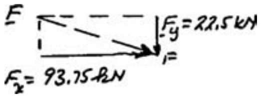
$$F_y = +22.5 \text{ kN}$$

$$+\circlearrowleft \Sigma M_F = 0: \quad D(75 \text{ mm}) - (22.5 \text{ kN})(100 \text{ mm}) - (56.25 \text{ kN})(160 \text{ mm}) = 0$$

$$D = +150 \text{ kN} \qquad \mathbf{D} = 150.0 \text{ kN} \longleftarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 56.25 \text{ kN} - 150 \text{ kN} + F_x = 0$$

$$F_x = 93.75 \text{ kN}$$



$$\mathbf{F} = 96.4 \text{ kN} \searrow 13.50° \blacktriangleleft$$

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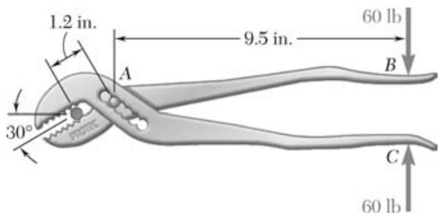


PROBLEM 6.145

The pliers shown are used to grip a 0.3-in.-diameter rod. Knowing that two 60-lb forces are applied to the handles, determine (*a*) the magnitude of the forces exerted on the rod, (*b*) the force exerted by the pin at *A* on portion *AB* of the pliers.

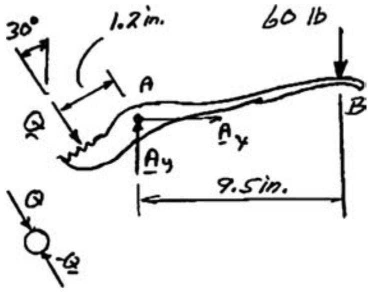
SOLUTION

<u>Free body: Portion *AB*</u>

(*a*)

$$+\circlearrowright \Sigma M_A = 0: \quad Q(1.2 \text{ in.}) - (60 \text{ lb})(9.5 \text{ in.}) = 0$$

$$Q = 475 \text{ lb} \blacktriangleleft$$



(*b*)

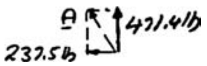
$$\xrightarrow{+} \Sigma F_x = 0: \quad Q(\sin 30°) + A_x = 0$$

$$(475 \text{ lb})(\sin 30°) + A_x = 0$$

$$A_x = -237.5 \text{ lb} \qquad \mathbf{A}_y = 237.5 \text{ lb} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad -Q(\cos 30°) + A_y - 60 \text{ lb} = 0$$

$$-(475 \text{ lb})(\cos 30°) + A_y - 60 \text{ lb} = 0$$



$$A_y = +471.4 \text{ lb} \qquad \mathbf{A}_y = 471.4 \text{ lb} \uparrow$$

$$\mathbf{A} = 528 \text{ lb} \searrow 63.3° \blacktriangleleft$$

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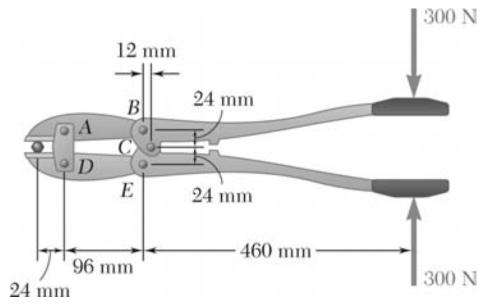
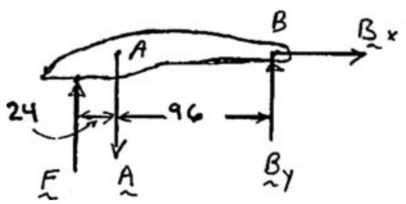


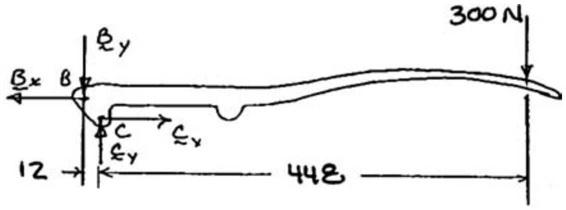
PROBLEM 6.146

In using the bolt cutter shown, a worker applies two 300-N forces to the handles. Determine the magnitude of the forces exerted by the cutter on the bolt.

SOLUTION

FBD cutter *AB*: **FBD handle *BC*:**





I Dimensions in mm **II**

FBD I: $\longrightarrow \Sigma F_x = 0$: $B_x = 0$

FBD II: $\circlearrowleft \Sigma M_C = 0$: $(12\text{ mm})B_y - (448\text{ mm})300\text{ N} = 0$

$$B_y = 11{,}200.0\text{ N}$$

Then FBD I: $\circlearrowleft \Sigma M_A = 0$: $(96\text{ mm})B_y - (24\text{ mm})F = 0$ $\quad F = 4B_y$

$$F = 44{,}800\text{ N} = 44.8\text{ kN} \blacktriangleleft$$

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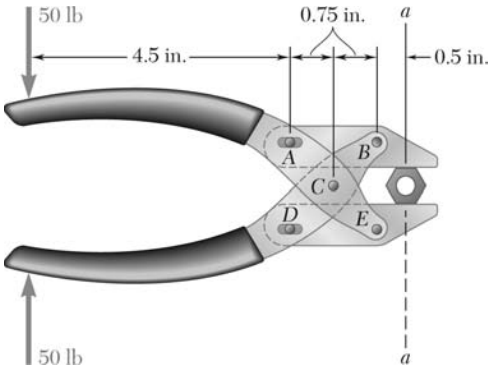
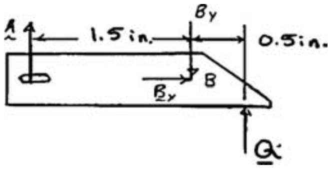


PROBLEM 6.147

Determine the magnitude of the gripping forces exerted along line *aa* on the nut when two 50-lb forces are applied to the handles as shown. Assume that pins *A* and *D* slide freely in slots cut in the jaws.

SOLUTION

FBD jaw *AB*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

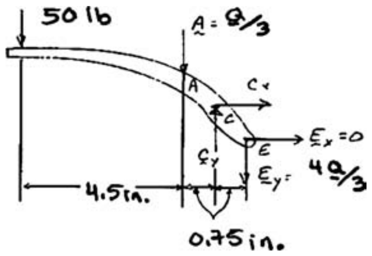
$$\circlearrowleft \Sigma M_B = 0: \quad (0.5 \text{ in.})Q - (1.5 \text{ in.})A = 0$$

$$A = \frac{Q}{3}$$

$$+\uparrow \Sigma F_y = 0: \quad A + Q - B_y = 0$$

$$B_y = A + Q = \frac{4Q}{3}$$

FBD handle *ACE*: By symmetry and FBD jaw *DE*:



$$D = A = \frac{Q}{3}$$

$$E_x = B_x = 0$$

$$E_y = B_y = \frac{4Q}{3}$$

$$\circlearrowleft \Sigma M_C = 0: \quad (5.25 \text{ in.})(50 \text{ lb}) + (0.75 \text{ in.})\frac{Q}{3} - (0.75 \text{ in.})\frac{4Q}{3} = 0$$

$$Q = 350 \text{ lb} \blacktriangleleft$$

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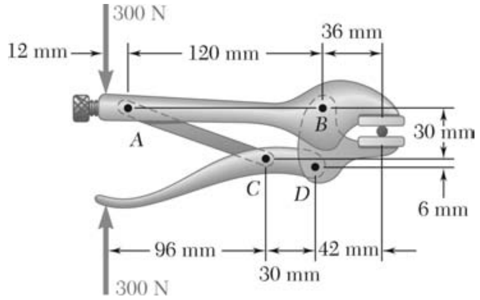


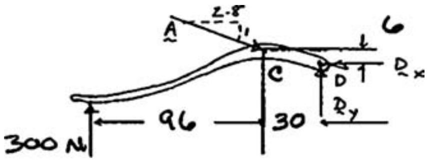
PROBLEM 6.148

Determine the magnitude of the gripping forces produced when two 300-N forces are applied as shown.

SOLUTION

We note that *AC* is a two-force member

FBD handle *CD*:

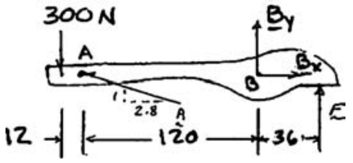


Dimensions in mm

$$\circlearrowleft \Sigma M_D = 0: \quad -(126 \text{ mm})(300 \text{ N}) - (6 \text{ mm})\frac{2.8}{\sqrt{8.84}}A + (30 \text{ mm})\left(\frac{1}{\sqrt{8.84}}A\right) = 0$$

$$A = 2863.6\sqrt{8.84} \text{ N}$$

FBD handle *AB*:



Dimensions in mm

$$\circlearrowleft \Sigma M_B = 0: \quad (132 \text{ mm})(300 \text{ N}) - (120 \text{ mm})\frac{1}{\sqrt{8.84}}(2863.6\sqrt{8.84} \text{ N}) + (36 \text{ mm})F = 0$$

$$F = 8.45 \text{ kN} \blacktriangleleft$$

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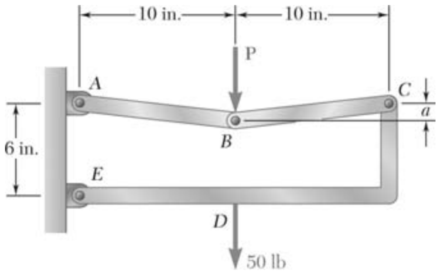
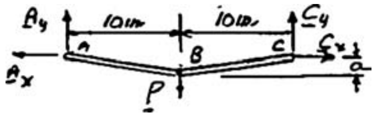


PROBLEM 6.149

Knowing that the frame shown has a sag at *B* of $a = 1$ in., determine the force **P** required to maintain equilibrium in the position shown.

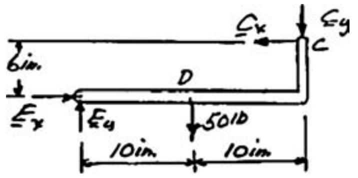
SOLUTION

We note that *AB* and *BC* are two-force members



Free body: Toggle

By symmetry:
$$C_y = \frac{P}{2}$$
$$\frac{C_x}{10\text{ in.}} = \frac{C_y}{a}$$
$$C_x = \frac{10}{a}C_y = \frac{10}{a}\cdot\frac{P}{2} = \frac{5P}{a}$$



Free body: Member *CDE*

$$+\circlearrowleft \Sigma M_E = 0: \quad C_x(6\text{ in.}) - C_y(20\text{ in.}) - (50\text{ lb})(10\text{ in.}) = 0$$
$$\frac{5P}{a}(b) - \frac{P}{2}(20) = 500$$
$$P\left(\frac{30}{a} - 10\right) = 500 \qquad (1)$$

For
$$a = 1.0\text{ in.}$$
$$P\left(\frac{30}{1} - 10\right) = 500$$
$$20P = 500$$

$\mathbf{P} = 25.0$ lb $\downarrow$ ◀

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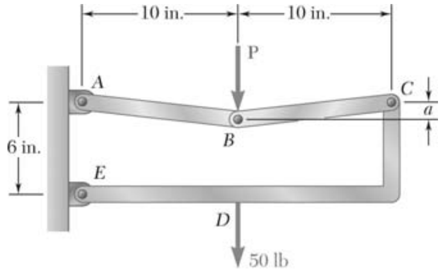
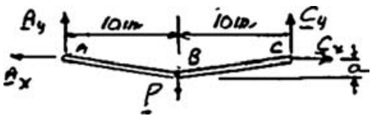


PROBLEM 6.150

Knowing that the frame shown has a sag at B of $a = 0.5$ in., determine the force **P** required to maintain equilibrium in the position shown.

SOLUTION

We note that AB and BC are two-force members



Free body: Toggle

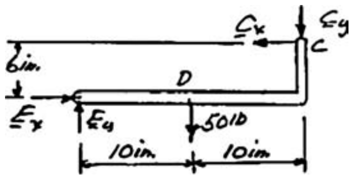
By symmetry:

$$C_y = \frac{P}{2}$$

$$\frac{C_x}{10\text{ in.}} = \frac{C_y}{a}$$

$$C_x = \frac{10}{a}C_y = \frac{10}{a}\cdot\frac{P}{2} = \frac{5P}{a}$$

Free body: Member CDE



$$+\circlearrowleft \Sigma M_E = 0: \quad C_x(6\text{ in.}) - C_y(20\text{ in.}) - (50\text{ lb})(10\text{ in.}) = 0$$

$$\frac{5P}{a}(6) - \frac{P}{2}(20) = 500$$

$$P\left(\frac{30}{a} - 10\right) = 500 \qquad (1)$$

For

$$a = 0.5\text{ in.}$$

$$P\left(\frac{30}{0.5} - 10\right) = 500$$

$$50P = 500$$

$\mathbf{P} = 10.00\text{ lb}\downarrow$ ◀

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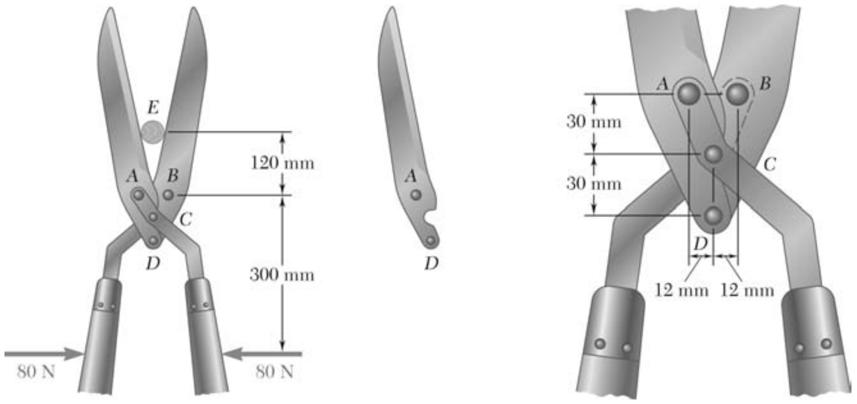
PROBLEM 6.151

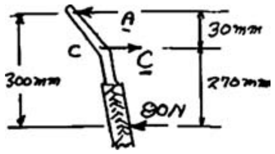
The garden shears shown consist of two blades and two handles. The two handles are connected by pin C and the two blades are connected by pin D. The left blade and the right handle are connected by pin A; the right blade and the left handle are connected by pin B. Determine the magnitude of the forces exerted on the small branch at E when two 80-N forces are applied to the handles as shown.



SOLUTION

By symmetry vertical components C_y, D_y, E_y are 0. Then by considering $\Sigma F_y = 0$ on the blades or handles, we find that A_y and B_y are 0.

Thus forces at A, B, C, D, and E are horizontal.



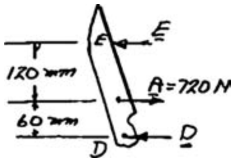
Free body: Right handle

$$+\circlearrowleft \Sigma M_C = 0: \quad A(30 \text{ mm}) - (80 \text{ N})(270 \text{ mm}) = 0$$

$$A = +720 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C - 720 \text{ N} - 80 \text{ N} = 0$$

$$C = +800 \text{ N}$$



Free body: Left blade

$$+\circlearrowleft \Sigma M_D = 0: \quad E(180 \text{ mm}) - (720 \text{ N})(60 \text{ mm}) = 0$$

$$E = 240 \text{ N} \blacktriangleleft$$

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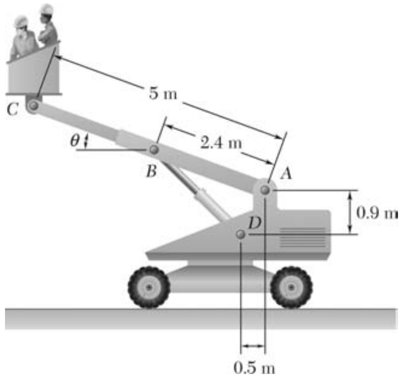
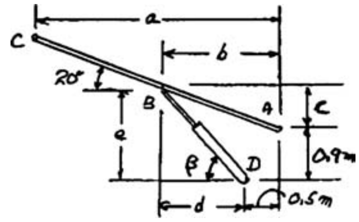


PROBLEM 6.152

The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The workers and the platform together have a mass of 200 kg and have a combined center of gravity located directly above *C*. For the position when $\theta = 20°$, determine (*a*) the force exerted at *B* by the single hydraulic cylinder *BD*, (*b*) the force exerted on the supporting carriage at *A*.

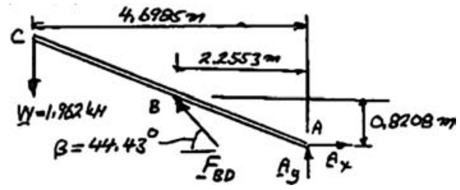
SOLUTION



Geometry:

$$a = (5\text{ m})\cos 20° = 4.6985\text{ m}$$
$$b = (2.4\text{ m})\cos 20° = 2.2553\text{ m}$$
$$c = (2.4\text{ m})\sin 20° = 0.8208\text{ m}$$
$$d = b - 0.5 = 1.7553\text{ m}$$
$$e = c + 0.9 = 1.7208\text{ m}$$

$$\tan\beta = \frac{e}{d} = \frac{1.7208}{1.7553}; \quad \beta = 44.43°$$



Free body: Arm *ABC*

We note that *BD* is a two-force member

$$W = (200\text{ kg})(9.81\text{ m/s}^2) = 1.962\text{ kN}$$

(*a*) $+\circlearrowleft \Sigma M_A = 0: \quad (1.962\text{ kN})(4.6985\text{ m}) - F_{BD}\sin 44.43°(2.2553\text{ m}) + F_{BD}\cos 44.43(0.8208\text{ m}) = 0$

$$9.2185 - F_{BD}(0.9927) = 0: \quad F_{BD} = 9.2867\text{ kN}$$

$$\mathbf{F}_{BD} = 9.29\text{ kN} \searrow 44.4° \blacktriangleleft$$

(*b*)

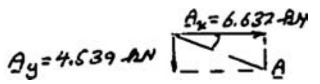
$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - F_{BD}\cos\beta = 0$

$$A_x = (9.2867\text{ kN})\cos 44.43° = 6.632\text{ kN} \qquad \mathbf{A}_x = 6.632\text{ kN} \longrightarrow$$

$+\uparrow \Sigma F_y = 0: \quad A_y - 1.962\text{ kN} + F_{BD}\sin\beta = 0$

$$A_y = 1.962\text{ kN} - (9.2867\text{ kN})\sin 44.43° = -4.539\text{ kN}$$

$$\mathbf{A}_y = 4.539\text{ kN} \downarrow$$



$$\mathbf{A} = 8.04\text{ kN} \nwarrow 34.4° \blacktriangleleft$$

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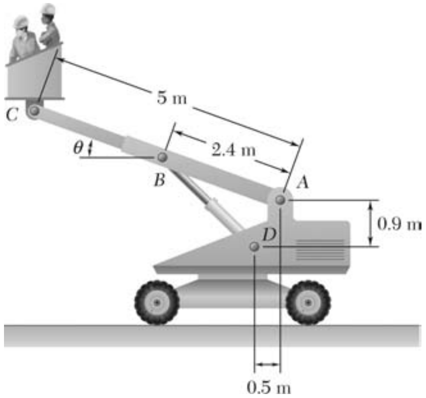
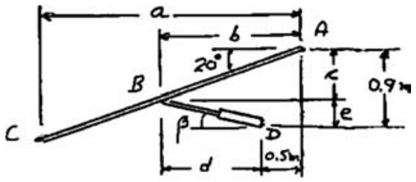


PROBLEM 6.153

The telescoping arm *ABC* can be lowered until end *C* is close to the ground, so that workers can easily board the platform. For the position when $\theta = -20°$, determine (*a*) the force exerted at *B* by the single hydraulic cylinder *BD*, (*b*) the force exerted on the supporting carriage at *A*.

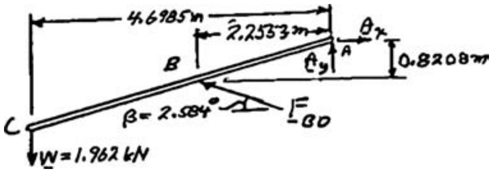
SOLUTION



Geometry:

$$a = (5\text{ m})\cos 20° = 4.6985\text{ m}$$
$$b = (2.4\text{ m})\cos 20° = 2.2552\text{ m}$$
$$c = (2.4\text{ m})\sin 20° = 0.8208\text{ m}$$
$$d = b - 0.5 = 1.7553\text{ m}$$
$$e = 0.9 - c = 0.0792\text{ m}$$

$$\tan\beta = \frac{e}{d} = \frac{0.0792}{1.7552}; \quad \beta = 2.584°$$



Free body: Arm *ABC*

We note that *BD* is a two-force member

$$W = (200\text{ kg})(9.81\text{ m/s}^2)$$
$$W = 1962\text{ N} = 1.962\text{ kN}$$

(*a*) $+\circlearrowright \Sigma M_A = 0: \quad (1.962\text{ kN})(4.6985\text{ m}) - F_{BD}\sin 2.584°(2.2553\text{ m}) - F_{BD}\cos 2.584°(0.8208\text{ m}) = 0$

$$9.2185 - F_{BD}(0.9216) = 0 \quad F_{BD} = 10.003\text{ kN}$$

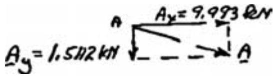
(*b*) $\mathbf{F}_{BD} = 10.00\text{ kN} \searrow 2.58°$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - F_{BD}\cos\beta = 0$

$$A_x = (10.003\text{ kN})\cos 2.583° = 9.993\text{ kN} \qquad \mathbf{A}_x = 9.993\text{ kN} \longrightarrow$$

$+\uparrow \Sigma F_y = 0: \quad A_y - 1.962\text{ kN} + F_{BD}\sin\beta = 0$

$$A_y = 1.962\text{ kN} - (10.003\text{ kN})\sin 2.583° = -1.5112\text{ kN}$$



$\mathbf{A}_y = 1.5112\text{ kN}\downarrow$

$\mathbf{A} = 10.11\text{ kN} \nwarrow 8.60°$ ◀

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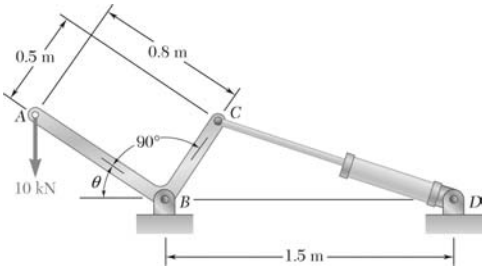
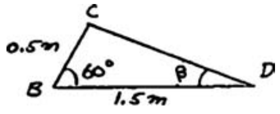


PROBLEM 6.154

The position of member *ABC* is controlled by the hydraulic cylinder *CD*. Knowing that $\theta = 30°$, determine for the loading shown (*a*) the force exerted by the hydraulic cylinder on pin *C*, (*b*) the reaction at *B*.

SOLUTION



Geometry: In ΔBCD

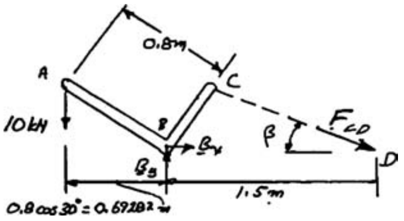
Law of cosines $\quad (CD)^2 = (0.5)^2 + (1.5)^2 - 2(0.5)(1.5)\cos 60°$

$$CD = 1.3229 \text{ m}$$

Law of sines

$$\frac{\sin\beta}{0.5 \text{ m}} = \frac{\sin 60°}{1.3229 \text{ m}}$$

$$\sin\beta = 0.3273 \qquad \beta = 19.107°$$



Free body: Entire system

Move force F_{CD} along its line of action so it acts at *D*.

(*a*) $\qquad +\circlearrowleft \Sigma M_B = 0: \quad (10 \text{ kN})(0.69282 \text{ m}) - F_{CD}\sin\beta(1.5 \text{ m}) = 0$

$$6.9282 \text{ kN}\cdot\text{m} - F_{CD}\sin 19.107°(1.5 \text{ m}) = 0$$

$$F_{CD} = 14.111 \text{ kN}$$

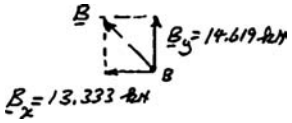
$$\mathbf{F}_{CD} = 14.11 \text{ kN} \searrow 19.11° \blacktriangleleft$$

(*b*) $\qquad \xrightarrow{+} \Sigma F_x = 0: \quad B_x + F_{CD}\cos\beta = 0$

$$B_x + (14.111 \text{ kN})\cos 19.107° = 0$$

$$B_x = -13.333 \text{ kN} \qquad \mathbf{B}_x = 13.333 \text{ kN} \longleftarrow$$

$+\uparrow \Sigma F_y = 0: \quad B_y - 10 \text{ kN} - F_{CD}\sin 19.107° = 0$



$$B_y - 10 \text{ kN} - (14.111 \text{ kN})\sin 19.107° = 0$$

$$B_y = +14.619 \text{ kN} \qquad \mathbf{B}_y = 14.619 \text{ kN} \uparrow$$

$$\mathbf{B} = 19.79 \text{ kN} \nwarrow 47.6° \blacktriangleleft$$

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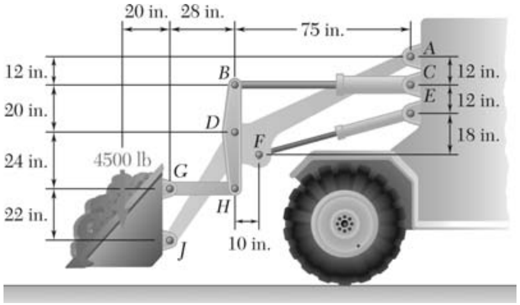
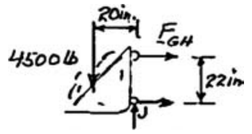


PROBLEM 6.155

The motion of the bucket of the front-end loader shown is controlled by two arms and a linkage that are pin-connected at *D*. The arms are located symmetrically with respect to the central, vertical, and longitudinal plane of the loader; one arm *AFJ* and its control cylinder *EF* are shown. The single linkage *GHDB* and its control cylinder *BC* are located in the plane of symmetry. For the position and loading shown, determine the force exerted (*a*) by cylinder *BC*, (*b*) by cylinder *EF*.

SOLUTION

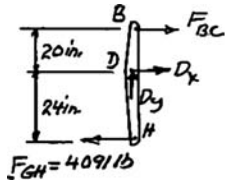
Free body: Bucket



$$+\circlearrowleft \Sigma M_J = 0: \quad (4500 \text{ lb})(20 \text{ in.}) - F_{GH}(22 \text{ in.}) = 0$$

$$F_{GH} = 4091 \text{ lb}$$

Free body: Arm *BDH*



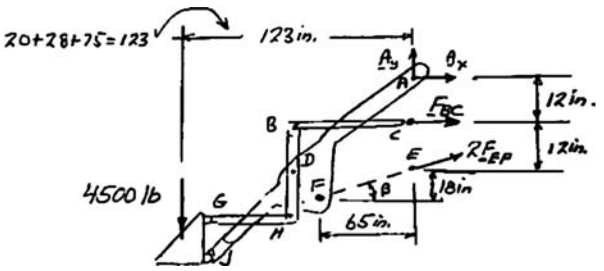
$$+\circlearrowleft \Sigma M_D = 0: \quad -(4091 \text{ lb})(24 \text{ in.}) - F_{BC}(20 \text{ in.}) = 0$$

$$F_{BC} = -4909 \text{ lb}$$

$$F_{BC} = 4.91 \text{ kips} \quad C \blacktriangleleft$$

Free body: Entire mechanism

(Two arms and cylinders *AFJE*)



Note: Two arms thus $2F_{EF}$

$$\tan\beta = \frac{18 \text{ in.}}{65 \text{ in.}}$$

$$\beta = 15.48°$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (4500 \text{ lb})(123 \text{ in.}) + F_{BC}(12 \text{ in.}) + 2F_{EF}\cos\beta(24 \text{ in.}) = 0$$

$$(4500 \text{ lb})(123 \text{ in.}) - (4909 \text{ lb})(12 \text{ in.}) + 2F_{EF}\cos 15.48°(24 \text{ in.}) = 0$$

$$F_{EF} = -10.690 \text{ lb} \qquad F_{EF} = 10.69 \text{ kips} \quad C \blacktriangleleft$$

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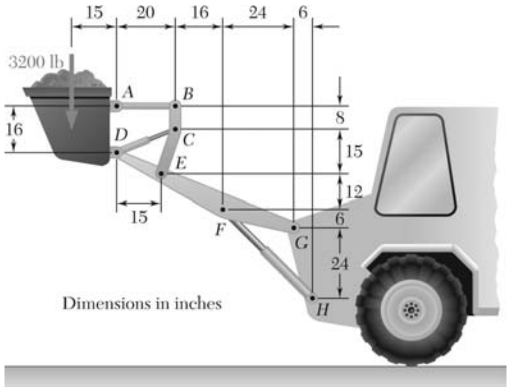
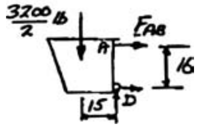


PROBLEM 6.156

The bucket of the front-end loader shown carries a 3200-lb load. The motion of the bucket is controlled by two identical mechanisms, only one of which is shown. Knowing that the mechanism shown supports one-half of the 3200-lb load, determine the force exerted (*a*) by cylinder *CD*, (*b*) by cylinder *FH*.

SOLUTION

Free body: Bucket (One mechanism)

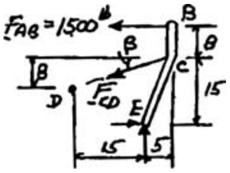


$$+\circlearrowleft \Sigma M_D = 0: \quad (1600 \text{ lb})(15 \text{ in.}) - F_{AB}(16 \text{ in.}) = 0$$

$$F_{AB} = 1500 \text{ lb}$$

Note: There are 2 identical support mechanisms.

Free body: One arm *BCE*



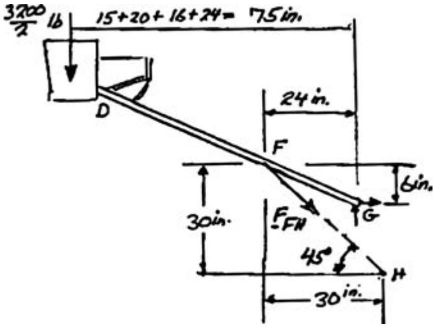
$$\tan \beta = \frac{8}{20}$$

$$\beta = 21.8°$$

$$+\circlearrowleft \Sigma M_E = 0: \quad (1500 \text{ lb})(23 \text{ in.}) + F_{CD} \cos 21.8°(15 \text{ in.}) - F_{CD} \sin 21.8°(5 \text{ in.}) = 0$$

$$F_{CD} = -2858 \text{ lb} \qquad F_{CD} = 2.86 \text{ kips} \quad C \blacktriangleleft$$

Free body: Arm *DFG*



$$+\circlearrowleft \Sigma M_G = 0: \quad (1600 \text{ lb})(75 \text{ in.}) + F_{FH} \sin 45°(24 \text{ in.}) - F_{FH} \cos 45°(6 \text{ in.}) = 0$$

$$F_{FH} = -9.428 \text{ kips} \qquad F_{FH} = 9.43 \text{ kips} \quad C \blacktriangleleft$$

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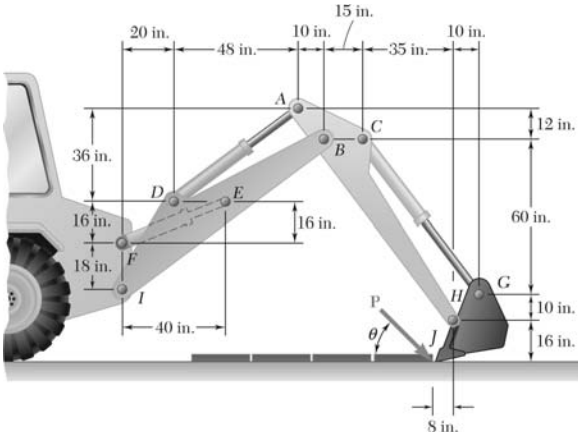
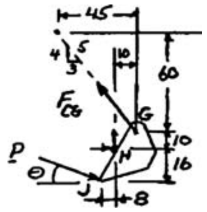


PROBLEM 6.157

The motion of the backhoe bucket shown is controlled by the hydraulic cylinders *AD*, *CG*, and *EF*. As a result of an attempt to dislodge a portion of a slab, a 2-kip force **P** is exerted on the bucket teeth at *J*. Knowing that $\theta = 45°$, determine the force exerted by each cylinder.

SOLUTION

<u>Free body: Bucket</u>



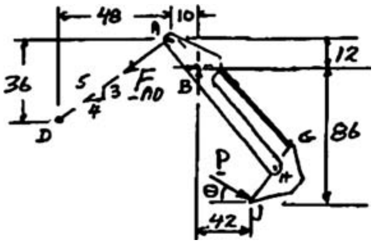
$$+\circlearrowleft \Sigma M_H = 0 \quad \text{(Dimensions in inches)}$$

$$\frac{4}{5}F_{CG}(10) + \frac{3}{5}F_{CG}(10) + P\cos\theta(16) + P\sin\theta(8) = 0$$

$$F_{CG} = -\frac{P}{14}(16\cos\theta + 8\sin\theta) \qquad (1)$$

<u>Free body: Arm *ABH* and bucket</u>

(Dimensions in inches)



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{4}{5}F_{AD}(12) + \frac{3}{5}F_{AD}(10) + P\cos\theta(86) - P\sin\theta(42) = 0$$

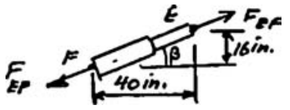
$$F_{AD} = -\frac{P}{15.6}(86\cos\theta - 42\sin\theta) \qquad (2)$$

<u>Free body: Bucket and arms *IEB* + *ABH*</u>

Geometry of cylinder *EF*

$$\tan\beta = \frac{16\text{ in.}}{40\text{ in.}}$$

$$\beta = 21.801°$$



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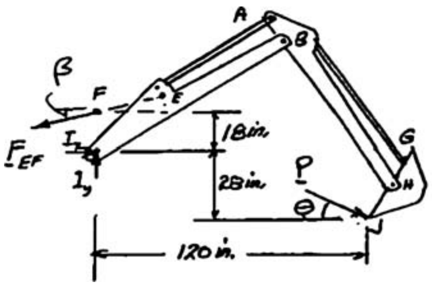
PROBLEM 6.157 (Continued)



$$+\circlearrowleft \Sigma M_I = 0: \quad F_{EF}\cos\beta(18 \text{ in.}) + P\cos\theta(28 \text{ in.}) - P\sin\theta(120 \text{ in.}) = 0$$

$$F_{EF} = \frac{P(120\sin\theta - 28\cos\theta)}{\cos 21.8°(18)}$$

$$= \frac{P}{16.7126}(120\sin\theta - 28\cos\theta) \qquad (3)$$

For $P = 2$ kips, $\theta = 45°$

Eq. (1): $F_{CG} = -\frac{2}{14}(16\cos 45° + 8\sin 45°) = -2.42$ kips $\qquad F_{CG} = 2.42$ kips C ◀

Eq. (2): $F_{AD} = -\frac{2}{15.6}(86\cos 45° - 42\sin 45°) = -3.99$ kips $\qquad F_{AD} = 3.99$ kips C ◀

Eq. (3): $F_{EF} = \frac{2}{16.7126}(120\sin 45° - 28\cos 45°) = +7.79$ kips $\qquad F_{EF} = 7.79$ kips T ◀

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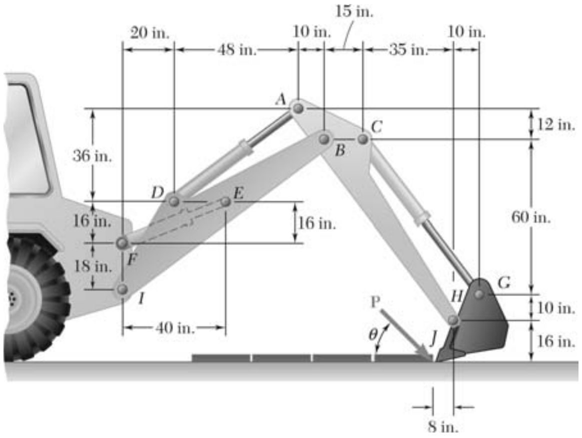


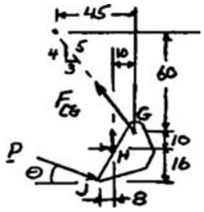
PROBLEM 6.158

Solve Problem 6.157 assuming that the 2-kip force **P** acts horizontally to the right ($\theta = 0$).

PROBLEM 6.157 The motion of the backhoe bucket shown is controlled by the hydraulic cylinders *AD*, *CG*, and *EF*. As a result of an attempt to dislodge a portion of a slab, a 2-kip force **P** is exerted on the bucket teeth at *J*. Knowing that $\theta = 45°$, determine the force exerted by each cylinder.

SOLUTION

Free body: Bucket



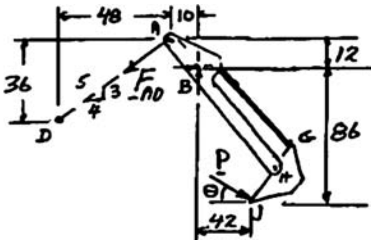
$$+\circlearrowleft \Sigma M_H = 0 \quad \text{(Dimensions in inches)}$$

$$\frac{4}{5}F_{CG}(10) + \frac{3}{5}F_{CG}(10) + P\cos\theta(16) + P\sin\theta(8) = 0$$

$$F_{CG} = -\frac{P}{14}(16\cos\theta + 8\sin\theta) \qquad (1)$$

Free body: Arm *ABH* and bucket

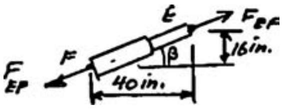
(Dimensions in inches)



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{4}{5}F_{AD}(12) + \frac{3}{5}F_{AD}(10) + P\cos\theta(86) - P\sin\theta(42) = 0$$

$$F_{AD} = -\frac{P}{15.6}(86\cos\theta - 42\sin\theta) \qquad (2)$$

Free body: Bucket and arms *IEB* + *ABH*



Geometry of cylinder *EF* $\qquad \tan\beta = \dfrac{16\text{ in.}}{40\text{ in.}}$

$$\beta = 21.801°$$

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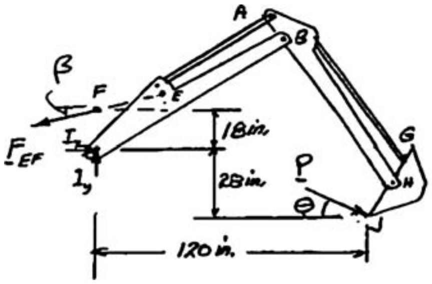
PROBLEM 6.158 (Continued)



$$+\circlearrowleft \Sigma M_I = 0: \quad F_{EF}\cos\beta(18\text{ in.}) + P\cos\theta(28\text{ in.}) - P\sin\theta(120\text{ in.}) = 0$$

$$F_{EF} = \frac{P(120\sin\theta - 28\cos\theta)}{\cos 21.8°(18)}$$

$$= \frac{P}{16.7126}(120\sin\theta - 28\cos\theta) \qquad (3)$$

For $P = 2$ kips, $\theta = 0$

Eq. (1): $F_{CG} = -\frac{2}{14}(16\cos 0 + 8\sin 0) = -2.29$ kips $\qquad F_{CG} = 2.29$ kips C ◀

Eq. (2): $F_{AD} = -\frac{2}{15.6}(86\cos 0 - 42\sin 0) = -11.03$ kips $\qquad F_{AD} = 11.03$ kips C ◀

Eq. (3): $F_{EF} = \frac{2}{16.7126}(120\sin 0 - 28\cos 0) = -3.35$ kips $\qquad F_{EF} = 3.35$ kips C ◀

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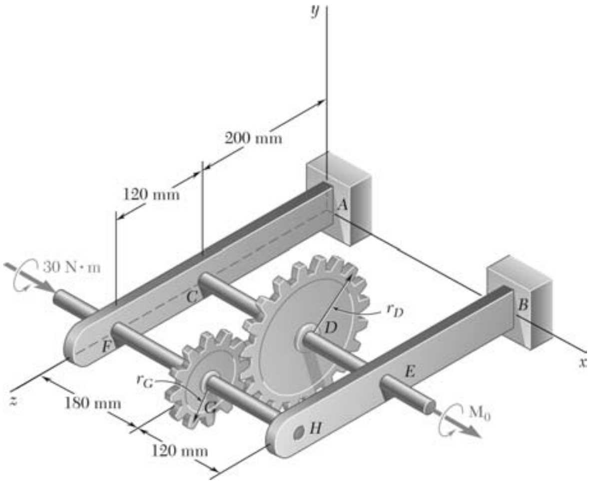
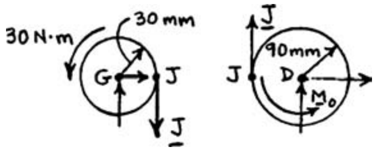


PROBLEM 6.159

The gears D and G are rigidly attached to shafts that are held by frictionless bearings. If $r_D = 90$ mm and $r_G = 30$ mm, determine (*a*) the couple $\mathbf{M}_0$ that must be applied for equilibrium, (*b*) the reactions at A and B.

SOLUTION



(*a*) Projections on *yz* plane

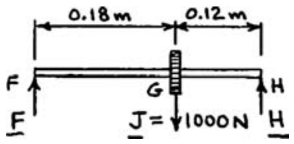
Free body: Gear *G*

$$+\circlearrowleft \Sigma M_G = 0: \quad 30 \text{ N}\cdot\text{m} - J(0.03 \text{ m}) = 0; \quad J = 1000 \text{ N}$$

Free body: Gear *D*

$$+\circlearrowleft \Sigma M_D = 0: \quad M_0 - (1000 \text{ N})(0.09 \text{ m}) = 0$$

$$M_0 = 90 \text{ N}\cdot\text{m} \qquad \mathbf{M}_0 = (90.0 \text{ N}\cdot\text{m})\mathbf{i} \blacktriangleleft$$



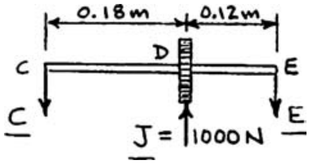
(*b*) Gear *G* and axle *FH*

$$+\circlearrowleft \Sigma M_F = 0: \quad H(0.3 \text{ m}) - (1000 \text{ N})(0.18 \text{ m}) = 0$$

$$H = 600 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad F + 600 - 1000 = 0$$

$$F = 400 \text{ N}$$



Gear *D* and axle *CE*

$$+\circlearrowleft \Sigma M_C = 0: \quad (1000 \text{ N})(0.18 \text{ m}) - E(0.3 \text{ m}) = 0$$

$$E = 600 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad 1000 - C - 600 = 0$$

$$C = 400 \text{ N}$$

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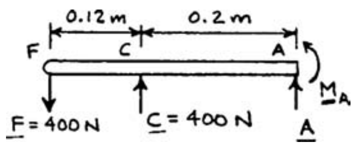
PROBLEM 6.159 (Continued)

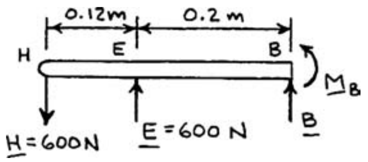


Free body: Bracket *AE*

$+\uparrow \Sigma F_y = 0: \quad A - 400 + 400 = 0$ $\qquad \mathbf{A} = 0$ ◀

$+\circlearrowleft \Sigma M_A = 0: \quad M_A + (400\text{ N})(0.32\text{ m}) - (400\text{ N})(0.2\text{ m}) = 0$

$M_A = -48\text{ N}\cdot\text{m} \qquad \mathbf{M}_A = -(48.0\text{ N}\cdot\text{m})\mathbf{i}$ ◀



Free body: Bracket *BH*

$+\uparrow \Sigma F_y = 0: \quad B - 600 + 600 = 0$ $\qquad \mathbf{B} = 0$ ◀

$+\circlearrowleft \Sigma M_B = 0: \quad M_B + (600\text{ N})(0.32\text{ m}) - (600\text{ N})(0.2\text{ m}) = 0$

$M_B = -72\text{ N}\cdot\text{m} \qquad \mathbf{M}_B = -(72.0\text{ N}\cdot\text{m})\mathbf{i}$ ◀

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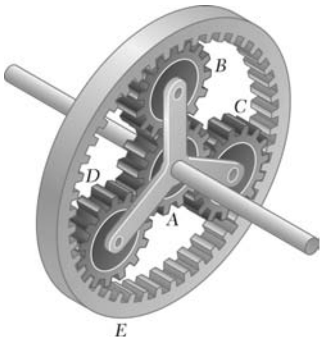
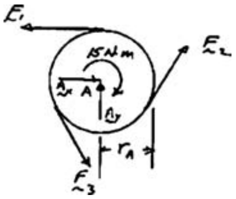


PROBLEM 6.160

In the planetary gear system shown, the radius of the central gear A is $a = 18$ mm, the radius of each planetary gear is b, and the radius of the outer gear E is $(a + 2b)$. A clockwise couple of magnitude $M_A = 10$ N · m is applied to the central gear A and a counterclockwise couple of magnitude $M_S = 50$ N · m is applied to the spider BCD. If the system is to be in equilibrium, determine (a) the required radius b of the planetary gears, (b) the magnitude M_E of the couple that must be applied to the outer gear E.

SOLUTION

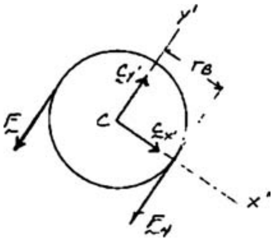
FBD Central Gear:



By symmetry: $F_1 = F_2 = F_3 = F$

$$\circlearrowleft \Sigma M_A = 0: \quad 3(r_A F) - 10 \text{ N}\cdot\text{m} = 0, \qquad F = \frac{10}{3r_A}\text{N}\cdot\text{m}$$

FBD Gear *C*:



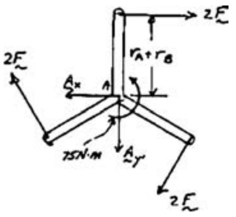
$$\circlearrowleft \Sigma M_C = 0: \quad r_B(F - F_4) = 0, \qquad F_4 = F$$

$$\searrow \Sigma F_{x'} = 0: \quad C_{x'} = 0$$

$$\nearrow \Sigma F_{y'} = 0: \quad C_{y'} - 2F = 0, \qquad C_{y'} = 2F$$

Gears B and D are analogous, each having a central force of $2F$

FBD Spider:



$$\circlearrowleft \Sigma M_A = 0: \quad 50 \text{ N}\cdot\text{m} - 3(r_A + r_B)2F = 0$$

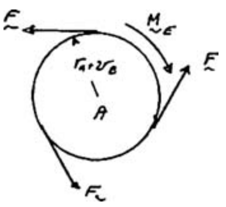
$$50 \text{ N}\cdot\text{m} - 3(r_A + r_B)\frac{20}{r_A}\text{N}\cdot\text{m} = 0$$

$$\frac{r_A + r_B}{r_A} = 2.5 = 1 + \frac{r_B}{r_A}, \qquad r_B = 1.5 r_A$$

Since $r_A = 18$ mm,

(a) $r_B = 27.0$ mm ◀

FBD Outer Gear:



$$\circlearrowleft \Sigma M_A = 0: \quad 3(r_A + 2r_B)F - M_E = 0$$

$$3(18 \text{ mm} + 54 \text{ mm})\frac{10 \text{ N}\cdot\text{m}}{54 \text{ mm}} - M_E = 0$$

(b) $\mathbf{M}_E = 40.0 \text{ N}\cdot\text{m} \circlearrowright$ ◀

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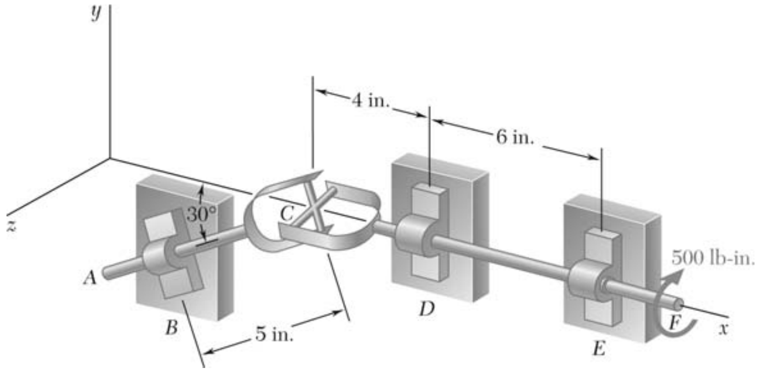
PROBLEM 6.161*

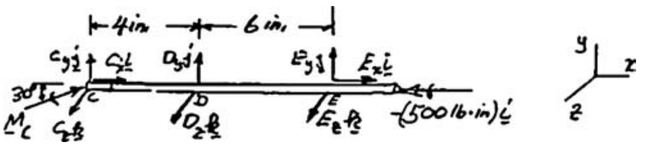
Two shafts *AC* and *CF*, which lie in the vertical *xy* plane, are connected by a universal joint at *C*. The bearings at *B* and *D* do not exert any axial force. A couple of magnitude 500 lb · in. (clockwise when viewed from the positive *x* axis) is applied to shaft *CF* at *F*. At a time when the arm of the crosspiece attached to shaft *CF* is horizontal, determine (*a*) the magnitude of the couple that must be applied to shaft *AC* at *A* to maintain equilibrium, (*b*) the reactions at *B*, *D*, and *E*. (*Hint:* The sum of the couples exerted on the crosspiece must be zero.)



SOLUTION

We recall from Figure 4.10, that a universal joint exerts on members it connects a force of unknown direction and a couple about an axis perpendicular to the crosspiece.

<u>Free body: Shaft *DF*</u>

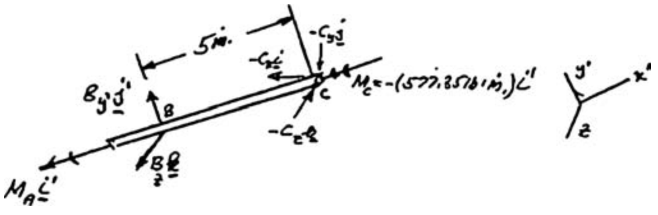


$$\Sigma M_x = 0: \quad M_C \cos 30° - 500 \text{ lb} \cdot \text{in.} = 0$$

$$M_C = 577.35 \text{ lb} \cdot \text{in.}$$

<u>Free body: Shaft *BC*</u>

We use here x', y', z with x' along BC



$$\Sigma M_C = 0: \quad -M_R i' - (577.35 \text{ lb} \cdot \text{in}) i' + (-5 \text{ in}) i' \times (B_y j' + B_z k) = 0$$

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PROBLEM 6.161* (Continued)

Equate coefficients of unit vectors to zero:

i: $M_A - 577.35 \text{ lb}\cdot\text{in} = 0$ $\qquad M_A = 577.35 \text{ lb}\cdot\text{in.}$

j: $B_z = 0$ $\qquad M_A = 577 \text{ lb}\cdot\text{in.}$ ◀

k: $B_y = 0$ → $B = 0$ $\qquad \mathbf{B} = 0$

$\Sigma\mathbf{F} = 0\!: \quad B + C = 0, \quad \text{since } B = 0,$ $\qquad \mathbf{C} = 0$

<u>Return to free body of shaft *DF*</u>

$\Sigma\mathbf{M}_D = 0$ (Note that $C = 0$ and $M_C = 577.35 \text{ lb}\cdot\text{in.}$)

$$(577.35 \text{ lb}\cdot\text{in.})(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) - (500 \text{ lb}\cdot\text{in.})\mathbf{i}$$

$$+(6 \text{ in.})\mathbf{i} \times (E_x\mathbf{i} + E_y\mathbf{j} + E_z\mathbf{k}) = 0$$

$$(500 \text{ lb}\cdot\text{in.})\mathbf{i} + (288.68 \text{ lb}\cdot\text{in.})\mathbf{j} - (500 \text{ lb}\cdot\text{in.})\mathbf{i}$$

$$+ (6 \text{ in.})E_y\mathbf{k} - (6 \text{ in.})E_z\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero:

j: $288.68 \text{ lb}\cdot\text{in.} - (6 \text{ in.})E_z = 0 \quad E_z = 48.1 \text{ lb}$

k: $E_y = 0$

$\Sigma\mathbf{F} = 0\!: \quad \mathbf{C} + \mathbf{D} + \mathbf{E} = 0$

$0 + D_y\mathbf{j} + D_z\mathbf{k} + E_x\mathbf{i} + (48.1 \text{ lb})\mathbf{k} = 0$

i: $E_x = 0$

j: $D_y = 0$

k: $D_z + 48.1 \text{ lb} = 0 \quad D_z = -48.1 \text{ lb}$

<u>Reactions are</u>:

$\mathbf{B} = 0$ ◀

$\mathbf{D} = -(48.1 \text{ lb})\mathbf{k}$ ◀

$\mathbf{E} = (48.1 \text{ lb})\mathbf{k}$ ◀

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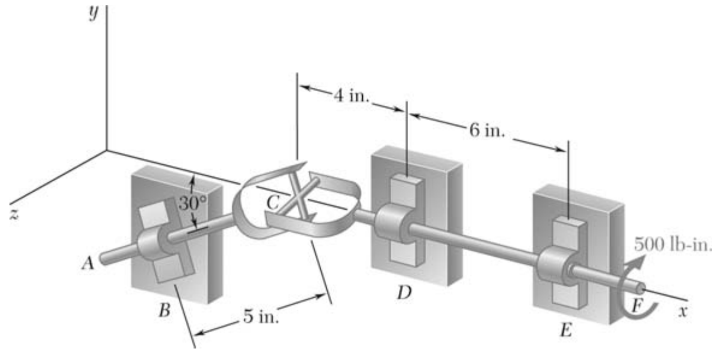
PROBLEM 6.162*

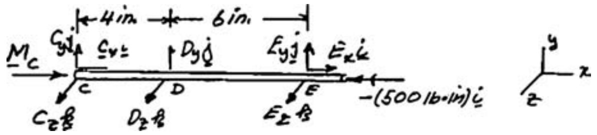
Solve Problem 6.161 assuming that the arm of the crosspiece attached to shaft *CF* is vertical.

PROBLEM 6.161 Two shafts *AC* and *CF*, which lie in the vertical *xy* plane, are connected by a universal joint at *C*. The bearings at *B* and *D* do not exert any axial force. A couple of magnitude 500 lb · in. (clockwise when viewed from the positive *x* axis) is applied to shaft *CF* at *F*. At a time when the arm of the crosspiece attached to shaft *CF* is horizontal, determine (*a*) the magnitude of the couple that must be applied to shaft *AC* at *A* to maintain equilibrium, (*b*) the reactions at *B*, *D*, and *E*. (*Hint:* The sum of the couples exerted on the crosspiece must be zero.)



SOLUTION

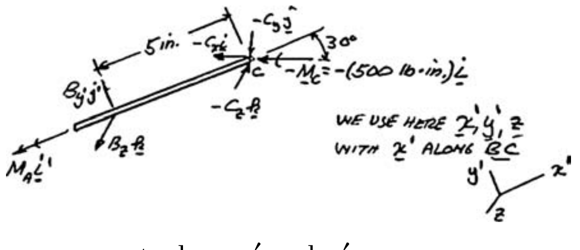
Free body: Shaft *DF*



$$\Sigma M_x = 0: \quad M_C - 500\text{ lb}\cdot\text{in.} = 0$$

$$M_C = 500\text{ lb}\cdot\text{in.}$$

Free body: Shaft *BC*



We resolve $-(520\text{ lb}\cdot\text{in.})\mathbf{i}$ into components along x' and y' axes:

$$-\mathbf{M}_C = -(500\text{ lb}\cdot\text{in.})(\cos 30°\mathbf{i}' + \sin 30°\mathbf{j}')$$

$$\Sigma\mathbf{M}_C = 0: \quad M_A\mathbf{i}' - (500\text{ lb}\cdot\text{in.})(\cos 30°\mathbf{i}' + \sin 30°\mathbf{j}') + (5\text{ in.})\mathbf{i}' \times (B_{y'}\mathbf{j}' + B_z\mathbf{k}) = 0$$

$$M_A\mathbf{i}' - (433\text{ lb}\cdot\text{in.})\mathbf{i}' - (250\text{ lb}\cdot\text{in.})\mathbf{j} + (5\text{ in.})B_{y'}\mathbf{k} - (5\text{ in.})B_z\mathbf{j}' = 0$$

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PROBLEM 6.162* (Continued)

Equate to zero coefficients of unit vectors:

$\mathbf{i}'$: $M_A - 433 \text{ lb}\cdot\text{in.} = 0$ $\qquad M_A = 433 \text{ lb}\cdot\text{in.}$ ◀

$\mathbf{j}'$: $-250 \text{ lb}\cdot\text{in.} - (5 \text{ in.})B_z = 0$ $\qquad B_z = -50 \text{ lb}$

$\mathbf{k}$: $\mathbf{B}_{y'} = 0$

Reactions at B: $\qquad \mathbf{B} = -(50 \text{ lb})\mathbf{k}$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{B} - \mathbf{C} = 0$$

$$-(50 \text{ lb})\mathbf{k} - C = 0 \qquad \mathbf{C} = -(50 \text{ lb})\mathbf{k}$$

Return to free body of shaft DF:

$$\Sigma\mathbf{M}_D = 0: \quad (6 \text{ in.})\mathbf{i} \times (E_x\mathbf{i} + E_y\mathbf{j} + E_z\mathbf{k}) - (4 \text{ in.})\mathbf{i} \times (-50 \text{ lb})\mathbf{k}$$
$$- (500 \text{ lb}\cdot\text{in.})\mathbf{i} + (500 \text{ lb}\cdot\text{in.})\mathbf{i} = 0$$
$$(6 \text{ in.})E_y\mathbf{k} - (6 \text{ in.})E_z\mathbf{j} - (200 \text{ lb}\cdot\text{in.})\mathbf{j} = 0$$

$\mathbf{k}$: $E_y = 0$

$\mathbf{j}$: $-(6 \text{ in.})E_z - 200 \text{ lb}\cdot\text{in.} = 0$ $\qquad E_z = -33.3 \text{ lb}$

$$\Sigma F = 0: \quad \mathbf{C} + \mathbf{D} + \mathbf{E} = 0$$

$$-(50 \text{ lb})\mathbf{k} + D_y\mathbf{j} + D_z\mathbf{k} + E_x\mathbf{i} - (33.3 \text{ lb})\mathbf{k} = 0$$

$\mathbf{i}$: $E_x = 0$

$\mathbf{k}$: $-50 \text{ lb} - 33.3 \text{ lb} + D_z = 0$

$$D_z = 83.3 \text{ lb}$$

Reactions are:

$\mathbf{B} = -(50 \text{ lb})\mathbf{k}$ ◀

$\mathbf{D} = (83.3 \text{ lb})\mathbf{k}$ ◀

$\mathbf{E} = -(33.3 \text{ lb})\mathbf{k}$ ◀

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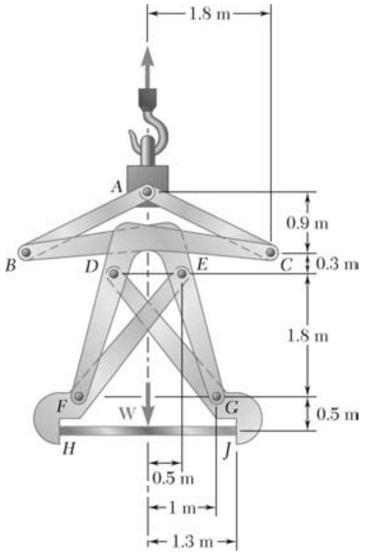
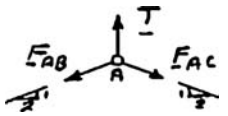


PROBLEM 6.163*

The large mechanical tongs shown are used to grab and lift a thick 7500-kg steel slab *HJ*. Knowing that slipping does not occur between the tong grips and the slab at *H* and *J*, determine the components of all forces acting on member *EFH*. (*Hint:* Consider the symmetry of the tongs to establish relationships between the components of the force acting at *E* on *EFH* and the components of the force acting at *D* on *CDF*.)

SOLUTION



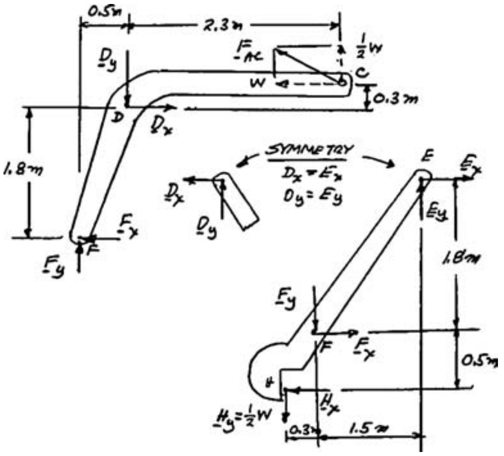
Free body: Pin *A*

$$T = W = mg = (7500\text{ kg})(9.81\text{ m/s}^2) = 73.575\text{ kN}$$

$$\Sigma F_x = 0: \quad (F_{AB})_x = (F_{AC})_x$$

$$\Sigma F_y = 0: \quad (F_{AB})_y = (F_{AC})_y = \frac{1}{2}W$$

Also: $(F_{AC})_x = 2(F_{AC})_y = W$



Free body: Member *CDF*

$$+\circlearrowleft \Sigma M_D = 0: \quad W(0.3) + \frac{1}{2}W(2.3) - F_x(1.8) - F_y(0.5\text{ m}) = 0$$

or $$1.8F_x + 0.5F_y = 1.45W \qquad (1)$$

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PROBLEM 6.163* (Continued)

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x - F_x - W = 0$$

or $$E_x - F_x = W \qquad (2)$$

$$+\uparrow \Sigma F_y = 0: \quad F_y - D_y + \frac{1}{2}W = 0$$

or $$E_y - F_y = \frac{1}{2}W \qquad (3)$$

Free body: Member *EFH*

$$+\circlearrowleft \Sigma M_E = 0: \quad F_x(1.8) + F_y(1.5) - H_x(2.3) + \frac{1}{2}W(1.8\text{ m}) = 0$$

or: $$1.8F_x + 1.5F_y = 2.3H_x - 0.9W \qquad (4)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad E_x + F_x - H_x = 0$$

or $$E_x + F_x = H_x \qquad (5)$$

Subtract (2) from (5): $$2F_x = H_x - W \qquad (6)$$

Subtract (4) from $3 \times (1)$: $$3.6F_x = 5.25W - 2.3H_x \qquad (7)$$

Add (7) to $2.3 \times (6)$: $$8.2F_x = 2.95W$$

$$F_x = 0.35976W \qquad (8)$$

Substitute from (8) into (1):

$$(1.8)(0.35976W) + 0.5F_y = 1.45W$$

$$0.5F_y = 1.45W - 0.64756W = 0.80244W$$

$$F_y = 1.6049W \qquad (9)$$

Substitute from (8) into (2): $E_x - 0.35976W = W; \quad E_x = 1.35976W$

Substitute from (9) into (3): $E_y - 1.6049W = \frac{1}{2}W \quad E_y = 2.1049W$

From (5): $H_x = E_x + F_x = 1.35976W + 0.35976W = 1.71952W$

Recall that: $H_y = \frac{1}{2}W$

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PROBLEM 6.163* (Continued)

Since all expressions obtained are positive, all forces are directed as shown on the free-body diagrams.

Substitute $W = 73.575\text{ kN}$

$\mathbf{E}_x = 100.0\text{ kN} \longrightarrow$ $\mathbf{E}_y = 154.9\text{ kN}\uparrow$ ◀

$\mathbf{F}_x = 26.5\text{ kN} \longrightarrow$ $\mathbf{F}_y = 118.1\text{ kN}\downarrow$ ◀

$\mathbf{H}_x = 126.5\text{ kN} \longleftarrow$ $\mathbf{H}_y = 36.8\text{ kN}\downarrow$ ◀

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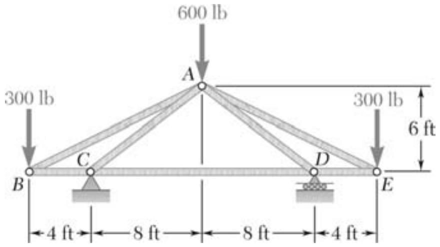


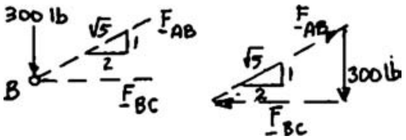
PROBLEM 6.164

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss.

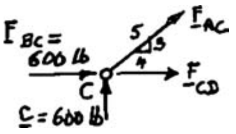
From the symmetry of the truss and loading, we find $\mathbf{C} = \mathbf{D} = 600 \text{ lb}\uparrow$



Free body: Joint *B* $\qquad \dfrac{F_{AB}}{\sqrt{5}} = \dfrac{F_{BC}}{2} = \dfrac{300 \text{ lb}}{1}$

$$F_{AB} = 671 \text{ lb} \quad T \qquad F_{BC} = 600 \text{ lb} \quad C \blacktriangleleft$$

Free body: Joint *C*



$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{AC} + 600 \text{ lb} = 0$$

$$F_{AC} = -1000 \text{ lb} \qquad F_{AC} = 1000 \text{ lb} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}(-1000 \text{ lb}) + 600 \text{ lb} + F_{CD} = 0 \qquad F_{CD} = 200 \text{ lb} \quad T \blacktriangleleft$$

From symmetry:

$$F_{AD} = F_{AC} = 1000 \text{ lb} \quad C, \quad F_{AE} = F_{AB} = 671 \text{ lb} \quad T, \quad F_{DE} = F_{BC} = 600 \text{ lb} \quad C \blacktriangleleft$$

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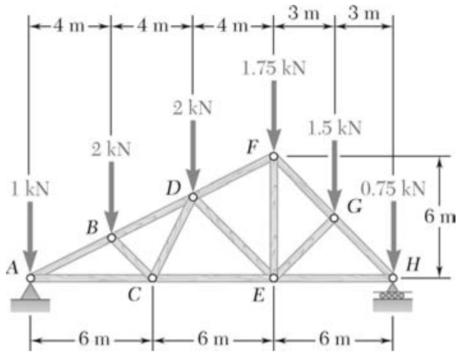
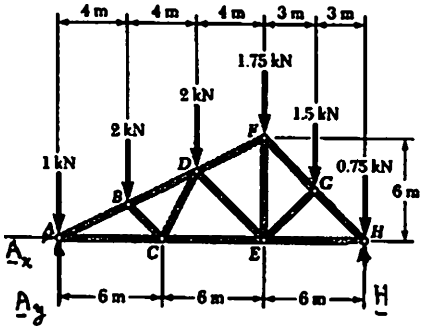


PROBLEM 6.165

Using the method of joints, determine the force in each member of the double-pitch roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss



$$+\circlearrowleft \Sigma M_A = 0: \quad H(18\text{ m}) - (2\text{ kN})(4\text{ m}) - (2\text{ kN})(8\text{ m}) - (1.75\text{ kN})(12\text{ m})$$
$$- (1.5\text{ kN})(15\text{ m}) - (0.75\text{ kN})(18\text{ m}) = 0$$

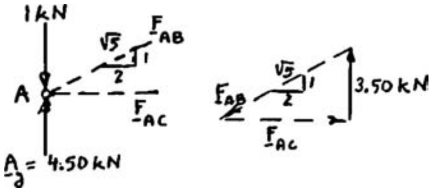
$$\mathbf{H} = 4.50\text{ kN}\uparrow$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$\Sigma F_y = 0: \quad A_y + H - 9 = 0$$

$$A_y = 9 - 4.50, \qquad A_y = 4.50\text{ kN}\uparrow$$

Free body: Joint *A*



$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{AC}}{2} = \frac{3.50\text{ kN}}{1}$$

$$F_{AB} = 7.8262\text{ kN} \quad C \qquad F_{AB} = 7.83\text{ kN} \quad C \blacktriangleleft$$

$$F_{AC} = 7.00\text{ kN} \quad T \blacktriangleleft$$

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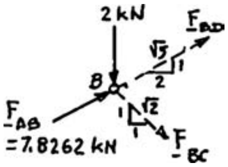
PROBLEM 6.165 (Continued)



Free body: Joint B

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{BD} + \frac{2}{\sqrt{5}}(7.8262 \text{ kN}) + \frac{1}{\sqrt{2}} F_{BC} = 0$$

or $$F_{BD} + 0.79057 F_{BC} = -7.8262 \text{ kN} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{BD} + \frac{1}{\sqrt{5}}(7.8262 \text{ kN}) - \frac{1}{\sqrt{2}} F_{BC} - 2 \text{ kN} = 0$$

or $$F_{BD} - 1.58114 F_{BC} = -3.3541 \qquad (2)$$

Multiply (1) by 2 and add (2):

$$3F_{BD} = -19.0065$$

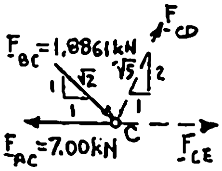
$$F_{BD} = -6.3355 \text{ kN} \qquad F_{BD} = 6.34 \text{ kN} \quad C \blacktriangleleft$$

Subtract (2) from (1):

$$2.37111 F_{BC} = -4.4721$$

$$F_{BC} = -1.8861 \text{ kN} \qquad F_{BC} = 1.886 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint C

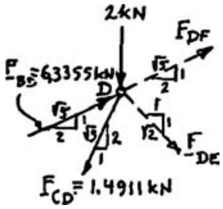


$$+\uparrow \Sigma F_y = 0: \quad \frac{2}{\sqrt{5}} F_{CD} - \frac{1}{\sqrt{2}}(1.8861 \text{ kN}) = 0$$

$$F_{CD} = +1.4911 \text{ kN} \qquad F_{CD} = 1.491 \text{ kN} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} - 7.00 \text{ kN} + \frac{1}{\sqrt{2}}(1.8861 \text{ kN}) + \frac{1}{\sqrt{5}}(1.4911 \text{ kN}) = 0$$

$$F_{CE} = 5.000 \text{ kN} \qquad F_{CE} = 5.00 \text{ kN} \quad T \blacktriangleleft$$



Free body: Joint D

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{DF} + \frac{1}{\sqrt{2}} F_{DE} + \frac{2}{\sqrt{5}}(6.3355 \text{ kN}) - \frac{1}{\sqrt{5}}(1.4911 \text{ kN}) = 0$$

or $$F_{DF} + 0.79057 F_{DE} = -5.5900 \text{ kN} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{DF} - \frac{1}{\sqrt{2}} F_{DE} + \frac{1}{\sqrt{5}}(6.3355 \text{ kN}) - \frac{2}{\sqrt{5}}(1.4911 \text{ kN}) - 2 \text{ kN} = 0$$

or $$F_{DF} - 0.79057 F_{DE} = -1.1188 \text{ kN} \ (2)$$

Add (1) and (2): $$2F_{DF} = -6.7088 \text{ kN}$$

$$F_{DF} = -3.3544 \text{ kN} \qquad F_{DF} = 3.35 \text{ kN} \quad C \blacktriangleleft$$

Subtract (2) from (1): $$1.58114 F_{DE} = -4.4712 \text{ kN}$$

$$F_{DE} = -2.8278 \text{ kN} \qquad F_{DE} = 2.83 \text{ kN} \quad C \blacktriangleleft$$

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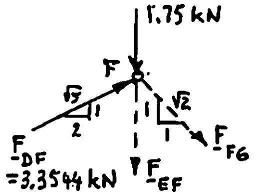
PROBLEM 6.165 (Continued)

Free body: Joint *F*



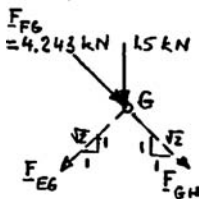
$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{2}} F_{FG} + \frac{2}{\sqrt{5}}(3.3544 \text{ kN}) = 0$$

$$F_{FG} = -4.243 \text{ kN} \qquad\qquad F_{FG} = 4.24 \text{ kN} \quad C \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad -F_{EF} - 1.75 \text{ kN} + \frac{1}{\sqrt{5}}(3.3544 \text{ kN}) - \frac{1}{\sqrt{2}}(-4.243 \text{ kN}) = 0$$

$$F_{EF} = 2.750 \text{ kN} \qquad\qquad F_{EF} = 2.75 \text{ kN} \quad T \blacktriangleleft$$

Free body: Joint *G*



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{2}} F_{GH} - \frac{1}{\sqrt{2}} F_{EG} + \frac{1}{\sqrt{2}}(4.243 \text{ kN}) = 0$$

or:
$$F_{GH} - F_{EG} = -4.243 \text{ kN} \qquad (1)$$

$$+\!\uparrow \Sigma F_y = 0: \quad -\frac{1}{\sqrt{2}} F_{GH} - \frac{1}{\sqrt{2}} F_{EG} - \frac{1}{\sqrt{2}}(4.243 \text{ kN}) - 1.5 \text{ kN} = 0$$

or:
$$F_{GH} + F_{EG} = -6.364 \text{ kN} \qquad (2)$$

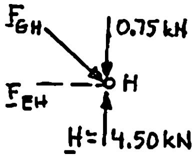
Add (1) and (2):
$$2F_{GH} = -10.607$$

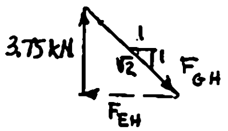
$$F_{GH} = -5.303 \qquad\qquad F_{GH} = 5.30 \text{ kN} \quad C \blacktriangleleft$$

Subtract (1) from (2):
$$2F_{EG} = -2.121 \text{ kN}$$

$$F_{EG} = -1.0605 \text{ kN} \qquad\qquad F_{EG} = 1.061 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint *H*





$$\frac{F_{EH}}{1} = \frac{3.75 \text{ kN}}{1} \qquad\qquad F_{EH} = 3.75 \text{ kN} \quad T \blacktriangleleft$$

We can also write:
$$\frac{F_{GH}}{\sqrt{2}} = \frac{3.75 \text{ kN}}{1} \qquad\qquad F_{GH} = 5.30 \text{ kN} \quad C \quad \text{(Checks)}$$

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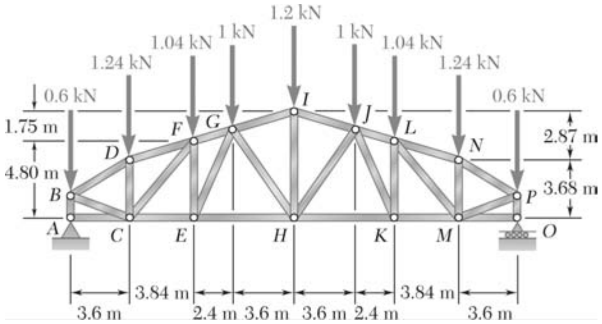


PROBLEM 6.166

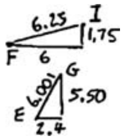
The truss shown was designed to support the roof of a food market. For the given loading, determine the force in members *FG*, *EG*, and *EH*.

SOLUTION

Reactions at supports. Because of the symmetry of the loading

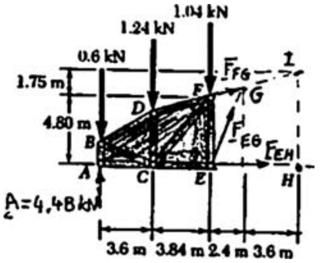
$$A_x = 0, \quad A_y = O = \frac{1}{2}\,(\text{Total load}) \qquad \mathbf{A} = \mathbf{O} = 4.48 \text{ kN}\uparrow ◀$$

We pass a section through members *FG*, *EG*, and *EH*, and use the Free body shown.



$$\text{Slope } FG = \text{Slope } FI = \frac{1.75 \text{ m}}{6 \text{ m}}$$

$$\text{Slope } EG = \frac{5.50 \text{ m}}{2.4 \text{ m}}$$



$$+\circlearrowleft \Sigma M_E = 0: \quad (0.6 \text{ kN})(7.44 \text{ m}) + (1.24 \text{ kN})(3.84 \text{ m}) - (4.48 \text{ kN})(7.44 \text{ m}) - \left(\frac{6}{6.25} F_{FG}\right)(4.80 \text{ m}) = 0$$

$$F_{FG} = -5.231 \text{ kN} \qquad F_{FG} = 5.23 \text{ kN} \quad C ◀$$

$$+\circlearrowleft \Sigma M_G = 0: \quad F_{EH}(5.50 \text{ m}) + (0.6 \text{ kN})(9.84 \text{ m}) + (1.24 \text{ kN})(6.24 \text{ m}) + (1.04 \text{ kN})(2.4 \text{ m}) - (4.48 \text{ kN})(9.84 \text{ m}) = 0$$

$$F_{EH} = 5.08 \text{ kN} \quad T ◀$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{5.50}{6.001} F_{EG} + \frac{1.75}{6.25}(-5.231 \text{ kN}) + 4.48 \text{ kN} - 0.6 \text{ kN} - 1.24 \text{ kN} - 1.04 \text{ kN} = 0$$

$$F_{EG} = -0.1476 \text{ kN} \qquad F_{EG} = 0.1476 \text{ kN} \quad C ◀$$

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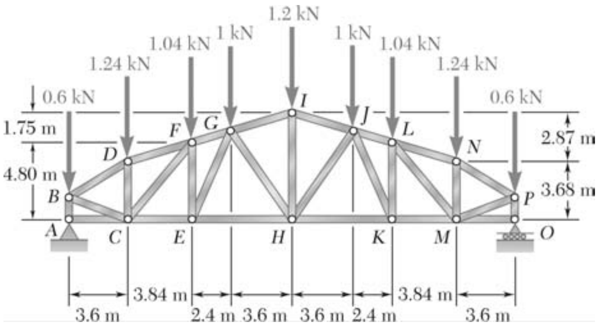


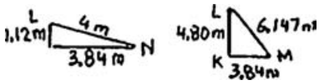
PROBLEM 6.167

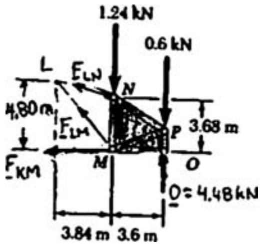
The truss shown was designed to support the roof of a food market. For the given loading, determine the force in members *KM*, *LM*, and *LN*.

SOLUTION

Because of symmetry of loading, $\mathbf{O} = \frac{1}{2}(\text{Load})$ $\qquad \mathbf{O} = 4.48 \text{ kN} \uparrow$ ◁

We pass a section through *KM*, *LM*, *LN*, and use free body shown





$$+\circlearrowleft \Sigma M_M = 0: \quad \left(\frac{3.84}{4} F_{LN}\right)(3.68 \text{ m}) + (4.48 \text{ kN} - 0.6 \text{ kN})(3.6 \text{ m}) = 0$$

$$F_{LN} = -3.954 \text{ kN} \qquad F_{LN} = 3.95 \text{ kN} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_L = 0: \quad -F_{KM}(4.80 \text{ m}) - (1.24 \text{ kN})(3.84 \text{ m}) + (4.48 \text{ kN} - 0.6 \text{ kN})(7.44 \text{ m}) = 0$$

$$F_{KM} = +5.022 \text{ kN} \qquad F_{KM} = 5.02 \text{ kN} \quad T \blacktriangleleft$$

$$+\Sigma F_y = 0: \quad \frac{4.80}{6.147} F_{LM} + \frac{1.12}{4}(-3.954 \text{ kN}) - 1.24 \text{ kN} - 0.6 \text{ kN} + 4.48 \text{ kN} = 0$$

$$F_{LM} = -1.963 \text{ kN} \qquad F_{LM} = 1.963 \text{ kN} \quad C \blacktriangleleft$$

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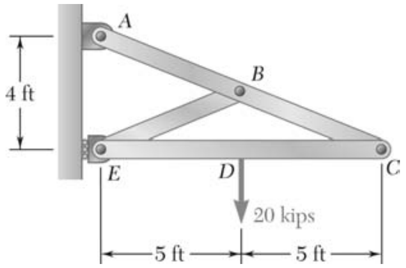
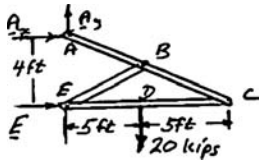


PROBLEM 6.168

For the frame and loading shown, determine the components of all forces acting on member *ABC*.

SOLUTION

Free body: Entire frame



$$+\circlearrowleft \Sigma M_E = 0: \quad -A_x(4) - (20 \text{ kips})(5) = 0$$

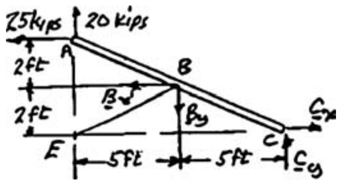
$$A_x = -25 \text{ kips}, \qquad \mathbf{A}_x = 25.0 \text{ kips} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 20 \text{ kips} = 0$$

$$A_y = 20 \text{ kips} \qquad \mathbf{A}_y = 20.0 \text{ kips} \uparrow \blacktriangleleft$$

Free body: Member *ABC*

Note: BE is a two-force member, thus **B** is directed along line *BE* and $B_y = \frac{2}{5}B_x$



$$+\circlearrowleft \Sigma M_C = 0: \quad (25 \text{ kips})(4 \text{ ft}) - (20 \text{ kips})(10 \text{ ft}) + B_x(2 \text{ ft}) + B_y(5 \text{ ft}) = 0$$

$$-100 \text{ kip} \cdot \text{ft} + B_x(2 \text{ ft}) + \frac{2}{5}B_x(5 \text{ ft}) = 0$$

$$B_x = 25 \text{ kips} \qquad \mathbf{B}_x = 25.0 \text{ kips} \leftarrow \blacktriangleleft$$

$$B_y = \frac{2}{5}(B_x) = \frac{2}{5}(25) = 10 \text{ kips} \qquad \mathbf{B}_y = 10.00 \text{ kips} \downarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 25 \text{ kips} - 25 \text{ kips} = 0$$

$$C_x = 50 \text{ kips} \qquad \mathbf{C}_x = 50.0 \text{ kips} \rightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 20 \text{ kips} - 10 \text{ kips} = 0$$

$$C_y = -10 \text{ kips} \qquad \mathbf{C}_y = 10.00 \text{ kips} \downarrow \blacktriangleleft$$

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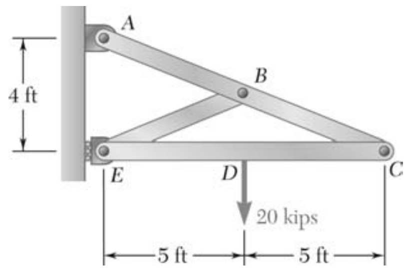


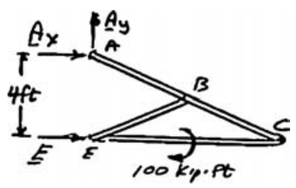
PROBLEM 6.169

Solve Problem 6.168 assuming that the 20-kip load is replaced by a clockwise couple of magnitude 100 kip · ft applied to member *EDC* at Point *D*.

PROBLEM 6.168 For the frame and loading shown, determine the components of all forces acting on member *ABC*.

SOLUTION

Free body: Entire frame



$$+\uparrow \Sigma F_y = 0: \quad A_y = 0$$

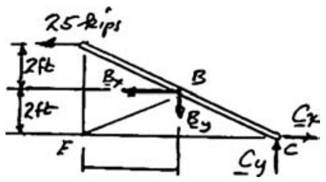
$$+\circlearrowleft \Sigma M_E = 0: \quad -A_x(4\text{ ft}) - 100\text{ kip}\cdot\text{ft} = 0$$

$$A_x = -25\text{ kips} \qquad \mathbf{A}_x = 25.0\text{ kips} \longleftarrow$$

$$\mathbf{A} = 25.0\text{ kips} \longleftarrow \blacktriangleleft$$

Free Body: Member *ABC*

Note: BE is a two-force member, thus **B** is directed along line *BE* and $B_y = \frac{2}{5}B_x$



$$+\circlearrowleft \Sigma M_C = 0: \quad (25\text{ kips})(4\text{ ft}) + B_x(2\text{ ft}) + B_y(5\text{ ft}) = 0$$

$$100\text{ kip}\cdot\text{ft} + B_x(2\text{ ft}) + \frac{2}{5}B_x(5\text{ ft}) = 0$$

$$B_x = -25\text{ kips} \qquad \mathbf{B}_x = 25.0\text{ kips} \longrightarrow \blacktriangleleft$$

$$B_y = \frac{2}{5}B_x = \frac{2}{5}(-25) = -10\text{ kips}; \qquad \mathbf{B}_y = 10.00\text{ kips}\uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -25\text{ kips} + 25\text{ kips} + C_x = 0 \qquad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad +10\text{ kips} + C_y = 0$$

$$C_y = -10\text{ kips} \qquad C_y = 10\text{ kips}\downarrow$$

$$\mathbf{C} = 10.00\text{ kips}\downarrow \blacktriangleleft$$

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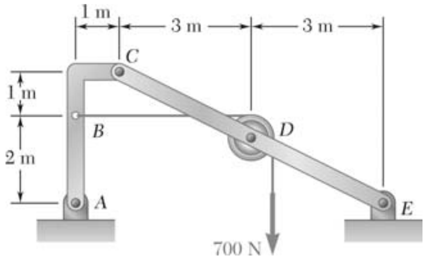
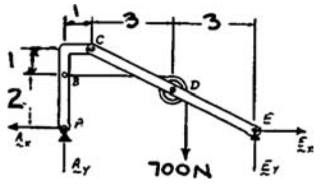


PROBLEM 6.170

Knowing that the pulley has a radius of 0.5 m, determine the components of the reactions at A and E.

SOLUTION

FBD Frame:



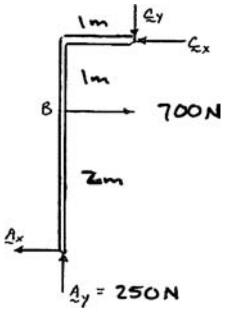
Dimensions in *m*

$$\circlearrowleft \Sigma M_A = 0: \quad (7\text{ m})E_y - (4.5\text{ m})(700\text{ N}) = 0 \qquad \mathbf{E}_y = 450\text{ N}\uparrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad A_y - 700\text{ N} + 450\text{ N} = 0 \qquad \mathbf{A}_y = 250\text{ N}\uparrow \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: \quad A_x - E_x = 0 \qquad A_x = E_x$$

FBD Member ABC:



$$\circlearrowleft \Sigma M_C = 0: \quad (1\text{ m})(700\text{ N}) - (1\text{ m})(250\text{ N}) - (3\text{ m})\,A_x = 0$$

$$\mathbf{A}_x = 150.0\text{ N} \leftarrow \blacktriangleleft$$

$$\text{so} \quad \mathbf{E}_x = 150.0\text{ N} \rightarrow \blacktriangleleft$$

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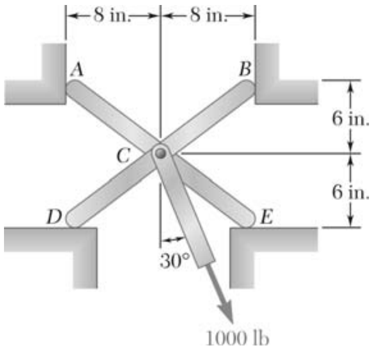
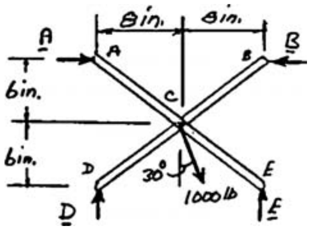


PROBLEM 6.171

For the frame and loading shown, determine the reactions at A, B, D, and E. Assume that the surface at each support is frictionless.

SOLUTION



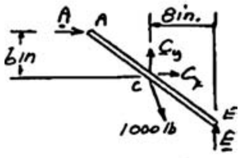
Free body: Entire frame

$$\xrightarrow{+} \Sigma F_x = 0: \quad A - B + (1000 \text{ lb})\sin 30° = 0$$

$$A - B + 500 = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad D + E - (1000 \text{ lb})\cos 30° = 0$$

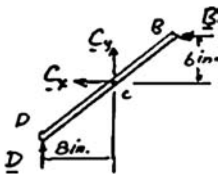
$$D + E - 866.03 = 0 \qquad (2)$$



Free body: Member ACE

$$+\circlearrowleft \Sigma M_C = 0: \quad -A(6 \text{ in.}) + E(8 \text{ in.}) = 0$$

$$E = \frac{3}{4}A \qquad (3)$$



Free body: Member BCD

$$+\circlearrowleft \Sigma M_C = 0: \quad -D(8 \text{ in.}) + B(6 \text{ in.}) = 0$$

$$D = \frac{3}{4}B \qquad (4)$$

Substitute E and D from (3) and (4) into (2):

$$-\frac{3}{4}A + \frac{3}{4}B - 866.06 = 0$$

$$A + B - 1154.71 = 0 \qquad (5)$$

(1)	$A - B + 500 = 0$			(6)
(5) + (6)	$2A - 654.71 = 0$	$A = 327.4$ lb	$\mathbf{A} = 327$ lb $\longrightarrow$ ◀	
(5) − (6)	$2B - 1654.71 = 0$	$B = 827.4$ lb	$\mathbf{B} = 827$ lb $\longleftarrow$ ◀	
(4)	$D = \frac{3}{4}(827.4)$	$D = 620.5$ lb	$\mathbf{D} = 621$ lb $\uparrow$ ◀	
(3)	$E = \frac{3}{4}(327.4)$	$E = 245.5$ lb	$\mathbf{E} = 246$ lb $\uparrow$ ◀	

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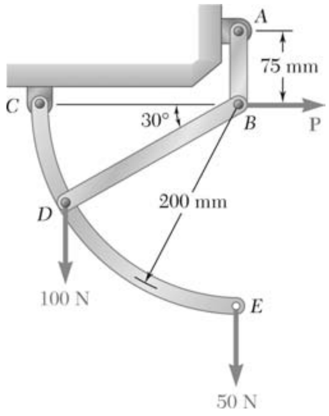


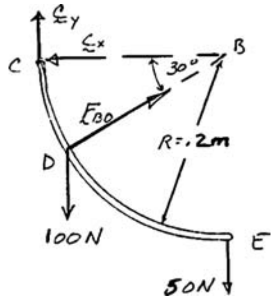
PROBLEM 6.172

For the system and loading shown, determine (*a*) the force **P** required for equilibrium, (*b*) the corresponding force in member *BD*, (*c*) the corresponding reaction at *C*.

SOLUTION

Member FBDs:

I:



FBD I:

$$\circlearrowleft \Sigma M_C = 0: \quad R(F_{BD} \sin 30°) - [R(1 - \cos 30°)](100 \text{ N}) - R(50 \text{ N}) = 0$$

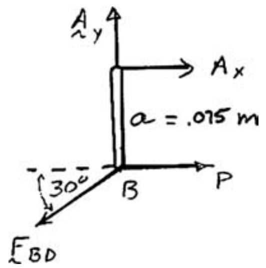
$$F_{BD} = 126.795 \text{ N} \qquad (b) \quad F_{BD} = 126.8 \text{ N} \quad T \blacktriangleleft$$

$$\xrightarrow{} \Sigma F_x = 0: \quad -C_x + (126.795 \text{ N}) \cos 30° = 0 \qquad \mathbf{C}_x = 109.808 \text{ N} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: \quad C_y + (126.795 \text{ N}) \sin 30° - 100 \text{ N} - 50 \text{ N} = 0$$

$$\mathbf{C}_y = 86.603 \text{ N} \uparrow \qquad (c) \quad \text{so} \quad \mathbf{C} = 139.8 \text{ N} \measuredangle 38.3° \blacktriangleleft$$

II:



FBD II:

$$\circlearrowleft \Sigma M_A = 0: \quad aP - a[(126.795 \text{ N}) \cos 30°] = 0 \qquad (a) \quad \mathbf{P} = 109.8 \text{ N} \longrightarrow \blacktriangleleft$$

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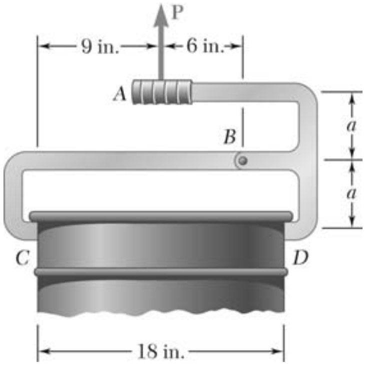
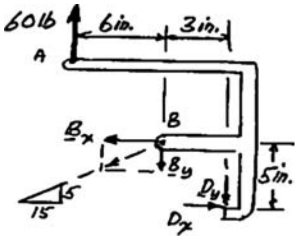


PROBLEM 6.173

A small barrel weighing 60 lb is lifted by a pair of tongs as shown. Knowing that $a = 5$ in., determine the forces exerted at B and D on tong ABD.

SOLUTION



We note that BC is a two-force member.

<u>Free body: Tong *ABD*</u>

$$\frac{B_x}{15} = \frac{B_y}{5} \quad B_x = 3B_y$$

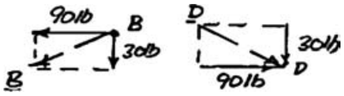
$$+\circlearrowleft \Sigma M_D = 0: \quad B_y(3\text{ in.}) + 3B_y(5\text{ in.}) - (60\text{ lb})(9\text{ in.}) = 0$$

$$B_y = 30\text{ lb}\downarrow$$

$$B_x = 3B_y: \quad B_x = 90\text{ lb} \leftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -90\text{ lb} + D_x = 0 \qquad \mathbf{D}_x = 90\text{ lb} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad 60\text{ lb} - 30\text{ lb} - D_y = 0 \qquad \mathbf{D}_y = 30\text{ lb}\downarrow$$



$\mathbf{B} = 94.9\text{ lb} \swarrow 18.43°$ ◀

$\mathbf{D} = 94.9\text{ lb} \searrow 18.43°$ ◀

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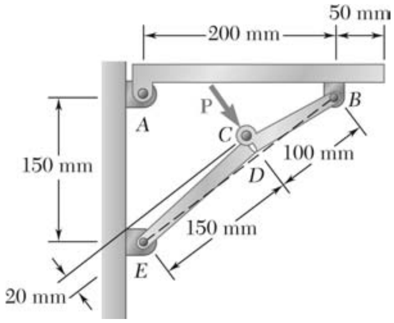
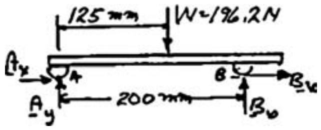


PROBLEM 6.174

A 20-kg shelf is held horizontally by a self-locking brace that consists of two Parts *EDC* and *CDB* hinged at *C* and bearing against each other at *D*. Determine the force **P** required to release the brace.

SOLUTION

Free body: Shelf

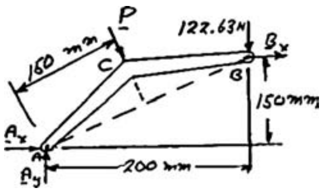


$$W = (20\text{ kg})(9.81\text{ m/s}^2) = 196.2\text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad B_y(200\text{ mm}) - (196.2\text{ N})(125\text{ mm}) = 0$$

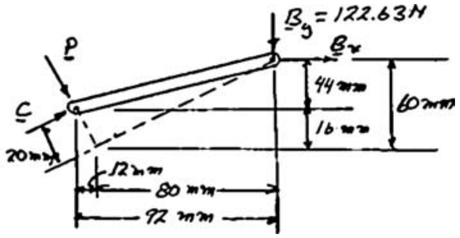
$$B_y = 122.63\text{ N}$$

Free body: Portion *ACB*



$$+\circlearrowleft \Sigma M_A = 0: \quad -B_x(150\text{ mm}) - P(150\text{ mm}) - (122.63\text{ N})(200\text{ mm}) = 0$$

$$B_x = -163.5 - P \qquad (1)$$



$$+\circlearrowleft \Sigma M_C = 0: \quad +(122.63\text{ N})(92\text{ mm}) + B_x(94\text{ mm}) = 0$$

$$+(122.63\text{ N})(92\text{ mm}) + (-163.5 - P)(44\text{ mm}) = 0$$

$$P = 92.9\text{ N} \qquad \mathbf{P} = 92.9\text{ N} \searrow \blacktriangleleft$$

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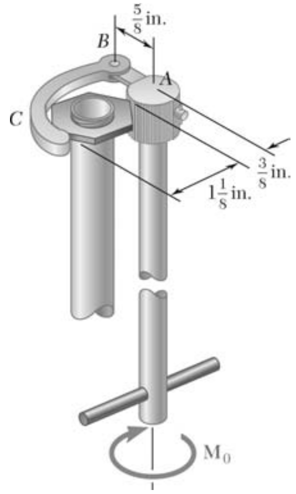
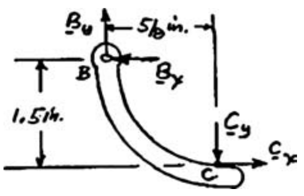


PROBLEM 6.175

The specialized plumbing wrench shown is used in confined areas (e.g., under a basin or sink). It consists essentially of a jaw *BC* pinned at *B* to a long rod. Knowing that the forces exerted on the nut are equivalent to a clockwise (when viewed from above) couple of magnitude 135 lb · in., determine (*a*) the magnitude of the force exerted by pin *B* on jaw *BC*, (*b*) the couple $\mathbf{M}_0$ that is applied to the wrench.

SOLUTION



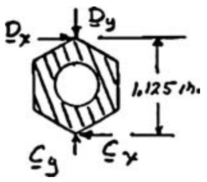
Free body: Jaw *BC*

This is a two-force member

$$\frac{C_y}{1.5\text{ in.}} = \frac{C_x}{\frac{5}{8}\text{ in.}} \quad C_y = 2.4C_x$$

$$\Sigma F_x = 0: \quad B_x = C_x \tag{1}$$

$$\Sigma F_y = 0: \quad B_y = C_y = 2.4\,C_x \tag{2}$$



Free body: Nut $\quad \Sigma F_x = 0: \quad C_x = D_x$

$$\curvearrowright \Sigma M = 135\text{ lb}\cdot\text{in.}$$

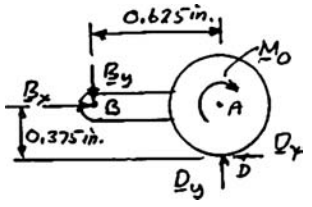
$$C_x(1.125\text{ in.}) = 135\text{ lb}\cdot\text{in.}$$

$$C_x = 120\text{ lb}$$

(*a*) Eq. (1): $\quad B_x = C_x = 120\text{ lb}$

Eq. (2): $\quad B_y = C_y = 2.4(120\text{ lb}) = 288\text{ lb}$

$$B = \left(B_x^2 + B_y^2\right)^{1/2} = (120^2 + 288^2)^{1/2} \qquad B = 312\text{ lb} \blacktriangleleft$$



(*b*) Free body: Rod

$$+\curvearrowright \Sigma M_D = 0: \quad -M_0 + B_y(0.625\text{ in.}) - B_x(0.375\text{ in.}) = 0$$

$$-M_0 + (288)(0.625) - (120)(0.375) = 0$$

$$\mathbf{M}_0 = 135.0\text{ lb}\cdot\text{in.} \curvearrowright \blacktriangleleft$$

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CHAPTER 7

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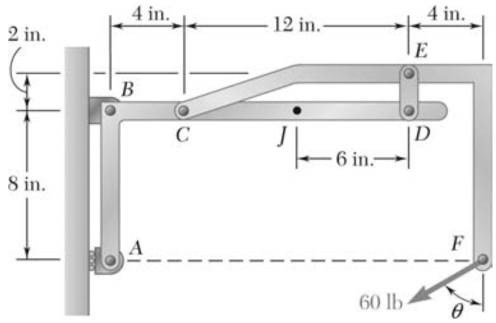


PROBLEM 7.1

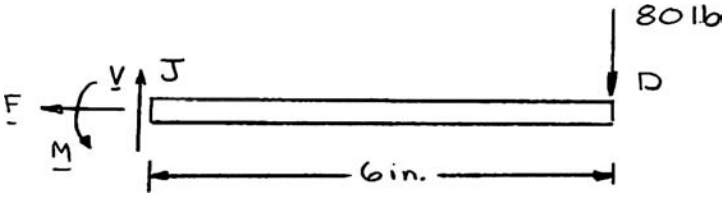
Determine the internal forces (axial force, shearing force, and bending moment) at Point *J* of the structure indicated.

Frame and loading of Problem 6.79.

SOLUTION

From Problem 6.79: On *JD* $\mathbf{D} = 80 \text{ lb}\downarrow$

FBD of *JD*:



$$+\uparrow \Sigma F_y = 0: \quad V - 80 \text{ lb} = 0 \qquad \mathbf{V} = 80.0 \text{ lb}\uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F = 0 \qquad F = 0 \blacktriangleleft$$

$$+\circlearrowright \Sigma M_J = 0: \quad M - (80 \text{ lb})(6 \text{ in.}) = 0 \qquad \mathbf{M} = 480 \text{ lb}\cdot\text{in.}\circlearrowright \blacktriangleleft$$

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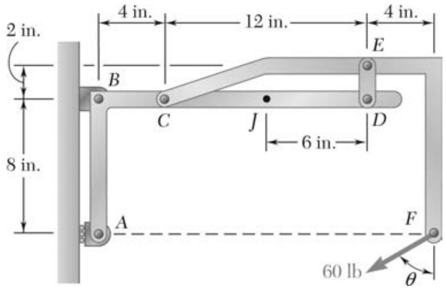


PROBLEM 7.2

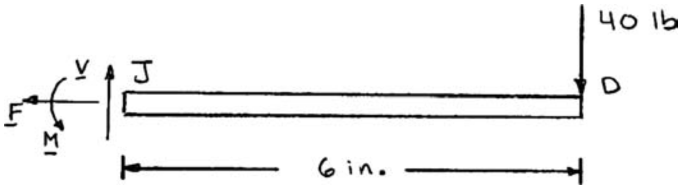
Determine the internal forces (axial force, shearing force, and bending moment) at Point *J* of the structure indicated.

Frame and loading of Problem 6.80.

SOLUTION

From Problem 6.80: On *JD* $\mathbf{D} = 40\text{ lb}\downarrow$

FBD of *JD*:



$+\uparrow \Sigma F_y = 0\!: \quad V - 40\text{ lb} = 0$ $\qquad \mathbf{V} = 40.0\text{ lb}\uparrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0\!: \quad F = 0$ $\qquad F = 0$ ◀

$+\circlearrowleft \Sigma M_J = 0\!: \quad M - (40\text{ lb})(6\text{ in.}) = 0$ $\qquad \mathbf{M} = 240\text{ lb}\cdot\text{in.}\circlearrowleft$ ◀

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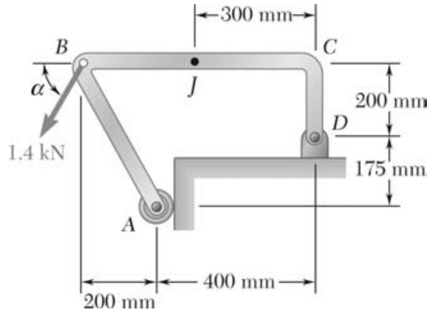
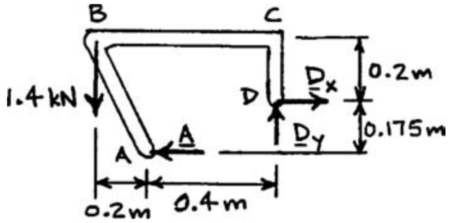


PROBLEM 7.3

Determine the internal forces at Point J when $\alpha = 90°$.

SOLUTION

Free body: Entire bracket



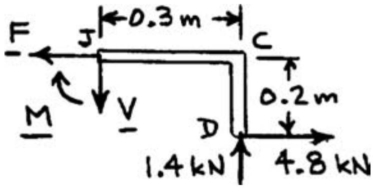
$$+\circlearrowright \Sigma M_D = 0: \quad (1.4\text{ kN})(0.6\text{ m}) - A(0.175\text{ m}) = 0$$

$$A = +4.8\text{ kN} \qquad \mathbf{A} = 4.8\text{ kN} \longleftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x - 4.8 = 0 \qquad \mathbf{D}_x = 4.8\text{ kN} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad D_y - 1.4 = 0 \qquad \mathbf{D}_y = 1.4\text{ kN} \uparrow$$

Free body: *JCD*



$$\xrightarrow{+} \Sigma F_x = 0: \quad 4.8\text{ kN} - F = 0 \qquad \mathbf{F} = 4.80\text{ kN} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 1.4\text{ kN} - V = 0 \qquad \mathbf{V} = 1.400\text{ kN} \downarrow \blacktriangleleft$$

$$+\circlearrowright \Sigma M_J = 0: \quad (4.8\text{ kN})(0.2\text{ m}) + (1.4\text{ kN})(0.3\text{ m}) - M = 0$$

$$M = +1.38\text{ kN}\cdot\text{m} \qquad \mathbf{M} = 1.380\text{ kN}\cdot\text{m} \circlearrowright \blacktriangleleft$$

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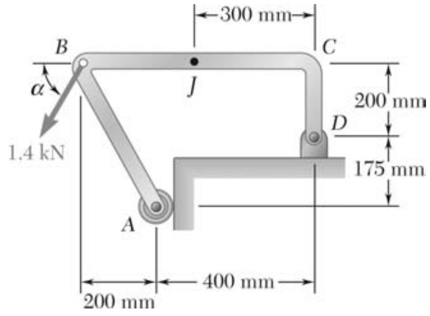
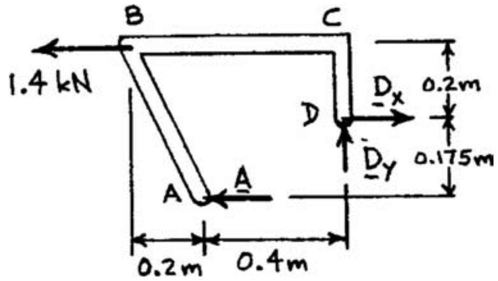


PROBLEM 7.4

Determine the internal forces at Point J when $\alpha = 0$.

SOLUTION

Free body: Entire bracket

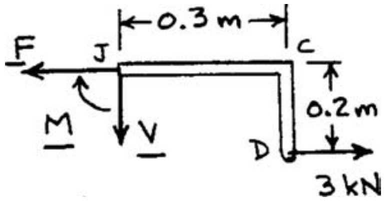


$$+\uparrow \Sigma F_y = 0: \quad D_y = 0 \qquad \mathbf{D}_y = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (1.4\text{ kN})(0.375\text{ m}) - D_x(0.175\text{ m}) = 0$$

$$D_x = +3\text{ kN} \qquad \mathbf{D}_x = 3\text{ kN} \longrightarrow$$

Free body: *JCD*



$$\xrightarrow{+} \Sigma F_x = 0: \quad 3\text{ kN} - F = 0 \qquad \mathbf{F} = 3.00\text{ kN} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -V = 0 \qquad V = 0 \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad (3\text{ kN})(0.2\text{ m}) - M = 0$$

$$M = +0.6\text{ kN}\cdot\text{m} \qquad \mathbf{M} = 0.600\text{ kN}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

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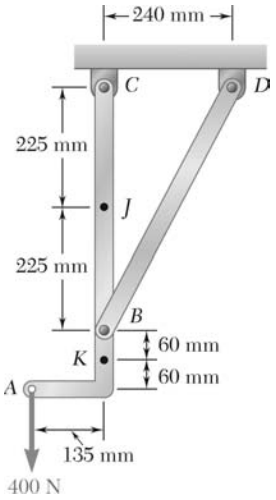
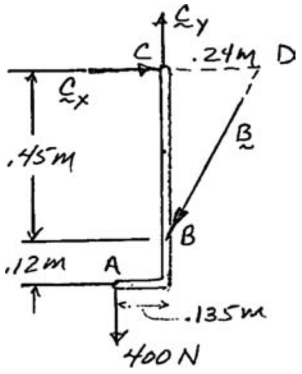


PROBLEM 7.5

Determine the internal forces at Point J of the structure shown.

SOLUTION

FBD ABC:



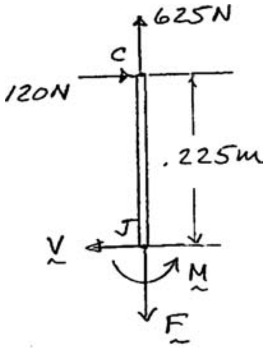
$$\circlearrowleft \Sigma M_D = 0: \quad (0.375 \text{ m})(400 \text{ N}) - (0.24 \text{ m})C_y = 0$$

$$\mathbf{C}_y = 625 \text{ N} \uparrow$$

$$\circlearrowleft \Sigma M_B = 0: \quad -(0.45 \text{ m})C_x + (0.135 \text{ m})(400 \text{ N}) = 0$$

$$\mathbf{C}_x = 120 \text{ N} \longrightarrow$$

FBD CJ:



$$\uparrow \Sigma F_y = 0: \quad 625 \text{ N} - F = 0 \qquad \mathbf{F} = 625 \text{ N} \downarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: \quad 120 \text{ N} - V = 0 \qquad \mathbf{V} = 120.0 \text{ N} \longleftarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad M - (0.225 \text{ m})(120 \text{ N}) = 0$$

$$\mathbf{M} = 27.0 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

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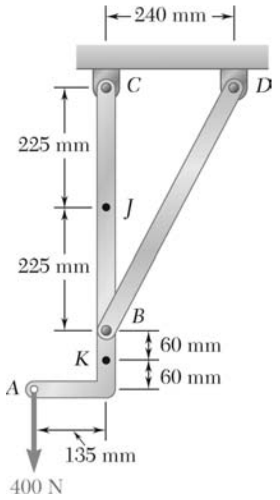
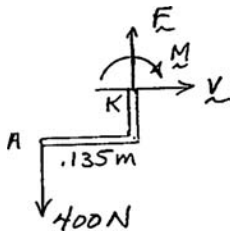


PROBLEM 7.6

Determine the internal forces at Point K of the structure shown.

SOLUTION

FBD AK:



$$\rightarrow \Sigma F_x = 0: \quad V = 0$$

$$\mathbf{V} = 0 \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad F - 400 \text{ N} = 0$$

$$\mathbf{F} = 400 \text{ N} \uparrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: \quad (0.135 \text{ m})(400 \text{ N}) - M = 0$$

$$\mathbf{M} = 54.0 \text{ N} \cdot \text{m} \circlearrowright \blacktriangleleft$$

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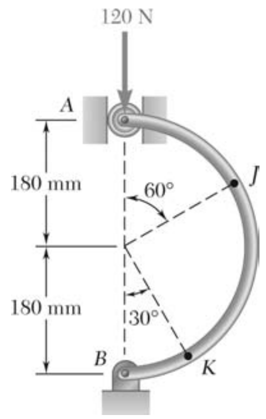
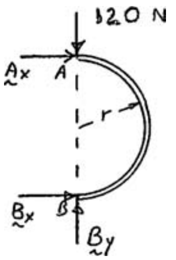


PROBLEM 7.7

A semicircular rod is loaded as shown. Determine the internal forces at Point *J*.

SOLUTION

FBD Rod:



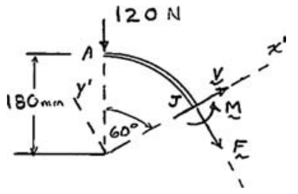
$$\circlearrowleft \Sigma M_B = 0: \quad A_x(2r) = 0$$

$$\mathbf{A}_x = 0$$

$$\nearrow \Sigma F_{x'} = 0: \quad V - (120\text{ N})\cos 60° = 0$$

$$\mathbf{V} = 60.0\text{ N} \nearrow \blacktriangleleft$$

FBD AJ:



$$\searrow \Sigma F_{y'} = 0: \quad F + (120\text{ N})\sin 60° = 0$$

$$F = -103.923\text{ N}$$

$$\mathbf{F} = 103.9\text{ N} \nwarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad M - [(0.180\text{ m})\sin 60°](120\text{ N}) = 0$$

$$M = 18.7061$$

$$\mathbf{M} = 18.71 \circlearrowright \blacktriangleleft$$

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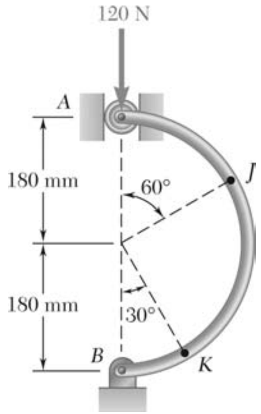
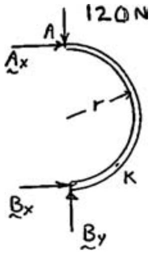


PROBLEM 7.8

A semicircular rod is loaded as shown. Determine the internal forces at Point *K*.

SOLUTION

FBD Rod:



$$\uparrow \Sigma F_y = 0: \quad B_y - 120\text{ N} = 0 \quad \mathbf{B}_y = 120\text{ N}\uparrow$$

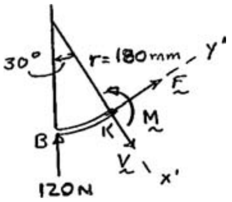
$$\circlearrowleft \Sigma M_A = 0: \quad 2rB_x = 0 \quad \mathbf{B}_x = 0$$

$$\searrow \Sigma F_{x'} = 0: \quad V - (120\text{ N})\cos 30° = 0$$

$$V = 103.923\text{ N}$$

$$\mathbf{V} = 103.9\text{ N} \searrow \blacktriangleleft$$

FBD BK:



$$\nearrow \Sigma F_{y'} = 0: \quad F + (120\text{ N})\sin 30° = 0$$

$$F = -60\text{ N}$$

$$\mathbf{F} = 60.0\text{ N} \swarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: \quad M - [(0.180\text{ m})\sin 30°](120\text{ N}) = 0$$

$$\mathbf{M} = 10.80\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

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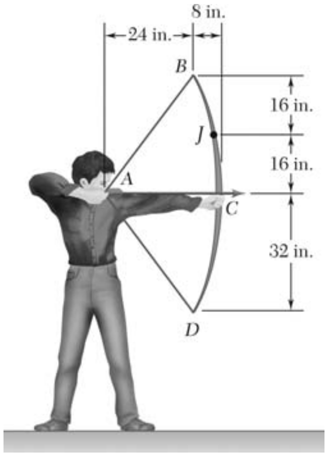
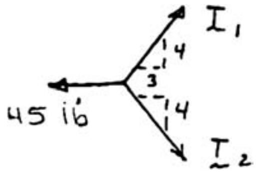


PROBLEM 7.9

An archer aiming at a target is pulling with a 45-lb force on the bowstring. Assuming that the shape of the bow can be approximated by a parabola, determine the internal forces at Point *J*.

SOLUTION

FBD Point A:

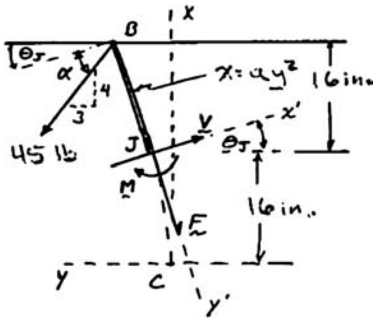


By symmetry $T_1 = T_2$

$$\xrightarrow{} \Sigma F_x = 0: \quad 2\left(\frac{3}{5}T_1\right) - 45\text{ lb} = 0 \quad T_1 = T_2 = 37.5\text{ lb}$$

Curve *CJB* is parabolic: $x = ay^2$

FBD BJ:



At B: $x = 8$ in. $y = 32$ in.

$$a = \frac{8\text{ in.}}{(32\text{ in.})^2} = \frac{1}{128\text{ in.}}$$

$$x = \frac{y^2}{128}$$

$$\text{Slope of parabola} = \tan\theta = \frac{dx}{dy} = \frac{2y}{128} = \frac{y}{64}$$

At J: $$\theta_J = \tan^{-1}\left[\frac{16}{64}\right] = 14.036°$$

So $$\alpha = \tan^{-1}\frac{4}{3} - 14.036° = 39.094°$$

$$\nearrow \Sigma F_{x'} = 0: \quad V - (37.5\text{ lb})\cos(39.094°) = 0$$

$\mathbf{V} = 29.1\text{ lb} \nearrow$ ◀

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PROBLEM 7.9 (Continued)

$$\searrow \Sigma F_{y'} = 0: \quad F + (37.5\text{ lb})\sin(39.094°) = 0$$

$$F = -23.647 \qquad \mathbf{F} = 23.6\text{ lb} \nwarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad M + (16\text{ in.})\left[\frac{3}{5}(37.5\text{ lb})\right] + [(8-2)\text{ in.}]\left[\frac{4}{5}(37.5\text{ lb})\right] = 0$$

$$\mathbf{M} = 540\text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

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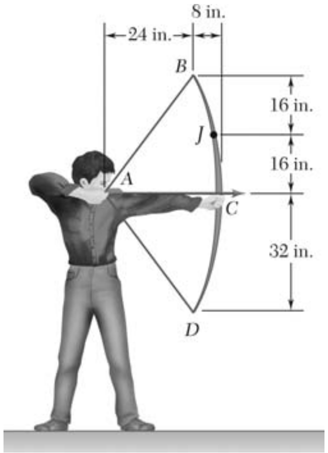


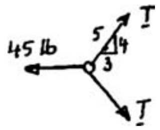
PROBLEM 7.10

For the bow of Problem 7.9, determine the magnitude and location of the maximum (*a*) axial force, (*b*) shearing force, (*c*) bending moment.

PROBLEM 7.9 An archer aiming at a target is pulling with a 45-lb force on the bowstring. Assuming that the shape of the bow can be approximated by a parabola, determine the internal forces at Point *J*.

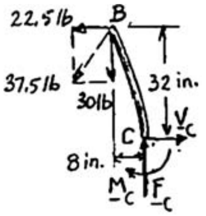
SOLUTION

<u>Free body: Point *A*</u>



$$\xrightarrow{+} \Sigma F_x = 0: \quad 2\left(\frac{3}{5}T\right) - 45 \text{ lb} = 0 \qquad T = 37.5 \text{ lb} \quad \triangleleft$$

<u>Free body: Portion of bow *BC*</u>



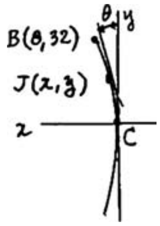
$$+\uparrow \Sigma F_y = 0: \quad F_C - 30 \text{ lb} = 0 \qquad \mathbf{F}_C = 30 \text{ lb} \uparrow \quad \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad V_C - 22.5 \text{ lb} = 0 \qquad \mathbf{V}_C = 22.5 \text{ lb} \longrightarrow \quad \triangleleft$$

$$\circlearrowleft \Sigma M_C = 0: \quad (22.5 \text{ lb})(32 \text{ in.}) + (30 \text{ lb})(8 \text{ in.}) - M_C = 0$$

$$\mathbf{M}_C = 960 \text{ lb} \cdot \text{in.} \circlearrowright \quad \triangleleft$$

<u>Equation of parabola</u>



$$x = ky^2$$

At *B*: $$8 = k(32)^2 \qquad k = \frac{1}{128}$$

Therefore, equation is $$x = \frac{y^2}{128} \qquad (1)$$

The slope at *J* is obtained by differentiating (1):

$$d_x = \frac{2y\,dy}{128}, \quad \tan\theta = \frac{dx}{dy} = \frac{y}{64} \qquad (2)$$

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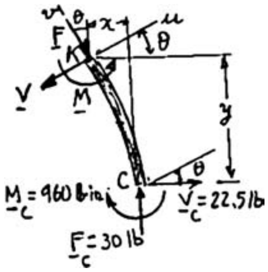
PROBLEM 7.10 (Continued)

(*a*) Maximum axial force

$$+\nwarrow \Sigma F_V = 0: \quad -F + (30\text{ lb})\cos\theta - (22.5\text{ lb})\sin\theta = 0$$



Free body: Portion bow *CK*

$$F = 30\cos\theta - 22.5\sin\theta$$

F is largest at $C(\theta = 0)$

$F_m = 30.0\text{ lb at } C$ ◀

(*b*) Maximum shearing force

$$+\nearrow \Sigma F_V = 0: \quad -V + (30\text{ lb})\sin\theta + (22.5\text{ lb})\cos\theta = 0$$

$$V = 30\sin\theta + 22.5\cos\theta$$

V is largest at *B* (and *D*)

Where $$\theta = \theta_{\max} = \tan^{-1}\left(\frac{1}{2}\right) = 26.56°$$

$$V_m = 30\sin 26.56° + 22.5\cos 26.56°$$

$V_m = 33.5\text{ lb at } B \text{ and } D$ ◀

(*c*) Maximum bending moment

$$+\circlearrowleft \Sigma M_K = 0: \quad M - 960\text{ lb}\cdot\text{in.} + (30\text{ lb})x + (22.5\text{ lb})y = 0$$

$$M = 960 - 30x - 22.5y$$

M is largest at *C*, where $x = y = 0$. $M_m = 960\text{ lb}\cdot\text{in. at } C$ ◀

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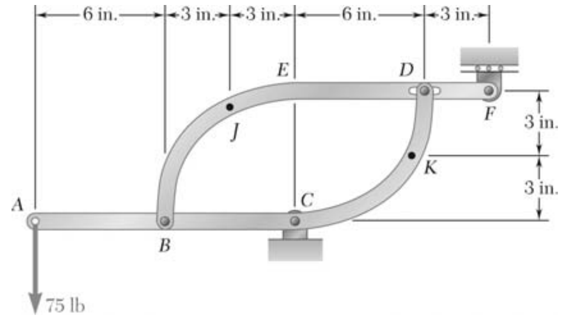
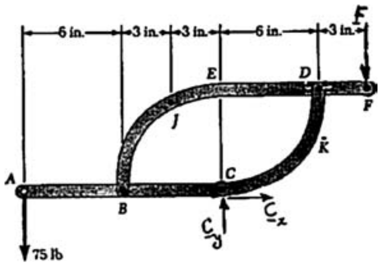


PROBLEM 7.11

Two members, each consisting of a straight and a quarter-circular portion of rod, are connected as shown and support a 75-lb load at A. Determine the internal forces at Point J.

SOLUTION



<u>Free body: Entire frame</u>

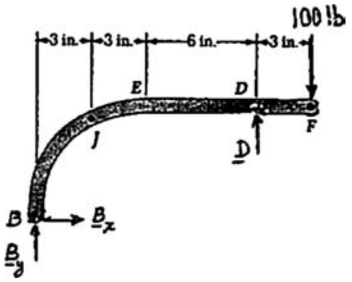
$$+\circlearrowleft \Sigma M_C = 0: \quad (75\text{ lb})(12\text{ in.}) - F(9\text{ in.}) = 0$$

$$\mathbf{F} = 100\text{ lb}\downarrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 75\text{ lb} - 100\text{ lb} = 0$$

$$C_y = +175\text{ lb} \qquad \mathbf{C} = 175\text{ lb}\uparrow \triangleleft$$



<u>Free body: Member *BEDF*</u>

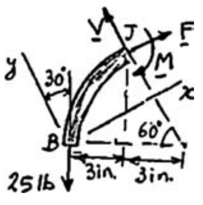
$$+\circlearrowleft \Sigma M_B = 0: \quad D(12\text{ in.}) - (100\text{ lb})(15\text{ in.}) = 0$$

$$\mathbf{D} = 125\text{ lb}\uparrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + 125\text{ lb} - 100\text{ lb} = 0$$

$$B_y = -25\text{ lb} \qquad \mathbf{B} = 25\text{ lb}\downarrow \triangleleft$$



<u>Free body: *BJ*</u>

$$\nearrow^{+} \Sigma F_x = 0: \quad F - (25\text{ lb})\sin 30° = 0$$

$$\mathbf{F} = 12.50\text{ lb} \measuredangle 30.0° \blacktriangleleft$$

$$+\nwarrow \Sigma F_y = 0: \quad V - (25\text{ lb})\cos 30° = 0$$

$$\mathbf{V} = 21.7\text{ lb} \searrow 60.0° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -M + (25\text{ lb})(3\text{ in.}) = 0$$

$$\mathbf{M} = 75.0\text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

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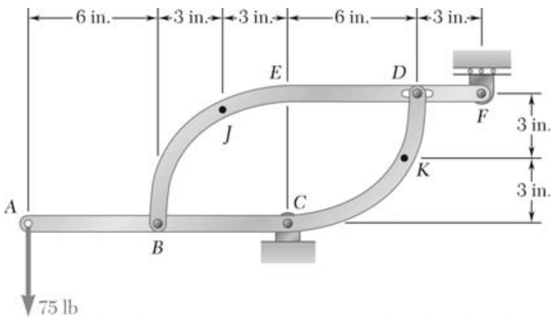
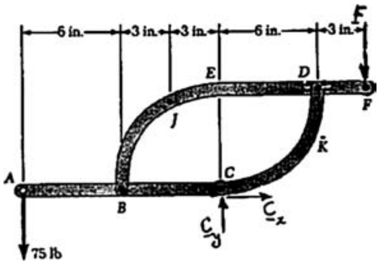


PROBLEM 7.12

Two members, each consisting of a straight and a quarter-circular portion of rod, are connected as shown and support a 75-lb load at *A*. Determine the internal forces at Point *K*.

SOLUTION



<u>Free body: Entire frame</u>

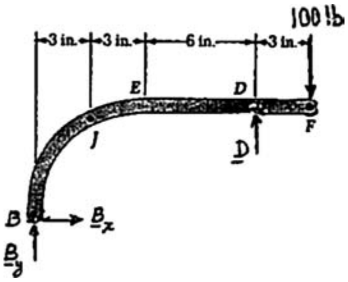
$$+\circlearrowleft \Sigma M_C = 0: \quad (75\text{ lb})(12\text{ in.}) - F(9\text{ in.}) = 0$$

$$\mathbf{F} = 100\text{ lb}\downarrow \;\triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 75\text{ lb} - 100\text{ lb} = 0$$

$$C_y = +175\text{ lb} \qquad \mathbf{C} = 175\text{ lb}\uparrow \;\triangleleft$$



<u>Free body: Member *BEDF*</u>

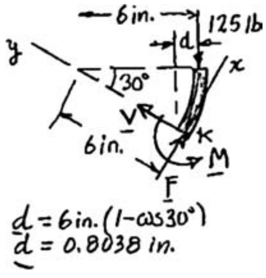
$$+\circlearrowleft \Sigma M_B = 0: \quad D(12\text{ in.}) - (100\text{ lb})(15\text{ in.}) = 0$$

$$\mathbf{D} = 125\text{ lb}\uparrow \;\triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + 125\text{ lb} - 100\text{ lb} = 0$$

$$B_y = -25\text{ lb} \qquad \mathbf{B} = 25\text{ lb}\downarrow \;\triangleleft$$



<u>Free body: *DK*</u>

We found in Problem 7.11 that

$$\mathbf{D} = 125\text{ lb}\uparrow \text{ on } BEDF.$$

Thus $\qquad \mathbf{D} = 125\text{ lb}\downarrow$ on *DK*. ◁

$$+\nearrow \Sigma F_x = 0: \quad F - (125\text{ lb})\cos 30° = 0$$

$$\mathbf{F} = 108.3\text{ lb} \measuredangle\; 60.0° \;\blacktriangleleft$$

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PROBLEM 7.12 (Continued)

$$\nwarrow + \Sigma F_y = 0: \quad V - (125 \text{ lb}) \sin 30° = 0$$

$$\mathbf{V} = 62.5 \text{ lb} \searrow 30.0° \blacktriangleleft$$

$$+\circlearrowright \Sigma M_K = 0: \quad M - (125 \text{ lb})d = 0$$

$$M = (125 \text{ lb})d = (125 \text{ lb})(0.8038 \text{ in.})$$
$$= 100.5 \text{ lb} \cdot \text{in.}$$

$$\mathbf{M} = 100.5 \text{ lb} \cdot \text{in.} \circlearrowright \blacktriangleleft$$

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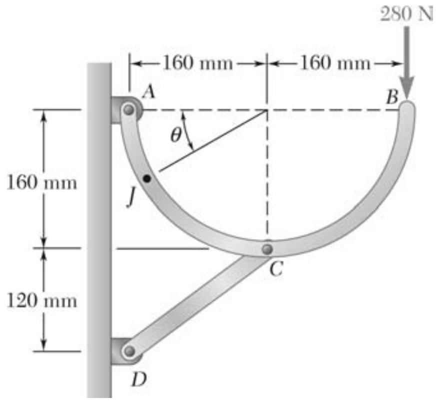
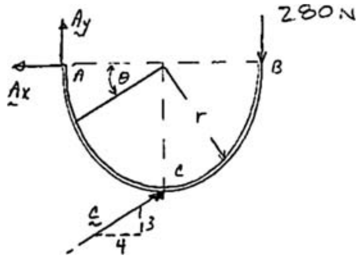


PROBLEM 7.13

A semicircular rod is loaded as shown. Determine the internal forces at Point J knowing that $\theta = 30°$.

SOLUTION

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: \quad r\left(\frac{4}{5}C\right) + r\left(\frac{3}{5}C\right) - 2r(280 \text{ N}) = 0$$

$$\mathbf{C} = 400 \text{ N} \nearrow$$

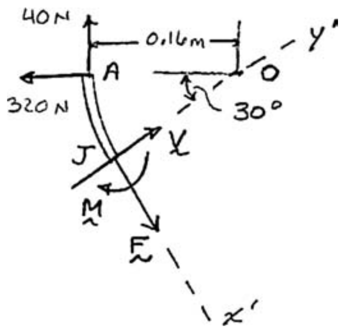
$$\xrightarrow{+} \Sigma F_x = 0: \quad -A_x + \frac{4}{5}(400 \text{ N}) = 0$$

$$\mathbf{A}_x = 320 \text{ N} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y + \frac{3}{5}(400 \text{ N}) - 280 \text{ N} = 0$$

$$\mathbf{A}_y = 40.0 \text{ N}\uparrow$$

FBD AJ:



$$\searrow \Sigma F_{x'} = 0: \quad F - (320 \text{ N})\sin 30° - (40.0 \text{ N})\cos 30° = 0$$

$$F = 194.641 \text{ N}$$

$$\mathbf{F} = 194.6 \text{ N} \searrow 60.0° \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0: \quad V - (320 \text{ N})\cos 30° + (40 \text{ N})\sin 30° = 0$$

$$V = 257.13 \text{ N}$$

$$\mathbf{V} = 257 \text{ N} \measuredangle 30.0° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_0 = 0: \quad (0.160 \text{ m})(194.641 \text{ N}) - (0.160 \text{ m})(40.0 \text{ N}) - M = 0$$

$$M = 24.743$$

$$\mathbf{M} = 24.7 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

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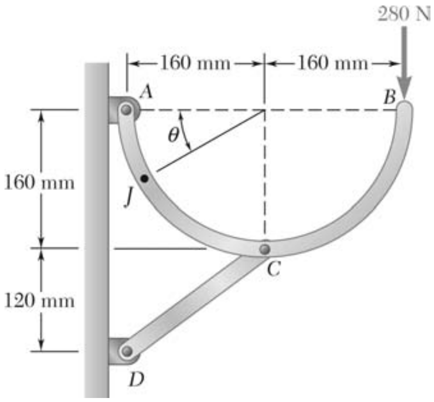
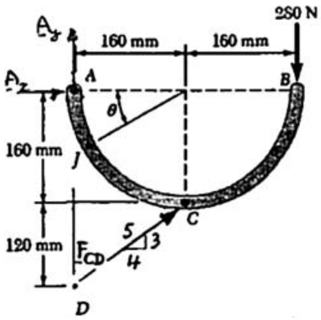


PROBLEM 7.14

A semicircular rod is loaded as shown. Determine the magnitude and location of the maximum bending moment in the rod.

SOLUTION



Free body: Rod *ACB*

$$+\circlearrowleft \Sigma M_A = 0: \quad \left(\frac{4}{5}F_{CD}\right)(0.16\text{ m}) + \left(\frac{3}{5}F_{CD}\right)(0.16\text{ m}) - (280\text{ N})(0.32\text{ m}) = 0$$

$$\mathbf{F}_{CD} = 400\text{ N} \nearrow \quad \triangleleft$$

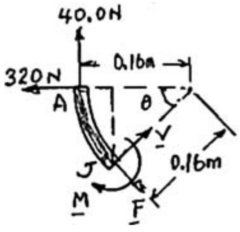
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + \frac{4}{5}(400\text{ N}) = 0$$

$$A_x = -320\text{ N} \qquad \mathbf{A}_x = 320\text{ N} \longleftarrow \quad \triangleleft$$

$$+\Sigma F_y = 0: \quad A_y + \frac{3}{5}(400\text{ N}) - 280\text{ N} = 0$$

$$A_y = +40.0\text{ N} \qquad \mathbf{A}_y = 40.0\text{ N} \uparrow \quad \triangleleft$$

Free body: *AJ* (For $\theta < 90°$)



$$+\circlearrowleft \Sigma M_J = 0: \quad (320\text{ N})(0.16\text{ m})\sin\theta - (40.0\text{ N})(0.16\text{ m})(1-\cos\theta) - M = 0$$

$$M = 51.2\sin\theta + 6.4\cos\theta - 6.4 \qquad (1)$$

For maximum value between *A* and *C*:

$$\frac{dM}{d\theta} = 0: \quad 51.2\cos\theta - 6.4\sin\theta = 0$$

$$\tan\theta = \frac{51.2}{6.4} = 8 \qquad \theta = 82.87° \quad \triangleleft$$

Carrying into (1):

$$M = 51.2\sin 82.87° + 6.4\cos 82.87° - 6.4 = +45.20\text{ N}\cdot\text{m} \quad \triangleleft$$

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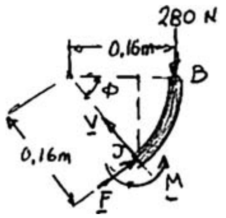
PROBLEM 7.14 (Continued)



Free body: BJ (For $\theta > 90°$)

$$+\circlearrowleft \Sigma M_J = 0: \quad M - (280 \text{ N})(0.16 \text{ m})(1 - \cos\phi) = 0$$

$$M = (44.8 \text{ N}\cdot\text{m})(1 - \cos\phi)$$

Largest value occurs for $\phi = 90°$, that is, at C, and is

$$M_C = 44.8 \text{ N}\cdot\text{m} \ \triangleleft$$

We conclude that

$$M_{\max} = 45.2 \text{ N}\cdot\text{m} \quad \text{for} \quad \theta = 82.9° \ \blacktriangleleft$$

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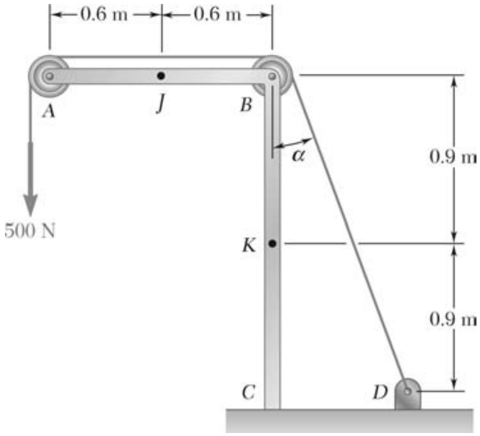


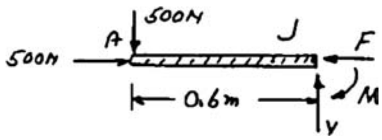
PROBLEM 7.15

Knowing that the radius of each pulley is 150 mm, that $\alpha = 20°$, and neglecting friction, determine the internal forces at (*a*) Point *J*, (*b*) Point *K*.

SOLUTION

Tension in cable = 500 N. Replace cable tension by forces at pins *A* and *B*. Radius does not enter computations: (cf. Problem 6.90)

(*a*) Free body: *AJ*



$$\xrightarrow{+} \Sigma F_x = 0: \quad 500\text{ N} - F = 0$$

$$F = 500\text{ N} \qquad \mathbf{F} = 500\text{ N} \longleftarrow \blacktriangleleft$$

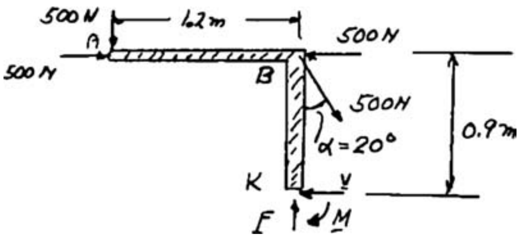
$$+\uparrow \Sigma F_y = 0: \quad V - 500\text{ N} = 0$$

$$V = 500\text{ N} \qquad \mathbf{V} = 500\text{ N}\uparrow \blacktriangleleft$$

$$+\circlearrowright \Sigma M_J = 0: \quad (500\text{ N})(0.6\text{ m}) = 0$$

$$M = 300\text{ N}\cdot\text{m} \qquad \mathbf{M} = 300\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

(*b*) Free body: *ABK*



$$\xrightarrow{+} \Sigma F_x = 0: \quad 500\text{ N} - 500\text{ N} + (500\text{ N})\sin 20° - V = 0$$

$$V = 171.01\text{ N} \qquad \mathbf{V} = 171.0\text{ N} \longleftarrow \blacktriangleleft$$

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PROBLEM 7.15 (Continued)

$$+\uparrow \Sigma F_y = 0: \quad -500\text{ N} - (500\text{ N})\cos 20° + F = 0$$

$$F = 969.8\text{ N} \qquad \mathbf{F} = 970\text{ N}\uparrow \blacktriangleleft$$

$$+\circlearrowright \Sigma M_K = 0: \quad (500\text{ N})(1.2\text{ m}) - (500\text{ N})\sin 20°(0.9\text{ m}) - M = 0$$

$$M = 446.1\text{ N}\cdot\text{m}$$

$$\mathbf{M} = 446\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

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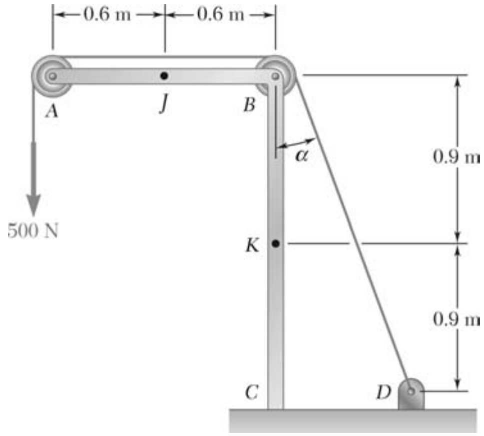


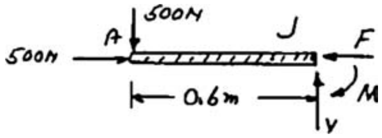
PROBLEM 7.16

Knowing that the radius of each pulley is 150 mm, that $\alpha = 30°$, and neglecting friction, determine the internal forces at (*a*) Point *J*, (*b*) Point *K*.

SOLUTION

Tension in cable = 500 N. Replace cable tension by forces at pins *A* and *B*. Radius does not enter computations: (cf. Problem 6.90)

(*a*) Free body: *AJ*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad 500\text{ N} - F = 0$$

$$F = 500\text{ N} \qquad \mathbf{F} = 500\text{ N} \longleftarrow ◀$$

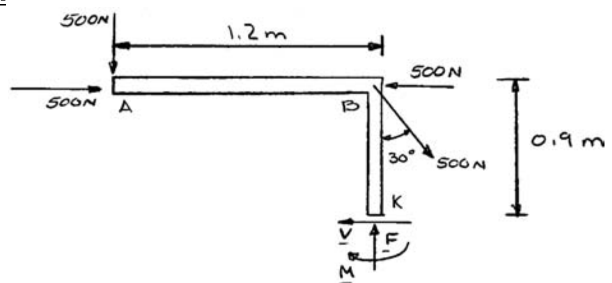
$$+\!\uparrow \Sigma F_y = 0: \quad V - 500\text{ N} = 0$$

$$V = 500\text{ N} \qquad \mathbf{V} = 500\text{ N}\uparrow ◀$$

$$+\circlearrowright \Sigma M_J = 0: \quad (500\text{ N})(0.6\text{ m}) = 0$$

$$M = 300\text{ N}\cdot\text{m} \qquad \mathbf{M} = 300\text{ N}\cdot\text{m} \circlearrowright ◀$$

(*b*) FBD: Portion *ABK*



$$\xrightarrow{+} \Sigma F_x = 0: \quad 500\text{ N} - 500\text{ N} + (500\text{ N})\sin 30° - V \qquad \mathbf{V} = 250\text{ N} \longleftarrow ◀$$

$$+\!\uparrow \Sigma F_y = 0: \quad -500\text{ N} - (500\text{ N})\cos 30° + F = 0 \qquad \mathbf{F} = 933\text{ N}\uparrow ◀$$

$$+\circlearrowright \Sigma M_K = 0: \quad (500\text{ N})(1.2\text{ m}) - (500\text{ N})\sin 30°(0.9\text{ m}) - M = 0 \qquad \mathbf{M} = 375\text{ N}\cdot\text{m} \circlearrowright ◀$$

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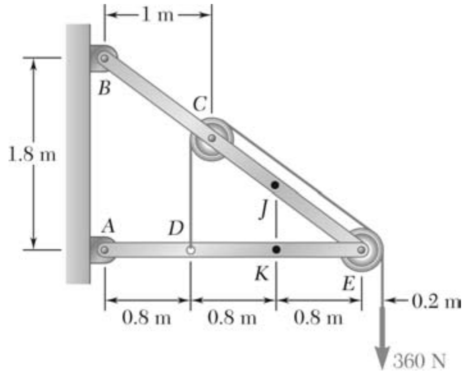
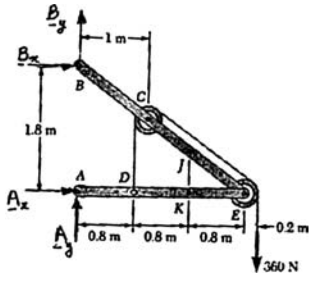


PROBLEM 7.17

Knowing that the radius of each pulley is 200 mm and neglecting friction, determine the internal forces at Point J of the frame shown.

SOLUTION

Free body: Frame and pulleys



$$+\circlearrowleft \Sigma M_A = 0: \quad -B_x(1.8\text{ m}) - (360\text{ N})(2.6\text{ m}) = 0$$

$$B_x = -520\text{ N} \qquad \mathbf{B}_x = 520\text{ N} \leftarrow \triangleleft$$

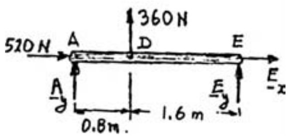
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 520\text{ N} = 0$$

$$A_x = +520\text{ N} \qquad \mathbf{A}_x = 520\text{ N} \rightarrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + B_y - 360\text{ N} = 0$$

$$A_y + B_y = 360\text{ N} \qquad (1)$$

Free body: Member AE



$$+\circlearrowleft \Sigma M_E = 0: \quad -A_y(2.4\text{ m}) - (360\text{ N})(1.6\text{ m}) = 0$$

$$A_y = -240\text{ N} \qquad \mathbf{A}_y = 240\text{ N}\downarrow \triangleleft$$

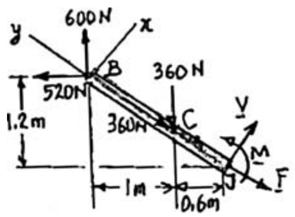
From (1):

$$B_y = 360\text{ N} + 240\text{ N}$$

$$B_y = +600\text{ N} \qquad \mathbf{B}_y = 600\text{ N}\uparrow \triangleleft$$

Free body: BJ

We recall that the forces applied to a pulley may be applied directly to its axle.



$$+\nwarrow \Sigma F_y = 0: \quad \frac{3}{5}(600\text{ N}) + \frac{4}{5}(520\text{ N})$$

$$-360\text{ N} - \frac{3}{5}(360\text{ N}) - F = 0$$

$$F = +200\text{ N} \qquad \mathbf{F} = 200\text{ N} \searrow \blacktriangleleft$$

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PROBLEM 7.17 (Continued)

$$+\nearrow \Sigma F_x = 0: \quad \frac{4}{5}(600 \text{ N}) - \frac{3}{5}(520 \text{ N}) - \frac{4}{5}(360 \text{ N}) + V = 0$$

$$V = +120.0 \text{ N} \qquad \mathbf{V} = 120.0 \text{ N} \nearrow \blacktriangleleft$$

$$+\circlearrowright \Sigma M_J = 0: \quad (520 \text{ N})(1.2 \text{ m}) - (600 \text{ N})(1.6 \text{ m}) + (360 \text{ N})(0.6 \text{ m}) + M = 0$$

$$M = +120.0 \text{ N}\cdot\text{m} \qquad \mathbf{M} = 120.0 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

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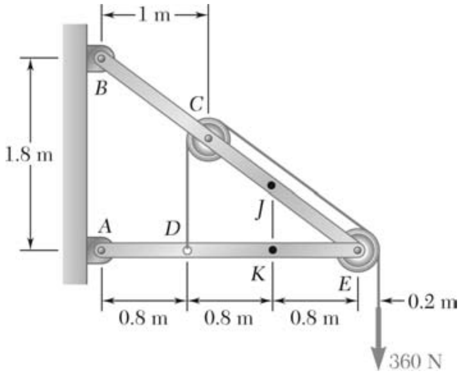
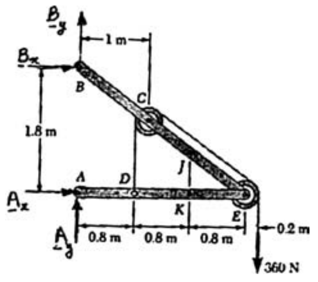


PROBLEM 7.18

Knowing that the radius of each pulley is 200 mm and neglecting friction, determine the internal forces at Point K of the frame shown.

SOLUTION

Free body: Frame and pulleys



$$+\circlearrowleft \Sigma M_A = 0: \quad -B_x(1.8\text{ m}) - (360\text{ N})(2.6\text{ m}) = 0$$

$$B_x = -520\text{ N} \qquad \mathbf{B}_x = 520\text{ N} \longleftarrow \triangleleft$$

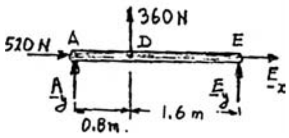
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 520\text{ N} = 0$$

$$A_x = +520\text{ N} \qquad \mathbf{A}_x = 520\text{ N} \longrightarrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + B_y - 360\text{ N} = 0$$

$$A_y + B_y = 360\text{ N} \qquad (1)$$

Free body: Member *AE*



$$+\circlearrowleft \Sigma M_E = 0: \quad -A_y(2.4\text{ m}) - (360\text{ N})(1.6\text{ m}) = 0$$

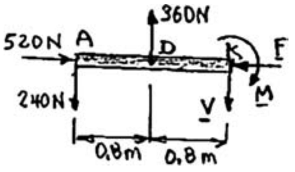
$$A_y = -240\text{ N} \qquad \mathbf{A}_y = 240\text{ N}\downarrow \triangleleft$$

From (1):

$$B_y = 360\text{ N} + 240\text{ N}$$

$$B_y = +600\text{ N} \qquad \mathbf{B}_y = 600\text{ N}\uparrow \triangleleft$$

Free body: *AK*



$$\xrightarrow{+} \Sigma F_x = 0: \quad 520\text{ N} - F = 0$$

$$F = +520\text{ N} \qquad \mathbf{F} = 520\text{ N} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 360\text{ N} - 240\text{ N} - V = 0$$

$$V = +120.0\text{ N} \qquad \mathbf{V} = 120.0\text{ N}\downarrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_K = 0: \quad (240\text{ N})(1.6\text{ m}) - (360\text{ N})(0.8\text{ m}) - M = 0$$

$$M = +96.0\text{ N}\cdot\text{m} \qquad \mathbf{M} = 96.0\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

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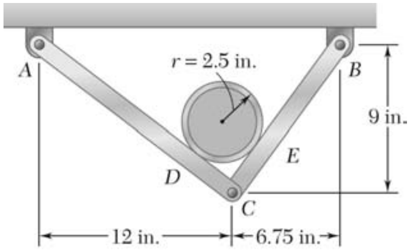
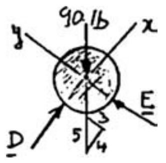


PROBLEM 7.19

A 5-in.-diameter pipe is supported every 9 ft by a small frame consisting of two members as shown. Knowing that the combined weight of the pipe and its contents is 10 lb/ft and neglecting the effect of friction, determine the magnitude and location of the maximum bending moment in member *AC*.

SOLUTION

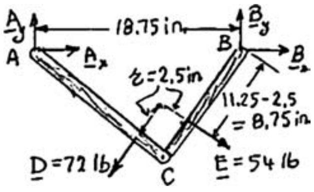


Free body: 10-ft section of pipe

$$+\nearrow \Sigma F_x = 0: \quad D - \frac{4}{5}(90 \text{ lb}) = 0 \qquad \mathbf{D} = 72 \text{ lb} \nearrow \triangleleft$$

$$+\nwarrow \Sigma F_y = 0: \quad E - \frac{3}{5}(90 \text{ lb}) = 0 \qquad \mathbf{E} = 54 \text{ lb} \nwarrow \triangleleft$$

Free body: Frame



$$+\circlearrowleft \Sigma M_B = 0: \quad -A_y(18.75 \text{ in.}) + (72 \text{ lb})(2.5 \text{ in.}) + (54 \text{ lb})(8.75 \text{ in.}) = 0$$

$$A_y = +34.8 \text{ lb} \qquad \mathbf{A}_y = 34.8 \text{ lb} \uparrow \triangleleft$$

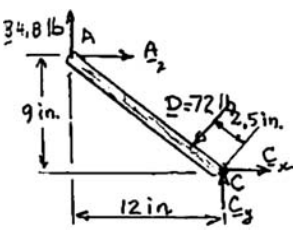
$$+\uparrow \Sigma F_y = 0: \quad B_y + 34.8 \text{ lb} - \frac{4}{5}(72 \text{ lb}) - \frac{3}{5}(54 \text{ lb}) = 0$$

$$B_y = +55.2 \text{ lb} \qquad \mathbf{B}_y = 55.2 \text{ lb} \uparrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + B_x - \frac{3}{5}(72 \text{ lb}) + \frac{4}{5}(54 \text{ lb}) = 0$$

$$A_x + B_x = 0 \qquad (1)$$

Free body: Member *AC*



$$+\circlearrowleft \Sigma M_C = 0: \quad (72 \text{ lb})(2.5 \text{ in.}) - (34.8 \text{ lb})(12 \text{ in.}) - A_x(9 \text{ in.}) = 0$$

$$A_x = -26.4 \text{ lb} \qquad \mathbf{A}_x = 26.4 \text{ lb} \leftarrow \triangleleft$$

From (1):

$$B_x = -A_x = +26.4 \text{ lb}$$

$$\mathbf{B}_x = 26.4 \text{ lb} \rightarrow \triangleleft$$

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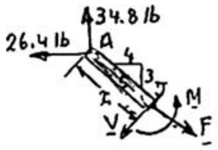
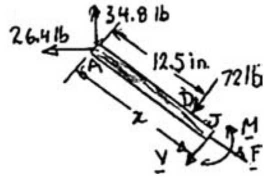
PROBLEM 7.19 (Continued)





Free body: Portion AJ

For $x \le 12.5$ in. $(AJ \le AD)$:

$$+\circlearrowleft \Sigma M_J = 0: \quad (26.4 \text{ lb})\frac{3}{5}x - (34.8 \text{ lb})\frac{4}{5}x + M = 0$$

$$M = 12x$$

$$M_{\max} = 150 \text{ lb} \cdot \text{in. for } x = 12.5 \text{ in.}$$

$$M_{\max} = 150.0 \text{ lb} \cdot \text{in. at } D \;\triangleleft$$

For $x > 12.5$ in. $(AJ > AD)$:

$$+\circlearrowleft \Sigma M_J = 0: \quad (26.4 \text{ lb})\frac{3}{5}x - (34.8 \text{ lb})\frac{4}{5}x + (72 \text{ lb})(x - 12.5) + M = 0$$

$$M = 900 - 60x$$

$$M_{\max} = 150 \text{ lb} \cdot \text{in. for } x = 12.5 \text{ in.}$$

Thus: $$M_{\max} = 150.0 \text{ lb} \cdot \text{in. at } D \;\blacktriangleleft$$

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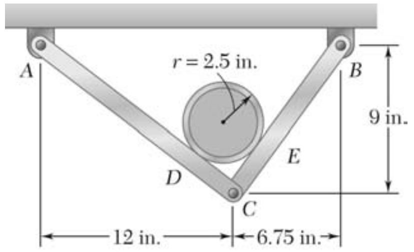
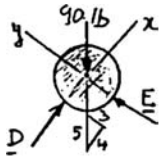


PROBLEM 7.20

For the frame of Problem 7.19, determine the magnitude and location of the maximum bending moment in member *BC*.

PROBLEM 7.19 A 5-in.-diameter pipe is supported every 9 ft by a small frame consisting of two members as shown. Knowing that the combined weight of the pipe and its contents is 10 lb/ft and neglecting the effect of friction, determine the magnitude and location of the maximum bending moment in member *AC*.

SOLUTION

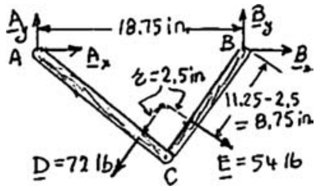


Free body: 10-ft section of pipe

$$+\nearrow \Sigma F_x = 0: \quad D - \frac{4}{5}(90 \text{ lb}) = 0 \qquad \mathbf{D} = 72 \text{ lb} \nearrow \triangleleft$$

$$+\nwarrow \Sigma F_y = 0: \quad E - \frac{3}{5}(90 \text{ lb}) = 0 \qquad \mathbf{E} = 54 \text{ lb} \nwarrow \triangleleft$$

Free body: Frame



$$+\circlearrowleft \Sigma M_B = 0: \quad -A_y(18.75 \text{ in.}) + (72 \text{ lb})(2.5 \text{ in.}) + (54 \text{ lb})(8.75 \text{ in.}) = 0$$

$$A_y = +34.8 \text{ lb} \qquad \mathbf{A}_y = 34.8 \text{ lb} \uparrow \triangleleft$$

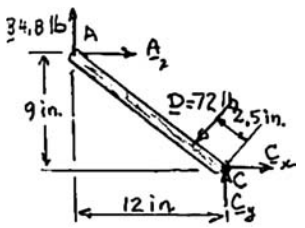
$$+\uparrow \Sigma F_y = 0: \quad B_y + 34.8 \text{ lb} - \frac{4}{5}(72 \text{ lb}) - \frac{3}{5}(54 \text{ lb}) = 0$$

$$B_y = +55.2 \text{ lb} \qquad \mathbf{B}_y = 55.2 \text{ lb} \uparrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + B_x - \frac{3}{5}(72 \text{ lb}) + \frac{4}{5}(54 \text{ lb}) = 0$$

$$A_x + B_x = 0 \qquad (1)$$

Free body: Member *AC*



$$+\circlearrowleft \Sigma M_C = 0: \quad (72 \text{ lb})(2.5 \text{ in.}) - (34.8 \text{ lb})(12 \text{ in.}) - A_x(9 \text{ in.}) = 0$$

$$A_x = -26.4 \text{ lb} \qquad \mathbf{A}_x = 26.4 \text{ lb} \longleftarrow \triangleleft$$

From (1): $B_x = -A_x = +26.4 \text{ lb}$

$$\mathbf{B}_x = 26.4 \text{ lb} \longrightarrow \triangleleft$$

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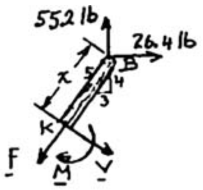
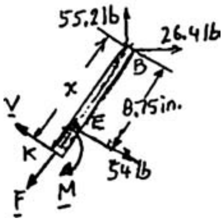
PROBLEM 7.20 (Continued)





Free body: Portion BK

For $x \le 8.75\text{ in.} (BK \le BE)$:

$$+\circlearrowleft \Sigma M_K = 0: \quad (55.2\text{ lb})\frac{3}{5}x - (26.4\text{ lb})\frac{4}{5}x - M = 0$$

$$M = 12x$$

$$M_{max} = 105.0\text{ lb}\cdot\text{in.} \quad \text{for} \quad x = 8.75\text{ in.}$$

$$M_{max} = 105.0\text{ lb}\cdot\text{in. at } E \ \triangleleft$$

For $x > 8.75\text{ in.} (BK > BE)$:

$$+\circlearrowleft \Sigma M_K = 0: \quad (55.2\text{ lb})\frac{3}{5}x - (26.4\text{ lb})\frac{4}{5}x - (54\text{ lb})(x - 8.75\text{ in.}) - M = 0$$

$$M = 472.5 - 42x$$

$$M_{max} = 105.0\text{ lb}\cdot\text{in.} \quad \text{for} \quad x = 8.75\text{ in.}$$

Thus $$M_{max} = 105.0\text{ lb}\cdot\text{in. at } E \ \blacktriangleleft$$

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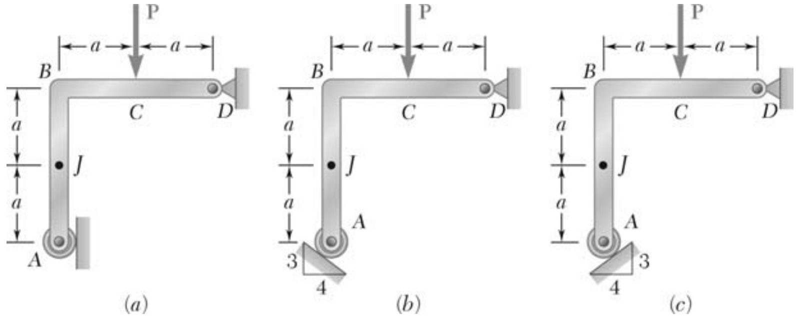
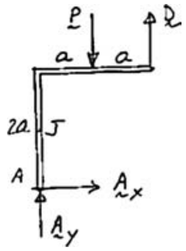
PROBLEM 7.21

A force **P** is applied to a bent rod that is supported by a roller and a pin and bracket. For each of the three cases shown, determine the internal forces at Point *J*.



SOLUTION

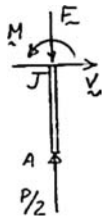
(*a*) **FBD Rod:**



$\xrightarrow{} \Sigma F_x = 0: \quad A_x = 0$

$\circlearrowleft \Sigma M_D = 0: \quad aP - 2aA_y = 0 \qquad A_y = \dfrac{P}{2}$

FBD AJ:

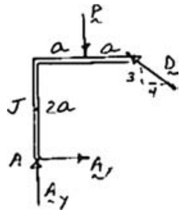


$\xrightarrow{} \Sigma F_x = 0: \quad \mathbf{V} = 0$ ◀

$\uparrow \Sigma F_y = 0: \quad \dfrac{P}{2} - F = 0 \qquad \mathbf{F} = \dfrac{P}{2}\downarrow$ ◀

$\circlearrowleft \Sigma M_J = 0: \quad \mathbf{M} = 0$ ◀

(*b*) **FBD Rod:**



$\circlearrowleft \Sigma M_A = 0: \quad 2a\left(\dfrac{4}{5}D\right) + 2a\left(\dfrac{3}{5}D\right) - aP = 0 \qquad D = \dfrac{5P}{14}$

$\xrightarrow{} \Sigma F_x = 0: \quad A_x - \dfrac{4}{5}\dfrac{5}{14}P = 0 \qquad A_x = \dfrac{2P}{7}$

$\uparrow \Sigma F_y = 0: \quad A_y - P + \dfrac{3}{5}\dfrac{5}{14}P = 0 \qquad A_y = \dfrac{11P}{14}$

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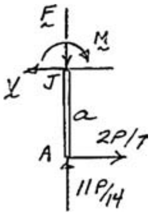
PROBLEM 7.21 (Continued)

FBD AJ:

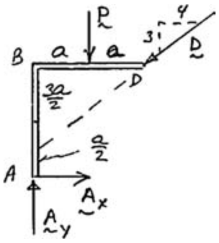


$$\rightarrow \Sigma F_x = 0: \quad \frac{2}{7}P - V = 0 \qquad \mathbf{V} = \frac{2P}{7} \leftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad \frac{11P}{14} - F = 0 \qquad \mathbf{F} = \frac{11P}{14} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad a\frac{2P}{7} - M = 0 \qquad \mathbf{M} = \frac{2}{7}aP \circlearrowright \blacktriangleleft$$

(*c*) **FBD Rod:**

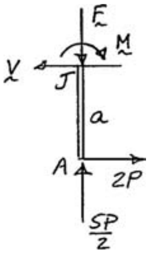


$$\circlearrowleft \Sigma M_A = 0: \quad \frac{a}{2}\left(\frac{4D}{5}\right) - aP = 0 \qquad D = \frac{5P}{2}$$

$$\rightarrow \Sigma F_x = 0: \quad A_x - \frac{4}{5}\frac{5P}{2} = 0 \qquad A_x = 2P$$

$$\uparrow \Sigma F_y = 0: \quad A_y - P - \frac{3}{5}\frac{5P}{2} = 0 \qquad A_y = \frac{5P}{2}$$

FBD AJ:



$$\rightarrow \Sigma F_x = 0: \quad 2P - V = 0 \qquad \mathbf{V} = 2P \leftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad \frac{5P}{2} - F = 0 \qquad \mathbf{F} = \frac{5P}{2} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad a(2P) - M = 0 \qquad \mathbf{M} = 2aP \circlearrowright \blacktriangleleft$$

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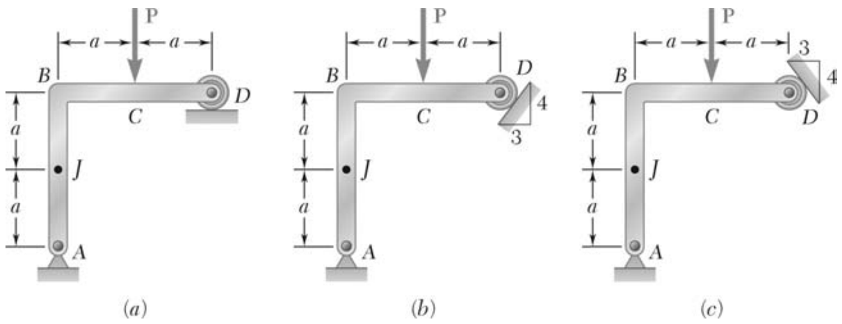
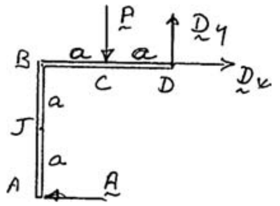
PROBLEM 7.22

A force **P** is applied to a bent rod that is supported by a roller and a pin and bracket. For each of the three cases shown, determine the internal forces at Point *J*.



SOLUTION

(*a*) **FBD Rod:**

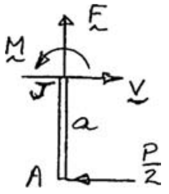


$\circlearrowleft \Sigma M_D = 0{:}\quad aP - 2aA = 0$

$$\mathbf{A} = \frac{P}{2} \longleftarrow$$

$\longrightarrow \Sigma F_x = 0{:}\quad V - \dfrac{P}{2} = 0$ $\qquad \mathbf{V} = \dfrac{P}{2} \longrightarrow$ ◀

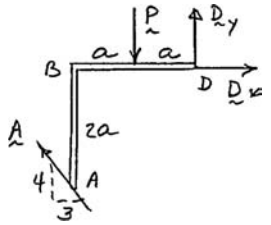
FBD AJ:



$\uparrow \Sigma F_y = 0{:}$ $\qquad \mathbf{F} = 0$ ◀

$\circlearrowleft \Sigma M_J = 0{:}\quad M - a\dfrac{P}{2} = 0$ $\qquad \mathbf{M} = \dfrac{aP}{2} \circlearrowright$ ◀

(*b*) **FBD Rod:**



$\circlearrowleft \Sigma M_D = 0{:}\quad aP - \dfrac{a}{2}\left(\dfrac{4}{5}A\right) = 0$

$$\mathbf{A} = \frac{5P}{2} \nearrow$$

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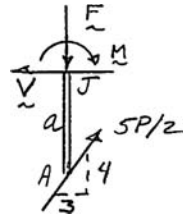
PROBLEM 7.22 (Continued)

FBD AJ:

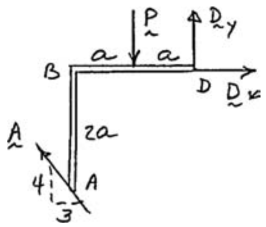


$\xrightarrow{} \Sigma F_x = 0: \quad \frac{3}{5}\frac{5P}{2} - V = 0$ $\qquad \mathbf{V} = \frac{3P}{2} \longleftarrow$ ◀

$\uparrow \Sigma F_y = 0: \quad \frac{4}{5}\frac{5P}{2} - F = 0$ $\qquad \mathbf{F} = 2P \downarrow$ ◀

$\mathbf{M} = \frac{3}{2}aP$ ↻ ◀

(*c*) **FBD Rod:**



$\circlearrowleft \Sigma M_D = 0: \quad aP - 2a\left(\frac{3}{5}A\right) - 2a\left(\frac{4}{5}A\right) = 0$

$A = \frac{5P}{14}$

$\xrightarrow{} \Sigma F_x = 0: \quad V - \left(\frac{3}{5}\frac{5P}{14}\right) = 0$ $\qquad \mathbf{V} = \frac{3P}{14} \longrightarrow$ ◀

$\uparrow \Sigma F_y = 0: \quad \frac{4}{5}\frac{5P}{14} - F = 0$ $\qquad \mathbf{F} = \frac{2P}{7} \downarrow$ ◀

$\circlearrowleft \Sigma M_J = 0: \quad M - a\left(\frac{3}{5}\frac{5P}{14}\right) = 0$ $\qquad \mathbf{M} = \frac{3}{14}aP$ ↻ ◀

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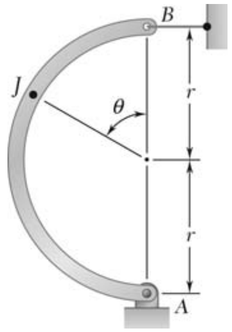
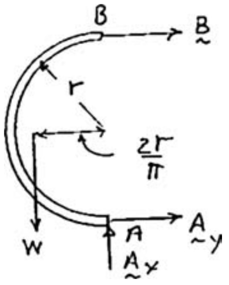


PROBLEM 7.23

A semicircular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 60°$.

SOLUTION

FBD Rod:



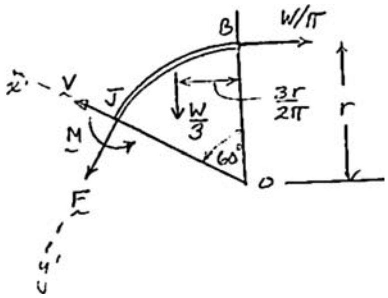
$$\circlearrowleft \Sigma M_A = 0: \quad \frac{2r}{\pi}W - 2rB = 0$$

$$\mathbf{B} = \frac{W}{\pi} \longrightarrow$$

$$\swarrow \Sigma F_{y'} = 0: \quad F + \frac{W}{3}\sin 60° - \frac{W}{\pi}\cos 60° = 0$$

$$F = -0.12952W$$

FBD BJ:



$$\circlearrowleft \Sigma M_0 = 0: \quad r\left(F - \frac{W}{\pi}\right) + \frac{3r}{2\pi}\left(\frac{W}{3}\right) + M = 0$$

$$M = Wr\left(0.12952 + \frac{1}{\pi} - \frac{1}{2\pi}\right) = 0.28868Wr$$

On BJ $\qquad \mathbf{M}_J = 0.289Wr \circlearrowright$ ◀

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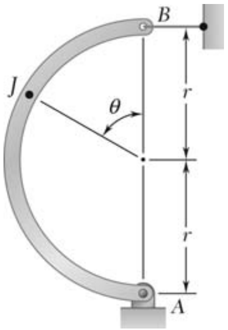
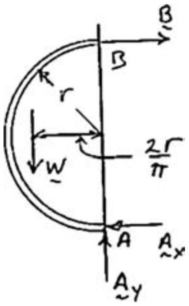


PROBLEM 7.24

A semicircular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 150°$.

SOLUTION

FBD Rod:

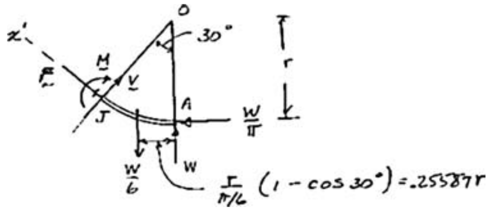


$$\uparrow \Sigma F_y = 0: \quad A_y - W = 0 \qquad \mathbf{A}_y = W \uparrow$$

$$\Sigma M_B = 0: \quad \frac{2r}{\pi} W - 2rA_x = 0$$

$$\mathbf{A}_x = \frac{W}{\pi} \longleftarrow$$

FBD AJ:



$$\nwarrow \Sigma F_{x'} = 0: \quad \frac{W}{\pi}\cos 30° + \frac{5W}{6}\sin 30° - F = 0 \quad F = 0.69233W \searrow$$

$$\circlearrowleft \Sigma M_0 = 0: \quad 0.25587r\left(\frac{W}{6}\right) + r\left(F - \frac{W}{\pi}\right) - M = 0$$

$$M = Wr\left[\frac{0.25587}{6} + 0.69233 - \frac{1}{\pi}\right]$$

$$M = Wr(0.4166)$$

On AJ $\quad \mathbf{M} = 0.417Wr$ ◀

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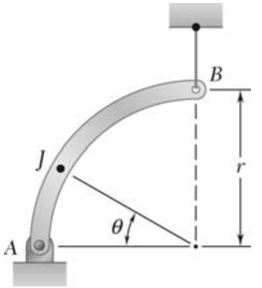
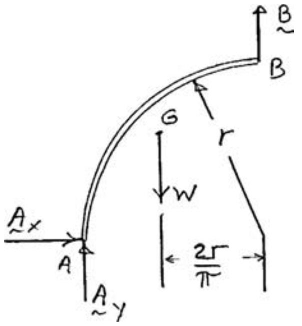


PROBLEM 7.25

A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30°$.

SOLUTION

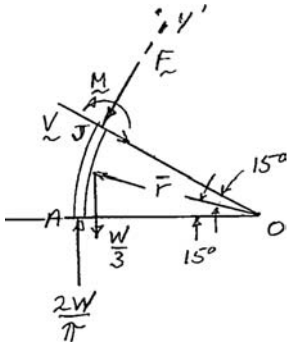
FBD Rod:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

$$\circlearrowleft \Sigma M_B = 0: \quad \frac{2r}{\pi} W - rA_y = 0 \qquad \mathbf{A}_y = \frac{2W}{\pi} \uparrow$$

FBD AJ:



$$\alpha = 15°, \text{ weight of segment} = W \frac{30°}{90°} = \frac{W}{3}$$

$$\overline{r} = \frac{r}{\alpha} \sin\alpha = \frac{r}{\frac{\pi}{12}} \sin 15° = 0.9886r$$

$$\nearrow \Sigma F_{y'} = 0: \quad \frac{2W}{\pi}\cos 30° - \frac{W}{3}\cos 30° - F = 0$$

$$\mathbf{F} = \frac{W\sqrt{3}}{2}\left(\frac{2}{\pi} - \frac{1}{3}\right) \swarrow$$

$$\circlearrowleft \Sigma M_0 = M + r\left(F - \frac{2W}{\pi}\right) + \overline{r}\cos 15° \frac{W}{3} = 0 \qquad \mathbf{M} = 0.0557Wr \circlearrowright \blacktriangleleft$$

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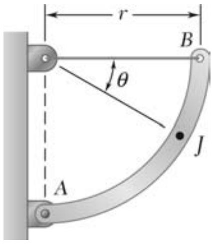
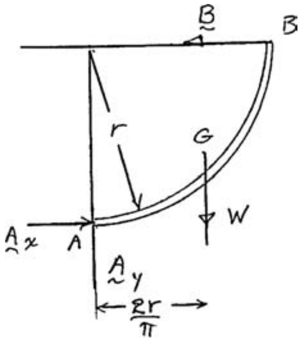


PROBLEM 7.26

A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30°$.

SOLUTION

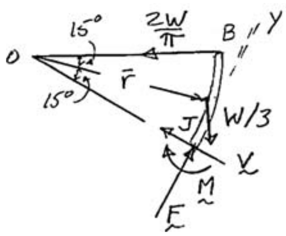
FBD Rod:



$$\circlearrowleft \Sigma M_A = 0: \quad rB - \frac{2r}{\pi} W = 0$$

$$\mathbf{B} = \frac{2W}{\pi} \longleftarrow$$

FBD BJ:



$$\alpha = 15° = \frac{\pi}{12}$$

$$\bar{r} = \frac{r}{\frac{\pi}{12}} \sin 15° = 0.98862r$$

$$\text{Weight of segment } = W \frac{30°}{90°} = \frac{W}{3}$$

$$\nearrow \Sigma F_{y'} = 0: \quad F - \frac{W}{3}\cos 30° - \frac{2W}{\pi}\sin 30° = 0$$

$$\mathbf{F} = \left(\frac{\sqrt{3}}{6} + \frac{1}{\pi}\right) W \nearrow$$

$$\circlearrowleft \Sigma M_0 = 0: \quad rF - (\bar{r}\cos 15°)\frac{W}{3} - M = 0$$

$$M = rW\left(\frac{\sqrt{3}}{6} + \frac{1}{\pi}\right) - \left(0.98862 \frac{\cos 15°}{3}\right) Wr \qquad \mathbf{M} = 0.289Wr \circlearrowright \blacktriangleleft$$

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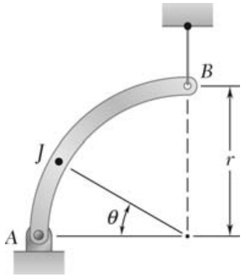


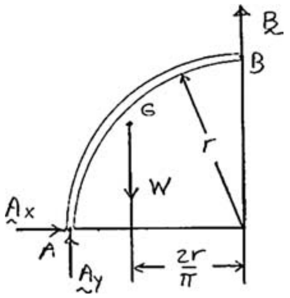
PROBLEM 7.27

For the rod of Problem 7.25, determine the magnitude and location of the maximum bending moment.

PROBLEM 7.25 A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30°$.

SOLUTION

FBD Rod:



$$\rightarrow \Sigma F_x = 0: \quad A_x = 0$$

$$\Sigma M_B = 0: \quad \frac{2r}{\pi}W - rA_y = 0 \qquad A_y = \frac{2W}{\pi}$$

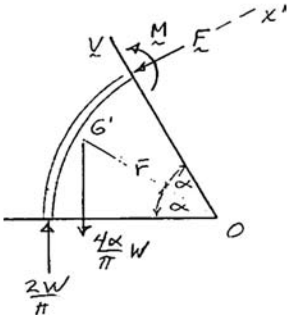
$$\alpha = \frac{\theta}{2}, \qquad \bar{r} = \frac{r}{\alpha}\sin\alpha$$

$$\text{Weight of segment } = W\frac{2\alpha}{\frac{\pi}{2}} = \frac{4\alpha}{\pi}W$$

$$\Sigma F_{x'} = 0: \quad -F - \frac{4\alpha}{\pi}W\cos 2\alpha + \frac{2W}{\pi}\cos 2\alpha = 0$$

$$F = \frac{2W}{\pi}(1-2\alpha)\cos 2\alpha = \frac{2W}{\pi}(1-\theta)\cos\theta$$

FBD AJ: $\Sigma M_0 = 0: \quad M + \left(F - \frac{2W}{\pi}\right)r + (\bar{r}\cos\alpha)\frac{4\alpha}{\pi}W = 0$



$$M = \frac{2W}{\pi}(1 + \theta\cos\theta - \cos\theta)r - \frac{4\alpha W}{\pi}\frac{r}{\alpha}\sin\alpha\cos\alpha$$

But, $\quad \sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha = \frac{1}{2}\sin\theta$

so $\quad M = \frac{2r}{\pi}W(1 - \cos\theta + \theta\cos\theta - \sin\theta)$

$$\frac{dM}{d\theta} = \frac{2rW}{\pi}(\sin\theta - \theta\sin\theta + \cos\theta - \cos\theta) = 0$$

for $\quad (1-\theta)\sin\theta = 0$

$$\frac{dM}{d\theta} = 0 \quad \text{for} \quad \theta = 0, 1, n\pi\,(n = 1, 2, \cdots)$$

Only 0 and 1 in valid range

At $\quad \theta = 0 \quad M = 0, \quad$ at $\quad \theta = 1\text{ rad}$

at $\quad \theta = 57.3°$ $\qquad M = M_{max} = 0.1009Wr$ ◀

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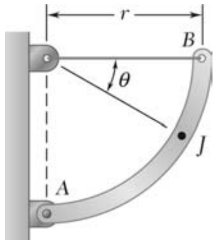


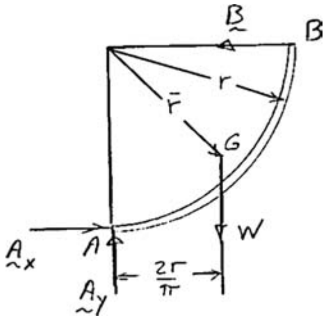
PROBLEM 7.28

For the rod of Problem 7.26, determine the magnitude and location of the maximum bending moment.

PROBLEM 7.26 A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30°$.

SOLUTION

FBD Bar:



$$\circlearrowleft \Sigma M_A = 0: \quad rB - \frac{2r}{\pi}W = 0 \qquad \mathbf{B} = \frac{2W}{\pi} \longleftarrow$$

$$\alpha = \frac{\theta}{2} \qquad \text{so} \qquad 0 \le \alpha \le \frac{\pi}{4}$$

$$\bar{r} = \frac{r}{\alpha}\sin\alpha$$

$$\text{Weight of segment } = W\frac{2\alpha}{\frac{\pi}{2}}$$

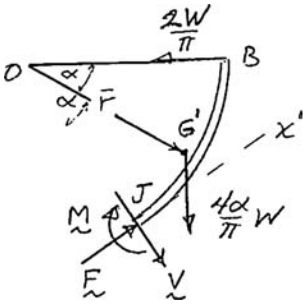
$$= \frac{4\alpha}{\pi}W$$

$$\nearrow \Sigma F_{x'} = 0: \quad F - \frac{4\alpha}{\pi}W\cos 2\alpha - \frac{2W}{\pi}\sin 2\alpha = 0$$

$$F = \frac{2W}{\pi}(\sin 2\alpha + 2\alpha\cos 2\alpha)$$

$$= \frac{2W}{\pi}(\sin\theta + \theta\cos\theta)$$

FBD BJ:



$$\circlearrowleft \Sigma M_0 = 0: \quad rF - (\bar{r}\cos\alpha)\frac{4\alpha}{\pi}W - M = 0$$

$$M = \frac{2}{\pi}Wr(\sin\theta + \theta\cos\theta) - \left(\frac{r}{\alpha}\sin\alpha\cos\alpha\right)\frac{4\alpha}{\pi}W$$

But, $\quad \sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha = \frac{1}{2}\sin\theta$

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PROBLEM 7.28 (Continued)

so $$M = \frac{2Wr}{\pi}(\sin\theta + \theta\cos\theta - \sin\theta)$$

or $$M = \frac{2}{\pi}Wr\theta\cos\theta$$

$$\frac{dM}{d\theta} = \frac{2}{\pi}Wr(\cos\theta - \theta\sin\theta) = 0 \quad \text{at } \theta\tan\theta = 1$$

Solving numerically $\theta = 0.8603\ \text{rad}$

and $\mathbf{M} = 0.357Wr$ ◀

at $\theta = 49.3°$ ◀

(Since $M = 0$ at both limits, this is the maximum)

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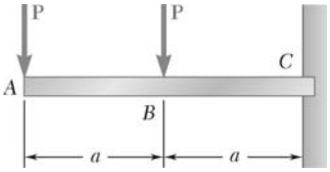
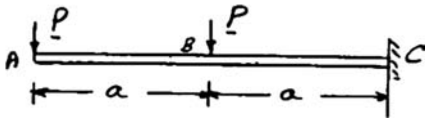


PROBLEM 7.29

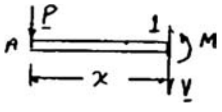
For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*)



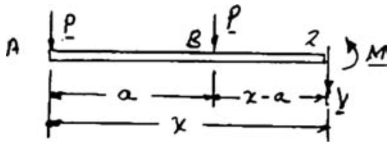
<u>From *A* to *B*:</u>



$+\uparrow \Sigma F_y = 0: \quad V = -P$

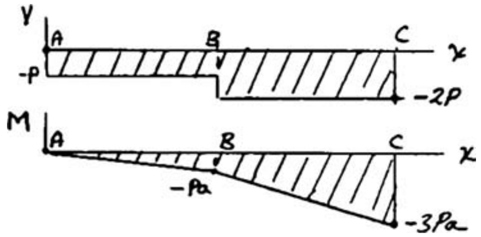
$+\circlearrowleft \Sigma M_1 = 0: \quad M = -P_x$

<u>From *B* to *C*:</u>



$+\uparrow \Sigma F_y = 0: \quad -P - P - V = 0 \qquad V = -2P$

$+\circlearrowleft \Sigma M_2 = 0: \quad P_x + P(x-a) + M = 0 \qquad M = -2P_x + P_a$



(*b*) $|V|_{\max} = 2P; \quad |M|_{\max} = 3P_a$ ◀

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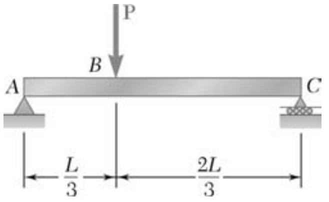


PROBLEM 7.30

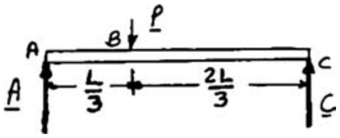
For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

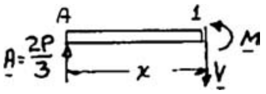
(*a*) Reactions:

$$\mathbf{A} = \frac{2P}{3} \uparrow$$

$$\mathbf{C} = \frac{P}{3} \uparrow$$



From *A* to *B*:



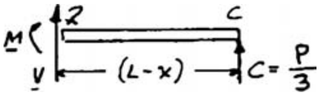
$$+\uparrow \Sigma F_y = 0: \quad V = +\frac{2P}{3}$$

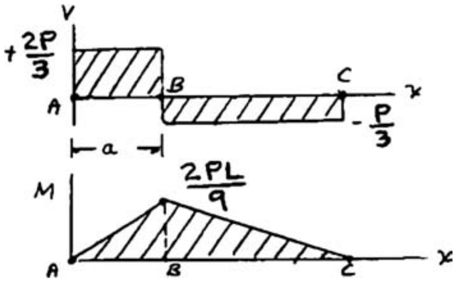
$$+\circlearrowleft \Sigma M_1 = 0: \quad M = +\frac{2P}{3}x$$

From *B* to *C*:

$$+\uparrow \Sigma F_y = 0: \quad V = -\frac{P}{3}$$

$$+\circlearrowleft \Sigma M_2 = 0: \quad M = +\frac{P}{3}(L-x)$$





(*b*) $$|V|_{\max} = \frac{2P}{3}; \quad |M|_{\max} = \frac{2PL}{9}$$ ◀

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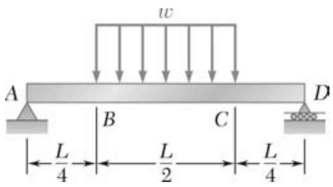


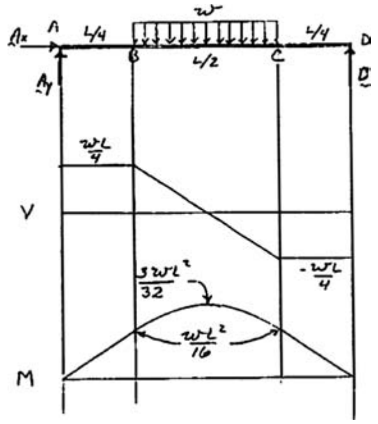
PROBLEM 7.31

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

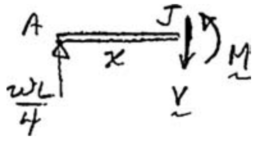
SOLUTION

FBD beam:

(*a*) By symmetry: $A_y = D = \frac{1}{2}(w)\frac{L}{2} \qquad \mathbf{A}_y = \mathbf{D} = \frac{wL}{4}\uparrow$



Along AB:

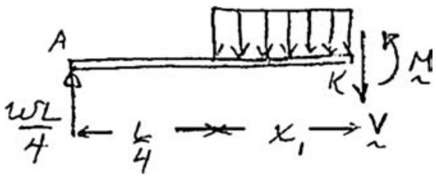


$$\uparrow \Sigma F_y = 0: \quad \frac{wL}{4} - V = 0 \qquad V = \frac{wL}{4}$$

$$\circlearrowleft \Sigma M_J = 0: \quad M - x\frac{wL}{4} = 0$$

$$M = \frac{wL}{4}x \text{ (straight)}$$

Along BC:



$$\uparrow \Sigma F_y = 0: \quad \frac{wL}{4} - wx_1 - V = 0$$

$$V = \frac{wL}{4} - wx_1$$

Straight with $\qquad V = 0 \quad \text{at} \quad x_1 = \frac{L}{4}$

$$\circlearrowleft \Sigma M_k = 0: \quad M + \frac{x_1}{2}wx_1 - \left(\frac{L}{4} + x_1\right)\frac{wL}{4} = 0$$

$$M = \frac{w}{2}\left(\frac{L^2}{8} + \frac{L}{2}x_1 - x_1^2\right)$$

Parabola with $\qquad M = \frac{3}{32}wL^2 \quad \text{at} \quad x_1 = \frac{L}{4}$

Section *CD* by symmetry

(*b*) From diagrams: $|V|_{max} = \frac{wL}{4}$ on *AB* and *CD* ◀

$$|M|_{max} = \frac{3wL^2}{32} \text{ at center} ◀$$

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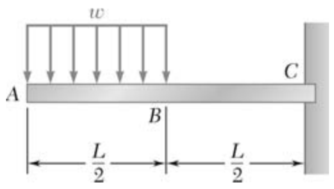
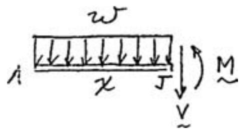


PROBLEM 7.32

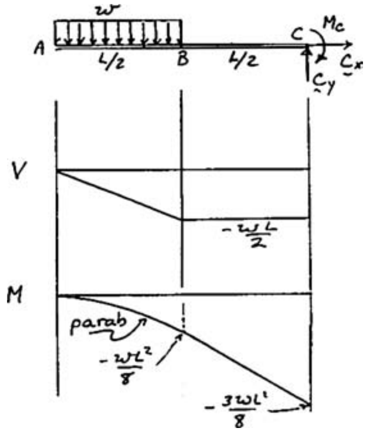
For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) **Along AB:**



$$\uparrow \Sigma F_y = 0: \quad -wx - V = 0 \qquad V = -wx$$

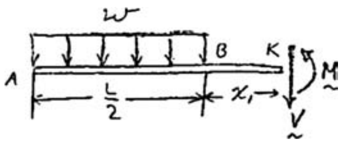


Straight with $V = -\dfrac{wL}{2}$ at $x = \dfrac{L}{2}$

$$\circlearrowleft \Sigma M_J = 0: \quad M + \frac{x}{2}wx = 0 \quad M = -\frac{1}{2}wx^2$$

Parabola with $M = -\dfrac{wL^2}{8}$ at $x = \dfrac{L}{2}$

Along BC:



$$\uparrow \Sigma F_y = 0: \quad -w\frac{L}{2} - V = 0 \qquad V = -\frac{1}{2}wL$$

$$\circlearrowleft \Sigma M_k = 0: \quad M + \left(x_1 + \frac{L}{4}\right)w\frac{L}{2} = 0$$

$$M = -\frac{wL}{2}\left(\frac{L}{4} + x_1\right)$$

Straight with $M = -\dfrac{3}{8}wL^2$ at $x_1 = \dfrac{L}{2}$

(*b*) From diagrams:

$$|V|_{max} = \frac{wL}{2} \text{ on } BC \blacktriangleleft$$

$$|M|_{max} = \frac{3wL^2}{8} \text{ at } C \blacktriangleleft$$

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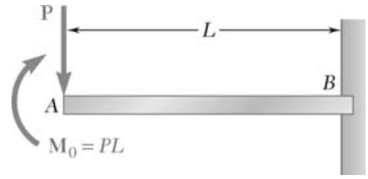
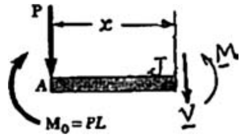


PROBLEM 7.33

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

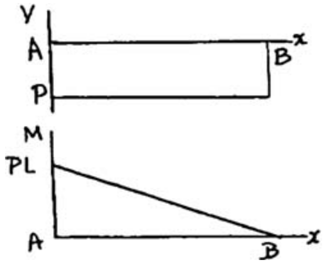
Free body: Portion *AJ*



$+\uparrow \Sigma F_y = 0: \quad -P - V = 0$ $\qquad V = -P$ ◁

$+\circlearrowleft \Sigma M_J = 0: \quad M + P_x - PL = 0$ $\qquad M = P(L - x)$ ◁

(*a*) The *V* and *M* diagrams are obtained by plotting the functions *V* and *M*.



(*b*)

$|V|_{max} = P$ ◀

$|M|_{max} = PL$ ◀

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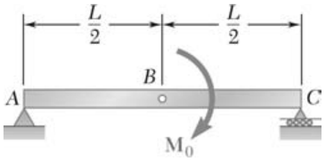
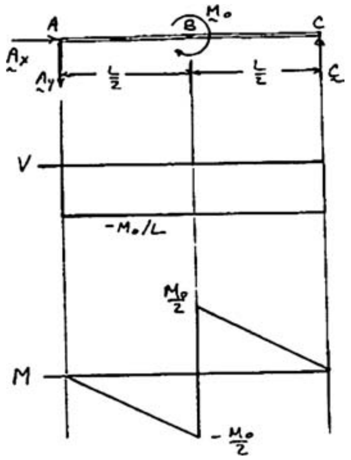


PROBLEM 7.34

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) **FBD Beam:**



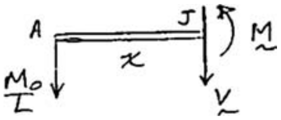
$$\circlearrowleft \Sigma M_C = 0: \quad LA_y - M_0 = 0$$

$$\mathbf{A}_y = \frac{M_0}{L} \downarrow$$

$$\uparrow \Sigma F_y = 0: \quad -A_y + C = 0$$

$$\mathbf{C} = \frac{M_0}{L} \uparrow$$

Along AB:



$$\uparrow \Sigma F_y = 0: \quad -\frac{M_0}{L} - V = 0$$

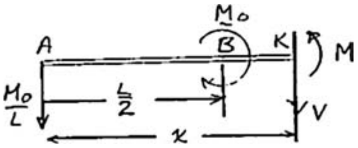
$$V = -\frac{M_0}{L}$$

$$\circlearrowleft \Sigma M_J = 0: \quad x\frac{M_0}{L} + M = 0$$

$$M = -\frac{M_0}{L}x$$

Straight with $M = -\dfrac{M_0}{2}$ at B

Along BC:



$$\uparrow \Sigma F_y = 0: \quad -\frac{M_0}{L} - V = 0 \qquad V = -\frac{M_0}{L}$$

$$\circlearrowleft \Sigma M_K = 0: \quad M + x\frac{M_0}{L} - M_0 = 0 \qquad M = M_0\left(1 - \frac{x}{L}\right)$$

Straight with $M = \dfrac{M_0}{2}$ at B $\qquad M = 0$ at C

(*b*) From diagrams: $|V|_{\max} = P$ everywhere ◀

$|M|_{\max} = \dfrac{M_0}{2}$ at B ◀

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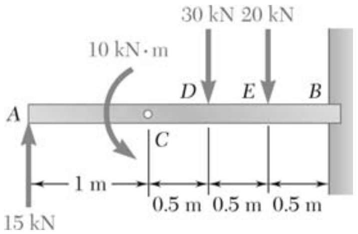
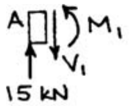


PROBLEM 7.35

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

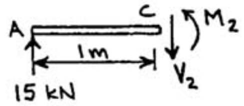
SOLUTION

(*a*) Just to the right of *A*:



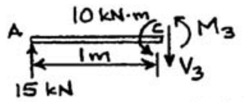
$$+\!\uparrow \Sigma F_y = 0 \quad V_1 = +15\text{ kN} \quad M_1 = 0$$

Just to the left of *C*:



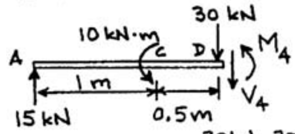
$$V_2 = +15\text{ kN} \quad M_2 = +15\text{ kN}\cdot\text{m}$$

Just to the right of *C*:



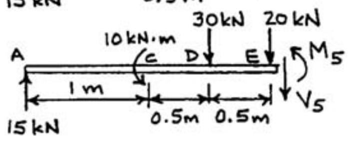
$$V_3 = +15\text{ kN} \quad M_3 = +5\text{ kN}\cdot\text{m}$$

Just to the right of *D*:



$$V_4 = -15\text{ kN} \quad M_4 = +12.5\text{ kN}\cdot\text{m}$$

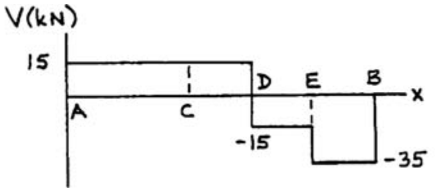
Just to the right of *E*:

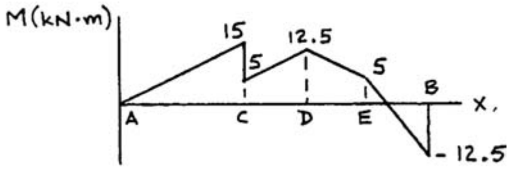


$$V_5 = -35\text{ kN} \quad M_5 = +5\text{ kN}\cdot\text{m}$$

At *B*:

$$M_B = -12.5\text{ kN}\cdot\text{m}$$





(*b*) $|V|_{max} = 35.0\text{ kN}$ $|M|_{max} = 12.50\text{ kN}\cdot\text{m}$ ◀

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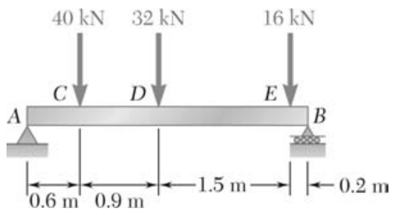
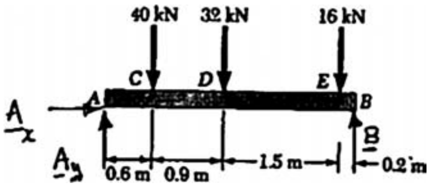


PROBLEM 7.36

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_A = 0: \quad B(3.2\text{ m}) - (40\text{ kN})(0.6\text{ m}) - (32\text{ kN})(1.5\text{ m}) - (16\text{ kN})(3\text{ m}) = 0$$

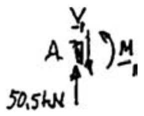
$$B = +37.5\text{ kN} \qquad \mathbf{B} = 37.5\text{ kN}\uparrow \triangleleft$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 37.5\text{ kN} - 40\text{ kN} - 32\text{ kN} - 16\text{ kN} = 0$$

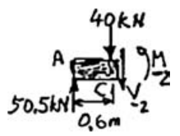
$$A_y = +50.5\text{ kN} \qquad \mathbf{A} = 50.5\text{ kN}\uparrow \triangleleft$$

(*a*) Shear and bending moment.



Just to the right of *A*: $\quad V_1 = 50.5\text{ kN} \qquad M_1 = 0 \triangleleft$

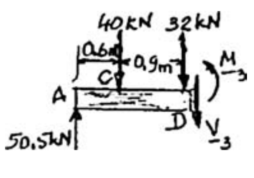
Just to the right of *C*:



$$+\uparrow \Sigma F_y = 0: \quad 50.5\text{ kN} - 40\text{ kN} - V_2 = 0 \qquad V_2 = +10.5\text{ kN} \triangleleft$$

$$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 - (50.5\text{ kN})(0.6\text{ m}) = 0 \qquad M_2 = +30.3\text{ kN}\cdot\text{m} \triangleleft$$

Just to the right of *D*:



$$+\uparrow \Sigma F_y = 0: \quad 50.5 - 40 - 32 - V_3 = 0 \qquad V_3 = -21.5\text{ kN} \triangleleft$$

$$+\circlearrowleft \Sigma M_3 = 0: \quad M_3 - (50.5)(1.5) + (40)(0.9) = 0 \qquad M_3 = +39.8\text{ kN}\cdot\text{m} \triangleleft$$

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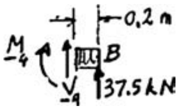
PROBLEM 7.36 (Continued)

Just to the right of *E*:

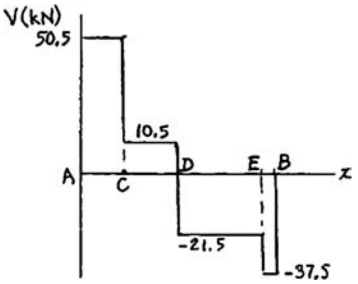


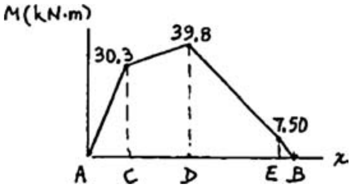
$+\uparrow \Sigma F_y = 0: \quad V_4 + 37.5 = 0 \qquad V_4 = -37.5 \text{ kN}$ ◁

$+\circlearrowleft \Sigma M_4 = 0: \quad -M_4 + (37.5)(0.2) = 0 \qquad M_4 = +7.50 \text{ kN}\cdot\text{m}$ ◁

At *B*: $V_B = M_B = 0$ ◁

(*b*) $|V|_{max} = 50.5 \text{ kN}$ ◀





$|M|_{max} = 39.8 \text{ kN}\cdot\text{m}$ ◀

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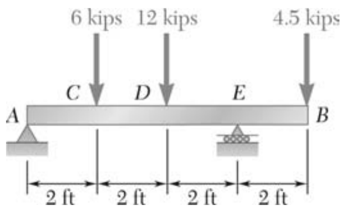
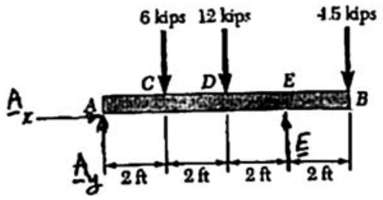


PROBLEM 7.37

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$+\circlearrowleft \Sigma M_A = 0$: $E(6 \text{ ft}) - (6 \text{ kips})(2 \text{ ft}) - (12 \text{ kips})(4 \text{ ft}) - (4.5 \text{ kips})(8 \text{ ft}) = 0$

$E = +16 \text{ kips}$ $\qquad \mathbf{E} = 16 \text{ kips} \uparrow \triangleleft$

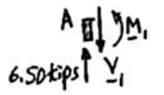
$\xrightarrow{+} \Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y + 16 \text{ kips} - 6 \text{ kips} - 12 \text{ kips} - 4.5 \text{ kips} = 0$

$A_y = +6.50 \text{ kips}$ $\qquad \mathbf{A} = 6.50 \text{ kips} \uparrow \triangleleft$

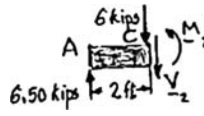
(*a*) Shear and bending moment

Just to the right of *A*:



$V_1 = +6.50 \text{ kips} \quad M_1 = 0$ $\qquad \triangleleft$

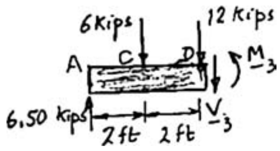
Just to the right of *C*:



$+\uparrow \Sigma F_y = 0$: $6.50 \text{ kips} - 6 \text{ kips} - V_2 = 0$ $\qquad V_2 = +0.50 \text{ kips} \triangleleft$

$+\circlearrowleft \Sigma M_2 = 0$: $M_2 - (6.50 \text{ kips})(2 \text{ ft}) = 0$ $\qquad M_2 = +13 \text{ kip} \cdot \text{ft} \triangleleft$

Just to the right of *D*:



$+\uparrow \Sigma F_y = 0$: $6.50 - 6 - 12 - V_3 = 0$ $\qquad V_3 = +11.5 \text{ kips} \triangleleft$

$+\circlearrowleft \Sigma M_3 = 0$: $M_3 - (6.50)(4) - (6)(2) = 0$ $\qquad M_3 = +14 \text{ kip} \cdot \text{ft} \triangleleft$

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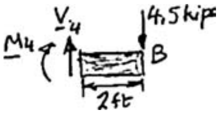
PROBLEM 7.37 (Continued)

Just to the right of E:

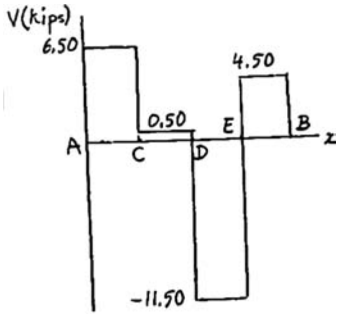


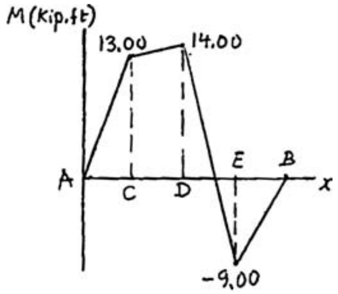
$+\uparrow \Sigma F_y = 0: \quad V_4 - 4.5 = 0 \qquad V_4 = +4.5 \text{ kips} \quad \triangleleft$

$+\circlearrowleft \Sigma M_4 = 0: \quad -M_4 - (4.5)2 = 0 \qquad M_4 = -9 \text{ kip} \cdot \text{ft} \quad \triangleleft$

At B: $\qquad V_B = M_B = 0 \quad \triangleleft$

(*b*) $\qquad |V|_{\max} = 11.50 \text{ kips} \quad \blacktriangleleft$





$|M|_{\max} = 14.00 \text{ kip} \cdot \text{ft} \quad \blacktriangleleft$

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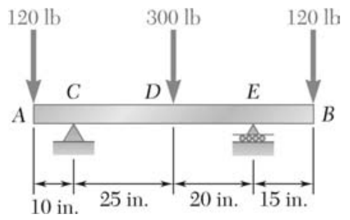
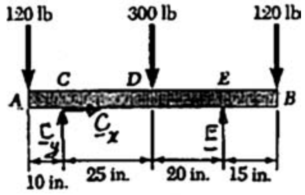


PROBLEM 7.38

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_C = 0: \quad (120 \text{ lb})(10 \text{ in.}) - (300 \text{ lb})(25 \text{ in.}) + E(45 \text{ in.}) - (120 \text{ lb})(60 \text{ in.}) = 0$$

$E = +300$ lb $\qquad \mathbf{E} = 300 \text{ lb} \uparrow$ ◁

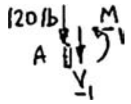
$\Sigma F_x = 0: \quad C_x = 0$

$+\uparrow \Sigma F_y = 0: \quad C_y + 300 \text{ lb} - 120 \text{ lb} - 300 \text{ lb} - 120 \text{ lb} = 0$

$C_y = +240$ lb $\qquad \mathbf{C} = 240 \text{ lb} \uparrow$ ◁

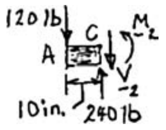
(*a*) Shear and bending moment

Just to the right of *A*:



$+\uparrow \Sigma F_y = 0: \quad -120 \text{ lb} - V_1 = 0 \qquad V_1 = -120 \text{ lb}, \quad M_1 = 0$ ◁

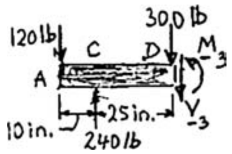
Just to the right of *C*:



$+\uparrow \Sigma F_y = 0: \quad 240 \text{ lb} - 120 \text{ lb} - V_2 = 0 \qquad V_2 = +120 \text{ lb}$ ◁

$+\circlearrowleft \Sigma M_C = 0: \quad M_2 + (120 \text{ lb})(10 \text{ in.}) = 0 \qquad M_2 = -1200 \text{ lb} \cdot \text{in.}$ ◁

Just to the right of *D*:



$+\uparrow \Sigma F_y = 0: \quad 240 - 120 - 300 - V_3 = 0 \qquad V_3 = -180 \text{ lb}$ ◁

$+\circlearrowleft \Sigma M_3 = 0: \quad M_3 + (120)(35) - (240)(25) = 0, \qquad M_3 = +1800 \text{ lb} \cdot \text{in.}$ ◁

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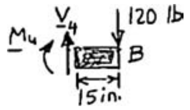
PROBLEM 7.38 (Continued)

Just to the right of E:

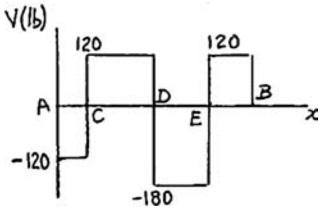


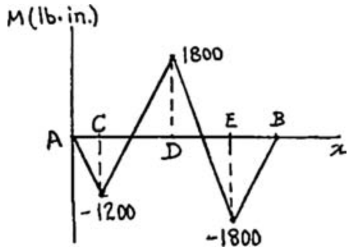
$+\Sigma F_y = 0\text{:}\quad V_4 - 120\text{ lb} = 0$ $\qquad V_4 = +120\text{ lb}$ ◁

$+\circlearrowleft \Sigma M_4 = 0\text{:}\quad -M_4 - (120\text{ lb})(15\text{ in.}) = 0$ $\qquad M_4 = -1800\text{ lb}\cdot\text{in.}$ ◁

At B: $\qquad V_B = M_B = 0$ ◁

(*b*) $\qquad |V|_{\max} = 180.0\text{ lb}$ ◀





$|M|_{\max} = 1800\text{ lb}\cdot\text{in.}$ ◀

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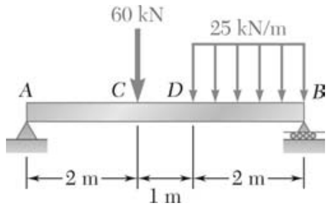
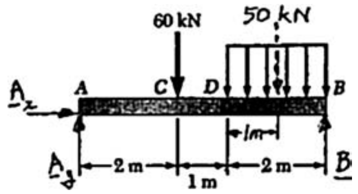


PROBLEM 7.39

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$+\circlearrowleft \Sigma M_A = 0: \quad B(5\text{ m}) - (60\text{ kN})(2\text{ m}) - (50\text{ kN})(4\text{ m}) = 0$

$B = +64.0\text{ kN}$ $\qquad \mathbf{B} = 64.0\text{ kN}\uparrow$ ◁

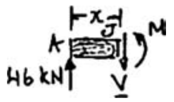
$\Sigma F_x = 0: \quad A_x = 0$

$+\uparrow \Sigma F_y = 0: \quad A_y + 64.0\text{ kN} - 6.0\text{ kN} - 50\text{ kN} = 0$

$A_y = +46.0\text{ kN}$ $\qquad \mathbf{A} = 46.0\text{ kN}\uparrow$ ◁

(*a*) Shear and bending-moment diagrams.

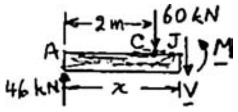
From *A* to *C*:



$+\uparrow \Sigma F_y = 0: \quad 46 - V = 0$ $\qquad V = +46\text{ kN}$ ◁

$+\circlearrowleft \Sigma M_y = 0: \quad M - 46x = 0$ $\qquad M = (46x)\text{kN}\cdot\text{m}$ ◁

From *C* to *D*:



$+\uparrow \Sigma F_y = 0: \quad 46 - 60 - V = 0$ $\qquad V = -14\text{ kN}$ ◁

$+\circlearrowleft \Sigma M_j = 0: \quad M - 46x + 60(x-2) = 0$

$M = (120 - 14x)\text{kN}\cdot\text{m}$

For $\quad x = 2\text{ m}: \quad M_C = +92.0\text{ kN}\cdot\text{m}$ ◁

For $\quad x = 3\text{ m}: \quad M_D = +78.0\text{ kN}\cdot\text{m}$ ◁

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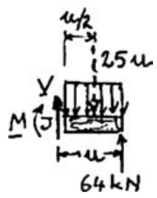
PROBLEM 7.39 (Continued)

From D to B:



$$+\uparrow \Sigma F_y = 0: \quad V + 64 - 25\mu = 0$$

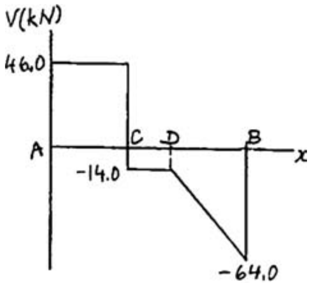
$$V = (25\mu - 64)\text{kN}$$

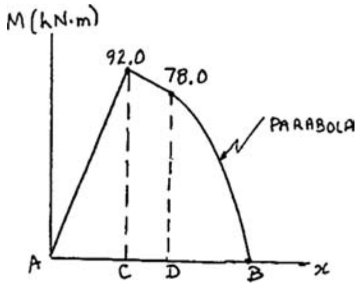
$$+\circlearrowleft \Sigma M_j = 0: \quad 64\mu - (25\mu)\left(\frac{\mu}{2}\right) - M = 0$$

$$M = (64\mu - 12.5\mu^2)\text{kN} \cdot \text{m}$$

For $\quad \mu = 0: \quad V_B = -64 \text{ kN} \qquad M_B = 0$ ◁

(*b*) $\qquad |V|_{max} = 64.0 \text{ kN}$ ◀





$|M|_{max} = 92.0 \text{ kN} \cdot \text{m}$ ◀

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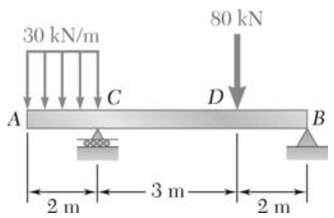
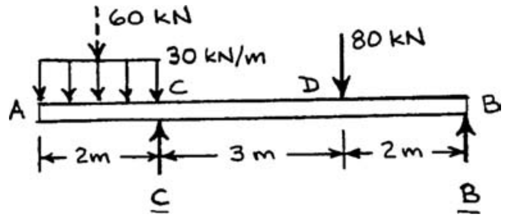


PROBLEM 7.40

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

<u>Free body: Entire beam</u>



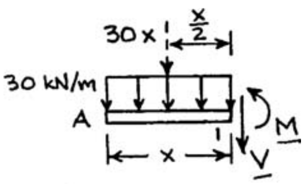
$$+\circlearrowleft \Sigma M_B = 0: \quad (60\text{ kN})(6\text{ m}) - C(5\text{ m}) + (80\text{ kN})(2\text{ m}) = 0$$

$$C = +104\text{ kN} \qquad \mathbf{C} = 104\text{ kN}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 104 - 60 - 80 + B = 0 \qquad \mathbf{B} = 36\text{ kN}\uparrow$$

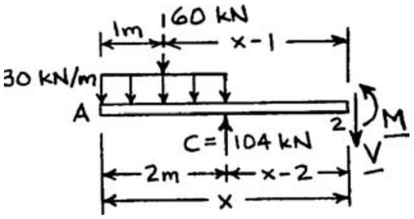
<u>From *A* to *C*</u>:

$$+\uparrow \Sigma F_y = 0: \quad -30x - V = 0 \qquad V = -30x$$



$$+\circlearrowleft \Sigma M_1 = 0: \quad (30x)\left(\frac{x}{2}\right) + M = 0 \qquad M = -15x^2$$

<u>From *C* to *D*</u>:



$$+\uparrow \Sigma F_y = 0: \quad 104 - 60 - V = 0 \qquad V = +44\text{ kN}$$

$$+\circlearrowleft \Sigma M_2 = 0: \quad (60)(x-1) - (104)(x-2) + M = 0 \qquad M = 44x - 148$$

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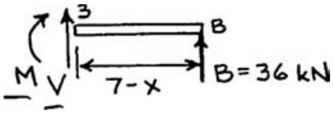
PROBLEM 7.40 (Continued)

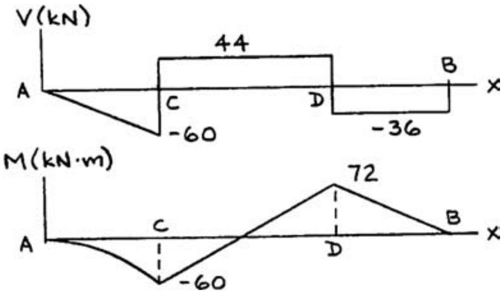
From D to B:



$+\uparrow \Sigma F_y = 0\colon \quad V = -36 \text{ kN}$

$+\circlearrowleft \Sigma M_3 = 0\colon \quad (36)(7-x) - M = 0$

$$M = -36x + 252$$



(*b*) $|V|_{max} = 60.0 \text{ kN}$ $|M|_{max} = 72.0 \text{ kN}\cdot\text{m.}$ ◀

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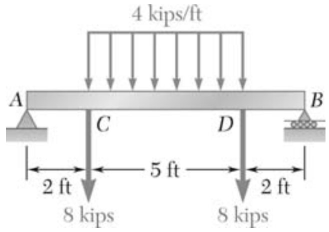


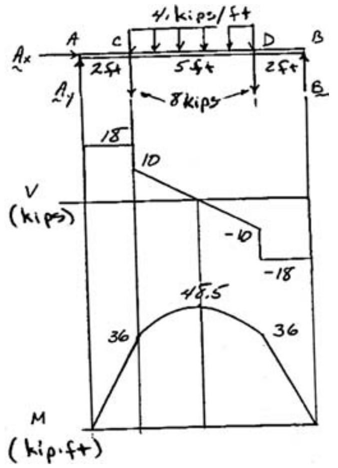
PROBLEM 7.41

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

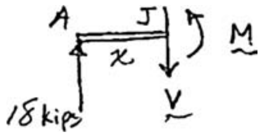
SOLUTION

(*a*) By symmetry:

$$A_y = B = 8 \text{ kips} + \frac{1}{2}(4 \text{ kips})(5 \text{ ft}) \quad \mathbf{A}_y = \mathbf{B} = 18 \text{ kips}\uparrow$$



Along AC:

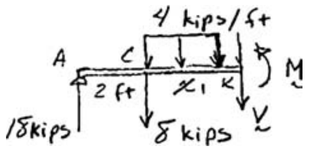


$$+\uparrow \Sigma F_y = 0: \quad 18 \text{ kips} - V = 0 \quad V = 18 \text{ kips}$$

$$\circlearrowleft \Sigma M_J = 0: \quad M - x(18 \text{ kips}) \quad M = (18 \text{ kips})x$$

$$M = 36 \text{ kip} \cdot \text{ft at } C(x = 2 \text{ ft})$$

Along CD:



$$+\uparrow \Sigma F_y = 0: \quad 18 \text{ kips} - 8 \text{ kips} - (4 \text{ kips/ft})x_1 - V = 0$$

$$V = 10 \text{ kips} - (4 \text{ kips/ft})x_1$$

$$V = 0 \text{ at } x_1 = 2.5 \text{ ft (at center)}$$

$$\circlearrowleft \Sigma M_K = 0: \quad M + \frac{x_1}{2}(4 \text{ kips/ft})x_1 + (8 \text{ kips})x_1 - (2 \text{ ft} + x_1)(18 \text{ kips}) = 0$$

$$M = 36 \text{ kip} \cdot \text{ft} + (10 \text{ kips/ft})x_1 - (2 \text{ kips/ft})x_1^2$$

$$M = 48.5 \text{ kip} \cdot \text{ft} \ \text{ at } \ x_1 = 2.5 \text{ ft}$$

Complete diagram by symmetry

(*b*) From diagrams:

$|V|_{\max} = 18.00$ kips on AC and DB ◀

$|M|_{\max} = 48.5$ kip · ft at center ◀

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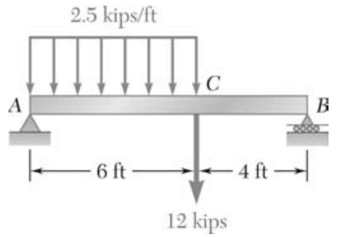
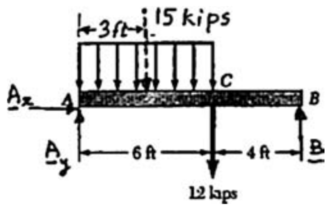


PROBLEM 7.42

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

<u>Free body: Entire beam</u>



$$+\circlearrowleft \Sigma M_A = 0: \quad B(10 \text{ ft}) - (15 \text{ kips})(3 \text{ ft}) - (12 \text{ kips})(6 \text{ ft}) = 0$$

$$B = +11.70 \text{ kips} \qquad \mathbf{B} = 11.70 \text{ kips} \uparrow \triangleleft$$

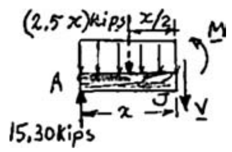
$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 15 - 12 + 11.70 = 0$$

$$A_y = +15.30 \text{ kips} \qquad \mathbf{A} = 15.30 \text{ kips} \uparrow \triangleleft$$

(*a*) <u>Shear and bending-moment diagrams</u>

<u>From *A* to *C*:</u>



$$+\uparrow \Sigma F_y = 0: \quad 15.30 - 2.5x - V = 0$$

$$V = (15.30 - 2.5x) \text{ kips}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + (2.5x)\left(\frac{x}{2}\right) - 15.30x = 0$$

$$M = 15.30x - 1.25x^2$$

For $x = 0$: $\quad V_A = +15.30 \text{ kips} \qquad M_A = 0 \triangleleft$

For $x = 6 \text{ ft}$: $\quad V_C = +0.300 \text{ kip} \qquad M_C = +46.8 \text{ kip} \cdot \text{ft} \triangleleft$

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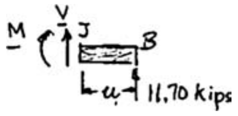
PROBLEM 7.42 (Continued)

From C to B:



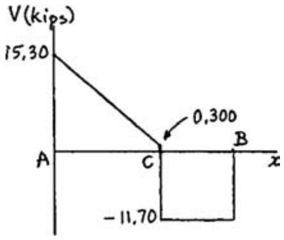
$$+\uparrow \Sigma F_y = 0: \quad V + 11.70 = 0 \qquad V = -11.70 \text{ kips} \ \triangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad 11.70\mu - M = 0$$

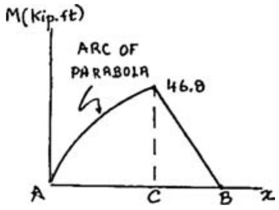
$$M = (11.70\mu) \text{ kip} \cdot \text{ft}$$

For $\mu = 4$ ft: $\qquad M_C = +46.8 \text{ kip} \cdot \text{ft} \ \triangleleft$

For $\mu = 0$: $\qquad M_B = 0 \ \triangleleft$



(b) $\qquad |V|_{\max} = 15.30 \text{ kips} \ \blacktriangleleft$



$|M|_{\max} = 46.8 \text{ kip} \cdot \text{ft} \ \blacktriangleleft$

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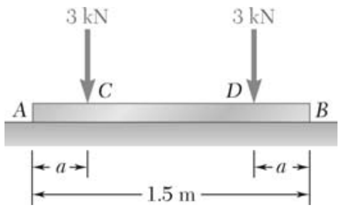


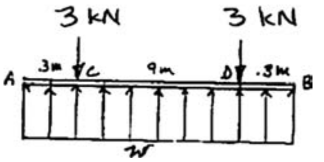
PROBLEM 7.43

Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed and knowing that $a = 0.3$ m, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

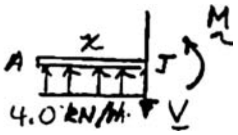
SOLUTION

(*a*) **FBD Beam:** $\uparrow \Sigma F_y = 0: \quad w(1.5\text{ m}) - 2(3.0\text{ kN}) = 0$

$$w = 4.0\text{ kN/m}$$



Along AC:

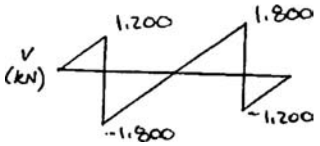


$$\uparrow \Sigma F_y = 0: \quad (4.0\text{ kN/m})x - V = 0$$

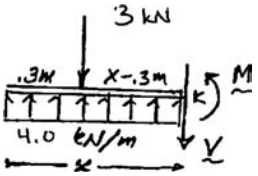
$$V = (4.0\text{ kN/m})x$$

$$\circlearrowleft \Sigma M_J = 0: \quad M - \frac{x}{2}(4.0\text{ kN/m})x = 0$$

$$M = (2.0\text{ kN/m})x^2$$



Along CD:

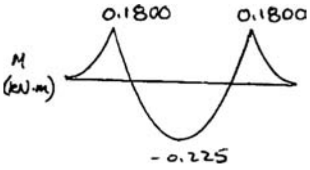


$$\uparrow \Sigma F_y = 0: \quad (4.0\text{ kN/m})x - 3.0\text{ kN} - V = 0$$

$$V = (4.0\text{ kN/m})x - 3.0\text{ kN}$$

$$\circlearrowleft \Sigma M_K = 0: \quad M + (x - 0.3\text{ m})(3.0\text{ kN}) - \frac{x}{2}(4.0\text{ kN/m})x = 0$$

$$M = 0.9\text{ kN}\cdot\text{m} - (3.0\text{ kN})x + (2.0\text{ kN/m})x^2$$



Note: $V = 0$ at $x = 0.75$ m, where $M = -0.225\text{ kN}\cdot\text{m}$

Complete diagrams using symmetry.

(*b*) $|V|_{\max} = 1.800\text{ kN}$ at C and D ◀

$|M|_{\max} = 0.225\text{ kN}\cdot\text{m}$ at center ◀

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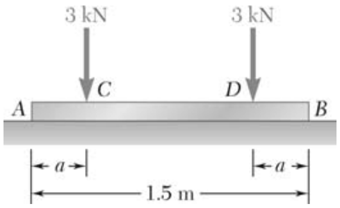


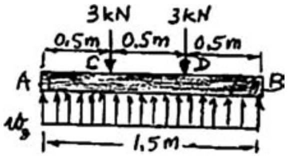
PROBLEM 7.44

Solve Problem 7.43 knowing that $a = 0.5$ m.

PROBLEM 7.43 Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed and knowing that *a* = 0.3 m, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

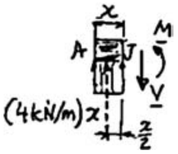
Free body: Entire beam



$+\!\uparrow \Sigma F_y = 0: \quad w_g(1.5\text{ m}) - 3\text{ kN} - 3\text{ kN} = 0 \qquad\qquad w_g = 4\text{ kN/m} \;\triangleleft$

(*a*) Shear and bending moment

From *A* to *C*:



$+\!\uparrow \Sigma F_y = 0: \quad 4x - V = 0 \quad V = (4x)\text{ kN}$

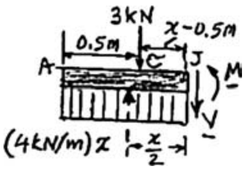
$+\circlearrowleft \Sigma M_J = 0: \quad M - (4x)\dfrac{x}{2} = 0, \quad M = (2x^2)\text{ kN}\cdot\text{m}$

For $x = 0$: $\qquad V_A = M_A = 0 \;\triangleleft$

For $x = 0.5$ m: $\qquad V_C = 2\text{ kN}, \qquad M_C = 0.500\text{ kN}\cdot\text{m} \;\triangleleft$

From *C* to *D*: $\quad +\!\uparrow \Sigma F_y = 0: \quad 4x - 3\text{ kN} - V = 0$

$V = (4x - 3)\text{ kN}$



$+\circlearrowleft \Sigma M_J = 0: \quad M + (3\text{ kN})(x - 0.5) - (4x)\dfrac{x}{2} = 0$

$M = (2x^2 - 3x + 1.5)\text{ kN}\cdot\text{m}$

For $x = 0.5$ m: $\qquad V_C = -1.00\text{ kN}, \quad M_C = 0.500\text{ kN}\cdot\text{m} \;\triangleleft$

For $x = 0.75$ m: $\qquad V_C = 0, \quad M_C = 0.375\text{ kN}\cdot\text{m} \;\triangleleft$

For $x = 1.0$ m: $\qquad V_C = 1.00\text{ kN}, \quad M_C = 0.500\text{ kN}\cdot\text{m} \;\triangleleft$

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PROBLEM 7.44 (Continued)

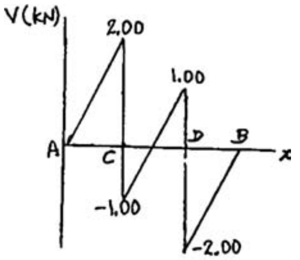
From D to B:

$+\uparrow \Sigma F_y = 0$: $V + 4\mu = 0 \quad V = -(4\mu)$ kN

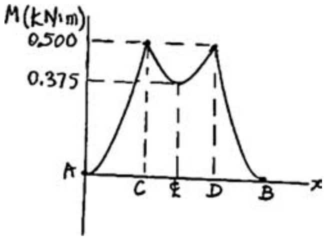
$+\circlearrowleft \Sigma M_J = 0$: $(4\mu)\dfrac{\mu}{2} - M = 0, \quad M = 2\mu^2$

For $\mu = 0$: $V_B = M_B = 0$ ◁

For $\mu = 0.5$ m: $V_D = -2$ kN, $M_D = 0.500$ kN·m ◁



(*b*) $|V|_{max} = 2.00$ kN ◀



$|M|_{max} = 0.500$ kN·m ◀

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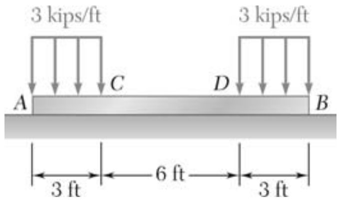
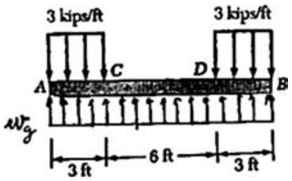


PROBLEM 7.45

Assuming the upward reaction of the ground on beam AB to be uniformly distributed, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

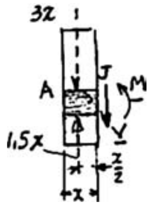
Free body: Entire beam



$+\uparrow \Sigma F_y = 0: \quad w_g(12\text{ ft}) - (3\text{ kips/ft})(6\text{ ft}) = 0$ $\qquad w_g = 1.5\text{ kips/ft} \triangleleft$

(a) Shear and bending-moment diagrams.

From A to C:



$+\uparrow \Sigma F_y = 0: \quad 1.5x - 3x - V = 0$

$V = (-1.5x)\text{ kips}$

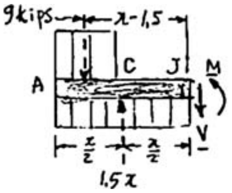
$+\circlearrowleft \Sigma M_J = 0: \quad M + (3x)\dfrac{x}{2} - (1.5x)\dfrac{x}{2} = 0$

$M = (-0.75x^2)\text{ kip}\cdot\text{ft}$

For $x = 0$: $\qquad V_A = M_A = 0 \triangleleft$

For $x = 3\text{ ft}$: $\qquad V_C = -4.5\text{ kips} \quad M_C = -6.75\text{ kip}\cdot\text{ft} \triangleleft$

From C to D:



$+\uparrow \Sigma F_y = 0: \quad 1.5x = 9 - V = 0, \quad V = (1.5x - 9)\text{ kips}$

$+\circlearrowleft \Sigma M_J = 0: \quad M + 9(x - 1.5) - (1.5x)\dfrac{x}{2} = 0$

$M = 0.75x^2 - 9x + 13.5$

For $x = 3\text{ ft}$: $\qquad V_C = -4.5\text{ kips}, \qquad M_C = -6.75\text{ kip}\cdot\text{ft} \triangleleft$

For $x = 6\text{ ft}$: $\qquad V_C = 0, \qquad M_C = -13.50\text{ kip}\cdot\text{ft} \triangleleft$

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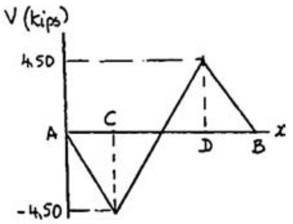
PROBLEM 7.45 (Continued)

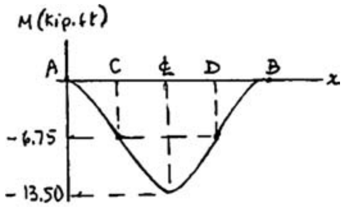
For $x = 9$ ft: $V_D = 4.5$ kips, $M_D = -6.75$ kip · ft ◁

At B: $V_B = M_B = 0$ ◁



(b) $|V|_{max} = 4.50$ kips ◀

Bending moment diagram consists of three distinct arcs of parabola.



$|M|_{max} = 13.50$ kip · ft ◀

Since entire diagram is below the x axis: $M \leq 0$ everywhere ◀

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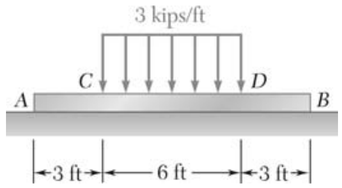
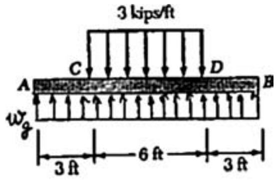


PROBLEM 7.46

Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

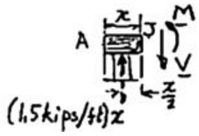


Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad w_g(12\text{ ft}) - (3\text{ kips/ft})(6\text{ ft}) = 0$$

$$w_g = 1.5\text{ kips/ft} \quad \triangleleft$$

(*a*) Shear and bending-moment diagrams from *A* to *C*:



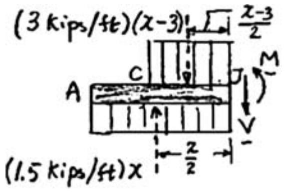
$$+\uparrow \Sigma F_y = 0: \quad 1.5x - V = 0 \qquad V = (1.5x)\text{kips}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M - (1.5x)\frac{x}{2} \qquad M = (0.75x^2)\text{kip}\cdot\text{ft}$$

For $x = 0$: $V_A = M_A = 0 \quad \triangleleft$

For $x = 3$ ft: $V_C = 4.5$ kips, $M_C = 6.75\text{ kip}\cdot\text{ft} \quad \triangleleft$

From *C* to *D*:



$$+\uparrow \Sigma F_y = 0: \quad 1.5x - 3(x-3) - V = 0$$

$$V = (9 - 1.5x)\text{kips}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + 3(x-3)\frac{x-3}{2} - (1.5x)\frac{x}{2} = 0$$

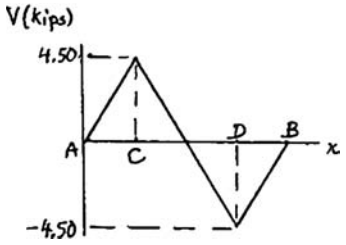
$$M = [0.75x^2 - 1.5(x-3)^2]\text{kip}\cdot\text{ft}$$

For $x = 3$ ft: $V_C = 4.5$ kips, $M_C = 6.75\text{ kip}\cdot\text{ft} \quad \triangleleft$

For $x = 6$ ft: $V_{℄} = 0$, $M_{℄} = 13.50\text{ kip}\cdot\text{ft} \quad \triangleleft$

For $x = 9$ ft: $V_D = -4.5$ kips, $M_D = 6.75\text{ kip}\cdot\text{ft} \quad \triangleleft$

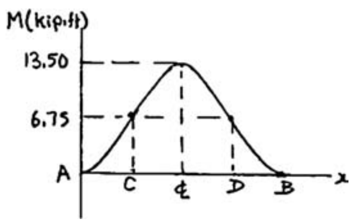
At *B*: $V_B = M_B = 0 \quad \triangleleft$



(*b*)

$$|V|_{max} = 4.50\text{ kips} \quad \blacktriangleleft$$

$$|M|_{max} = 13.50\text{ kip}\cdot\text{ft} \quad \blacktriangleleft$$



Bending-moment diagram consists or three distinct arcs of parabola, all located above the *x* axis.

Thus: $M \geq 0$ everywhere $\blacktriangleleft$

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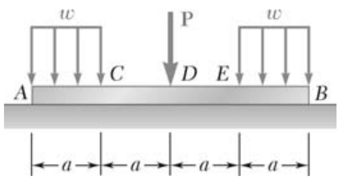
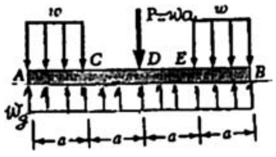


PROBLEM 7.47

Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $P = wa$, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION



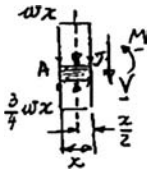
Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad w_g(4a) - 2wa - wa = 0$$

$$w_g = \frac{3}{4}w \quad \triangleleft$$

(a) Shear and bending-moment diagrams

From A to C:



$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{4}wx - wx - V = 0$$

$$V = -\frac{1}{4}wx$$

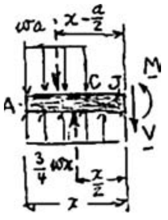
$$+\circlearrowleft \Sigma M_J = 0: \quad M + (wx)\frac{x}{2} - \left(\frac{3}{4}wx\right)\frac{x}{2} = 0$$

$$M = -\frac{1}{8}wx^2$$

For $x = 0$: $\quad V_A = M_A = 0 \quad \triangleleft$

For $x = a$: $\quad V_C = -\frac{1}{4}wa \qquad M_C = -\frac{1}{8}wa^2 \quad \triangleleft$

From C to D:



$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{4}wx - wa - V = 0$$

$$V = \left(\frac{3}{4}x - a\right)w$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + wa\left(x - \frac{a}{2}\right) - \frac{3}{4}wx\left(\frac{x}{2}\right) = 0$$

$$M = \frac{3}{8}wx^2 - wa\left(x - \frac{a}{2}\right) \qquad (1)$$

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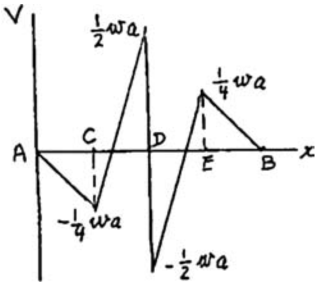
PROBLEM 7.47 (Continued)

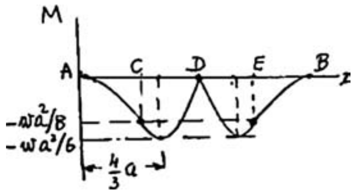
For $x = a$: $V_C = -\frac{1}{4}wa$ $M_C = -\frac{1}{8}wa^2$ ◁

For $x = 2a$: $V_D = +\frac{1}{2}wa$ $M_D = 0$ ◁

Because of the symmetry of the loading, we can deduce the values of V and M for the right-hand half of the beam from the values obtained for its left-hand half.



(b) $|V|_{\max} = \frac{1}{2}wa$ ◀



To find $|M|_{\max}$, we differentiate Eq. (1) and set $\frac{dM}{dx} = 0$:

$$\frac{dM}{dx} = \frac{3}{4}wx - wa = 0, \quad x = \frac{4}{3}a$$

$$M = \frac{3}{8}w\left(\frac{4}{3}a\right)^2 - wa^2\left(\frac{4}{3} - \frac{1}{2}\right) = -\frac{wa^2}{6}$$

$|M|_{\max} = \frac{1}{6}wa^2$ ◀

Bending-moment diagram consists of four distinct arcs of parabola.

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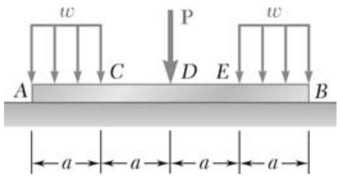
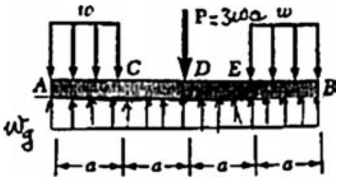


PROBLEM 7.48

Solve Problem 7.47 knowing that $P = 3wa$.

PROBLEM 7.47 Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed and knowing that $P = wa$, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



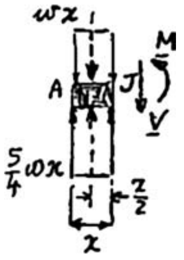
Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad w_g(4a) - 2wa - 3wa = 0$$

$$w_g = \frac{5}{4}w \quad \triangleleft$$

(*a*) Shear and bending-moment diagrams

From *A* to *C*:



$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{4}wx - wx - V = 0$$

$$V = +\frac{1}{4}wx$$

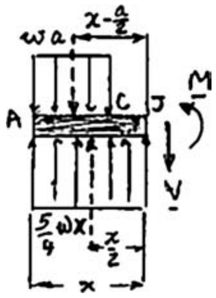
$$+\circlearrowleft \Sigma M_J = 0: \quad M + (wx)\frac{x}{2} - \left(\frac{5}{4}wx\right)\frac{x}{2} = 0$$

$$M = +\frac{1}{8}wx^2$$

For $x = 0$: $\qquad V_A = M_A = 0 \quad \triangleleft$

For $x = a$: $\qquad V_C = +\frac{1}{4}wa \qquad M_C = +\frac{1}{8}wa^2 \quad \triangleleft$

From *C* to *D*:



$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{4}wx - wa - V = 0$$

$$V = \left(\frac{5}{4}x - a\right)w$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + wa\left(x - \frac{a}{2}\right) - \frac{5}{4}wx\left(\frac{x}{2}\right) = 0$$

$$M = \frac{5}{8}wx^2 - wa\left(x - \frac{a}{2}\right) \qquad (1)$$

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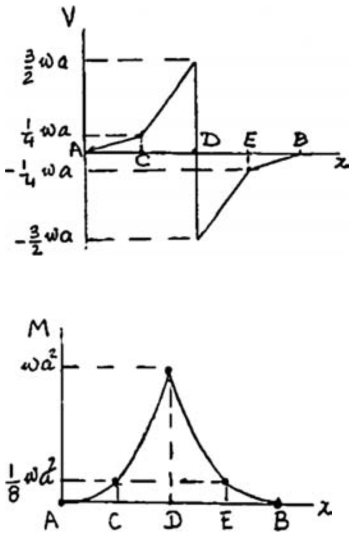
PROBLEM 7.48 (Continued)

For $x = a$: $V_C = +\frac{1}{4}wa,\ M_C = +\frac{1}{8}wa^2$ ◁

For $x = 2a$: $V_D = +\frac{3}{2}wa,\ M_D = +wa^2$ ◁

Because of the symmetry of the loading, we can deduce the values of V and M for the right-hand half of the beam from the values obtained for its left-hand half.



(*b*) $|V|_{\max} = \frac{3}{2}wa$ ◀

To find $|M|_{\max}$, we differentiate Eq. (1) and set $\frac{dM}{dx} = 0$:

$$\frac{dM}{dx} = \frac{5}{4}wx - wa = 0$$

$$x = \frac{4}{5}a < a \quad \text{(outside portion } CD\text{)}$$

The maximum value of $|M|$ occurs at D:

$$|M|_{\max} = wa^2 \ ◀$$

Bending-moment diagram consists of four distinct arcs of parabola.

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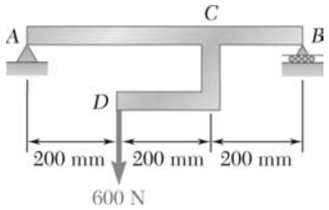
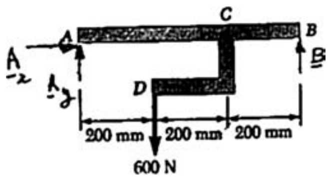


PROBLEM 7.49

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the shear and bending moment (*a*) just to the left of *C*, (*b*) just to the right of *C*.

SOLUTION



Free body: Entire beam

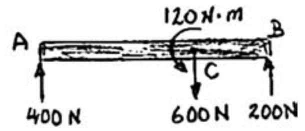
$$+\circlearrowleft \Sigma M_A = 0: \quad B(0.6 \text{ m}) - (600 \text{ N})(0.2 \text{ m}) = 0$$

$$B = +200 \text{ N} \qquad \mathbf{B} = 200 \text{ N}\uparrow \triangleleft$$

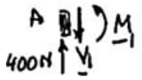
$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 600 \text{ N} + 200 \text{ N} = 0$$

$$A_y = +400 \text{ N} \qquad \mathbf{A} = 400 \text{ N}\uparrow \triangleleft$$

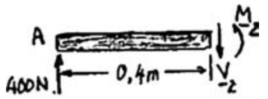


We replace the 600-N load by an equivalent force-couple system at *C*



Just to the right of *A*:

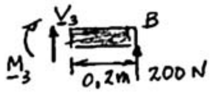
$$V_1 = +400 \text{ N}, \quad M_1 = 0 \triangleleft$$



(*a*) Just to the left of *C*:

$$V_2 = +400 \text{ N} \blacktriangleleft$$

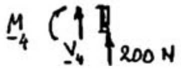
$$M_2 = (400 \text{ N})(0.4 \text{ m}) \qquad M_2 = +160.0 \text{ N}\cdot\text{m} \blacktriangleleft$$



(*b*) Just to the right of *C*:

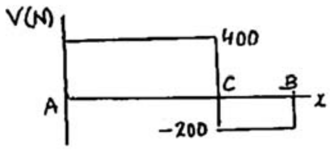
$$V_3 = -200 \text{ N} \blacktriangleleft$$

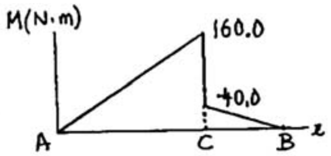
$$M_3 = (200 \text{ N})(0.2 \text{ m}) \qquad M_3 = +40.0 \text{ N}\cdot\text{m} \blacktriangleleft$$



Just to the left of *B*:

$$V_4 = -200 \text{ N}, \quad M_4 = 0 \triangleleft$$





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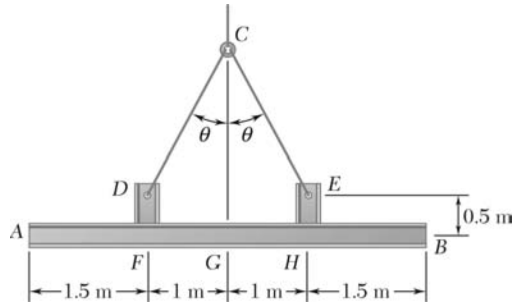
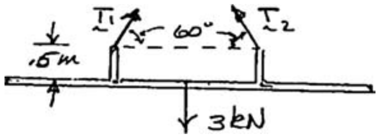


PROBLEM 7.50

Two small channel sections *DF* and *EH* have been welded to the uniform beam *AB* of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at *D* and *E*. Knowing that $\theta = 30°$ and neglecting the weight of the channel sections, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION



FBD Beam + channels:

(*a*) By symmetry: $T_1 = T_2 = T$

$$+\uparrow \Sigma F_y = 0: \quad 2T \sin 60° - 3 \text{ kN} = 0$$

$$T = \frac{3}{\sqrt{3}} \text{ kN}$$

$$T_{1x} = \frac{3}{2\sqrt{3}}$$

$$T_{1y} = \frac{3}{2} \text{ kN}$$

FBD Beam:

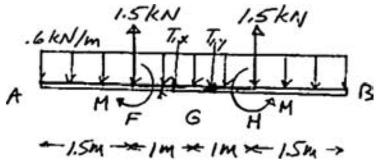
$$M = (0.5 \text{ m})\frac{3}{2\sqrt{3}} \text{ kN}$$

$$= 0.433 \text{ kN} \cdot \text{m}$$

With cable force replaced by equivalent force-couple system at *F* and *G*

Shear Diagram: *V* is piecewise linear

$$\left(\frac{dV}{dx} = -0.6 \text{ kN/m}\right) \text{with } 1.5 \text{ kN}$$



discontinuities at *F* and *H*.

$$V_{F^-} = -(0.6 \text{ kN/m})(1.5 \text{ m}) = 0.9 \text{ kN}$$

V increases by 1.5 kN to $+0.6$ kN at F^+

$$V_G = 0.6 \text{ kN} - (0.6 \text{ kN/m})(1 \text{ m}) = 0$$

Finish by invoking symmetry

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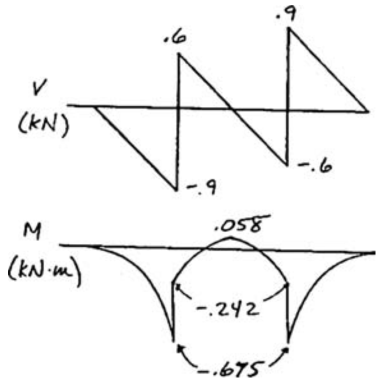
PROBLEM 7.50 (Continued)

Moment diagram: M is piecewise parabolic

$$\left(\frac{dM}{dx} \text{ decreasing with } V\right)$$

with discontinuities of .433 kN at F and H.



$$M_{F^-} = -\frac{1}{2}(0.9 \text{ kN})(1.5 \text{ m})$$
$$= -0.675 \text{ kN} \cdot \text{m}$$

M increases by 0.433 kN m to –0.242 kN · m at F^+

$$M_G = -0.242 \text{ kN} \cdot \text{m} + \frac{1}{2}(0.6 \text{ kN})(1 \text{ m})$$
$$= 0.058 \text{ kN} \cdot \text{m}$$

Finish by invoking symmetry

(*b*) $|V|_{max} = 900 \text{ N}$ ◀

at F^- and G^+

$|M|_{max} = 675 \text{ N} \cdot \text{m}$ ◀

at F and G

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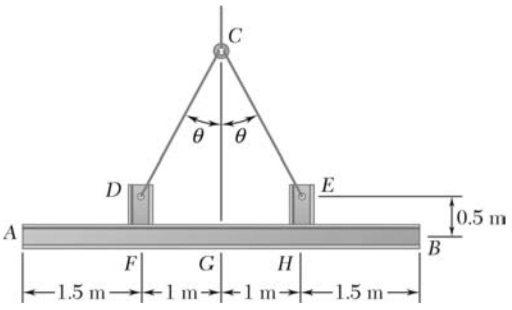


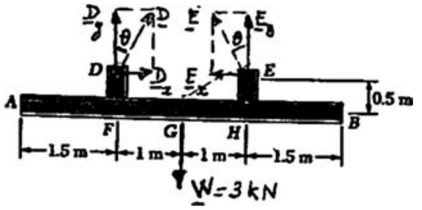
PROBLEM 7.51

Solve Problem 7.50 when $\theta = 60°$.

PROBLEM 7.50 Two small channel sections *DF* and *EH* have been welded to the uniform beam *AB* of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at *D* and *E*. Knowing that $\theta = 30°$ and neglecting the weight of the channel sections, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION

Free body: Beam and channels



From symmetry:

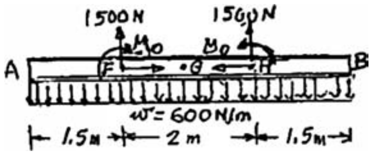
$$E_y = D_y$$

Thus: $$E_x = D_x = D_y \tan\theta \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad D_y + E_y - 3 \text{ kN} = 0 \qquad \mathbf{D}_y = \mathbf{E}_y = 1.5 \text{ kN}\uparrow \triangleleft$$

From (1): $$\mathbf{D}_x = (1.5 \text{ kN})\tan\theta \longrightarrow \qquad \mathbf{E} = (1.5 \text{ kN})\tan\theta \longleftarrow \triangleleft$$

We replace the forces at *D* and *E* by equivalent force-couple systems at *F* and *H*, where



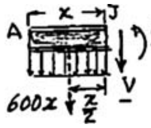
$$M_0 = (1.5 \text{ kN} \tan\theta)(0.5 \text{ m}) = (750 \text{ N}\cdot\text{m})\tan\theta \qquad (2)$$

We note that the weight of the beam per unit length is

$$w = \frac{W}{L} = \frac{3 \text{ kN}}{5 \text{ m}} = 0.6 \text{ kN/m} = 600 \text{ N/m}$$

(*a*) Shear and bending moment diagrams

From *A* to *F*:



$$+\uparrow \Sigma F_y = 0: \quad -V - 600x = 0 \quad V = (-600x)\text{N}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + (600x)\frac{x}{2} = 0, \quad M = (-300x^2)\text{N}\cdot\text{m}$$

For $x = 0$: $$V_A = M_A = 0 \triangleleft$$

For $x = 1.5$ m: $$V_F = -900 \text{ N}, \quad M_F = -675 \text{ N}\cdot\text{m} \triangleleft$$

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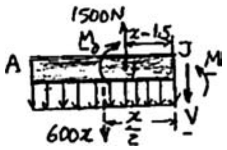
PROBLEM 7.51 (Continued)

<u>From F to H</u>:



$$+\uparrow \Sigma F_y = 0: \quad 1500 - 600x - V = 0$$

$$V = (1500 - 600x)\text{N}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + (600x)\frac{x}{2} - 1500(x - 1.5) - M_0 = 0$$

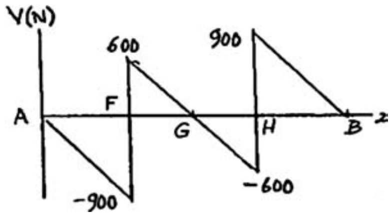
$$M = M_0 - 300x^2 + 1500(x - 1.5)\text{N}\cdot\text{m}$$

For $x = 1.5$ m: $\quad V_F = +600$ N, $\quad M_F = (M_0 - 675)$ N·m ◁

For $x = 2.5$ m: $\quad V_G = 0, \quad M_G = (M_0 - 375)$ N·m ◁

<u>From G To B</u>, The V and M diagrams will be obtained by symmetry,

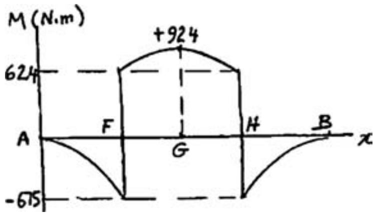
(*b*) $\quad |V|_{\max} = 900$ N ◀



Making $\theta = 60°$ in Eq. (2): $\quad M_0 = 750 \tan 60° = 1299$ N·m

Thus, just to the right of F: $\quad M = 1299 - 675 = 624$ N·m ◁

and $\quad M_G = 1299 - 375 = 924$ N·m ◁



(*b*) $\quad |V|_{\max} = 900$ N ◀

$|M|_{\max} = 924$ N·m ◀

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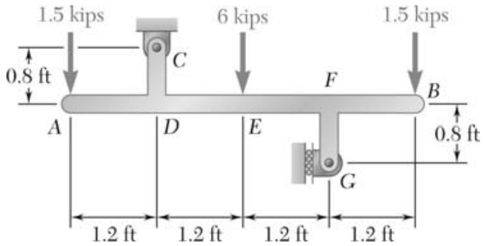
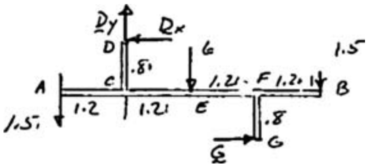


PROBLEM 7.52

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD whole:



$$\circlearrowleft \Sigma M_D = 0: \quad (1.2\text{ ft})(1.5\text{ kips}) - (1.2\text{ ft})(6\text{ kips}) - (3.6\text{ ft})(1.5\text{ kips}) + (1.6\text{ ft})G = 0$$

$$\mathbf{G} = 6.75\text{ kips} \longrightarrow$$

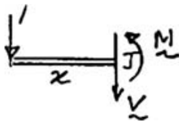
(Dimensions in ft., loads in kips, moments in kips · ft)

$$\longrightarrow \Sigma F_x = 0: \quad -D_x + G = 0 \qquad \mathbf{D}_x = 6.75\text{ kips} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: \quad D_y - 1.5\text{ kips} - 6\text{ kips} - 1.5\text{ kips} = 0 \qquad \mathbf{D}_y = 9\text{ kips} \uparrow$$

Beam AB, with forces **D** and **G** replaced by equivalent force/couples at *C* and *F*

Along AD:

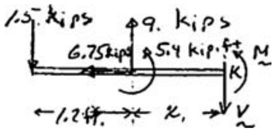


$$\uparrow \Sigma F_y = 0: \quad -1.5\text{ kips} - V = 0 \quad V = -1.5\text{ kips}$$

$$\circlearrowleft \Sigma M_J = 0: \quad x(1.5\text{ kips}) + M = 0 \quad M = -(1.5\text{ kips})x$$

$$M = -1.8\text{ kips at } x = 1.2\text{ ft}$$

Along DE:



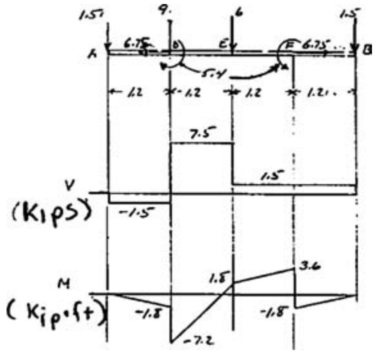
$$\uparrow \Sigma F_y = 0: \quad -1.5\text{ kips} + 9\text{ kips} - V = 0$$

$$V = 7.5\text{ kips}$$

$$\circlearrowleft \Sigma M_K = 0: \quad M + 5.4\text{ kip}\cdot\text{ft} - x_1(9\text{ kips}) + (1.2\text{ ft} + x_1)(1.5\text{ kips}) = 0$$

$$M = 7.2\text{ kip}\cdot\text{ft} + (7.5\text{ kips})x_1$$

$$M = 1.8\text{ kip}\cdot\text{ft} \;\text{ at }\; x_1 = 1.2\text{ ft}$$



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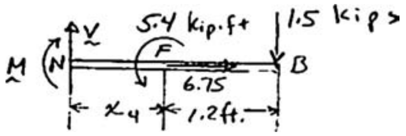
PROBLEM 7.52 (Continued)

Along EF:



$$+\uparrow \Sigma F_y = 0: \quad V - 1.5 \text{ kips} = 0 \quad V = 1.5 \text{ kips}$$

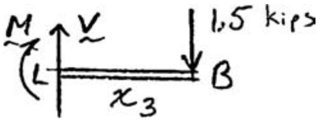
$$+\circlearrowleft \Sigma M_N = 0: \quad -M + 5.4 \text{ kip}\cdot\text{ft} - (x_4 + 1.2 \text{ ft})(1.5 \text{ kips})$$

$$M = 3.6 \text{ kip}\cdot\text{ft} - (1.5 \text{ kips})x_4$$

$$M = 1.8 \text{ kip}\cdot\text{ft} \;\text{ at }\; x_4 = 1.2 \text{ ft}$$

$$M = 3.6 \text{ kip}\cdot\text{ft} \;\text{ at }\; x_4 = 0$$

Along FB:



$$+\uparrow \Sigma F_y = 0: \quad V - 1.5 \text{ kips} = 0$$

$$V = 1.5 \text{ kips}$$

$$+\circlearrowleft \Sigma M_L = 0: \quad -M - x_3(1.5 \text{ kips}) = 0$$

$$M = (-1.5 \text{ kips})x_3$$

$$M = -1.8 \text{ kip}\cdot\text{ft at } x_3 = 1.2 \text{ ft}$$

From diagrams: $|V|_{max} = 7.50 \text{ kips on } DE$ ◀

$|M|_{max} = 7.20 \text{ kip}\cdot\text{ft} \;\text{ at }\; D^+$ ◀

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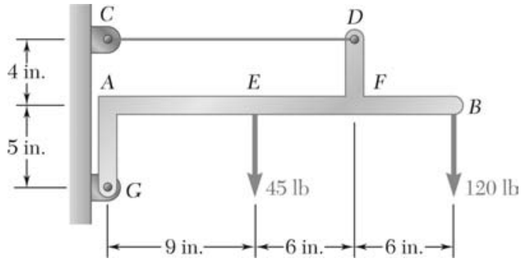
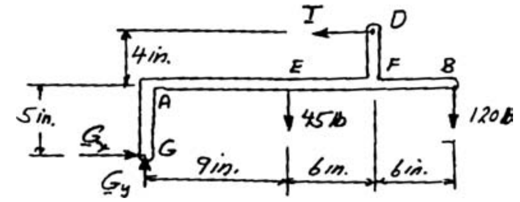


PROBLEM 7.53

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

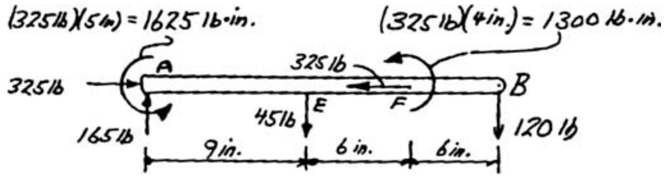


$$+\circlearrowleft \Sigma M_G = 0: \quad T(9 \text{ in.}) - (45 \text{ lb})(9 \text{ in.}) - (120 \text{ lb})(21 \text{ in.}) = 0 \qquad T = 325 \text{ lb}$$

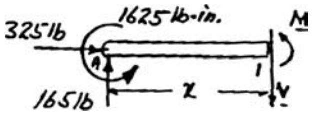
$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad -325 \text{ lb} + G_x = 0 \qquad \mathbf{G}_x = 325 \text{ lb} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad G_y - 45 \text{ lb} - 120 \text{ lb} = 0 \qquad \mathbf{G}_y = 165 \text{ lb} \uparrow$$

Equivalent loading on straight part of beam *AB*



From *A* to *E*:

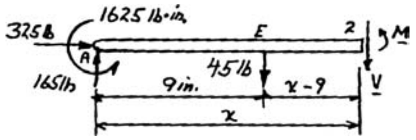


$$\Sigma F_y = 0: \quad V = +165 \text{ lb}$$

$$+\circlearrowleft \Sigma M_1 = 0: \quad +1625 \text{ lb} \cdot \text{in.} - (165 \text{ lb})x + M = 0$$

$$M = -1625 + 165x$$

From *E* to *F*:



$$+\uparrow \Sigma F_y = 0: \quad 165 - 45 - V = 0$$

$$V = +120 \text{ lb}$$

$$+\circlearrowleft \Sigma M_2 = 0: \quad +1625 \text{ lb} \cdot \text{in.} - (165 \text{ lb})x + (45 \text{ lb})(x - 9) + M = 0$$

$$M = -1220 + 120x$$

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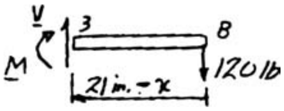
PROBLEM 7.53 (Continued)

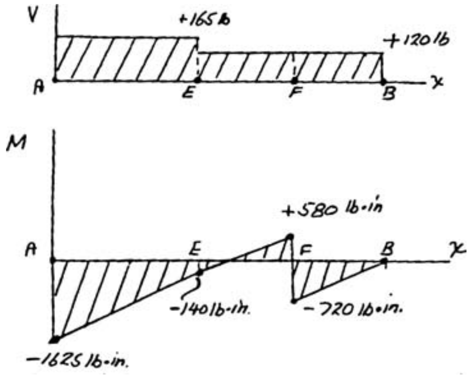
From F to B:



$$\Sigma F_y = 0 \quad V = +120 \text{ lb}$$

$$+\circlearrowleft \Sigma M_3 = 0: \quad -(120)(21 - x) - M = 0$$

$$M = -2520 + 120x$$



$$|V|_{max} = 165.0 \text{ lb} \blacktriangleleft$$

$$|M|_{max} = 1625 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

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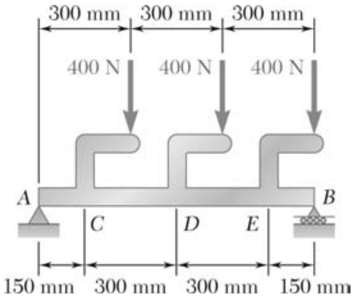
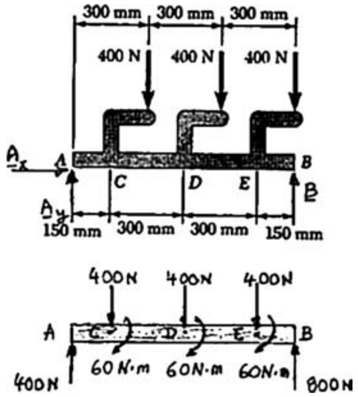


PROBLEM 7.54

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_A = 0: \quad B(0.9\text{ m}) - (400\text{ N})(0.3\text{ m}) - (400\text{ N})(0.6\text{ m}) - (400\text{ N})(0.9\text{ m}) = 0$$

$$B = +800\text{ N} \qquad \mathbf{B} = 800\text{ N}\uparrow \triangleleft$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 800\text{ N} - 3(400\text{ N}) = 0$$

$$A_y = +400\text{ N} \qquad \mathbf{A} = 400\text{ N}\uparrow \triangleleft$$

We replace the loads by equivalent force-couple systems at *C*, *D*, and *E*.

We consider successively the following *F-B* diagrams.

$V_1 = +400\text{ N}$
$M_1 = 0$

$V_2 = +400\text{ N}$
$M_2 = +60\text{ N}\cdot\text{m}$

$V_3 = 0$
$M_3 = +120\text{ N}\cdot\text{m}$

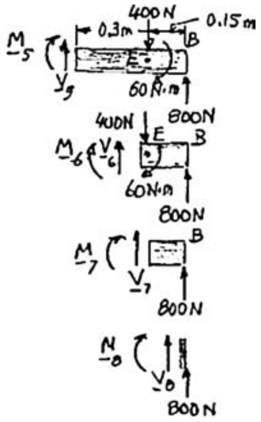
$V_4 = 0$
$M_4 = +120\text{ N}\cdot\text{m}$

$V_5 = -400\text{ N}$
$M_5 = +180\text{ N}\cdot\text{m}$

$V_6 = -400\text{ N}$
$M_6 = +60\text{ N}\cdot\text{m}$

$V_7 = -800\text{ N}$
$M_7 = +120\text{ N}\cdot\text{m}$

$V_8 = -800\text{ N}$
$M_8 = 0$



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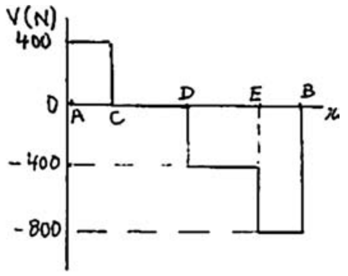
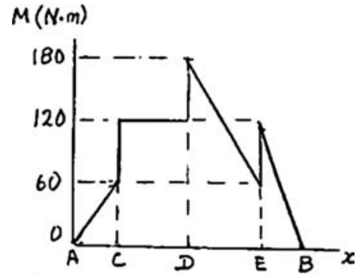
PROBLEM 7.54 (Continued)



(*b*) $|V|_{max} = 800 \text{ N}$ ◀



$|M|_{max} = 180.0 \text{ N} \cdot \text{m}$ ◀

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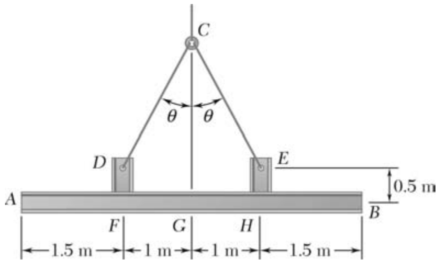


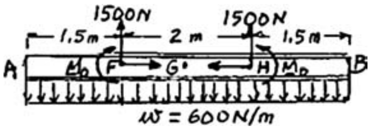
PROBLEM 7.55

For the structural member of Problem 7.50, determine (*a*) the angle θ for which the maximum absolute value of the bending moment in beam *AB* is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (*Hint:* Draw the bending-moment diagram and then equate the absolute values of the largest positive and negative bending moments obtained.)

PROBLEM 7.50 Two small channel sections *DF* and *EH* have been welded to the uniform beam *AB* of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at *D* and *E*. Knowing that $\theta = 30°$ and neglecting the weight of the channel sections, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION

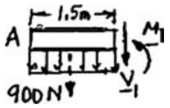
See solution of Problem 7.50 for reduction of loading or beam *AB* to the following:



where $M_0 = (750\ \text{N}\cdot\text{m})\tan\theta$ ◁

[Equation (2)]

The largest negative bending moment occurs Just to the left of *F*:

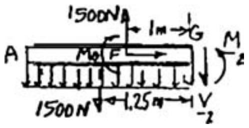


$$+\circlearrowleft \Sigma M_1 = 0: \quad M_1 + (900\ \text{N})\left(\frac{1.5\ \text{m}}{2}\right) = 0$$

$M_1 = -675\ \text{N}\cdot\text{m}$ ◁

The largest positive bending moment occurs

At *G*:



$$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 - M_0 + (1500\ \text{N})(1.25\ \text{m} - 1\ \text{m}) = 0$$

$M_2 = M_0 - 375\ \text{N}\cdot\text{m}$ ◁

Equating M_2 and $-M_1$:

$$M_0 - 375 = +675$$

$$M_0 = 1050\ \text{N}\cdot\text{m}$$

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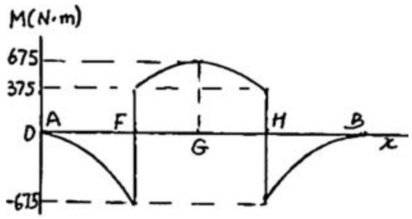
PROBLEM 7.55 (Continued)

(*a*) From Equation (2): $\tan\theta = \dfrac{1050}{750} = 1.400$ $\theta = 54.5°$ ◀



(*b*) $|M|_{\max} = 675\ \text{N}\cdot\text{m}$ ◀

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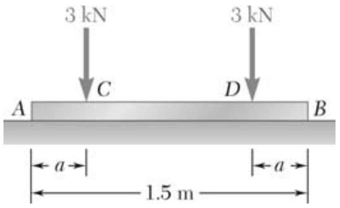


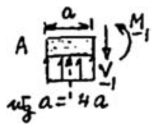
PROBLEM 7.56

For the beam of Problem 7.43, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Problem 7.55.)

PROBLEM 7.43 Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed and knowing that $a = 0.3$ m, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Force per unit length exerted by ground:

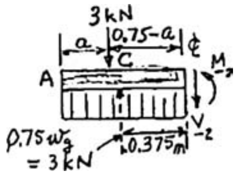


$$w_g = \frac{6\text{ kN}}{1.5\text{ m}} = 4\text{ kN/m}$$

The largest positive bending moment occurs Just to the left of *C*:

$$+\circlearrowleft \Sigma M_1 = 0: \quad M_1 = (4a)\frac{a}{2} \qquad M_1 = 2a^2 \triangleleft$$

The largest negative bending moment occurs



At the center line:

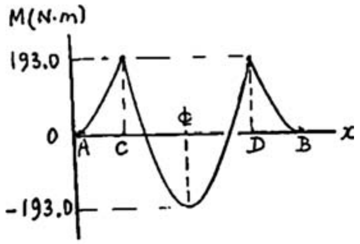
$$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 + 3(0.75 - a) - 3(0.375) = 0 \qquad M_2 = -(1.125 - 3a) \triangleleft$$

Equating M_1 and $-M_2$: $\quad 2a^2 = 1.125 - 3a$

$$a^2 + 1.5a - 0.5625 = 0$$

(*a*) Solving the quadratic equation: $a = 0.31066,$ $\qquad a = 0.311\text{ m}$ ◀

(*b*) Substituting: $|M|_{max} = M_1 = 2(0.31066)^2$ $\qquad |M|_{max} = 193.0\text{ N}\cdot\text{m}$ ◀



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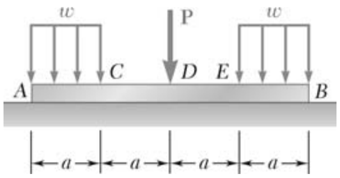
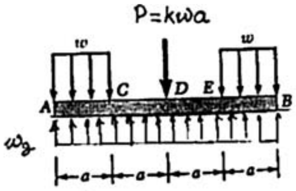


PROBLEM 7.57

For the beam of Problem 7.47, determine (*a*) the ratio $k = P/wa$ for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Problem 7.55.)

PROBLEM 7.47 Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed and knowing that $P = wa$, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Free body: Entire beam

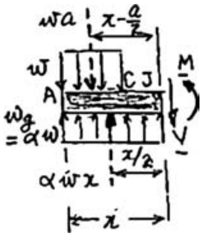
$$+\uparrow \Sigma F_y = 0: \quad w_g(4a) - 2wa - kwa = 0$$

$$w_g = \frac{w}{4}(2+k)$$

Setting $$\frac{w_g}{w} = \alpha \qquad (1)$$

We have $$k = 4\alpha - 2 \qquad (2)$$

Minimum value of B.M. For *M* to have negative values, we must have $w_g < w.$ We verify that *M* will then be negative and keep decreasing in the portion *AC* of the beam. Therefore, M_{min} will occur between *C* and *D*.



From *C* to *D*:

$$+\circlearrowleft \Sigma M_J = 0: \quad M + wa\left(x - \frac{a}{2}\right) - \alpha wx\left(\frac{x}{2}\right) = 0$$

$$M = \frac{1}{2}w(\alpha x^2 - 2ax + a^2) \qquad (3)$$

We differentiate and set $\frac{dM}{dx} = 0$: $$\alpha x - a = 0 \qquad x_{min} = \frac{a}{\alpha} \qquad (4)$$

Substituting in (3):

$$M_{min} = \frac{1}{2}wa^2\left(\frac{1}{\alpha} - \frac{2}{\alpha} + 1\right)$$

$$M_{min} = -wa^2\frac{1-\alpha}{2\alpha} \qquad (5)$$

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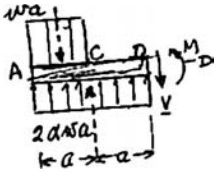
PROBLEM 7.57 (Continued)

Maximum value of bending moment occurs at *D*



$$+\circlearrowleft \Sigma M_D = 0: \quad M_D + wa\left(\frac{3a}{2}\right) - (2\alpha wa)a = 0$$

$$M_{max} = M_D = wa^2\left(2\alpha - \frac{3}{2}\right) \qquad (6)$$

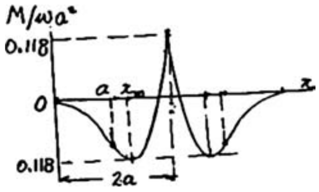
Equating $-M_{min}$ and M_{max}:

$$wa^2 \frac{1-\alpha}{2\alpha} = wa^2\left(2\alpha - \frac{3}{2}\right)$$

$$4\alpha^2 - 2\alpha - 1 = 0$$

$$\alpha = \frac{2+\sqrt{20}}{8} \qquad \alpha = \frac{1+\sqrt{5}}{4} = 0.809$$

(*a*) Substitute in (2): $k = 4(0.809) - 2$ $\qquad k = 1.236$ ◀



(*b*) Substitute for α in (5):

$$|M|_{max} = -M_{min} = -wa^2 \frac{1-0.809}{2(0.809)} \qquad |M|_{max} = 0.1180wa^2 \blacktriangleleft$$

Substitute for α in (4): $\qquad x_{min} = \frac{a}{0.809} 1.236a$ ◁

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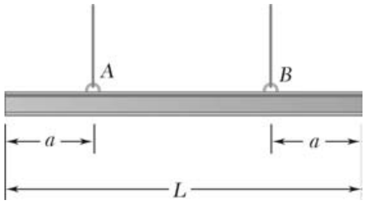
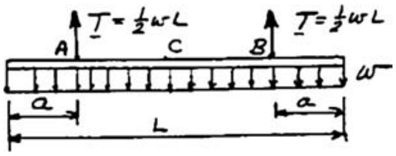
PROBLEM 7.58



A uniform beam is to be picked up by crane cables attached at A and B. Determine the distance a from the ends of the beam to the points where the cables should be attached if the maximum absolute value of the bending moment in the beam is to be as small as possible. (*Hint:* Draw the bending-moment diagram in terms of a, L, and the weight w per unit length, and then equate the absolute values of the largest positive and negative bending moments obtained.)

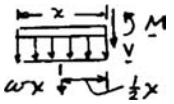
SOLUTION

w = weight per unit length



To the left of A:

$$+\circlearrowleft \Sigma M_1 = 0: \quad M + wx\left(\frac{x}{2}\right) = 0$$

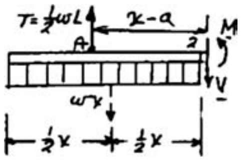


$$M = -\frac{1}{2}wx^2$$

$$M_A = -\frac{1}{2}wa^2$$

Between A and B:

$$+\circlearrowleft \Sigma M_2 = 0: \quad M - \left(\frac{1}{2}wL\right)(x-a) + (wx)\left(\frac{1}{2}x\right) = 0$$

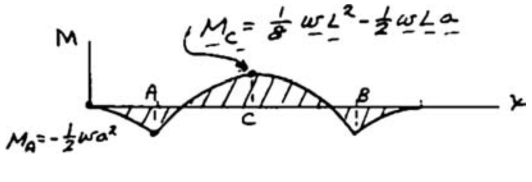


$$M = -\frac{1}{2}wx^2 + \frac{1}{2}wLx - \frac{1}{2}wLa$$

At center C:

$$x = \frac{L}{2}$$

$$M_C = -\frac{1}{2}w\left(\frac{L}{2}\right)^2 + \frac{1}{2}wL\left(\frac{L}{2}\right) - \frac{1}{2}wLa$$



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PROBLEM 7.58 (Continued)

We set $|M_A| = |M_C|$: $\left|-\frac{1}{2}wa^2\right| = \left|\frac{1}{8}wL^2 - \frac{1}{2}wLa\right| + \frac{1}{2}wa^2 = \frac{1}{8}wL^2 - \frac{1}{2}wLa$

$$a^2 + La - 0.25L^2 = 0$$

$$a = \frac{1}{2}(L \pm \sqrt{L^2 + L^2}) = \frac{1}{2}(\sqrt{2} - 1)L$$

$$M_{\max} = \frac{1}{2}w(0.207L)^2 = 0.0214wL^2$$

$a = 0.207L$ ◀

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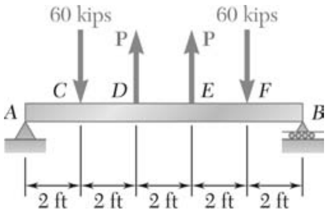
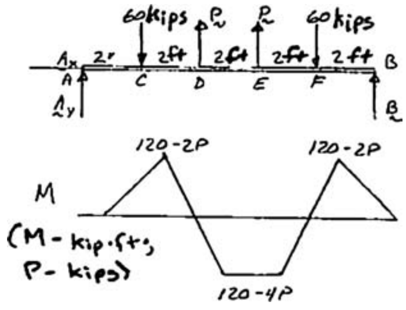


PROBLEM 7.59

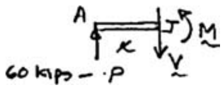
For the beam shown, determine (*a*) the magnitude *P* of the two upward forces for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{\max}$. (See hint for Problem 7.55.)

SOLUTION

By symmetry: $A_y = B = 60 \text{ kips} - P$



Along AC:



$$\circlearrowleft \Sigma M_J = 0: \quad M - x(60 \text{ kips} - P) = 0$$

$$M = (60 \text{ kips} - P)x$$

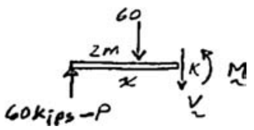
$$M = 120 \text{ kips} \cdot \text{ft} - (2 \text{ ft})P \quad \text{at} \quad x = 2 \text{ ft}$$

Along CD:

$$\circlearrowleft \Sigma M_K = 0: \quad M + (x - 2 \text{ ft})(60 \text{ kips}) - x(60 \text{ kips} - P) = 0$$

$$M = 120 \text{ kip} \cdot \text{ft} - Px$$

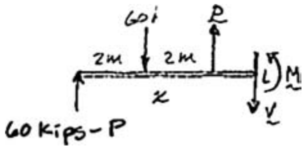
$$M = 120 \text{ kip} \cdot \text{ft} - (4 \text{ ft})P \quad \text{at} \quad x = 4 \text{ ft}$$



Along DE:

$$\circlearrowleft \Sigma M_L = 0: \quad M - (x - 4 \text{ ft})P + (x - 2 \text{ ft})(60 \text{ kips}) - x(60 \text{ kips} - P) = 0$$

$$M = 120 \text{ kip} \cdot \text{ft} - (4 \text{ ft})P \quad \text{(const)}$$



Complete diagram by symmetry

For minimum $|M|_{\max}$, set $M_{\max} = -M_{\min}$

$$120 \text{ kip} \cdot \text{ft} - (2 \text{ ft})P = -[120 \text{ kip} \cdot \text{ft} - (4 \text{ ft})P]$$

(*a*) $P = 40.0 \text{ kips}$ ◀

$$M_{\min} = 120 \text{ kip} \cdot \text{ft} - (4 \text{ ft})P$$

(*b*) $|M|_{\max} = 40.0 \text{ kip} \cdot \text{ft}$ ◀

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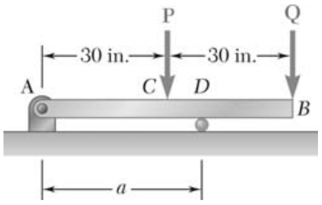
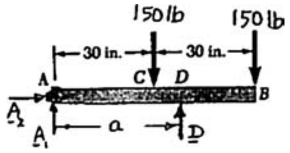


PROBLEM 7.60

Knowing that $P = Q = 150$ lb, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in beam *AB* is as small as possible, (*b*) the corresponding value of $|M|_{\max}$. (See hint for Problem 7.55.)

SOLUTION

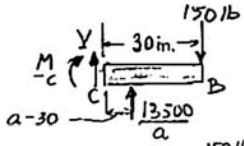


Free body: Entire beam

$$+\circlearrowleft \Sigma M_A = 0: \quad Da - (150)(30) - (150)(60) = 0$$

$$D = \frac{13,500}{a} \quad \triangleleft$$

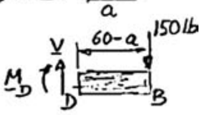
Free body: *CB*



$$+\circlearrowleft \Sigma M_C = 0: \quad -M_C - (150)(30) + \frac{13,500}{a}(a-30) = 0$$

$$M_C = 9000\left(1 - \frac{45}{a}\right) \quad \triangleleft$$

Free body: *DB*



$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D - (150)(60-a) = 0$$

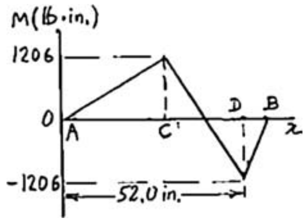
$$M_D = -150(60-a) \quad \triangleleft$$

(*a*) We set

$$M_{\max} = |M_{\min}| \quad \text{or} \quad M_C = -M_D: \quad 9000\left(1 - \frac{45}{a}\right) = 150(60-a)$$

$$60 - \frac{2700}{a} = 60 - a$$

$$a^2 = 2700 \quad a = 51.96 \text{ in.} \qquad a = 52.0 \text{ in.} \blacktriangleleft$$



(*b*) $|M|_{\max} = -M_D = 150(60 - 51.96)$

$$|M|_{\max} = 1206 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

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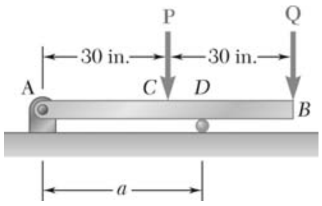
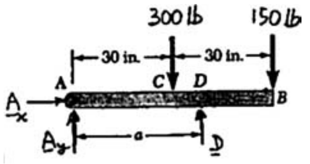


PROBLEM 7.61

Solve Problem 7.60 assuming that $P = 300$ lb and $Q = 150$ lb.

PROBLEM 7.60 Knowing that $P = Q = 150$ lb, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in beam *AB* is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Problem 7.55.)

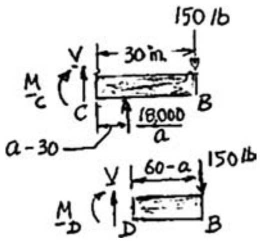
SOLUTION



Free body: Entire beam

$$+\circlearrowleft \Sigma M_A = 0: \quad Da - (300)(30) - (150)(60) = 0$$

$$D = \frac{18,000}{a} \triangleleft$$



Free body: *CB*

$$+\circlearrowleft \Sigma M_C = 0: \quad -M_C - (150)(30) + \frac{18,000}{a}(a - 30) = 0$$

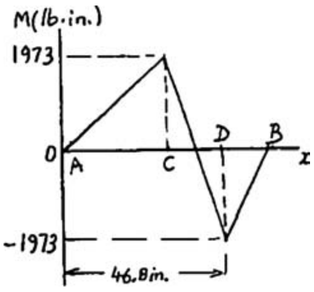
$$M_C = 13,500\left(1 - \frac{40}{a}\right) \triangleleft$$

Free body: *DB*

$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D - (150)(60 - a) = 0$$

$$M_D = -150(60 - a) \triangleleft$$

(*a*) We set



$$M_{max} = |M_{min}| \quad \text{or} \quad M_C = -M_D: \quad 13,500\left(1 - \frac{40}{a}\right) = 150(60 - a)$$

$$90 - \frac{3600}{a} = 60 - a$$

$$a^2 + 30a - 3600 = 0$$

$$a = \frac{-30 + \sqrt{15.300}}{2} = 46.847$$

$$a = 46.8 \text{ in.} \blacktriangleleft$$

(*b*) $|M|_{max} = -M_D = 150(60 - 46.847)$

$$|M|_{max} = 1973 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

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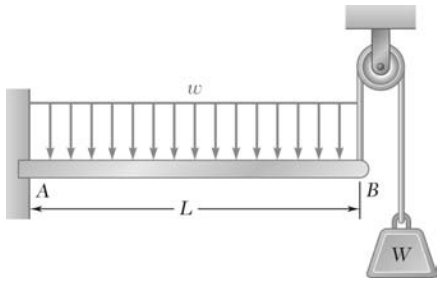
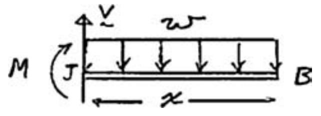


PROBLEM 7.62*

In order to reduce the bending moment in the cantilever beam *AB*, a cable and counterweight are permanently attached at end *B*. Determine the magnitude of the counterweight for which the maximum absolute value of the bending moment in the beam is as small as possible and the corresponding value of $|M|_{\max}$. Consider (*a*) the case when the distributed load is permanently applied to the beam, (*b*) the more general case when the distributed load may either be applied or removed.

SOLUTION

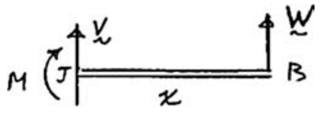
M due to distributed load:



$$\circlearrowleft \Sigma M_J = 0: \quad -M - \frac{x}{2} wx = 0$$

$$M = -\frac{1}{2} wx^2$$

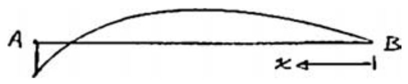
M due to counter weight:



$$\circlearrowleft \Sigma M_J = 0: \quad -M + xw = 0$$

$$M = wx$$

(*a*) **Both applied:**



$$M = W_x - \frac{w}{2} x^2$$

$$\frac{dM}{dx} = W - wx = 0 \text{ at } x = \frac{W}{w}$$

And here $M = \dfrac{W^2}{2w} > 0$ so $M_{\max}$; $M_{\min}$ must be at $x = L$

So $M_{\min} = WL - \frac{1}{2} wL^2$. For minimum $|M|_{\max}$ set $M_{\max} = -M_{\min}$,

so $$\frac{W^2}{2w} = -WL + \frac{1}{2} wL^2 \quad \text{or} \quad W^2 + 2wLW - w^2L^2 = 0$$

$$W = -wL \pm \sqrt{2w^2L^2} \text{ (need+)} \qquad W = (\sqrt{2} - 1)wL = 0.414\, wL \blacktriangleleft$$

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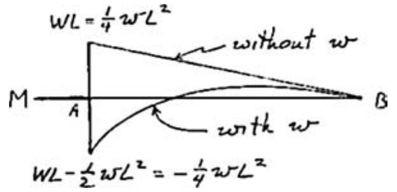
PROBLEM 7.62* (Continued)

(*b*) ***w* may be removed**

$$M_{\max} = \frac{W^2}{2w} = \frac{(\sqrt{2}-1)^2}{2} wL^2 \qquad M_{\max} = 0.858\, wL^2 \blacktriangleleft$$



Without *w*, $M = Wx$

$$M_{\max} = WL \text{ at } A$$

With *w* (see Part *a*) $M = Wx - \frac{w}{2}x^2$

$$M_{\max} = \frac{W^2}{2w} \text{ at } x = \frac{W}{w}$$

$$M_{\min} = WL - \frac{1}{2}wL^2 \text{ at } x = L$$

For minimum $M_{\max}$, set $M_{\max}$ (no w) $= -M_{\min}$ (with w)

$$WL = -WL + \frac{1}{2}wL^2 \rightarrow W = \frac{1}{4}wL \rightarrow \qquad M_{\max} = \frac{1}{4}wL^2 \blacktriangleleft$$

With $\qquad W = \frac{1}{4}wL \blacktriangleleft$

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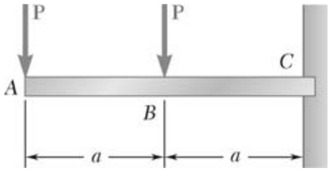
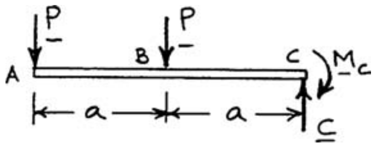


PROBLEM 7.63

Using the method of Section 7.6, solve Problem 7.29.

PROBLEM 7.29 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



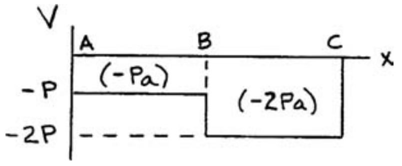
Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad C - P - P = 0$$

$$\mathbf{C} = 2P \uparrow$$

$$+\circlearrowright \Sigma M_C = 0: \quad P(2a) + P(a) - M_C = 0$$

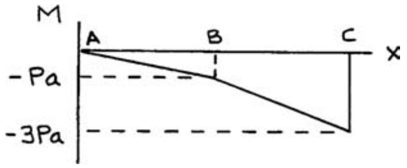
$$\mathbf{M}_C = 3Pa \circlearrowright$$



Shear diagram

At A: $V_A = -P$

$$|V|_{\max} = 2P \blacktriangleleft$$



Bending-moment diagram

At A: $M_A = 0$

$$|M|_{\max} = 3Pa \blacktriangleleft$$

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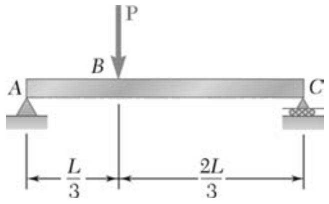
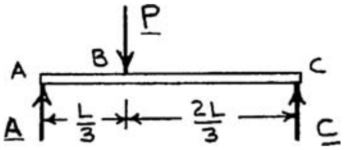


PROBLEM 7.64

Using the method of Section 7.6, solve Problem 7.30.

PROBLEM 7.30 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



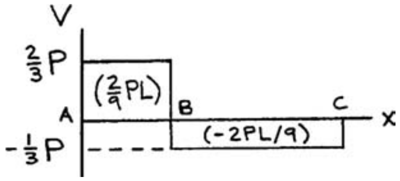
Free body: Entire beam

$$+\circlearrowleft \Sigma M_C = 0: \quad P\left(\frac{2L}{3}\right) - A(L) = 0$$

$$\mathbf{A} = \frac{2}{3}P \uparrow$$

$$+\uparrow \Sigma F_Y = 0: \quad \frac{2}{3}P - P + C = 0$$

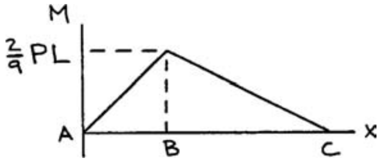
$$\mathbf{C} = \frac{1}{3}P \uparrow$$



Shear diagram

At A: $V_A = \frac{2}{3}P$

$$|V|_{max} = \frac{2}{3}P \blacktriangleleft$$



Bending-moment diagram

At A: $M_A = 0$

$$|M|_{max} = \frac{2}{9}PL \blacktriangleleft$$

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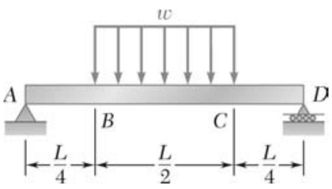
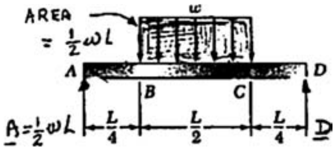


PROBLEM 7.65

Using the method of Section 7.6, solve Problem 7.31.

PROBLEM 7.31 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

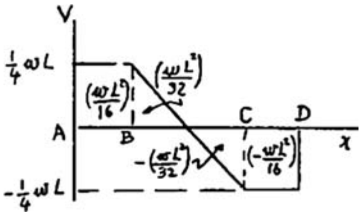


Reactions at *A* and *D*

Because of the symmetry of the supports and loading.

$$A = D = \frac{1}{2}\left(w\frac{L}{2}\right) = \frac{1}{4}wL$$

$$\mathbf{A} = \mathbf{D} = \frac{1}{4}wL \uparrow \triangleleft$$

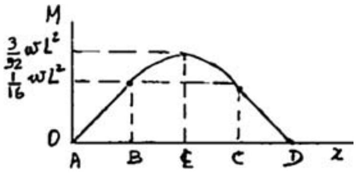


Shear diagram

At *A*: $V_A = +\frac{1}{4}wL$

From *B* to *C*:

Oblique straight line $|V|_{max} = \frac{1}{4}wL$ ◀



Bending-moment diagram

At *A*: $M_A = 0$

From *B* to *C*: *ARC* of parabola $|M|_{max} = \frac{3}{32}wL^2$ ◀

Since *V* has no discontinuity at *B* nor *C*, the slope of the parabola at these points is the same as the slope of the adjoining straight line segment.

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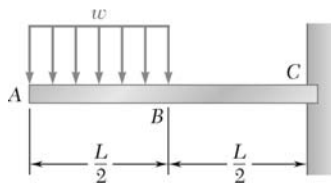
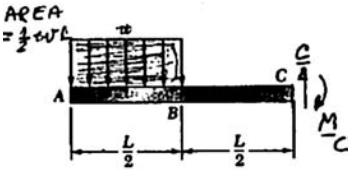


PROBLEM 7.66

Using the method of Section 7.6, solve Problem 7.32.

PROBLEM 7.32 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



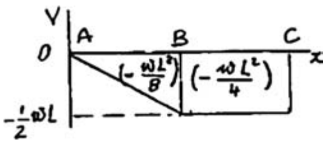
Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad C - w\frac{L}{2} = 0$$

$$\mathbf{C} = \frac{1}{2}wL \uparrow$$

$$+\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{1}{2}wL\right)\left(\frac{3L}{4}\right) = M_C = 0$$

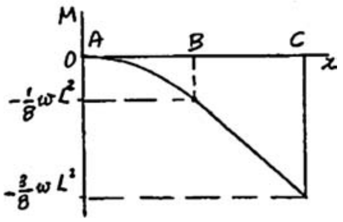
$$\mathbf{M}_C = \frac{3}{8}wL^2 \circlearrowright$$



Shear diagram

At A: $\quad V_A = 0$

$$|V|_{max} = \frac{1}{2}wL \blacktriangleleft$$



Bending-moment diagram

At A: $\quad M = 0 \frac{dM}{dx} = V = 0$

$$|M|_{max} = \frac{3}{8}wL^2 \blacktriangleleft$$

From A to B: *ARC* of parabola

Since V has no discontinuity at B, the slope of the parabola at B is equal to the slope of the straightline segment.

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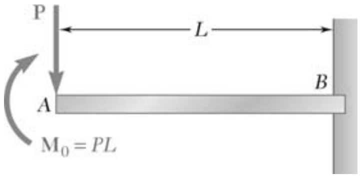
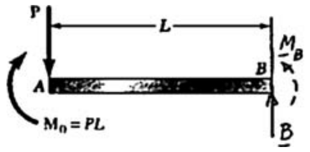


PROBLEM 7.67

Using the method of Section 7.6, solve Problem 7.33.

PROBLEM 7.33 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



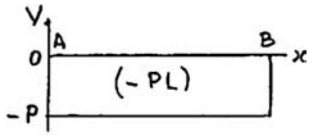
Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad B - P = 0$$

$$\mathbf{B} = P \uparrow$$

$$+\circlearrowleft \Sigma M_B = 0: \quad M_B - M_0 + PL = 0$$

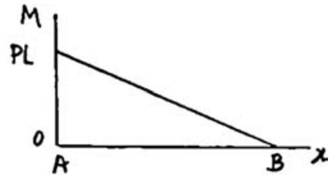
$$\mathbf{M}_B = 0$$



Shear diagram

At A: $\quad V_A = -P$

$$|V|_{\max} = P \blacktriangleleft$$



Bending-moment diagram

At A: $\quad M_A = M_0 = PL$

$$|M|_{\max} = PL \blacktriangleleft$$

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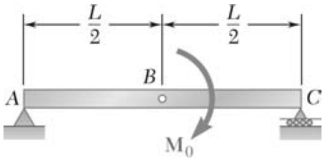
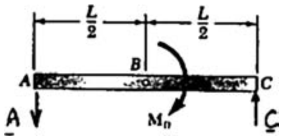


PROBLEM 7.68

Using the method of Section 7.6, solve Problem 7.34.

PROBLEM 7.34 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

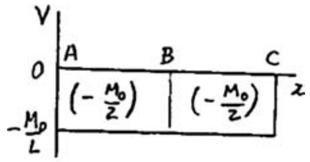


Free body: Entire beam

$$\Sigma F_y = 0: \quad A = C$$

$$+\circlearrowleft \Sigma M_C = 0: \quad Al - M_0 = 0$$

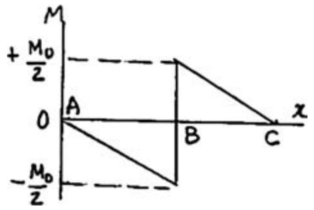
$$A = C = \frac{M_0}{L}$$



Shear diagram

At A: $$V_A = -\frac{M_0}{L}$$

$$|V|_{\max} = \frac{M_0}{L} \blacktriangleleft$$



Bending-moment diagram

At A: $$M_A = 0$$

At B, M increases by M_0 on account of applied couple.

$$|M|_{\max} = M_0/2 \blacktriangleleft$$

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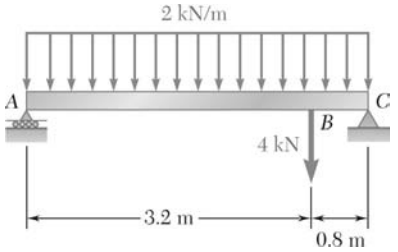
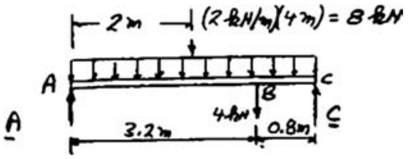


PROBLEM 7.69

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

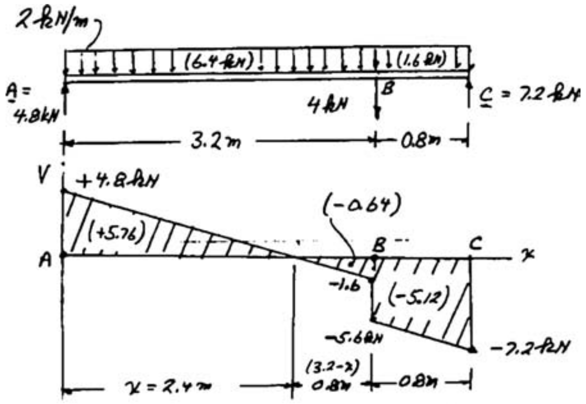


$$+\circlearrowleft \Sigma M_A = 0: \quad (8)(2) + (4)(3.2) - 4C = 0$$

$$\mathbf{C} = 7.2 \text{ kN} \uparrow$$

$$\Sigma F_y = 0: \quad \mathbf{A} = 4.8 \text{ kN} \uparrow$$

(*a*) Shear diagram

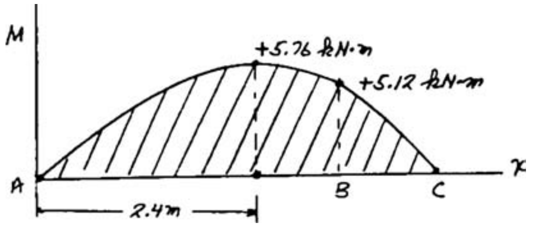


Similar Triangles: $\dfrac{x}{4.8} = \dfrac{3.2 - x}{1.6} = \dfrac{3.2}{6.4}; \quad x = 2.4 \text{ m}$

↑

Add num. & den.

<u>Bending-moment diagram</u>



(*b*)

$$|V|_{\max} = 7.20 \text{ kN} \blacktriangleleft$$

$$|M|_{\max} = 5.76 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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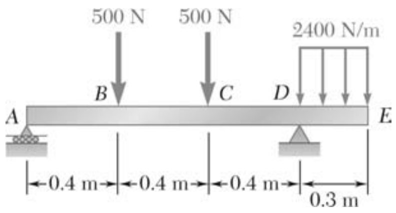
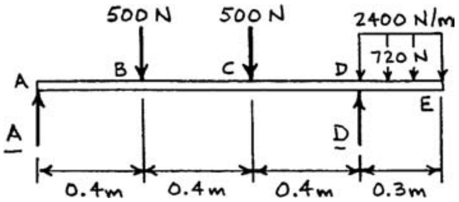


PROBLEM 7.70

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



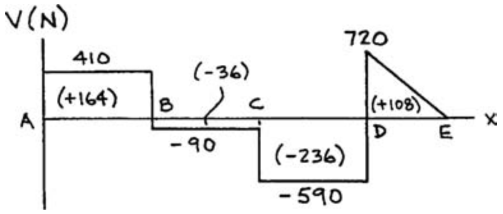
$$+\circlearrowleft \Sigma M_D = 0: \quad (500 \text{ N})(0.8 \text{ m}) + (500 \text{ N})(0.4 \text{ m})$$
$$-(2400 \text{ N/m})(0.3 \text{ m})(0.15 \text{ m}) - A(1.2 \text{ m}) = 0$$

$$\mathbf{A} = 410 \text{ N} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 410 - 2(500) - 2400(0.3) + D = 0$$

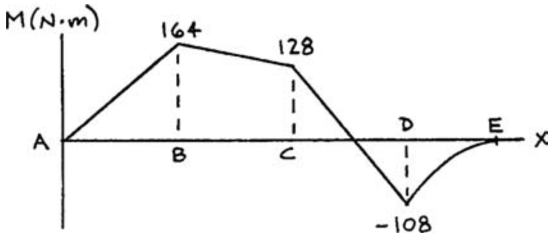
$$\mathbf{D} = 1310 \text{ N} \uparrow$$

Shear diagram



At A: $V_A = +410 \text{ N}$ $|V|_{max} = 720 \text{ N}$ ◀

Bending-moment diagram



At A: $M_A = 0$ $|M|_{max} = 164.0 \text{ N} \cdot \text{m}$ ◀

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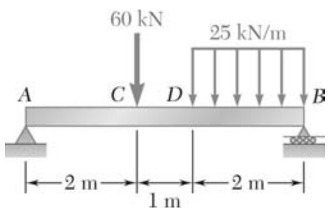
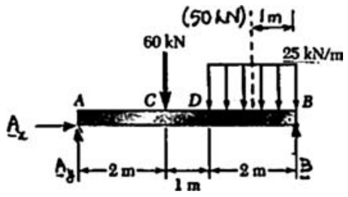


PROBLEM 7.71

Using the method of Section 7.6, solve Problem 7.39.

PROBLEM 7.39 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Free body: Beam

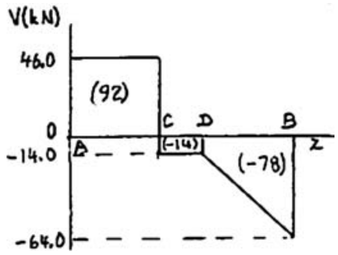
$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (60\text{ kN})(3\text{ m}) + (50\text{ kN})(1\text{ m}) - A_y(5\text{ m}) = 0$$

$$A_y = +46.0\text{ kN} \quad \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad B + 46.0\text{ kN} - 60\text{ kN} - 50\text{ kN} = 0$$

$$B = +64.0\text{ kN} \quad \triangleleft$$



Shear diagram

At A: $\quad V_A = A_y = +46.0\text{ kN}$

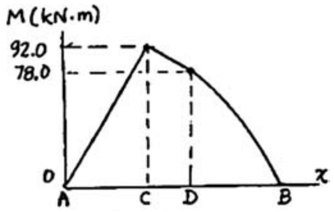
$$|V|_{max} = 64.0\text{ kN} \quad \blacktriangleleft$$

Bending-moment diagram

At A: $\quad M_A = 0$

$$|M|_{max} = 92.0\text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

Parabola from D to B. Its slope at D is same as that of straight-line segment CD since V has no discontinuity at D.



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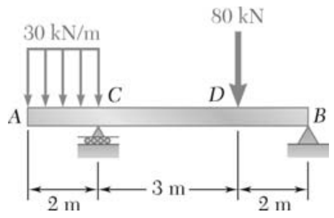
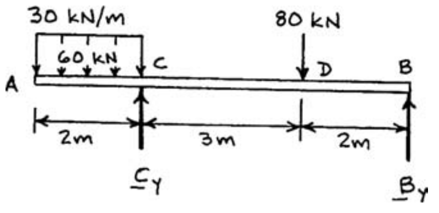


PROBLEM 7.72

Using the method of Section 7.6, solve Problem 7.40.

PROBLEM 7.40 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



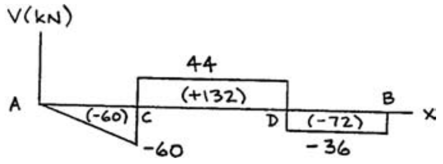
Free body: Entire beam

$$+\circlearrowleft \Sigma M_B = 0: \quad (30\text{ kN/m})(2\text{ m})(6\text{ m}) - C(5\text{ m}) + (80\text{ kN})(2\text{ m}) = 0$$

$$\mathbf{C} = 104\text{ kN}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 104 - 30(2) - 80 + B = 0$$

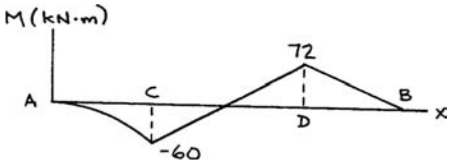
$$\mathbf{B} = 36\text{ kN}\uparrow$$



Shear diagram

At A: $V_A = 0$

$$|V|_{max} = 60.0\text{ kN} \blacktriangleleft$$



Bending-moment diagram

At A: $M_A = 0$

$$|M|_{max} = 72.0\text{ kN}\cdot\text{m} \blacktriangleleft$$

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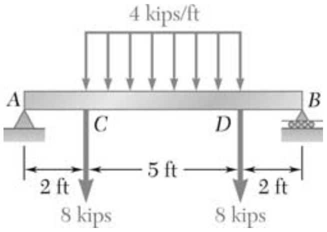
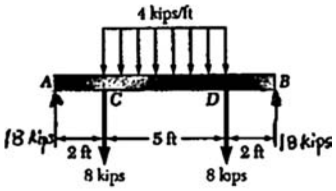


PROBLEM 7.73

Using the method of Section 7.6, solve Problem 7.41.

PROBLEM 7.41 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

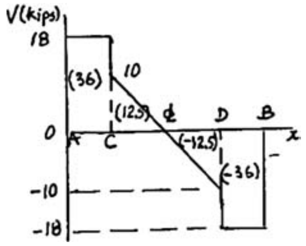


<u>Reactions at supports</u>.

Because of the symmetry:

$$A = B = \frac{1}{2}(8 + 8 + 4 \times 5)\,\text{kips}$$

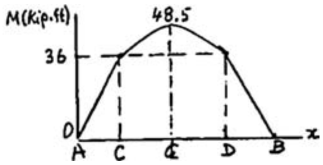
$$\mathbf{A} = \mathbf{B} = 18\text{ kips}\uparrow \;\triangleleft$$



<u>Shear diagram</u>

At A: $V_A = +18\text{ kips}$

$$|V|_{\max} = 18.00\text{ kips} \;\blacktriangleleft$$



<u>Bending-moment diagram</u>

At A: $M_A = 0$

$$|M|_{\max} = 48.5\text{ kip}\cdot\text{ft} \;\blacktriangleleft$$

Discontinuities in slope at C and D, due to the discontinuities of V.

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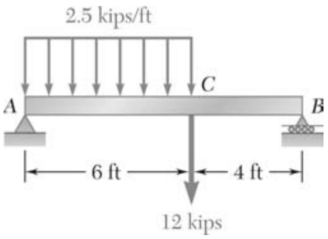
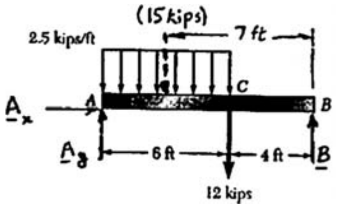


PROBLEM 7.74

Using the method of Section 7.6, solve Problem 7.42.

PROBLEM 7.42 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Free body: Beam

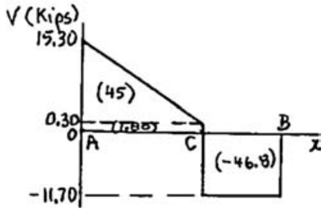
$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (12\text{ kips})(4\text{ ft}) + (15\text{ kips})(7\text{ ft}) - A_y(10\text{ ft}) = 0$$

$$A_y = +15.3\text{ kips} \quad \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad B + 15.3 - 15 - 12 = 0$$

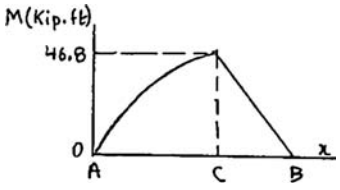
$$B = +11.7\text{ kips} \quad \triangleleft$$



Shear diagram

At A: $\quad V_A = A_y = 15.3\text{ kips}$

$$|V|_{\max} = 15.30\text{ kips} \quad \blacktriangleleft$$



Bending-moment diagram

At A: $\quad M_A = 0$

$$|M|_{\max} = 46.8\text{ kip}\cdot\text{ft} \quad \blacktriangleleft$$

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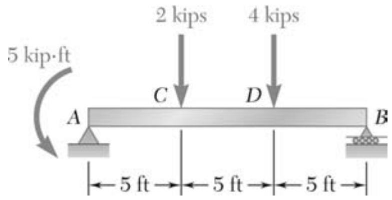
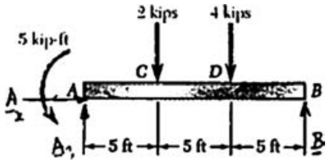


PROBLEM 7.75

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

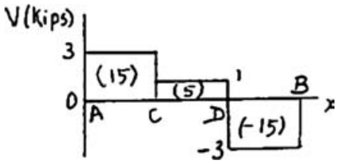


Free body: Beam

$$+\circlearrowleft \Sigma M_B = 0: \quad 5\text{ kip}\cdot\text{ft} + (1\text{ kips})(10\text{ ft}) + (4\text{ kips})(5\text{ ft}) - A_y(15\text{ ft}) = 0$$

$$A_y = +3.00\text{ kips} \;\triangleleft$$

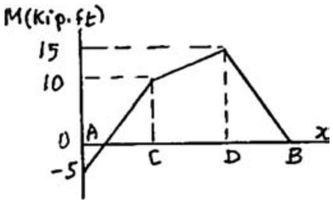
$$\Sigma F_x = 0: \quad A_x = 0$$



Shear diagram

At A: $\quad V_A = A_y = +3.00$ kips

$$|V|_{\max} = 3.00\text{ kips} \;\blacktriangleleft$$



Bending-moment diagram

At A: $\quad M_A = -5$ kip · ft

$$|M|_{\max} = 15.00\text{ kip}\cdot\text{ft} \;\blacktriangleleft$$

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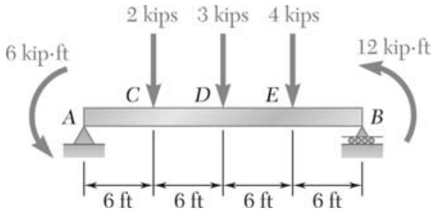
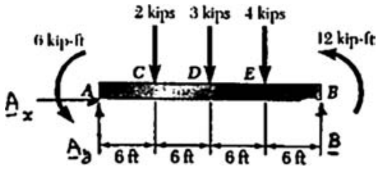


PROBLEM 7.76

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

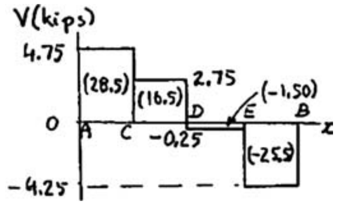


Free body: Beam

$$+\circlearrowleft \Sigma M_B = 0: \quad 6 \text{ kip}\cdot\text{ft} + 12 \text{ kip}\cdot\text{ft} + (2 \text{ kips})(18 \text{ ft}) + (3 \text{ kips})(12 \text{ ft}) + (4 \text{ kips})(6 \text{ ft}) - A_y(24 \text{ ft}) = 0$$

$$A_y = +4.75 \text{ kips} \quad \triangleleft$$

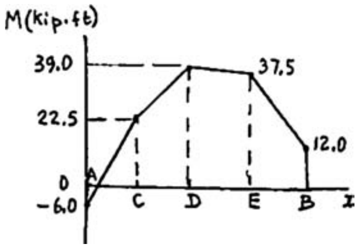
$$\Sigma F_x = 0: \quad A_x = 0$$



Shear diagram

At A: $\quad V_A = A_y = +4.75 \text{ kips}$

$$|V|_{max} = 4.75 \text{ kips} \quad \blacktriangleleft$$



Bending-moment diagram

At A: $\quad M_A = -6 \text{ kip}\cdot\text{ft}$

$$|M|_{max} = 39.0 \text{ kip}\cdot\text{ft} \quad \blacktriangleleft$$

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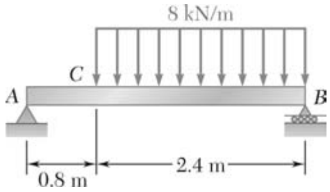
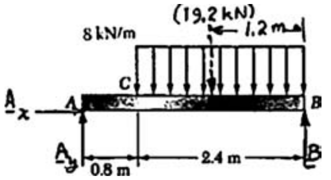


PROBLEM 7.77

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Free body: Beam

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (19.2\text{ kN})(1.2\text{ m}) - A_y(3.2\text{ m}) = 0$$

$$A_y = +7.20\text{ kN} \ \triangleleft$$

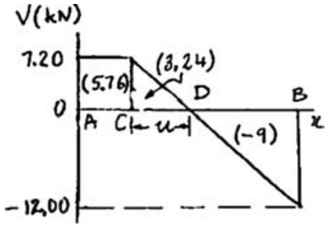
Shear diagram

$$V_A = V_C = A_y = +7.20\text{ kN}$$

To determine Point D where $V = 0$, we write

$$V_D - V_C = wu$$

$$0 - 7.20\text{ kN} = -(8\text{ kN/m})u \qquad u = 0.9\text{ m} \ \triangleleft$$



We next compute all areas

Bending-moment diagram

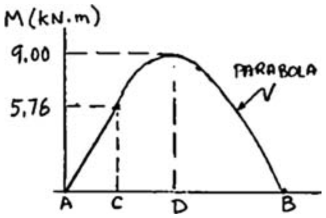
At A: $\qquad M_A = 0$

Largest value occurs at D with

$$AD = 0.8 + 0.9 = 1.700\text{ m}$$

$$|M|_{\max} = 9.00\text{ kN}\cdot\text{m} \ \blacktriangleleft$$

$$1.700\text{ m from } A \ \blacktriangleleft$$



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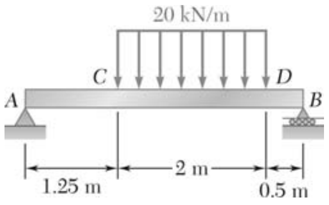
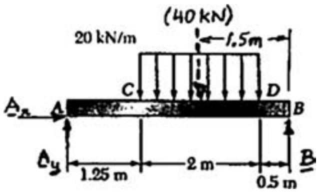


PROBLEM 7.78

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION



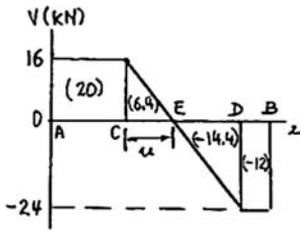
<u>Free body: Beam</u>

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (40 \text{ kN})(1.5 \text{ m}) - A_y(3.75 \text{ m}) = 0$$

$$A_y = +16.00 \text{ kN} \quad \triangleleft$$

<u>Shear diagram</u>



$$V_A = V_C = A_y = +16.00 \text{ kN}$$

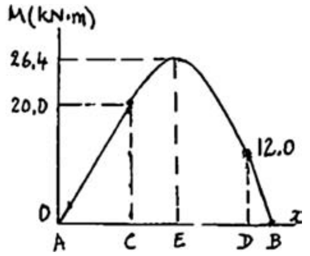
To determine Point E where $V = 0$, we write

$$V_E - V_C = -wu$$

$$0 - 16 \text{ kN} = -(20 \text{ kN/m})u \qquad u = 0.800 \text{ m} \quad \triangleleft$$

We next compute all areas

<u>Bending-moment diagram</u>



At A: $\qquad M_A = 0$

Largest value occurs at E with

$$AE = 1.25 + 0.8 = 2.05 \text{ m} \qquad |M|_{\max} = 26.4 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

$$2.05 \text{ m from } A \quad \blacktriangleleft$$

From A to C and D to B: Straight line segments. From C to D: Parabola

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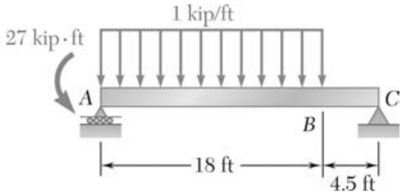
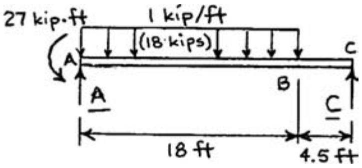


PROBLEM 7.79

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION



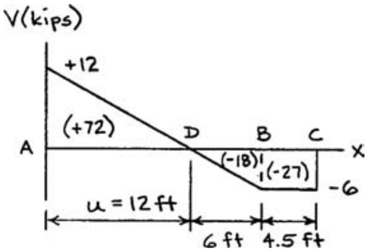
Free body: Entire beam

$$+\circlearrowleft \Sigma M_C = 0: \quad -(27 \text{ kip} \cdot \text{ft}) + A(22.5 \text{ ft}) - (1 \text{ kip/ft})(18 \text{ ft})(13.5 \text{ ft}) = 0$$

$$\mathbf{A} = 12 \text{ kips} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 12 - 1(18) + C = 0$$

$$\mathbf{C} = 6 \text{ kips} \uparrow$$

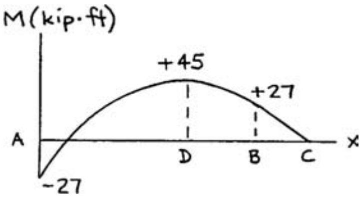


Shear diagram

At A: $\quad V_A = +12 \text{ kips}$

To locate Point D (where $V = 0$)

$$V_D - V_A = -wu$$
$$0 - 12 \text{ kips} = -(1 \text{ kip/ft})u$$
$$u = 12 \text{ ft}$$



Bending-moment diagram

At A: $\quad M_A = 0$

$|M|_{max} = 45.0 \text{ kip} \cdot \text{ft}$ ◀

12.00 ft from A ◀

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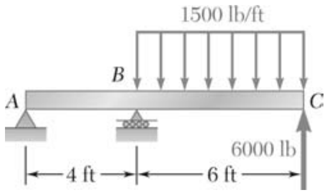
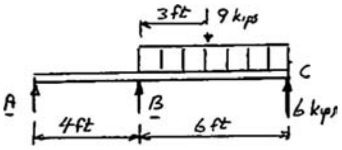


PROBLEM 7.80

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION



Free body: Entire beam

$$+\circlearrowleft \Sigma M_A = 0: \quad (6 \text{ kips})(10 \text{ ft}) - (9 \text{ kips})(7 \text{ ft}) + 8(4 \text{ ft}) = 0$$

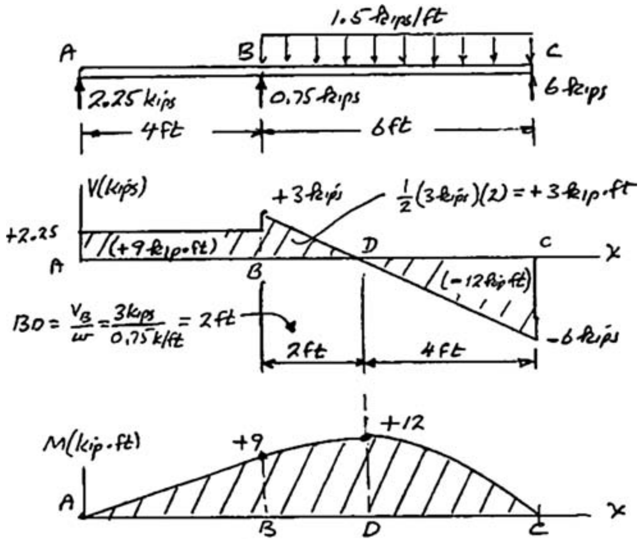
$$B = +0.75 \text{ kips}$$

$$\mathbf{B} = 0.75 \text{ kips} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad A + 0.75 \text{ kips} - 9 \text{ kips} + 6 \text{ kips} = 0$$

$$A = +2.25 \text{ kips}$$

$$\mathbf{A} = 2.25 \text{ kips} \uparrow$$



$$M_{max} = 12.00 \text{ kip} \cdot \text{ft} \blacktriangleleft$$

6.00 ft from A

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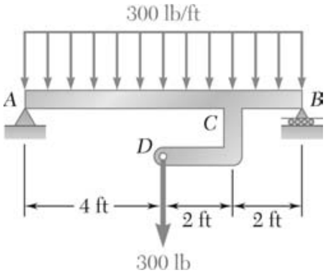
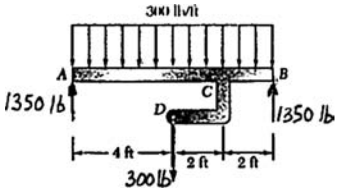


PROBLEM 7.81

(*a*) Draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

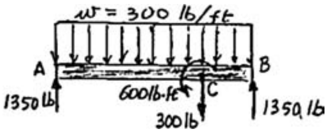


Reactions at supports

Because of symmetry of load

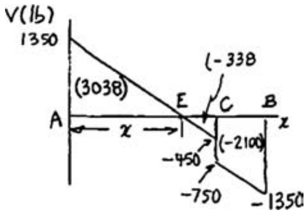
$$A = B = \frac{1}{2}(300 \times 8 + 300)$$

$\mathbf{A} = \mathbf{B} = 1350 \text{ lb} \uparrow$ ◁



Load diagram for *AB*

The 300-lb force at *D* is replaced by an equivalent force-couple system at *C*.



Shear diagram

At *A*: $V_A = A = 1350 \text{ lb}$

To determine Point *E* where $V = 0$:

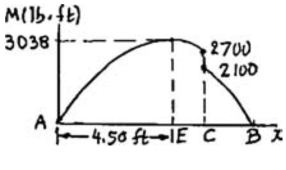
$$V_E - V_A = -wx$$
$$0 - 1350 \text{ lb} = -(300 \text{ lb/ft})x$$

$x = 4.50 \text{ ft}$ ◁

We compute all areas

Bending-moment diagram

At *A*: $M_A = 0$



Note $600 - \text{lb} \cdot \text{ft}$ drop at *C* due to couple

$|M|_{max} = 3040 \text{ lb} \cdot \text{ft}$ ◀

4.50 ft from *A* ◀

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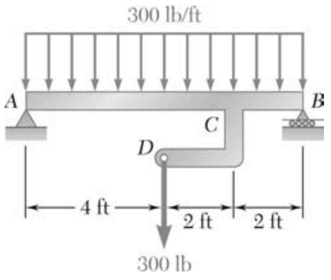
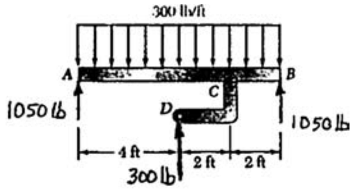


PROBLEM 7.82

Solve Problem 7.81 assuming that the 300-lb force applied at *D* is directed upward.

PROBLEM 7.81 (*a*) Draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

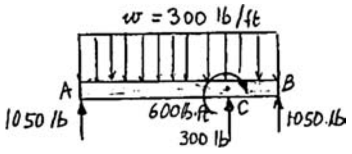


Reactions at supports

Because of symmetry of load:

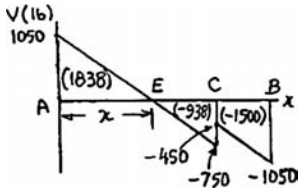
$$A = B = \frac{1}{2}(300 \times 8 - 300)$$

$\mathbf{A} = \mathbf{B} = 1050 \text{ lb}\uparrow$ ◁



Load diagram

The 300-lb force at *D* is replaced by an equivalent force-couple system at *C*.



Shear diagram

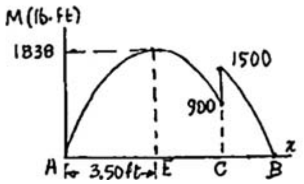
At *A*: $V_A = A = 1050 \text{ lb}$

To determine Point *E* where $V = 0$:

$$V_E - V_A = -wx$$
$$0 - 1050 \text{ lb} = -(300 \text{ lb/ft})x$$

$x = 3.50 \text{ ft}$ ◁

We compute all areas



Bending-moment diagram

At *A*: $M_A = 0$

Note $600 - \text{lb} \cdot \text{ft}$ increase at *C* due to couple

$|M|_{\max} = 1838 \text{ lb} \cdot \text{ft}$ ◀

3.50 ft from *A* ◀

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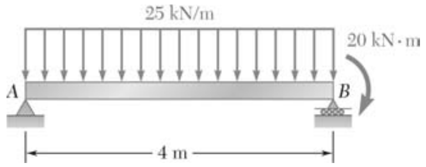
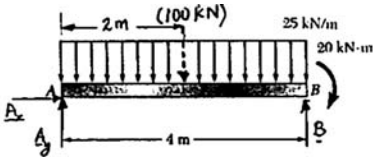


PROBLEM 7.83

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION



Free body: Beam

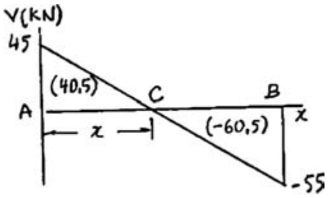
$$+\circlearrowleft \Sigma M_A = 0: \quad B(4\text{ m}) - (100\text{ kN})(2\text{ m}) - 20\text{ kN}\cdot\text{m} = 0$$

$$B = +55\text{ kN} \quad \triangleleft$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 55 - 100 = 0$$

$$A_y = +45\text{ kN} \quad \triangleleft$$



Shear diagram

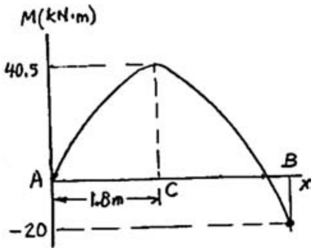
At A: $V_A = A_y = +45\text{ kN}$

To determine Point C where $V = 0$:

$$V_C - V_A = -wx$$
$$0 - 45\text{ kN} = -(25\text{ kN}\cdot\text{m})x$$

$$x = 1.8\text{ m} \quad \triangleleft$$

We compute all areas bending-moment



Bending-moment diagram

At A: $M_A = 0$

At B: $M_B = -20\text{ kN}\cdot\text{m}$

$$|M|_{max} = 40.5\text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

$$1.800\text{ m from } A \quad \blacktriangleleft$$

Single arc of parabola

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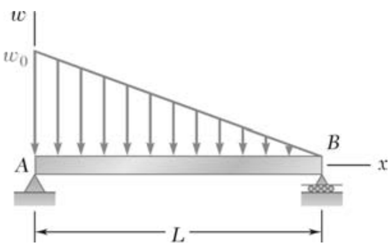
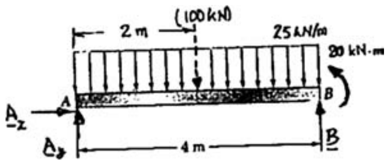


PROBLEM 7.84

Solve Problem 7.83 assuming that the 20-kN · m couple applied at B is counterclockwise.

PROBLEM 7.83 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION



Free body: Beam

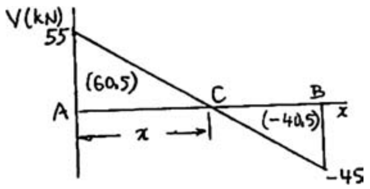
$$+\circlearrowleft \Sigma M_A = 0: \quad B(4\text{ m}) - (100\text{ kN})(2\text{ m}) - 20\text{ kN}\cdot\text{m} = 0$$

$$B = +45\text{ kN} \quad \triangleleft$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 45 - 100 = 0$$

$$A_y = +55\text{ kN} \quad \triangleleft$$



Shear diagram

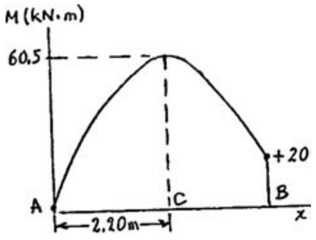
At A: $\quad V_A = A_y = +55\text{ kN}$

To determine Point C where $V = 0$:

$$V_C - V_A = -wx$$
$$0 - 55\text{ kN} = -(25\text{ kN/m})x$$

$$x = 2.20\text{ m} \quad \triangleleft$$

We compute all areas bending-moment



Bending-moment diagram

At A: $\quad M_A = 0$

At B: $\quad M_B = +20\text{ kN}\cdot\text{m}$

$$|M|_{max} = 60.5\text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

$$2.20\text{ m from } A \quad \blacktriangleleft$$

Single arc of parabola

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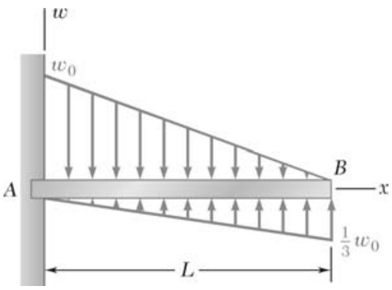
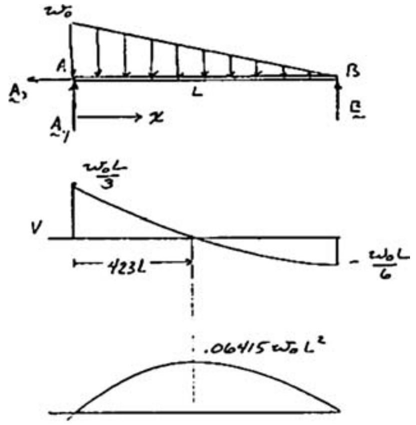


PROBLEM 7.85

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION



Distributed load

$$w = w_0\left(1 - \frac{x}{L}\right) \quad \left(\text{Total} = \frac{1}{2}w_0L\right)$$

$$\circlearrowleft \Sigma M_A = 0: \quad \frac{L}{3}\left(\frac{1}{2}w_0L\right) - LB = 0 \qquad \mathbf{B} = \frac{w_0L}{6}\uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - \frac{1}{2}w_0L + \frac{w_0L}{6} = 0 \quad \mathbf{A}_y = \frac{w_0L}{3}\uparrow$$

Shear: $\quad V_A = A_y = \dfrac{w_0L}{3}$

Then $\quad \dfrac{dV}{dx} = -w \to V$

$$= V_A - \int_0^x w_0\left(1 - \frac{x}{L}\right)dx$$

$$V = \left(\frac{w_0L}{3}\right) - w_0x + \frac{1}{2}\frac{w_0}{L}x^2$$

$$= w_0L\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right]$$

Note: At $\quad x = L$

$$V = -\frac{w_0L}{6}$$

$$V = 0 \text{ at } \left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right) + \frac{2}{3}$$

$$= 0 \to \frac{x}{L} = 1 - \sqrt{\frac{1}{3}}$$

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PROBLEM 7.85 (Continued)

Moment: $M_A = 0$

Then $\left(\dfrac{dM}{dx}\right) = V \rightarrow M = \int_0^x V dx = L\int_0^{x/L} V\left(\dfrac{x}{L}\right) d\left(\dfrac{x}{L}\right)$

$$M = w_0 L^2 \int_0^{x/L}\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right] d\left(\frac{x}{L}\right)$$

$$M = w_0 L^2 \left[\frac{1}{3}\left(\frac{x}{L}\right) - \frac{1}{2}\left(\frac{x}{L}\right)^2 + \frac{1}{6}\left(\frac{x}{L}\right)^3\right]$$

$$M_{\max}\left(\text{at } \frac{x}{L} = 1 - \sqrt{\frac{1}{3}}\right) = 0.06415 w_0 L^2$$

(*a*) $V = w_0 L\left[\dfrac{1}{3} - \dfrac{x}{L} + \dfrac{1}{2}\left(\dfrac{x}{L}\right)^2\right]$ ◀

$M = w_0 L^2 \left[\dfrac{1}{3}\left(\dfrac{x}{L}\right) - \dfrac{1}{2}\left(\dfrac{x}{L}\right)^2 + \dfrac{1}{6}\left(\dfrac{x}{L}\right)^3\right]$ ◀

(*c*) $M_{\max} = 0.0642\, w_0 L^2$ ◀

at $x = 0.423L$ ◀

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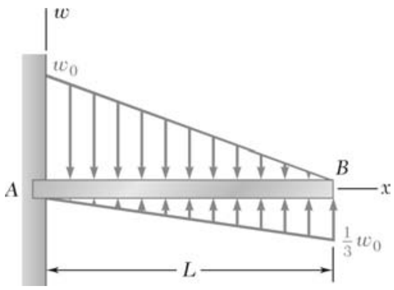

PROBLEM 7.86

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION

(*a*) We note that at $B(x=L)$: $V_B = 0, \quad M_B = 0$ (1)

Load: $$w(x) = w_0\left(1-\frac{x}{L}\right) - \frac{1}{3}w_0\left(\frac{x}{L}\right) = w_0\left(1-\frac{4x}{3L}\right)$$

Shear: We use Eq. (7.2) between $C(x=x)$ and $B(x=L)$:

$$V_B - V_C = -\int_x^L w(x)dx \quad 0 - V(x) = -\int_x^L w(x)dx$$

$$V(x) = w_0\int_x^L\left(1-\frac{4x}{3L}\right)dx$$

$$= w_0\left[x - \frac{2x^2}{3L}\right]_x^L = w_0\left(L - \frac{2L}{3} - x + \frac{2x^2}{3L}\right)$$

$$V(x) = \frac{w_0}{3L}(2x^2 - 3Lx + L^2) \quad (2) ◀$$

Bending moment: We use to Eq. (7.4) between $C(x=x)$ and $B(x=L)$:

$$M_B - M_C = \int_x^L V(x)dx \quad 0 - M(x)$$

$$= \frac{w_0}{3L}\int_x^L (2x^2 - 3Lx + L^2)dx$$

$$M(x) = -\frac{w_0}{3L}\left[\frac{2}{3}x^3 - \frac{3}{2}Lx^2 + L^2x\right]^L$$

$$= -\frac{w_0}{18L}[4x^3 - 9Lx^2 + 6L^2x]_x^L$$

$$= -\frac{w_0}{18L}[(4L^3 - 9L^3 + 6L^3) - (4x^3 - 9Lx^2 + 6L^2x)]$$

$$M(x) = \frac{w_0}{18L}(4x^3 - 9Lx^2 + 6L^2x - L^3) \quad (3) ◀$$

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PROBLEM 7.86 (Continued)

(*b*) Maximum bending moment

$$\frac{dM}{dx} = V = 0$$

Eq. (2): $2x^2 - 3Lx + L^2 = 0$

$$x = \frac{3 - \sqrt{9-8}}{4} L = \frac{L}{2}$$

Carrying into (3): $M_{\max} = \dfrac{w_0 L^2}{72}$, At $x = \dfrac{L}{2}$ ◀

Note: $|M|_{\max} = \dfrac{w_0 L^2}{18}$ At $x = 0$

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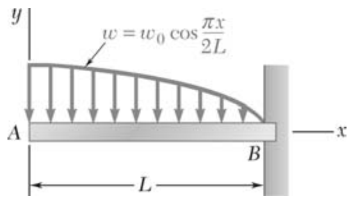
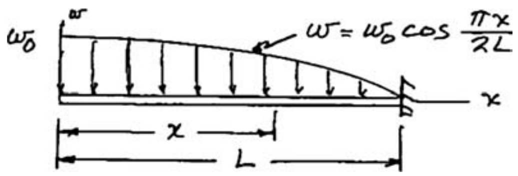


PROBLEM 7.87

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION



$$\frac{dv}{dx} = -w = w_0 \cos\frac{\pi}{2}\frac{x}{L}$$

$$V = -\int w\,dx = -w_0\left(\frac{2L}{\pi}\right)\sin\frac{\pi x}{2L} + C_1 \qquad (1)$$

$$\frac{dM}{dx} = V = -w_0\left(\frac{2L}{\pi}\right)\sin\frac{\pi x}{2L} + C_1$$

$$M = \int V\,dx = +w_0\left(\frac{2L}{\pi}\right)^2 \cos\frac{\pi x}{2L} + C_1 x + C_2 \qquad (2)$$

<u>Boundary conditions</u>

At $x = 0$: $\quad V = C_1 = 0 \quad C_1 = 0$

At $x = 0$: $\quad M = +w_0\left(\frac{2L}{\pi}\right)^2 \cos(0) + C_2 = 0$

$$C_2 = -w_0\left(\frac{2L}{\pi}\right)^2$$

Eq. (1) $\quad V = -w_0\left(\frac{2L}{\pi}\right)\sin\frac{\pi x}{2L}$ ◀

$$M = w_0\left(\frac{2L}{\pi}\right)^2\left(-1 + \cos\frac{\pi x}{2L}\right) \blacktriangleleft$$

$M_{\max}$ at $x = L$: $\quad |M_{\max}| = w_0\left(\frac{2L}{\pi}\right)^2 |-1 + 0| = \frac{4}{\pi^2} w_0 L^2$ ◀

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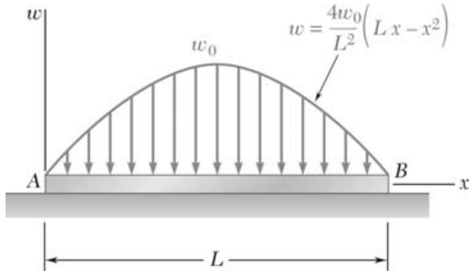
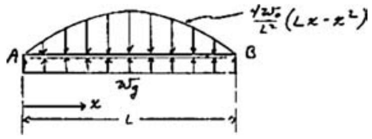


PROBLEM 7.88

The beam *AB*, which lies on the ground, supports the parabolic load shown. Assuming the upward reaction of the ground to be uniformly distributed, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the maximum bending moment.

SOLUTION



(*a*) $$+\uparrow \Sigma F_y = 0: \quad w_g L - \int_0^L \frac{4w_0}{L^2}(Lx - x^2)dx = 0$$

$$w_g L = \frac{4w_0}{L^2}\left(\frac{1}{2}LL^2 - \frac{1}{3}L^3\right) = \frac{2}{3}w_0 L$$

$$w_g = \frac{2w_0}{3}$$

Define $$\xi = \frac{x}{L} \text{ so } d\xi = \frac{dx}{L} \longrightarrow \text{net load } w = 4w_0\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right] - \frac{2}{3}w_0$$

or $$w = 4w_0\left(-\frac{1}{6} + \xi - \xi^2\right)$$

$$V = V(0) - \int_0^\xi 4w_0 L\left(-\frac{1}{6} + \xi - \xi^2\right)d\xi$$

$$= 0 + 4w_0 L\left(\frac{1}{6}\xi + \frac{1}{2}\xi^2 - \frac{1}{3}\xi^3\right) \qquad V = \frac{2}{3}w_0 L\,(\xi - 3\xi^2 + 2\xi^3) \blacktriangleleft$$

$$M = M_0 + \int_0^x V dx = 0 + \frac{2}{3}w_0 L^2 \int_0^\xi (\xi - 3\xi^2 + 2\xi^3)d\xi$$

$$= \frac{2}{3}w_0 L^2\left(\frac{1}{2}\xi^2 - \xi^3 + \frac{1}{2}\xi^4\right) = \frac{1}{3}w_0 L^2(\xi^2 - 2\xi^3 + \xi^4) \blacktriangleleft$$

(*b*) Max *M* occurs where $$V = 0 \longrightarrow 1 - 3\xi + 2\xi^2 = 0 \longrightarrow \xi = \frac{1}{2}$$

$$M\left(\xi = \frac{1}{2}\right) = \frac{1}{3}w_0 L^2\left(\frac{1}{4} - \frac{2}{8} + \frac{1}{16}\right) = \frac{w_0 L^2}{48}$$

$$M_{max} = \frac{w_0 L^2}{48} \text{ at center of beam} \blacktriangleleft$$

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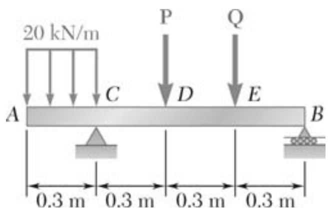
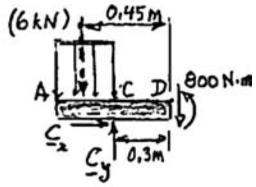


PROBLEM 7.89

The beam *AB* is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is $+800\ \text{N}\cdot\text{m}$ at *D* and $+1300\ \text{N}\cdot\text{m}$ at *E*, (*a*) determine **P** and **Q**, (*b*) draw the shear and bending-moment diagrams for the beam.

SOLUTION

(*a*) Free body: Portion *AD*

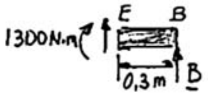


$$\Sigma F_x = 0: \quad C_x = 0$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -C_y(0.3\ \text{m}) + 0.800\ \text{kN}\cdot\text{m} + (6\ \text{kN})(0.45\ \text{m}) = 0$$

$$C_y = +11.667\ \text{kN} \qquad \mathbf{C} = 11.667\ \text{kN}\uparrow \ \triangleleft$$

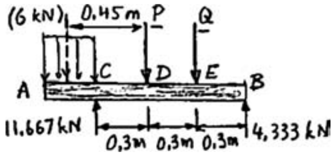
Free body: Portion *EB*



$$+\circlearrowleft \Sigma M_E = 0: \quad B(0.3\ \text{m}) - 1.300\ \text{kN}\cdot\text{m} = 0$$

$$\mathbf{B} = 4.333\ \text{kN}\uparrow \ \triangleleft$$

Free body: Entire beam



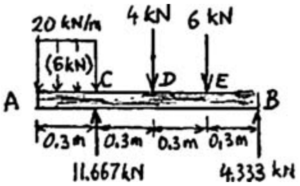
$$+\circlearrowleft \Sigma M_D = 0: \quad (6\ \text{kN})(0.45\ \text{m}) - (11.667\ \text{kN})(0.3\ \text{m})$$
$$-Q(0.3\ \text{m}) + (4.333\ \text{kN})(0.6\ \text{m}) = 0$$

$$\mathbf{Q} = 6.00\ \text{kN}\downarrow \ \blacktriangleleft$$

$$+\uparrow \Sigma M_y = 0: \quad 11.667\ \text{kN} + 4.333\ \text{kN}$$
$$-6\ \text{kN} - P - 6\ \text{kN} = 0$$

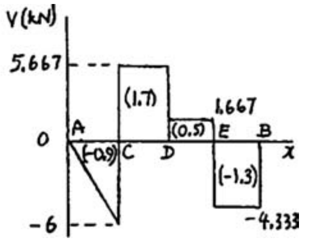
$$\mathbf{P} = 4.00\ \text{kN}\downarrow \ \blacktriangleleft$$

Load diagram



(*b*) Shear diagram

At *A*: $V_A = 0$



$$|V|_{\max} = 6\ \text{kN} \ \triangleleft$$

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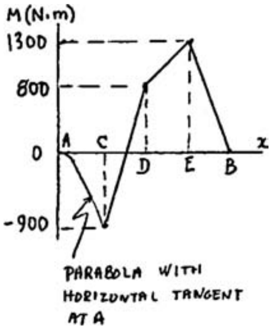
PROBLEM 7.89 (Continued)



Bending-moment diagram

At A: $M_A = 0$

$$|M|_{max} = 1300 \text{ N} \cdot \text{m} \triangleleft$$

We check that

$$M_D = +800 \text{ N} \cdot \text{m} \quad \text{and} \quad M_E = +1300 \text{ N} \cdot \text{m}$$

As given:

At C: $M_C = -900 \text{ N} \cdot \text{m}$ ◀

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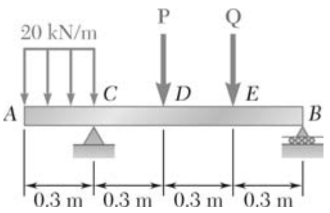
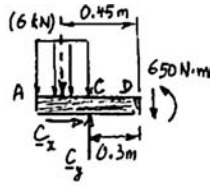
PROBLEM 7.90



Solve Problem 7.89 assuming that the bending moment was found to be $+650\ \text{N}\cdot\text{m}$ at D and $+1450\ \text{N}\cdot\text{m}$ at E.

PROBLEM 7.89 The beam AB is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is $+800\ \text{N}\cdot\text{m}$ at D and $+1300\ \text{N}\cdot\text{m}$ at E, (a) determine **P** and **Q**, (b) draw the shear and bending-moment diagrams for the beam.

SOLUTION

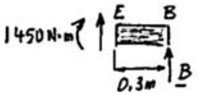


(a) Free body: Portion AD

$$\Sigma F_x = 0: \quad C_x = 0$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -C(0.3\ \text{m}) + 0.650\ \text{kN}\cdot\text{m} + (6\ \text{kN})(0.45\ \text{m}) = 0$$

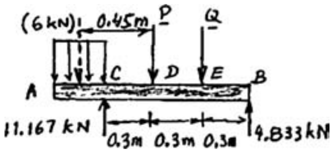
$$C_y = +11.167\ \text{kN} \qquad \mathbf{C} = 11.167\ \text{kN}\uparrow \triangleleft$$



Free body: Portion EB

$$+\circlearrowleft \Sigma M_E = 0: \quad B(0.3\ \text{m}) - 1.450\ \text{kN}\cdot\text{m} = 0$$

$$\mathbf{B} = 4.833\ \text{kN}\uparrow \triangleleft$$



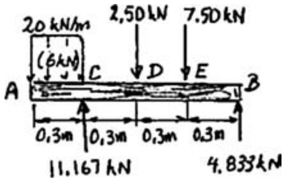
Free body: Entire beam

$$+\circlearrowleft \Sigma M_D = 0: \quad (6\ \text{kN})(0.45\ \text{m}) - (11.167\ \text{kN})(0.3\ \text{m})$$
$$-Q(0.3\ \text{m}) + (4.833\ \text{kN})(0.6\ \text{m}) = 0$$

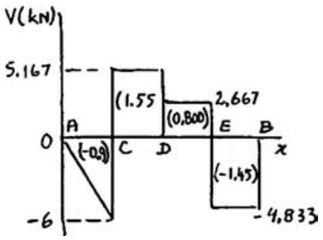
$$\mathbf{Q} = 7.50\ \text{kN}\downarrow \blacktriangleleft$$

$$+\uparrow \Sigma M_y = 0: \quad 11.167\ \text{kN} + 4.833\ \text{kN}$$
$$-6\ \text{kN} - P - 7.50\ \text{kN} = 0$$

$$\mathbf{P} = 2.50\ \text{kN}\downarrow \blacktriangleleft$$



Load diagram



(b) Shear diagram

At A: $\qquad V_A = 0$

$$|V|_{\max} = 6\ \text{kN} \triangleleft$$

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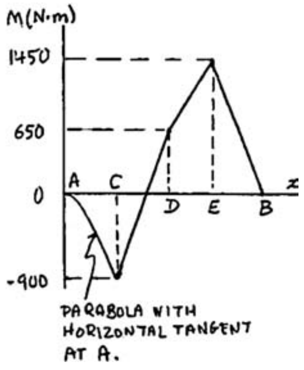
PROBLEM 7.90 (Continued)



Bending-moment diagram

At *A*: $M_A = 0$

$|M|_{max} = 1450 \text{ N} \cdot \text{m}$ ◁

We check that

$$M_D = +650 \text{ N} \cdot \text{m} \quad \text{and} \quad M_E = +1450 \text{ N} \cdot \text{m}$$

As given:

At *C*: $M_C = -900 \text{ N} \cdot \text{m}$ ◀

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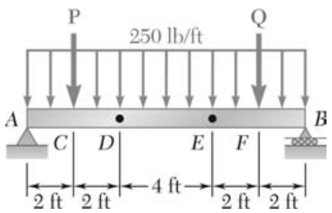
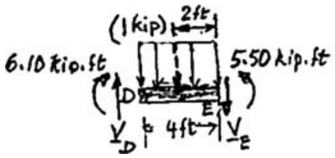


PROBLEM 7.91*

The beam *AB* is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is $+6.10 \text{ kip}\cdot\text{ft}$ at *D* and $+5.50 \text{ kip}\cdot\text{ft}$ at *E*, (*a*) determine **P** and **Q**, (*b*) draw the shear and bending-moment diagrams for the beam.

SOLUTION

(*a*) Free body: Portion *DE*



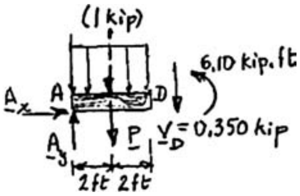
$$+\circlearrowleft \Sigma M_E = 0: \quad 5.50 \text{ kip}\cdot\text{ft} - 6.10 \text{ kip}\cdot\text{ft} + (1 \text{ kip})(2 \text{ ft}) - V_D(4 \text{ ft}) = 0$$

$$V_D = +0.350 \text{ kip}$$

$$+\uparrow \Sigma F_y = 0: \quad 0.350 \text{ kip} - 1 \text{ kip} - V_E = 0$$

$$V_E = -0.650 \text{ kip}$$

Free body: Portion *AD*



$$+\circlearrowleft \Sigma M_A = 0: \quad 6.10 \text{ kip}\cdot\text{ft} - P(2 \text{ ft}) - (1 \text{ kip})(2 \text{ ft}) - (0.350 \text{ kip})(4 \text{ ft}) = 0$$

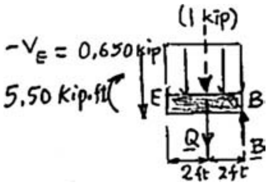
$$\mathbf{P} = 1.350 \text{ kips}\downarrow \blacktriangleleft$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 1 \text{ kip} - 1.350 \text{ kip} - 0.350 \text{ kip} = 0$$

$$A_y = +2.70 \text{ kips} \qquad \mathbf{A} = 2.70 \text{ kips}\uparrow \blacktriangleleft$$

Free body: Portion *EB*



$$+\circlearrowleft \Sigma M_B = 0: \quad (0.650 \text{ kip})(4 \text{ ft}) + (1 \text{ kip})(2 \text{ ft}) + Q(2 \text{ ft}) - 5.50 \text{ kip}\cdot\text{ft} = 0$$

$$\mathbf{Q} = 0.450 \text{ kip}\downarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad B - 0.450 - 1 - 0.650 = 0$$

$$\mathbf{B} = 2.10 \text{ kips}\uparrow \blacktriangleleft$$

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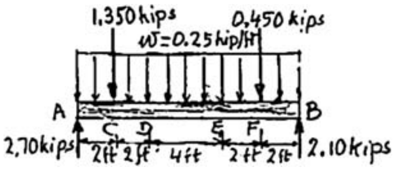
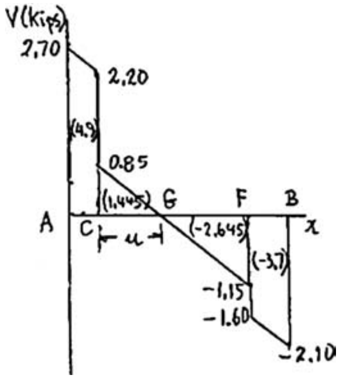
PROBLEM 7.91* (Continued)

(*b*) Load diagram





Shear diagram

At A: $V_A = A = +2.70\text{ kips}$

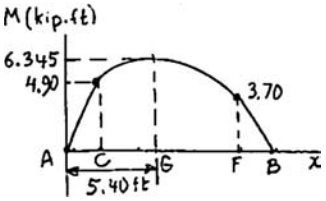
To determine Point G where $V = 0$, we write

$$V_G - V_C = -w\mu$$
$$0 - 0.85\text{ kips} = -(0.25\text{ kip/ft})\mu$$

$\mu = 3.40\text{ ft}$ ◀

We next compute all areas

$|V|_{max} = 2.70\text{ kips at } A$ ◀



Bending-moment diagram

At A: $M_A = 0$

Largest value occurs at G with

$$AG = 2 + 3.40 = 5.40\text{ ft}$$

$|M|_{max} = 6.345\text{ kip}\cdot\text{ft}$ ◀

5.40 ft from A ◀

Bending-moment diagram consists of 3 distinct arcs of parabolas.

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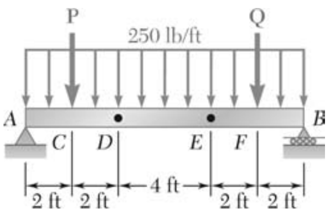


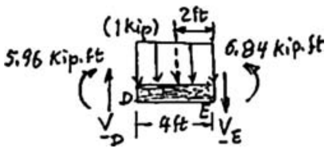
PROBLEM 7.92*

Solve Problem 7.91 assuming that the bending moment was found to be $+5.96\text{ kip}\cdot\text{ft}$ at *D* and $+6.84\text{ kip}\cdot\text{ft}$ at *E*.

PROBLEM 7.91* The beam *AB* is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is $+6.10\text{ kip}\cdot\text{ft}$ at *D* and $+5.50\text{ kip}\cdot\text{ft}$ at *E*, (*a*) determine **P** and **Q**, (*b*) draw the shear and bending-moment diagrams for the beam.

SOLUTION

(*a*) <u>Free body: Portion *DE*</u>



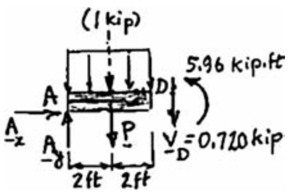
$$+\circlearrowleft \Sigma M_E = 0: \quad 6.84\text{ kip}\cdot\text{ft} - 5.96\text{ kip}\cdot\text{ft} + (1\text{ kip})(2\text{ ft}) - V_D(4\text{ ft}) = 0$$

$$V_D = +0.720\text{ kip}$$

$$+\uparrow \Sigma F_y = 0: \quad 0.720\text{ kip} - 1\text{kip} - V_E = 0$$

$$V_E = -0.280\text{ kip}$$

<u>Free body: Portion *AD*</u>



$$+\circlearrowleft \Sigma M_A = 0: \quad 5.96\text{ kip}\cdot\text{ft} - P(2\text{ ft}) - (1\text{ kip})(2\text{ ft}) - (0.720\text{ kip})(4\text{ ft}) = 0$$

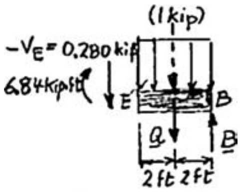
$$\mathbf{P} = 0.540\text{ kip}\downarrow \blacktriangleleft$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 1\text{ kip} - 0.540\text{ kip} - 0.720\text{ kip} = 0$$

$$A_y = +2.26\text{ kips} \qquad \mathbf{A} = 2.26\text{ kips}\uparrow \triangleleft$$

<u>Free body: Portion *EB*</u>



$$+\circlearrowleft \Sigma M_B = 0: \quad (0.280\text{ kip})(4\text{ ft}) + (1\text{ kip})(2\text{ ft}) + Q(2\text{ ft}) - 6.84\text{ kip}\cdot\text{ft} = 0$$

$$\mathbf{Q} = 1.860\text{ kips}\downarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad B - 1.860 - 1 - 0.280 = 0$$

$$\mathbf{B} = 3.14\text{ kips}\uparrow \blacktriangleleft$$

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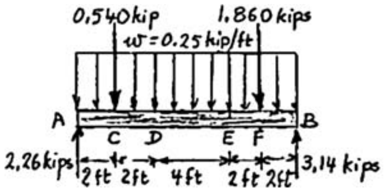
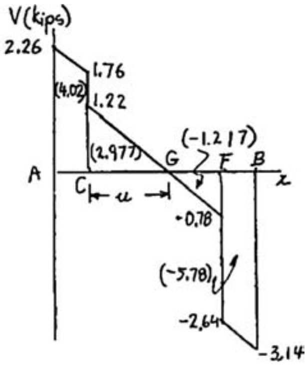
PROBLEM 7.92* (Continued)

(*b*) Load diagram



Shear diagram



At A: $V_A = A = +2.26$ kips

To determine Point G where $V = 0$, we write

$$V_G - V_C = -w\mu$$

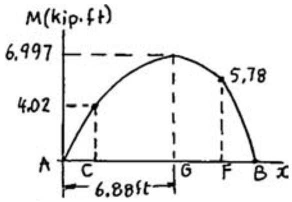
$$0 - (1.22 \text{ kips}) = -(0.25 \text{ kip/ft})\mu$$

$\mu = 4.88$ ft ◁

We next compute all areas

$|V|_{\max} = 3.14$ kips at B ◀

Bending-moment diagram



At A: $M_A = 0$

Largest value occurs at G with

$$AG = 2 + 4.88 = 6.88 \text{ ft}$$

$|M|_{\max} = 6.997$ kip·ft ◀

6.88 ft from A ◀

Bending-moment diagram consists of 3 distinct arcs of parabolas.

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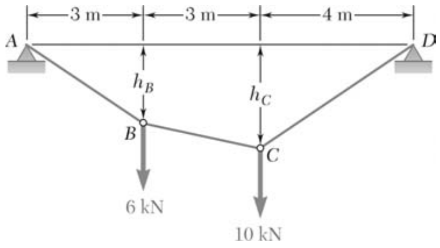
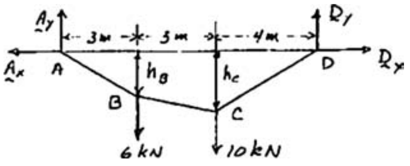


PROBLEM 7.93

Two loads are suspended as shown from the cable *ABCD*. Knowing that $h_B = 1.8\text{ m}$, determine (*a*) the distance h_C, (*b*) the components of the reaction at *D*, (*c*) the maximum tension in the cable.

SOLUTION

FBD Cable:



$$\xrightarrow{+} \Sigma F_x = 0: \quad -A_x + D_x = 0 \quad A_x = D_x$$

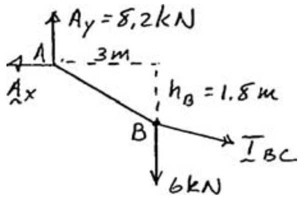
$$\circlearrowleft \Sigma M_A = 0: \quad (10\text{ m})D_y - (6\text{ m})(10\text{ kN}) - (3\text{ m})(6\text{ kN}) = 0$$

$$\mathbf{D}_y = 7.8\text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - 6\text{ kN} - 10\text{ kN} + 7.8\text{ kN} = 0$$

$$\mathbf{A}_y = 8.2\text{ kN}\uparrow$$

FDB AB:

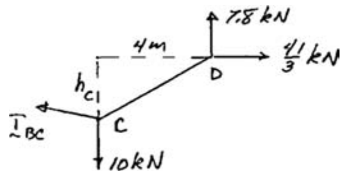


$$\circlearrowleft \Sigma M_B = 0: \quad (1.8\text{ m})A_x - (3\text{ m})(8.2\text{ kN}) = 0$$

$$\mathbf{A}_x = \frac{41}{3}\text{ kN}\leftarrow$$

From above $\quad D_x = A_x = \dfrac{41}{3}\text{ kN}$

FBD CD:



$$\circlearrowleft \Sigma M_C = 0: \quad (4\text{ m})(7.8\text{ kN}) - h_C\left(\frac{41}{3}\text{ kN}\right) = 0$$

$$h_C = 2.283\text{ m}$$

(*a*) $\quad h_C = 2.28\text{ m}$ ◀

(*b*) $\quad \mathbf{D}_x = 13.67\text{ kN}\rightarrow$ ◀

$\mathbf{D}_y = 7.80\text{ kN}\uparrow$ ◀

Since $A_x = B_x$ and $A_y > B_y$, max T is T_{AB}

$$T_{AB} = \sqrt{A_x^2 + A_y^2} = \sqrt{\left(\frac{41}{3}\text{ kN}\right)^2 + (8.2\text{ kN})^2}$$

(*c*) $\quad T_{\max} = 15.94\text{ kN}$ ◀

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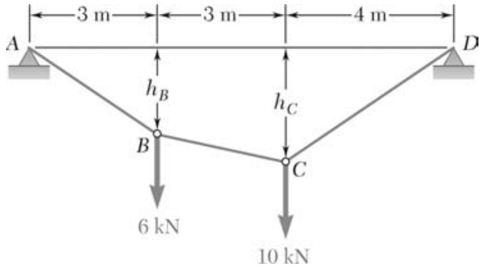
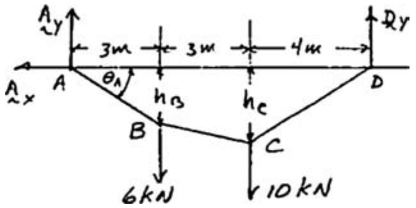


PROBLEM 7.94

Knowing that the maximum tension in cable *ABCD* is 15 kN, determine (*a*) the distance h_B, (*b*) the distance h_C.

SOLUTION

FBD Cable:



$$\rightarrow \Sigma F_x = 0: \quad -A_x + D_x = 0 \quad A_x = D_x$$

$$\circlearrowleft \Sigma M_A = 0: \quad (10\text{ m})\,D_y - (6\text{ m})(10\text{ kN}) - (3\text{ m})(6\text{ kN}) = 0$$

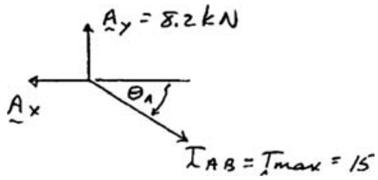
$$\mathbf{D}_y = 7.8\text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - 6\text{ kN} - 10\text{ kN} + 7.8\text{ kN} = 0$$

$$\mathbf{A}_y = 8.2\text{ kN}\uparrow$$

Since $\quad A_x = D_x \quad$ and $\quad A_y > D_y, \quad T_{\max} = T_{AB}$

FBD Pt A:



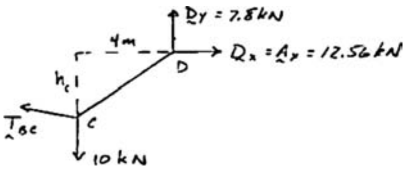
$$\uparrow \Sigma F_y = 0: \quad 8.2\text{ kN} - (15\text{ kN})\sin\theta_A = 0$$

$$\theta_A = \sin^{-1}\frac{8.2\text{ kN}}{15\text{ kN}} = 33.139°$$

$$\rightarrow \Sigma F_x = 0: \quad -A_x + (15\text{ kN})\cos\theta_A = 0$$

$$A_x = (15\text{ kN})\cos(33.139°) = 12.56\text{ kN}$$

FBD CD:



From *FBD* cable:

$$h_B = (3\text{ m})\tan\theta_A$$
$$= (3\text{ m})\tan(33.139°)$$

(*a*) $\qquad h_B = 1.959\text{ m}$ ◀

$$\circlearrowleft \Sigma M_C = 0: \quad (4\text{ m})(7.8\text{ kN}) - h_C(12.56\text{ kN}) = 0$$

(*b*) $\qquad h_C = 2.48\text{ m}$ ◀

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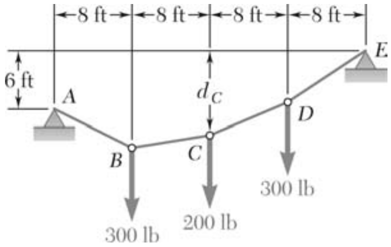
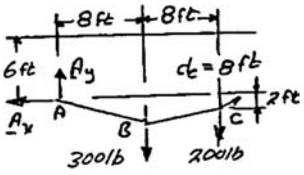


PROBLEM 7.95

If $d_C = 8$ ft, determine (*a*) the reaction at *A*, (*b*) the reaction at *E*.

SOLUTION

Free body: Portion *ABC*

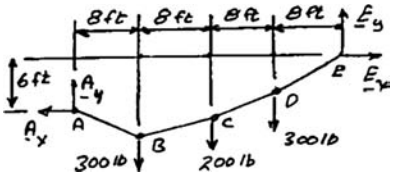


$$+\circlearrowleft \Sigma M_C = 0$$

$$2A_x - 16A_y + 300(8) = 0$$

$$A_x = 8A_y - 1200 \qquad (1)$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_E = 0: \quad +6A_x + 32A_y - (300\text{ lb} + 200\text{ lb} + 300\text{ lb})16\text{ ft} = 0$$

$$3A_x + 16A_y - 6400 = 0$$

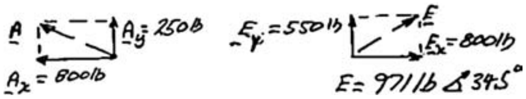
Substitute from Eq. (1):

$$3(8A_y - 1200) + 16A_y - 6400 = 0 \qquad \mathbf{A}_y = 250\text{ lb}\uparrow$$

Eq. (1) $\quad A_x = 8(250) - 1200 \qquad \mathbf{A}_x = 800\text{ lb}\longleftarrow$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -A_x + E_x = 0 \quad -800\text{ lb} + E_x = 0 \qquad \mathbf{E}_x = 800\text{ lb}\longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad 250 + E_y - 300 - 200 - 300 = 0 \qquad \mathbf{E}_y = 550\text{ lb}\uparrow$$



$\mathbf{A} = 838\text{ lb} \searrow 17.4°$ ◀

$\mathbf{E} = 971\text{ lb} \nearrow 34.5°$ ◀

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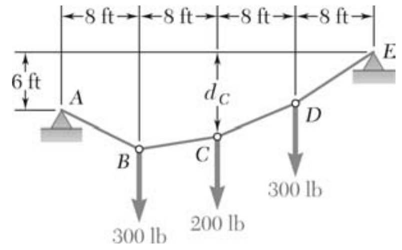
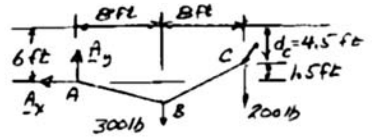


PROBLEM 7.96

If $d_C = 4.5$ ft, determine (*a*) the reaction at *A*, (*b*) the reaction at *E*.

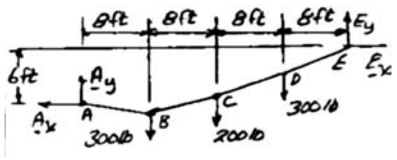
SOLUTION

Free body: Portion *ABC*



$$+\circlearrowleft \Sigma M_C = 0: \quad -1.5A_x - 16A_y + 300 \times 8 = 0$$

$$A_x = \frac{(2400 - 16A_y)}{1.5} \qquad (1)$$



Free body: Entire cable

$$+\circlearrowleft \Sigma M_E = 0: \quad +6A_x + 32A_y - (300 \text{ lb} + 200 \text{ lb} + 300 \text{ lb})16 \text{ ft} = 0$$

$$3A_x + 16A_y - 6400 = 0$$

Substitute from Eq. (1):

$$\frac{3(2400 - 16A_y)}{1.5} + 16A_y - 6400 = 0$$

$$A_y = -100 \text{ lb}$$

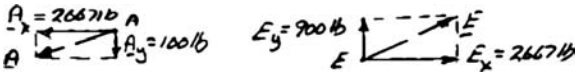
Thus A_y acts downward $\qquad \mathbf{A}_y = 100 \text{ lb} \downarrow$

Eq. (1) $\qquad A_x = \dfrac{(2400 - 16(-100))}{1.5} = 2667 \text{ lb} \qquad \mathbf{A}_x = 2667 \text{ lb} \leftarrow$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -A_x + E_x = 0 \quad -2667 + E_x = 0 \qquad \mathbf{E}_x = 2667 \text{ lb} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + E_y - 300 - 200 - 300 = 0$$

$$-100 \text{ lb} + E_y - 800 \text{ lb} = 0 \qquad \mathbf{E}_y = 900 \text{ lb} \uparrow$$



$\mathbf{A} = 2670 \text{ lb} \measuredangle 2.10°$ ◀

$\mathbf{E} = 2810 \text{ lb} \measuredangle 18.6°$ ◀

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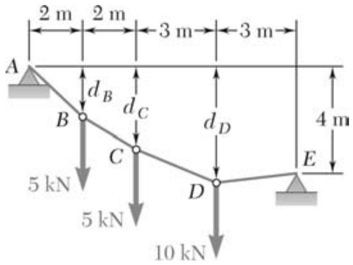
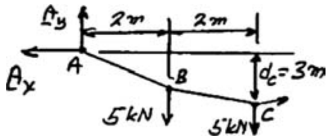


PROBLEM 7.97

Knowing that $d_C = 3$ m, determine (*a*) the distances d_B and d_D (*b*) the reaction at *E*.

SOLUTION

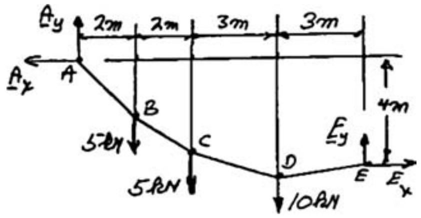
Free body: Portion *ABC*



$$+\circlearrowleft \Sigma M_C = 0: \quad 3A_x - 4A_y + (5\text{ kN})(2\text{ m}) = 0$$

$$A_x = \frac{4}{3}A_y - \frac{10}{3} \qquad (1)$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_E = 0: \quad 4A_x - 10A_y + (5\text{ kN})(8\text{ m}) + (5\text{ kN})(6\text{ m}) + (10\text{ kN})(3\text{ m}) = 0$$

$$4A_x - 10A_y + 100 = 0$$

Substitute from Eq. (1):

$$4\left(\frac{4}{3}A_y - \frac{10}{3}\right) - 10A_y + 100 = 0$$

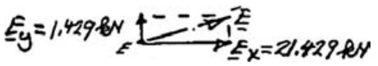
$$A_y = +18.571\text{ kN} \qquad \mathbf{A}_y = 18.571\text{ kN}\uparrow$$

Eq. (1)

$$A_x = \frac{4}{3}(18.511) - \frac{10}{3} = +21.429\text{ kN} \qquad \mathbf{A}_x = 21.429\text{ kN}\leftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -A_x + E_x = 0 \quad -21.429 + E_x = 0 \qquad \mathbf{E}_x = 21.429\text{ kN}\rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad 18.571\text{ kN} + E_y + 5\text{ kN} + 5\text{ kN} + 10\text{ kN} = 0 \qquad \mathbf{E}_y = 1.429\text{ kN}\uparrow$$



$$\mathbf{E} = 21.5\text{ kN} \measuredangle 3.81° \blacktriangleleft$$

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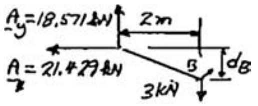
PROBLEM 7.97 (Continued)

Portion *AB*

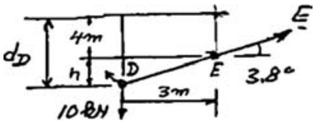


$$+\circlearrowleft \Sigma M_B = 0: \quad (18.571 \text{ kN})(2 \text{ m}) - (21.429 \text{ kN})d_B = 0$$

$d_B = 1.733$ m ◀

Portion *DE*

Geometry



$$h = (3 \text{ m}) \tan 3.8°$$
$$= 0.199 \text{ m}$$
$$d_D = 4 \text{ m} + 0.199 \text{ m}$$

$d_D = 4.20$ m ◀

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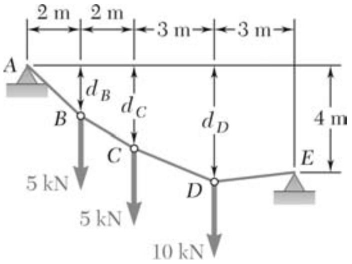
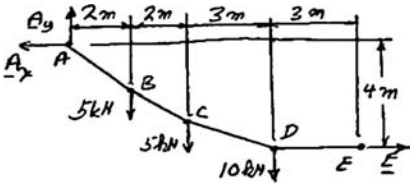


PROBLEM 7.98

Determine (*a*) distance d_C for which portion *DE* of the cable is horizontal, (*b*) the corresponding reactions at *A* and *E*.

SOLUTION

<u>Free body: Entire cable</u>

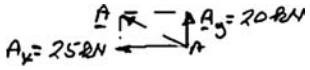


$$+\uparrow \Sigma F_y = 0: \quad A_y - 5\text{ kN} - 5\text{ kN} - 10\text{ kN} = 0 \qquad \mathbf{A}_y = 20\text{ kN}\uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad E(4\text{ m}) - (5\text{ kN})(2\text{ m}) - (5\text{ kN})(4\text{ m}) - (10\text{ kN})(7\text{m}) = 0$$

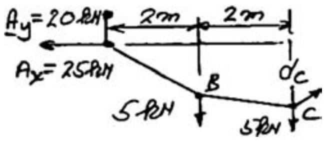
$$E = +25\text{ kN} \qquad \mathbf{E} = 25.0\text{ kN} \longrightarrow \blacktriangleleft$$

$$\Sigma F_x = 0: \quad -A_x + 25\text{ kN} = 0 \qquad \mathbf{A}_x = 25\text{ kN} \longleftarrow$$



$$\mathbf{A} = 32.0\text{ kN} \nwarrow 38.7° \blacktriangleleft$$

<u>Free body: Portion *ABC*</u>



$$+\circlearrowleft \Sigma M_C = 0: \quad (25\text{ kN})d_C - (20\text{ kN})(4\text{ m}) + (5\text{ kN})(2\text{ m}) = 0$$

$$25d_C - 70 = 0 \qquad d_C = 2.80\text{ m} \blacktriangleleft$$

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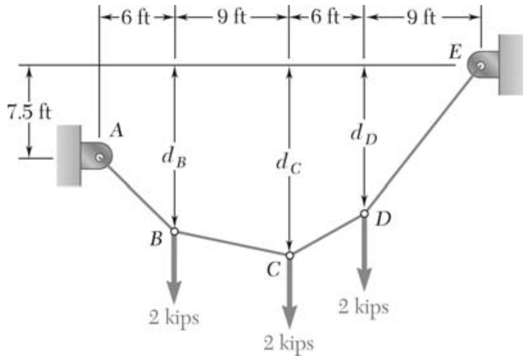
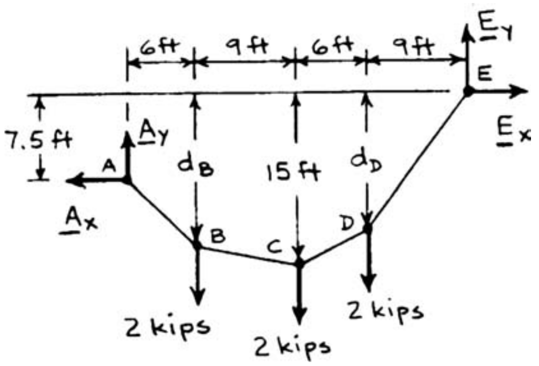


PROBLEM 7.99

If $d_C = 15$ ft, determine (*a*) the distances d_B and d_D, (*b*) the maximum tension in the cable.

SOLUTION

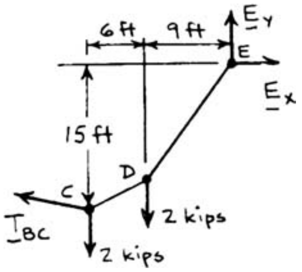
<u>Free body: Entire cable</u>



$$+\circlearrowleft \Sigma M_A = 0: \quad E_y(30\text{ ft}) - E_x(7.5\text{ ft}) - (2\text{ kips})(6\text{ ft}) - (2\text{ kips})(15\text{ ft}) - (2\text{ kips})(21\text{ ft}) = 0$$

$$7.5E_x - 30E_y + 84 = 0 \qquad (1)$$

<u>Free body: Portion *CDE*</u>



$$+\circlearrowleft \Sigma M_C = 0: \quad E_y(15\text{ ft}) - E_x(15\text{ ft}) - (2\text{ kips})(6\text{ ft}) = 0$$

$$15E_x - 15E_y + 12 = 0 \qquad (2)$$

$$\text{Eq. (1)} \times \frac{1}{2}: \quad 3.75E_x - 15E_y + 42 = 0 \qquad (3)$$

$$(2) - (3): \quad 11.25E_x - 30 = 0$$

$$E_x = 2.6667\text{ kips}$$

Eq. (1): $\quad 7.5(2.6667) - 30E_y + 84 = 0 \quad E_y = 3.4667\text{ kips}$

$$T_m = \sqrt{E_x^2 + E_y^2} = \sqrt{(2.6667)^2 + (3.4667)^2} \qquad T_m = 4.37\text{ kips} \blacktriangleleft$$

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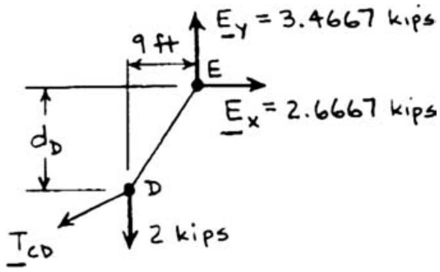
PROBLEM 7.99 (Continued)

Free body: Portion *DE*



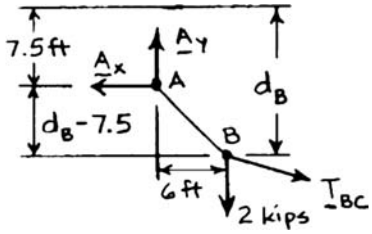
$$+\circlearrowleft \Sigma M_D = 0: \quad (3.4667 \text{ kips})(9 \text{ ft}) - (2.6667 \text{ kips})d_D = 0 \qquad d_D = 11.70 \text{ ft} \blacktriangleleft$$

Return to free body of entire cable (with $E_x = 2.6667$ kips, $E_y = 3.4667$ kips)

$$+\uparrow \Sigma F_y = 0: \quad A_y - 3(2 \text{ kips}) + 3.4667 \text{ kips} = 0 \qquad A_y = 2.5333 \text{ kips}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 2.6667 \text{ kips} - A_x = 0 \qquad A_x = 2.6667 \text{ kips}$$

Free body: Portion *AB*



$$+\circlearrowleft \Sigma M_B = 0: \quad A_x(d_B - 7.5) - A_y(6) = 0$$

$$(2.6667)(d_B - 7.5) - (2.5333)(6) = 0 \qquad d_B = 13.20 \text{ ft} \blacktriangleleft$$

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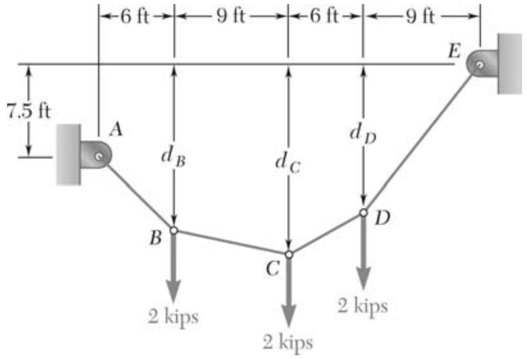
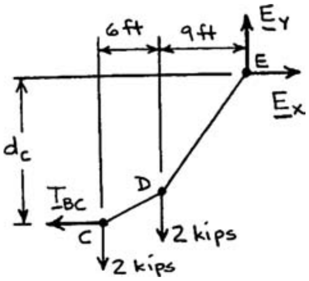


PROBLEM 7.100

Determine (*a*) the distance d_C for which portion *BC* of the cable is horizontal, (*b*) the corresponding components of the reaction at *E*.

SOLUTION

Free body: Portion *CDE*

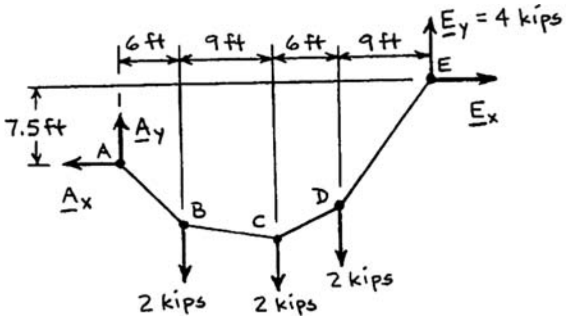


$$+\uparrow \Sigma F_y = 0: \quad E_y - 2(2\text{ kips}) = 0 \quad E_y = 4\text{ kips}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad (4\text{ kips})(15\text{ ft}) - E_x d_C - (2\text{ kips})(6\text{ ft}) = 0$$

$$E_x d_C = 48\text{ kip}\cdot\text{ft} \qquad (1)$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad (4\text{ kips})(30\text{ ft}) - E_x(7.5\text{ ft}) - (2\text{ kips})(6\text{ ft}) - (2\text{ kips})(15\text{ ft}) - (2\text{ kips})(21\text{ ft}) = 0$$

$$E_x = 4.8\text{ kips}$$

From Eq. (1): $$d_C = \frac{48}{E_x} = \frac{48}{4.8}$$ $d_C = 10.00\text{ ft}$ ◀

Components of reaction at *E*: $\mathbf{E}_x = 4.80\text{ kips} \longrightarrow; \quad E_y = 4.00\text{ kips}\uparrow$ ◀

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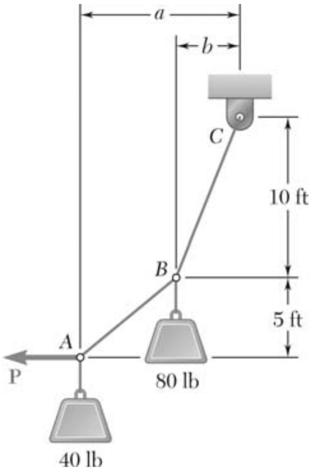
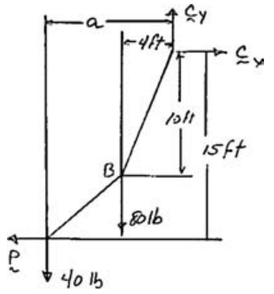


PROBLEM 7.101

Cable *ABC* supports two loads as shown. Knowing that $b = 4$ ft, determine (*a*) the required magnitude of the horizontal force **P**, (*b*) the corresponding distance *a*.

SOLUTION

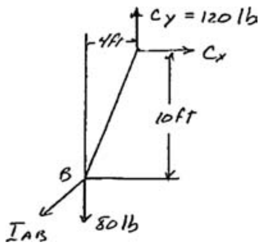
FBD ABC:



$$+\uparrow \Sigma F_y = 0: \quad -40 \text{ lb} - 80 \text{ lb} + C_y = 0$$

$$\mathbf{C}_y = 120 \text{ lb} \uparrow$$

FBD BC:



$$\circlearrowleft \Sigma M_B = 0: \quad (4 \text{ ft})(120 \text{ lb}) - (10 \text{ ft})C_x = 0$$

$$\mathbf{C}_x = 48 \text{ lb} \longrightarrow$$

From *ABC*: $\quad \xrightarrow{+} \Sigma F_x = 0: \quad -P + C_x = 0$

$$P = C_x = 48 \text{ lb}$$

(*a*) $P = 48.0$ lb ◀

$$\circlearrowleft \Sigma M_C = 0: \quad (4 \text{ ft})(80 \text{ lb}) + a(40 \text{ lb}) - (15 \text{ ft})(48 \text{ lb}) = 0$$

(*b*) $a = 10.00$ ft ◀

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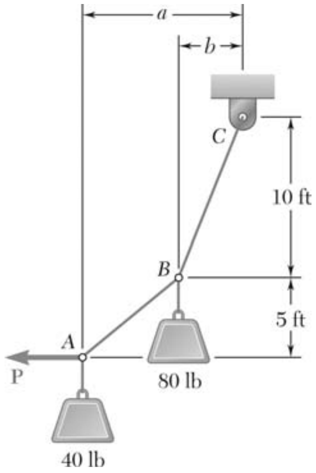
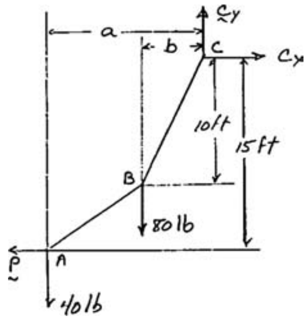


PROBLEM 7.102

Cable *ABC* supports two loads as shown. Determine the distances *a* and *b* when a horizontal force **P** of magnitude 60 lb is applied at *A*.

SOLUTION

FBD ABC:

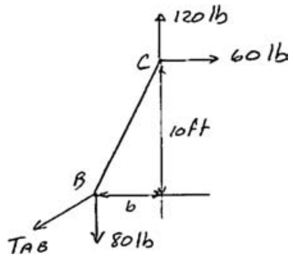


$$\rightarrow \Sigma F_x = 0: \quad C_x - P = 0 \qquad C_x = 60 \text{ lb} \rightarrow$$

$$\uparrow \Sigma F_y = 0: \quad C_y - 40 \text{ lb} - 80 \text{ lb} = 0$$

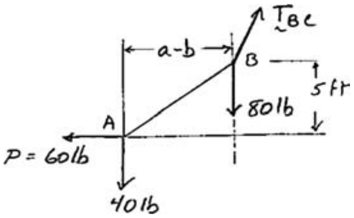
$$\mathbf{C}_y = 120 \text{ lb} \uparrow$$

FBD BC:



$$\circlearrowleft \Sigma M_B = 0: \quad b(120 \text{ lb}) - (10 \text{ ft})(60 \text{ lb}) = 0 \qquad b = 5.00 \text{ ft} \blacktriangleleft$$

FBD AB:



$$\Sigma M_B = 0: \quad (a-b)(40 \text{ lb}) - (5 \text{ ft})60 \text{ lb} = 0$$

$$a - b = 7.5 \text{ ft}$$

$$a = b + 7.5 \text{ ft}$$

$$= 5 \text{ ft} + 7.5 \text{ ft}$$

$$a = 12.50 \text{ ft} \blacktriangleleft$$

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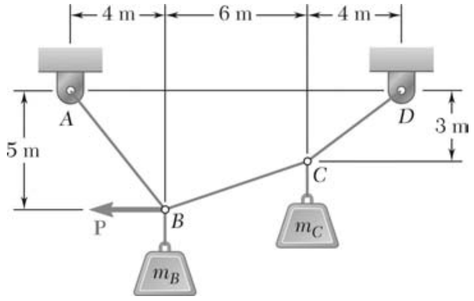
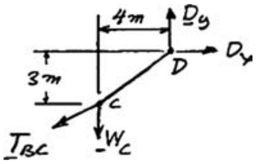


PROBLEM 7.103

Knowing that m_B = 70 kg and m_C = 25 kg, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

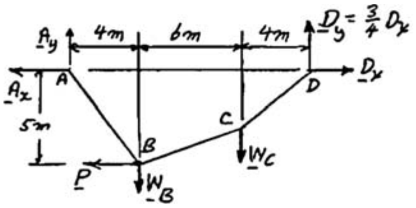
Free body: Portion *CD*



$$+\circlearrowleft \Sigma M_C = 0: \quad D_y(4\text{ m}) - D_x(3\text{ m}) = 0$$

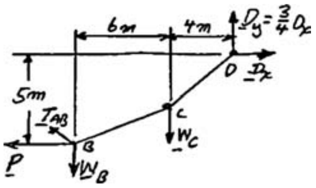
$$D_y = \frac{3}{4}D_x$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{3}{4}D_x(14\text{ m}) - W_B(4\text{ m}) - W_C(10\text{ m}) - P(5\text{ m}) = 0 \qquad (1)$$

Free body: Portion *BCD*



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{3}{4}D_x(10\text{ m}) - D_x(5\text{ m}) - W_C(6\text{ m}) = 0$$

$$D_x = 2.4W_C \qquad (2)$$

For $m_B = 70\text{ kg} \quad m_C = 25\text{ kg}$

$g = 9.81\text{ m/s}^2$: $W_B = 70g \quad W_C = 25g$

Eq. (2): $D_x = 2.4W_C = 2.4(25g) = 60g$

Eq. (1): $\frac{3}{4}60g(14) - 70g(4) - 25g(10) - 5P = 0$

$$100g - 5P = 0: \quad P = 20g$$

$$P = 20(9.81) = 196.2\text{ N}$$

$P = 196.2$ N ◀

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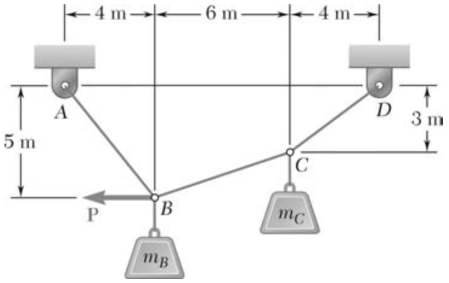


PROBLEM 7.104

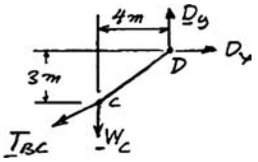
Knowing that $m_B = 18$ kg and $m_C = 10$ kg, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

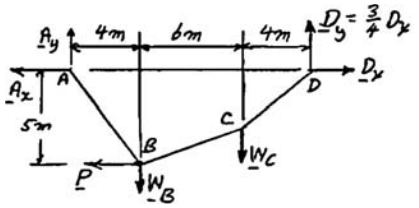
Free body: Portion *CD*

$$+\circlearrowleft \Sigma M_C = 0: \quad D_y(4\text{ m}) - D_x(3\text{ m}) = 0$$

$$D_y = \frac{3}{4}D_x$$

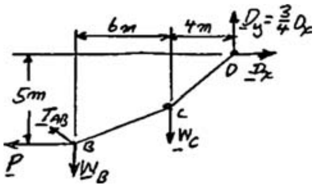


Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{3}{4}D_x(14\text{ m}) - W_B(4\text{ m}) - W_C(10\text{ m}) - P(5\text{ m}) = 0 \tag{1}$$

Free body: Portion *BCD*



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{3}{4}D_x(10\text{ m}) - D_x(5\text{ m}) - W_C(6\text{ m}) = 0$$

$$D_x = 2.4W_C \tag{2}$$

For $\quad m_B = 18\text{ kg} \quad m_C = 10\text{ kg}$

$g = 9.81\text{ m/s}^2$: $\quad W_B = 18g \quad W_C = 10g$

Eq. (2): $\quad D_x = 2.4W_C = 2.4(10g) = 24g$

Eq. (1): $\quad \frac{3}{4}24g(14) - (18g)(4) - (10g)(10) - 5P = 0$

$80g - 5P: \quad P = 16g$

$P = 16(9.81) = 156.96$ N $\qquad P = 157.0$ N ◀

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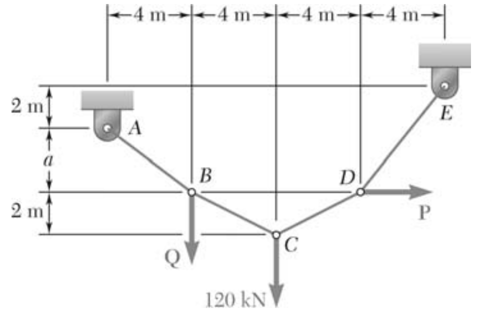
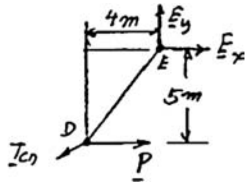


PROBLEM 7.105

If a = 3 m, determine the magnitudes of **P** and **Q** required to maintain the cable in the shape shown.

SOLUTION

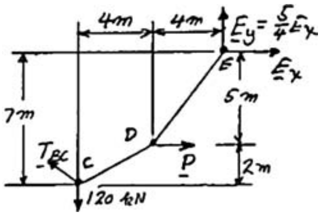
Free body: Portion *DE*



$$+\circlearrowleft \Sigma M_D = 0: \quad E_y(4\text{ m}) - E_x(5\text{ m}) = 0$$

$$E_y = \frac{5}{4}E_x$$

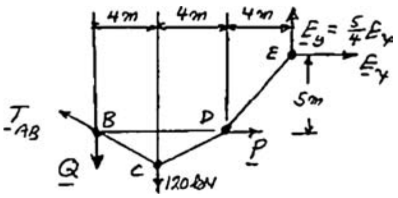
Free body: Portion *CDE*



$$+\circlearrowleft \Sigma M_C = 0: \quad \frac{5}{4}E_x(8\text{ m}) - E_x(7\text{ m}) - P(2\text{ m}) = 0$$

$$E_x = \frac{2}{3}P \qquad (1)$$

Free body: Portion *BCDE*



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{5}{4}E_x(12\text{ m}) - E_x(5\text{ m}) - (120\text{ kN})(4\text{ m}) = 0$$

$$10E_x - 480 = 0; \quad E_x = 48\text{ kN}$$

Eq. (1):

$$48\text{ kN} = \frac{2}{3}P$$

$P = 72.0$ kN ◀

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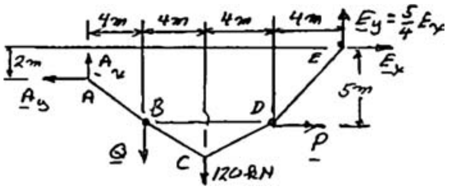
PROBLEM 7.105 (Continued)

Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{5}{4}E_x(16\text{ m}) - E_x(2\text{ m}) + P(3\text{ m}) - Q(4\text{ m}) - (120\text{ kN})(8\text{ m}) = 0$$

$$(48\text{ kN})(20\text{ m} - 2\text{ m}) + (72\text{ kN})(3\text{ m}) - Q(4\text{ m}) - 960\text{ kN}\cdot\text{m} = 0$$

$$4Q = 120 \qquad Q = 30.0\text{ kN} \blacktriangleleft$$

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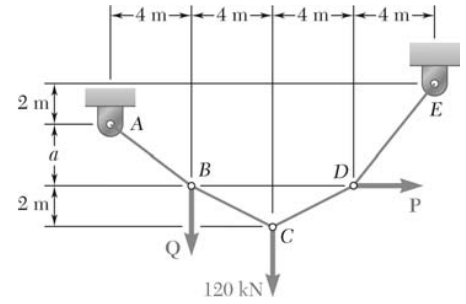
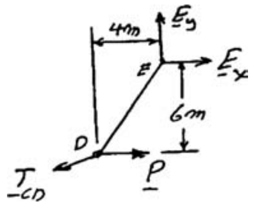


PROBLEM 7.106

If $a = 4$ m, determine the magnitudes of **P** and **Q** required to maintain the cable in the shape shown.

SOLUTION

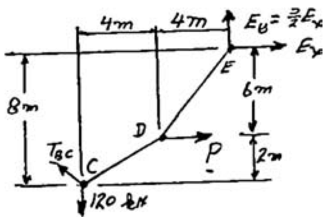
Free body: Portion *DE*



$$+\circlearrowleft \Sigma M_D = 0: \quad E_y(4 \text{ m}) - E_x(6 \text{ m}) = 0$$

$$E_y = \frac{3}{2}E_x$$

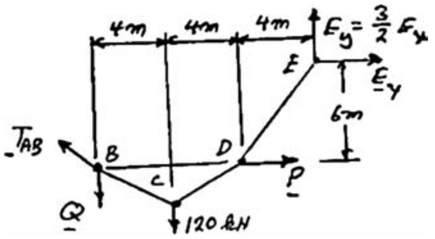
Free body: Portion *CDE*



$$+\circlearrowleft \Sigma M_C = 0: \quad \frac{3}{2}E_x(8 \text{ m}) - E_x(8 \text{ m}) - P(2 \text{ m}) = 0$$

$$E_x = \frac{1}{2}P \qquad (1)$$

Free body: Portion *BCDE*



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{3}{2}E_x(12 \text{ m}) - E_x(6 \text{ m}) + (120 \text{ kN})(4 \text{ m}) = 0$$

$$12E_x = 480 \qquad E_x = 40 \text{ kN}$$

Eq (1): $E_x = \frac{1}{2}P; \qquad 40 \text{ kN} = \frac{1}{2}P$

$$P = 80.0 \text{ kN} \blacktriangleleft$$

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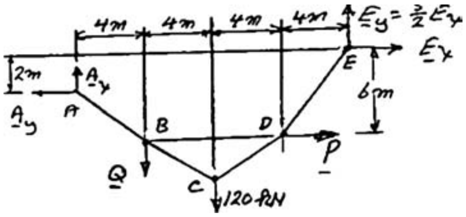
PROBLEM 7.106 (Continued)

Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{3}{2}E_x(16\text{ m}) - E_x(2\text{ m}) + P(4\text{ m}) - Q(4\text{ m}) - (120\text{ kN})(8\text{ m}) = 0$$

$$(40\text{ kN})(24\text{ m} - 2\text{ m}) + (80\text{ kN})(4\text{ m}) - Q(4\text{ m}) - 960\text{ kN}\cdot\text{m} = 0$$

$$4Q = 240 \qquad Q = 60.0\text{ kN} \blacktriangleleft$$

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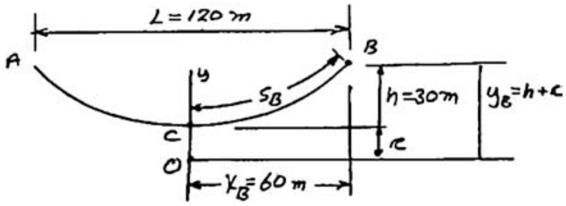
PROBLEM 7.107

A wire having a mass per unit length of 0.65 kg/m is suspended from two supports at the same elevation that are 120 m apart. If the sag is 30 m, determine (*a*) the total length of the wire, (*b*) the maximum tension in the wire.

SOLUTION



Eq. 7.16: $y_B = c\cosh\dfrac{x_B}{c}$

$$30m + c = c\cosh\frac{60}{c}$$

Solve by trial and error: $c = 64.459\text{ m}$

Eq. 7.15: $s_B = c\sin h\dfrac{x_B}{c}$

$$s_B = (64.456\text{ m})\sinh\frac{60\text{ m}}{64.459\text{ m}}$$

$$s_B = 69.0478\text{ m}$$

Length $= 2s_B = 2(69.0478\text{ m}) = 138.0956\text{ m}$ $\quad L = 138.1\text{ m}$ ◀

Eq. 7.18: $T_m = wy_B = w(h+c)$

$$= (0.65\text{ kg/m})(9.81\text{ m/s}^2)(30\text{ m} + 64.459\text{ m})$$

$$T_m = 602.32\text{ N}$$ $\quad T_m = 602\text{ N}$ ◀

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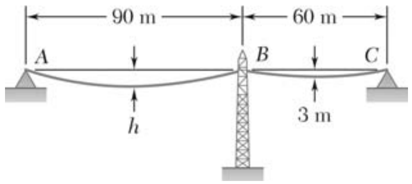
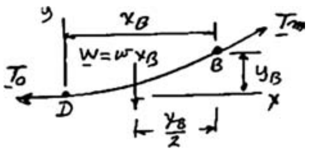


PROBLEM 7.108

Two cables of the same gauge are attached to a transmission tower at *B*. Since the tower is slender, the horizontal component of the resultant of the forces exerted by the cables at *B* is to be zero. Knowing that the mass per unit length of the cables is 0.4 kg/m, determine (*a*) the required sag *h*, (*b*) the maximum tension in each cable.

SOLUTION



$$W = wx_B$$

$$+\circlearrowleft \Sigma M_B = 0: \quad T_0 \; y_B - (wx_B)\frac{y_B}{2} = 0$$

(*a*) Horiz. comp. $= T_0 = \dfrac{wx_B^2}{2y_B}$

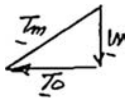
<u>Cable *AB*</u> $x_B = 45 \text{ m}$

$$T_0 = \frac{w(45 \text{ m})^2}{2h}$$

<u>Cable *BC*</u> $x_B = 30 \text{ m}, \quad y_B = 3 \text{ m}$

$$T_0 = \frac{w(30 \text{ m})^2}{2(3 \text{ m})}$$

Equate $T_0 = T_0$ $\quad \dfrac{w(45 \text{ m})^2}{2h} = \dfrac{w(30 \text{ m})^2}{2(3 \text{ m})}$ $\quad h = 6.75 \text{ m}$ ◀



(*b*) $T_m^2 = T_0^2 + W^2$

<u>Cable *AB*</u>: $w = (0.4 \text{ kg/m})(9.81 \text{ m/s}) = 3.924 \text{ N/m}$

$$x_B = 45 \text{ m}, \quad y_B = h = 6.75 \text{ m}$$

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(3.924 \text{ N/m})(45 \text{ m})^2}{2(6.75 \text{ m})} = 588.6 \text{ N}$$

$$W = wx_B = (3.924 \text{ N/m})(45 \text{ m}) = 176.58 \text{ N}$$

$$T_m^2 = (588.6 \text{ N})^2 + (176.58 \text{ N})^2$$

<u>For *AB*</u>: $T_m = 615 \text{ N}$ ◀

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PROBLEM 7.108 (Continued)

Cable *BC* $x_B = 30\text{ m}, \quad y_B = 3\text{ m}$

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(3.924\text{ N/m})(30\text{ m})^2}{2(3\text{ m})} = 588.6\text{ N} \quad \text{(Checks)}$$

$$W = wx_B = (3.924\text{ N/m})(30\text{ m}) = 117.72\text{ N}$$

$$T_m^2 = (588.6\text{ N})^2 + (117.72\text{ N})^2$$

For *BC* $T_m = 600\text{ N}$ ◀

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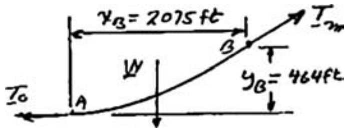
PROBLEM 7.109

Each cable of the Golden Gate Bridge supports a load $w = 11.1$ kips/ft along the horizontal. Knowing that the span L is 4150 ft and that the sag h is 464 ft, determine (*a*) the maximum tension in each cable, (*b*) the length of each cable.

SOLUTION

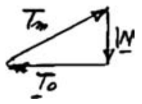
Eq. (7.8) Page 386:



At *B*: $$y_B = \frac{wx_B^2}{2T_0}$$

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(11.1\text{ kip/ft})(2075\text{ ft})^2}{2(464\text{ ft})}$$

(*a*)



$$T_0 = 51.500\text{ kips}$$

$$W = wx_B = (11.1\text{ kips/ft})(2075\text{ ft}) = 23.033\text{ kips}$$

$$T_m = \sqrt{T_0^2 + W^2} = \sqrt{(51.500\text{ kips})^2 + (23.033\text{ kips})^2}$$

$$T_m = 56,400\text{ kips} \blacktriangleleft$$

(*b*)

$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{y_B}\right)^4 + \cdots\right] \qquad \frac{y_B}{x_B} = \frac{464\text{ ft}}{2075\text{ ft}} = 0.22361$$

$$s_B = (2075\text{ ft})\left[1 + \frac{2}{3}(0.22361)^2 - \frac{2}{5}(0.22361)^4 + \cdots\right] = 2142.1\text{ ft}$$

$$\text{Length} = 2s_B = 2(2142.1\text{ ft})$$

$$\underline{\text{Length}} = 4280\text{ ft} \blacktriangleleft$$

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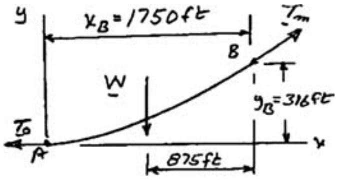
PROBLEM 7.110

The center span of the George Washington Bridge, as originally constructed, consisted of a uniform roadway suspended from four cables. The uniform load supported by each cable was $w = 9.75$ kips/ft along the horizontal. Knowing that the span L is 3500 ft and that the sag h is 316 ft, determine for the original configuration (*a*) the maximum tension in each cable, (*b*) the length of each cable.

SOLUTION



$$W = wx_B = (9.75\text{ kips/ft})(1750\text{ ft})$$

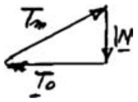
$$W = 17{,}063\text{ kips}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad T_0(316\text{ ft}) - (17063\text{ kips})(875\text{ ft}) = c$$

$$T_0 = 47{,}247\text{ kips}$$

(*a*)

$$T_m = \sqrt{T_0^2 + W^2}$$



$$= \sqrt{(47{,}247\text{ kips})^2 + (17{,}063\text{ kips})^2}$$

$$T_m = 50{,}200\text{ kips} \blacktriangleleft$$

(*b*)

$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4 + \cdots\right]$$

$$\frac{y_B}{x_B} = \frac{316\text{ ft}}{1750\text{ ft}} = 0.18057$$

$$= (1750\text{ ft})\left[1 + \frac{2}{3}(0.18057)^2 - \frac{2}{5}(0.18057)^4 + \cdots\right]$$

$$s_B = 1787.3\text{ ft; Length} = 2s_B = 3579.6\text{ ft} \qquad \text{Length} = 3580\text{ ft} \blacktriangleleft$$

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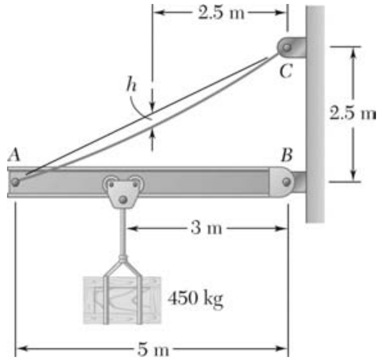
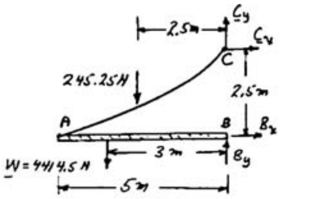


PROBLEM 7.111

The total mass of cable AC is 25 kg. Assuming that the mass of the cable is distributed uniformly along the horizontal, determine the sag h and the slope of the cable at A and C.

SOLUTION



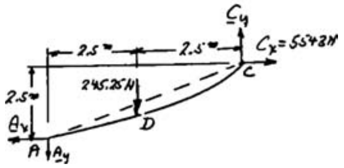
Cable: $m = 25\text{ kg}$

$W = 25\,(9.81)$

$= 245.25\text{ N}$

Block: $m = 450\text{ kg}$

$W = 4414.5\text{ N}$



$+\circlearrowleft \Sigma M_B = 0\text{:}\quad (245.25)(2.5) + (4414.5)(3) - C_x(2.5) = 0$

$C_x = 5543\text{ N}$

$\Sigma F_x = 0\text{:}\quad A_x = C_x = 5543\text{ N}$

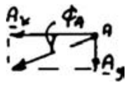
$+\circlearrowleft \Sigma M_A = 0\text{:}\quad C_y(5) - (5543)(2.5) - (245.25)(2.5) = 0$

$\mathbf{C}_y = 2894\text{ N}\uparrow$

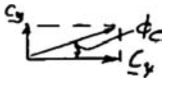
$+\uparrow \Sigma F_y = 0\text{:}\quad C_y - A_y - 245.25\text{N} = 0$

$2894\ N - A_y - 245.25\text{ N} = 0 \qquad \mathbf{A}_y = 2649\text{ N}\downarrow$

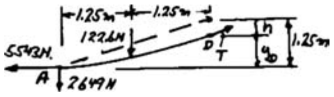
Point A: $\tan\phi_A = \dfrac{A_y}{A_x} = \dfrac{2649}{5543} = 0.4779;$ $\qquad \phi_A = 25.5°$ ◀



Point C: $\tan\phi_C = \dfrac{C_y}{C_x} = \dfrac{2894}{5543} = 0.5221;$ $\qquad \phi_C = 27.6°$ ◀



Free body: Half cable $\qquad W = (12.5\text{ kg})\,g = 122.6$



$+\circlearrowleft \Sigma M_0 = 0\text{:}\quad (122.6\text{ N})(1.25\text{ M}) + (2649\text{ N})(2.5\text{ m}) - (5543\text{ N})\,y_d = 0$

$y_d = 1.2224\text{ m; sag} = h = 1.25\text{ m} - 1.2224\text{ m}$

$h = 0.0276\text{ m} = 27.6\text{ mm}$ ◀

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PROBLEM 7.112

A 50.5-m length of wire having a mass per unit length of 0.75 kg/m is used to span a horizontal distance of 50 m. Determine (*a*) the approximate sag of the wire, (*b*) the maximum tension in the wire. [*Hint:* Use only the first two terms of Eq. (7.10).]

SOLUTION

First two terms of Eq. 7.10

(*a*)

$$s_B = \frac{1}{2}(50.5 \text{ m}) = 25.25 \text{ m},$$

$$x_B = \frac{1}{2}(50 \text{ m}) = 25 \text{ m}$$

$$y_B = h$$

$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right]$$

$$25.25 \text{ m} = 25 \text{ m}\left[1 - \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right]$$

$$\left(\frac{y_B}{x_B}\right)^2 = 0.01\left(\frac{3}{2}\right)^2 = \sqrt{0.015}$$

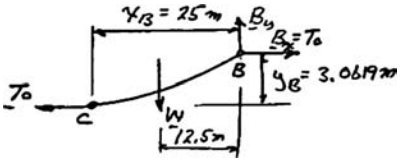
$$\frac{y_B}{x_B} = 0.12247$$

$$\frac{h}{25\text{m}} = 0.12247$$

$$h = 3.0619 \text{ m}$$

$h = 3.06$ m ◁

(*b*) Free body: Portion *CB*



$$w = (0.75 \text{ kg/m})(9.81\text{m}) = 7.3575 \text{ N/m}$$

$$W = s_B\ w = (25.25 \text{ m})(7.3575 \text{ N/m})$$

$$W = 185.78 \text{ N}$$

$$+\circlearrowleft \Sigma M_0 = 0: \quad T_0(3.0619 \text{ m}) - (185.78 \text{ N})(12.5 \text{ m}) = 0$$

$$T_0 = 758.4 \text{ N}$$

$$B_x = T_0 = 758.4 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad B_y - 185.78 \text{ N} = 0 \quad B_y = 185.78 \text{ N}$$

$$T_m = \sqrt{B_x^2 + B_y^2} = \sqrt{(758.4 \text{ N})^2 + (185.78 \text{ N})^2}$$

$T_m = 781$ N ◁

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PROBLEM 7.113

A cable of length $L + \Delta$ is suspended between two points that are at the same elevation and a distance L apart. (*a*) Assuming that Δ is small compared to L and that the cable is parabolic, determine the approximate sag in terms of L and Δ. (*b*) If $L = 100$ ft and $\Delta = 4$ ft, determine the approximate sag. [*Hint:* Use only the first two terms of Eq. (7.10).

SOLUTION

Eq. 7.10 (First two terms)

(*a*)

$$s_B = x_B\left[1+\frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right]$$

$$x_B = L/2$$

$$s_B = \frac{1}{2}(L+\Delta)$$

$$y_B = h$$

$$\frac{1}{2}(L+\Delta) = \frac{L}{2}\left[1+\frac{2}{3}\left(\frac{h}{\frac{L}{2}}\right)^2\right]$$

$$\frac{\Delta}{2} = \frac{4}{3}\frac{h^2}{L}; \quad h^2 = \frac{3}{8}L\Delta; \qquad h = \sqrt{\frac{3}{8}L\Delta} \blacktriangleleft$$

(*b*) $L = 100$ ft, $h = 4$ ft. $h = \sqrt{\frac{3}{8}(100)(4)}$; $h = 12.25$ ft ◀

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PROBLEM 7.114

The center span of the Verrazano-Narrows Bridge consists of two uniform roadways suspended from four cables. The design of the bridge allows for the effect of extreme temperature changes that cause the sag of the center span to vary from $h_w = 386$ ft in winter to $h_s = 394$ ft in summer. Knowing that the span is $L = 4260$ ft, determine the change in length of the cables due to extreme temperature changes.

SOLUTION

Eq. 7.10.

$$s_B = x_B\left[1+\frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4 + \cdots\right]$$

Winter:

$$y_B = h = 386 \text{ ft}, \quad x_B = \frac{1}{2}L = 2130 \text{ ft}$$

$$s_B = (2130)\left[1+\frac{2}{3}\left(\frac{386}{2130}\right)^2 - \frac{2}{5}\left(\frac{386}{2130}\right)^4 + \cdots\right] = 2175.715 \text{ ft}$$

Summer:

$$y_B = h = 394 \text{ ft}, \quad x_B = \frac{1}{2}L = 2130 \text{ ft}$$

$$s_B = (2130)\left[1+\frac{2}{3}\left(\frac{394}{2130}\right)^2 - \frac{2}{5}\left(\frac{394}{2130}\right)^4 + \cdots\right] = 2177.59 \text{ ft}$$

$$\Delta = 2(\Delta s_B) = 2(2177.59 \text{ ft} - 2175.715 \text{ ft}) = 2(1.875 \text{ ft})$$

Change in length $= 3.75$ ft ◀

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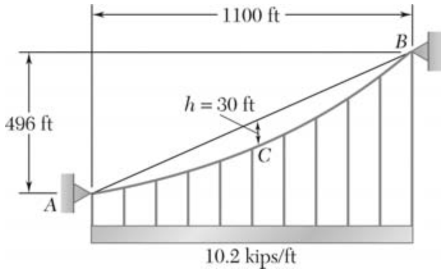
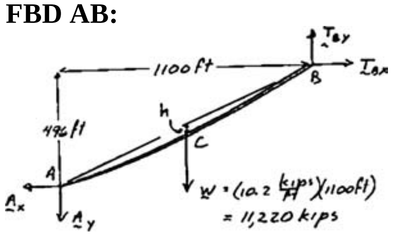


PROBLEM 7.115

Each cable of the side spans of the Golden Gate Bridge supports a load w = 10.2 kips/ft along the horizontal. Knowing that for the side spans the maximum vertical distance h from each cable to the chord AB is 30 ft and occurs at midspan, determine (*a*) the maximum tension in each cable, (*b*) the slope at B.

SOLUTION

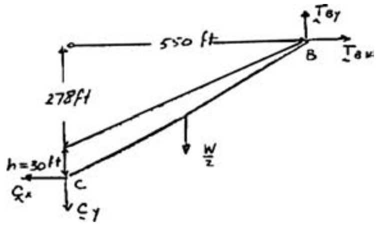
FBD AB:



$$\circlearrowleft \Sigma M_A = 0: \quad (1100 \text{ ft})T_{By} - (496 \text{ ft})T_{Bx} - (550 \text{ ft})W = 0$$

$$11T_{By} - 4.96T_{Bx} = 5.5W \qquad (1)$$

FBD CB:



$$\circlearrowleft \Sigma M_C = 0: \quad (550 \text{ ft})T_{By} - (278 \text{ ft})T_{Bx} - (275 \text{ ft})\frac{W}{2} = 0$$

$$11T_{By} - 5.56T_{Bx} = 2.75W \qquad (2)$$

Solving (1) and (2) $\qquad T_{By} = 28,798 \text{ kips}$

$$T_{Bx} = 51,425 \text{ kips}$$

$$T_{\max} = T_B = \sqrt{T_{B_x}^2 + T_{B_y}^2} \qquad \tan\theta_B = \frac{T_{B_y}}{T_{B_x}}$$

So that

(*a*) $T_{\max} = 58,940 \text{ kips}$ ◀

(*b*) $\theta_B = 29.2°$ ◀

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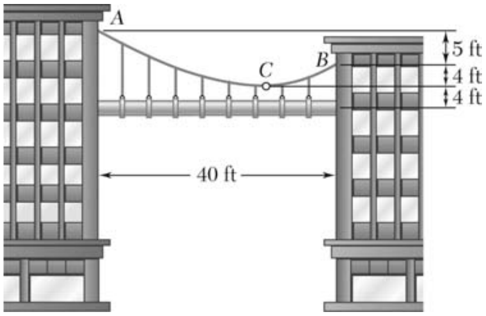
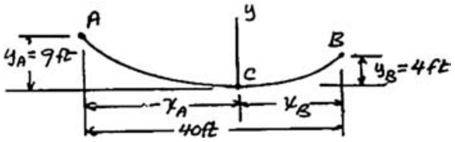


PROBLEM 7.116

A steam pipe weighting 45 lb/ft that passes between two buildings 40 ft apart is supported by a system of cables as shown. Assuming that the weight of the cable system is equivalent to a uniformly distributed loading of 5 lb/ft, determine (*a*) the location of the lowest Point *C* of the cable, (*b*) the maximum tension in the cable.

SOLUTION



Note: $x_B - x_A = 40\text{ ft}$

or $x_A = x_B - 40\text{ ft}$

(*a*) Use Eq. 7.8

Point *A*: $$y_A = \frac{wx_A^2}{2T_0}; \quad 9 = \frac{w(x_B - 40)^2}{2T_0} \qquad (1)$$

Point *B*: $$y_B = \frac{wx_B^2}{2T_0}; \quad 4 = \frac{wx_B^2}{2T_0} \qquad (2)$$

Dividing (1) by (2): $$\frac{9}{4} = \frac{(x_B - 40)^2}{x_B^2}; \quad x_B = 16\text{ ft}$$

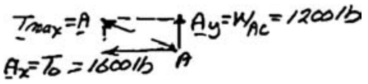
Point *C* is 16 ft to left of *B* ◀

(*b*) Maximum slope and thus T_{max} is at *A*

$$x_A = x_B - 40 = 16 - 40 = -24\text{ ft}$$

$$y_A = \frac{wx_A^2}{2T_0}; \quad 9\text{ ft} = \frac{(50\text{ lb/ft})(-24\text{ ft})^2}{2T_0}; \quad T_0 = 1600\text{ lb}$$

$$W_{AC} = (50\text{ lb/ft})(24\text{ ft}) = 1200\text{ lb}$$



$T_{max} = 2000\text{ lb}$ ◀

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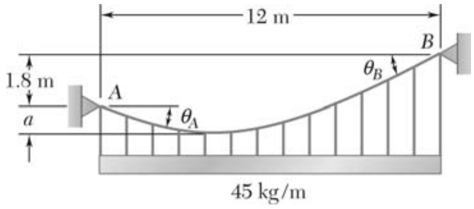


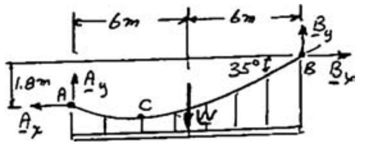
PROBLEM 7.117

Cable *AB* supports a load uniformly distributed along the horizontal as shown. Knowing that at *B* the cable forms an angle $\theta_B = 35°$ with the horizontal, determine (*a*) the maximum tension in the cable, (*b*) the vertical distance *a* from *A* to the lowest point of the cable.

SOLUTION

<u>Free body: Entire cable</u>

$$B_y = B_x \tan 35°$$



$$W = (45 \text{ kg/m})(12 \text{ m})(9.81 \text{ m/s}^2)$$
$$W = 5297.4 \text{ N}$$

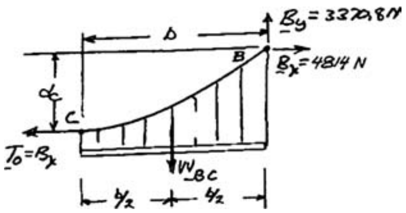
$$+\circlearrowleft \Sigma M_A = 0: \quad W(6 \text{ m}) + B_x(1.8 \text{ m}) - B_y(12 \text{ m}) = 0$$

$$(5297.4)(6) + 1.8B_x - B_x \tan 35°(12) = 0$$

$$B_x = 4814 \text{ N} \quad B_y = (4814 \text{ N})\tan 35° = 3370.8 \text{ N}$$

<u>Free body: Portion *CB*</u>

$$+\uparrow \Sigma F_y = 0: \quad B_y - W_{BC} = 0$$



$$W_{BC} = B_y = 3370.8 \text{ N}$$

$$W_{BC} = (45 \text{ kg/m})(9.81 \text{ m/s}^2)b$$

$$3370.8 \text{ N} = (441.45 \text{ N/m})b$$

$$b = 7.6357 \text{ m}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad T_0 d_C - W_{BC}\left(\frac{1}{2}b\right) = 0$$

$$(4814 \text{ N})d_C - (3370.8 \text{ N})\frac{1}{2}(7.6357 \text{ m}) = 0$$

$$d_C = 2.6733 \text{ m}$$

(*a*) $d_C = 1.8 \text{ m} + a; \quad 2.6733 \text{ m} = 1.8 \text{ m} + a;$ $\qquad a = 0.873 \text{ m}$ ◀

(*b*) $T_m = B = \sqrt{B_x^2 + B_y^2} = \sqrt{(4814 \text{ N})^2 + (3370.8 \text{ N})}$

$T_m = 5877 \text{ N}$ $\qquad T_m = 5880 \text{ N}$ ◀

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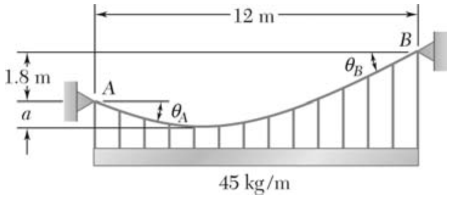
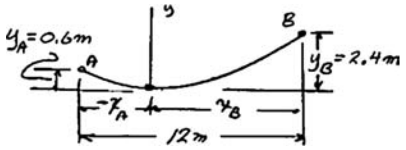


PROBLEM 7.118

Cable AB supports a load uniformly distributed along the horizontal as shown. Knowing that the lowest point of the cable is located at a distance $a = 0.6$ m below A, determine (*a*) the maximum tension in the cable, (*b*) the angle θ_B that the cable forms with the horizontal at B.

SOLUTION



Note: $x_B - x_A = 12$ m

or $x_A = x_B - 12$ m

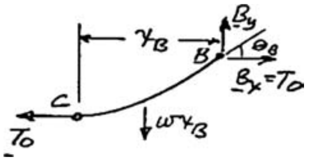
Point A: $y_A = \dfrac{wx_A^2}{2T_0}; \quad 0.6 = \dfrac{w(x_B - 12)^2}{2T_0}$ (1)

Point B: $y_B = \dfrac{wx_B^2}{2T_0}; \quad 2.4 = \dfrac{wx_B^2}{2T_0}$ (2)

Dividing (1) by (2): $\dfrac{0.6}{2.4} = \dfrac{(x_B - 12)^2}{x_B^2}; \quad x_B = 8$ m

(*a*) Eq. (2): $2.4 = \dfrac{w(8)^2}{2T_0}; \quad T_0 = 13.333w$

Free body: Portion CB



$\Sigma F_y = 0 \quad B_y = wx_B$

$B_y = 8w$

$T_m^2 = B_x^2 + B_y^2; \quad T_m^2 = (13.333w)^2 + (8w)^2$

$T_m = 15.549w = 15.549(45)(9.81)$ $\quad T_m = 6860$ N ◀

$\theta_B = \tan^{-1} B_y/B_x = \tan^{-1} 8w/13.333w$ $\quad \theta_B = 31.0°$ ◀

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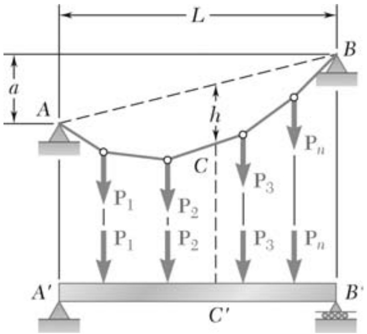


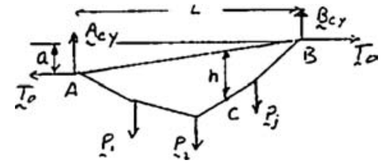
PROBLEM 7.119*

A cable *AB* of span *L* and a simple beam *A′B′* of the same span are subjected to identical vertical loadings as shown. Show that the magnitude of the bending moment at a point *C′* in the beam is equal to the product T_0h, where T_0 is the magnitude of the horizontal component of the tension force in the cable and *h* is the vertical distance between Point *C* and the chord joining the points of support *A* and *B*.

SOLUTION

$$\circlearrowleft \Sigma M_B = 0: \quad LA_{Cy} + aT_0 - \Sigma M_{B\ \text{loads}} = 0 \qquad (1)$$

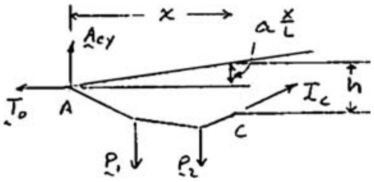
FBD Cable:



(Where $\Sigma M_{B\ \text{loads}}$ includes all applied loads)

$$\circlearrowleft \Sigma M_C = 0: \quad xA_{Cy} - \left(h - a\frac{x}{L}\right)T_0 - \Sigma M_{C\ \text{left}} = 0 \qquad (2)$$

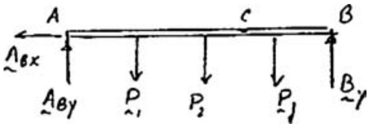
FBD AC:



(Where $\Sigma M_{C\ \text{left}}$ includes all loads left of *C*)

$$\frac{x}{L}(1) - (2): \quad hT_0 - \frac{x}{L}\Sigma M_{B\ \text{loads}} + \Sigma M_{C\ \text{left}} = 0 \qquad (3)$$

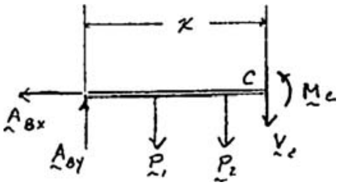
FBD Beam:



$$\circlearrowleft \Sigma M_B = 0: \quad LA_{By} - \Sigma M_{B\ \text{loads}} = 0 \qquad (4)$$

$$\circlearrowleft \Sigma M_C = 0: \quad xA_{By} - \Sigma M_{C\ \text{left}} - M_C = 0 \qquad (5)$$

FBD AC:



$$\frac{x}{L}(4) - (5): \quad -\frac{x}{L}\Sigma M_{B\ \text{loads}} + \Sigma M_{C\ \text{left}} + M_C = 0 \qquad (6)$$

Comparing (3) and (6) $\qquad M_C = hT_0 \qquad$ Q.E.D.

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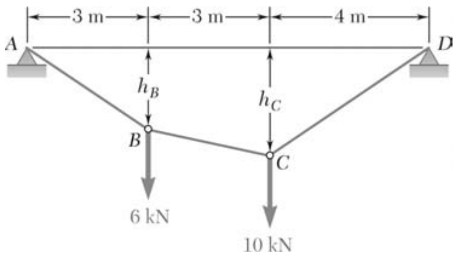
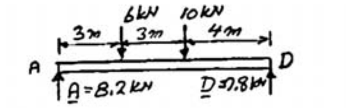


PROBLEM 7.120

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

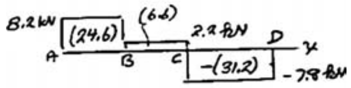
PROBLEM 7.94 (*a*) Knowing that the maximum tension in cable *ABCD* is 15 kN, determine the distance h_B.

SOLUTION



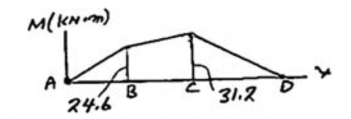
$$+\circlearrowleft \Sigma M_B = 0: \quad A(10\text{ m}) - (6\text{ kN})(7\text{ m}) - (10\text{ kN})(4\text{ m}) = 0$$

$$A = 8.2\text{ kN}$$



$$+\uparrow \Sigma F_y = 0: \quad 8.2\text{ kN} - 6\text{ kN} - 10\text{ kN} + B = 0$$

$$B = 7.8\text{ kN}$$



<u>At *A*</u>:

$$T_m^2 = T_0^2 + A^2$$

$$15^2 = T_0^2 + 8.2$$

$$T_0 = 12.56\text{ kN}$$

<u>At *B*</u>: $M_B = T_0 h_B$; $\quad 24.6\text{ kN}\cdot\text{m} = (12.56\text{ kN})h_B$; $\qquad h_B = 1.959\text{ m}$ ◀

<u>At *C*</u>: $M_C = T_0 h_C$; $\quad 31.2\text{ kN}\cdot\text{m} = (12.56\text{ kN})h_C$; $\qquad h_C = 2.48\text{ m}$ ◀

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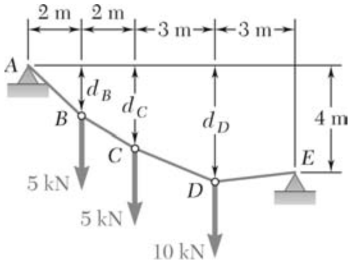
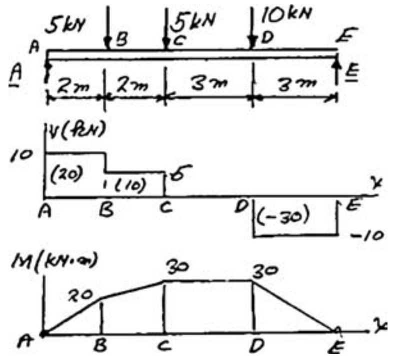


PROBLEM 7.121

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

PROBLEM 7.97 (*a*) Knowing that $d_C = 3\text{ m}$, determine the distances d_B and d_D.

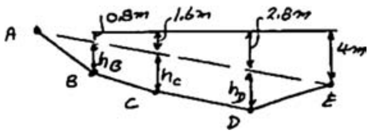
SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad A(10\text{ m}) - (5\text{ kN})(8\text{ m}) - (5\text{ kN})(6\text{ m}) - (10\text{ kN})(3\text{ m}) = 0$$

$$A = 10\text{ kN}$$

<u>Geometry</u>:



$$d_C = 1.6\text{ m} + h_C$$
$$3\text{ m} = 1.6\text{ m} + h_C$$
$$h_C = 1.4\text{ m}$$

Since $M = T_0 h$, h is proportional to M, thus

$$\frac{h_B}{M_B} = \frac{h_C}{M_C} = \frac{h_D}{M_D}; \quad \frac{h_B}{20\text{ kN}\cdot\text{m}} = \frac{1.4\text{ m}}{30\text{ kN}\cdot\text{m}} = \frac{h_D}{30\text{ kN}\cdot\text{m}}$$

$$h_B = 1.4\left(\frac{20}{30}\right) = 0.9333\text{ m} \qquad \| \qquad h_D = 1.4\left(\frac{30}{30}\right) = 1.4\text{ m}$$

$$d_B = 0.8\text{ m} + 0.9333\text{ m} \qquad \| \qquad d_D = 2.8\text{ m} + 1.4\text{ m}$$

$$d_B = 1.733\text{ m} \blacktriangleleft \qquad d_D = 4.20\text{ m} \blacktriangleleft$$

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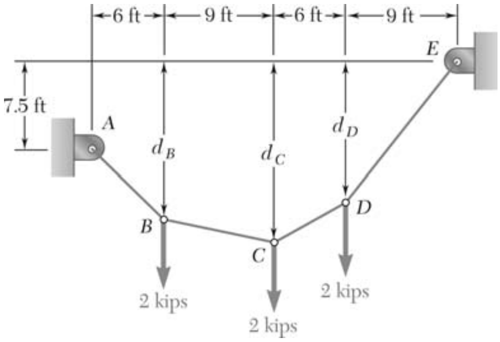


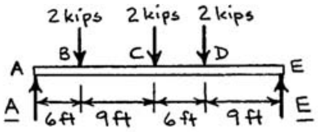
PROBLEM 7.122

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

PROBLEM 7.99 (*a*) If $d_C = 15$ ft, determine the distances d_B and d_D.

SOLUTION

Free body: Beam *AE*

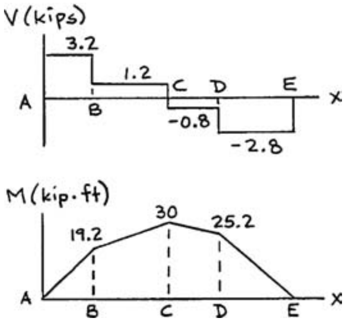


$$+\circlearrowleft \Sigma M_E = 0: \quad -A(30) + 2(24) + 2(15) + 2(9) = 0$$

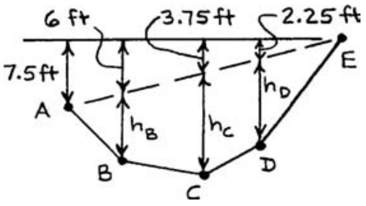
$$\mathbf{A} = 3.2 \text{ kips} \uparrow$$

$$+\Sigma F_y = 0: \quad 3.2 - 3(2) + B = 0$$

$$\mathbf{B} = 2.8 \text{ kips} \uparrow$$



Geometry:



Given: $d_C = 15$ ft

Then, $h_C = d_C - 3.75 \text{ ft} = 11.25 \text{ ft}$

Since $M = T_0\, h$, h is proportional to M. Thus,

$$\frac{h_B}{M_B} = \frac{h_C}{M_C} = \frac{h_D}{M_D}$$

or, $$\frac{h_B}{19.2} = \frac{11.25 \text{ ft}}{30} = \frac{h_D}{25.2}$$

or, $h_B = 7.2$ ft $\quad h_D = 9.45$ ft

Then, $d_B = 6 + h_B = 6 + 7.2$ $\qquad d_B = 13.20 \text{ ft}$ ◀

$d_D = 2.25 + h_D = 2.25 + 9.45$ $\qquad d_D = 11.70 \text{ ft}$ ◀

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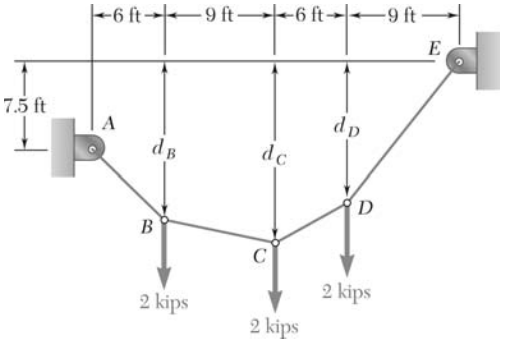


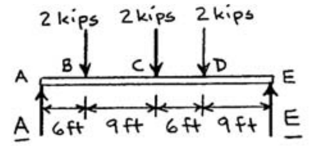
PROBLEM 7.123

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

PROBLEM 7.100 (*a*) Determine the distance d_C for which portion *BC* of the cable is horizontal.

SOLUTION

Free body: Beam *AE*

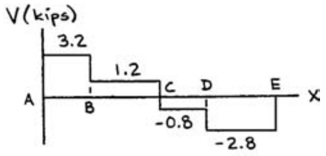


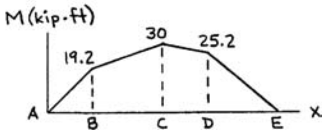
$$+\circlearrowleft \Sigma M_E = 0: \quad -A(30) + 2(24) + 2(15) + 2(9) = 0$$

$$\mathbf{A} = 3.2 \text{ kips} \uparrow$$

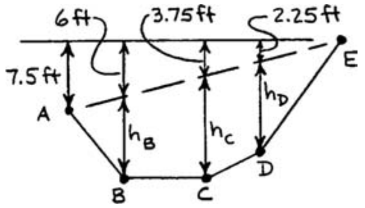
$$+\uparrow \Sigma F_y = 0: \quad 3.2 - 3(2) + B = 0$$

$$\mathbf{B} = 2.8 \text{ kips} \uparrow$$





Geometry:



Given: $$d_C = d_B$$

Then, $$3.75 + h_C = 6 + h_B$$

$$h_C = 2.25 + h_B \qquad (1)$$

Since $M = T_0 h$, h is proportional to M. Thus,

$$\frac{h_B}{M_B} = \frac{h_C}{M_C} \quad \text{or,} \quad \frac{h_B}{19.2} = \frac{h_C}{30}$$

$$h_B = 0.64 h_C \qquad (2)$$

Substituting (2) into (1):

$$h_C = 2.25 + 0.64 h_C \qquad h_C = 6.25 \text{ ft}$$

Then, $$d_C = 3.75 + h_C = 3.75 + 6.25 \qquad d_C = 10.00 \text{ ft} \blacktriangleleft$$

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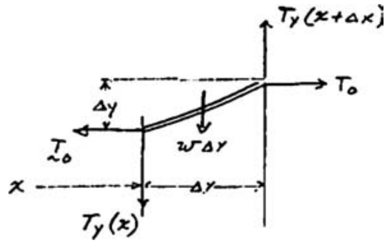
PROBLEM 7.124*

Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION

FBD Elemental segment:



$$\uparrow \Sigma F_y = 0: \quad T_y(x+\Delta x) - T_y(x) - w(x)\Delta x = 0$$

So
$$\frac{T_y(x+\Delta x)}{T_0} - \frac{T_y(x)}{T_0} = \frac{w(x)}{T_0}\Delta x$$

But
$$\frac{T_y}{T_0} = \frac{dy}{dx}$$

So
$$\frac{\left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}}{\Delta x} = \frac{w(x)}{T_0}$$

In $\lim_{\Delta x \to 0}$:
$$\frac{d^2 y}{dx^2} = \frac{w(x)}{T_0} \qquad \text{Q.E.D.}$$

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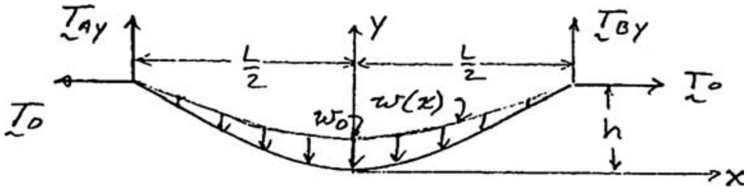
PROBLEM 7.125*

Using the property indicated in Problem 7.124, determine the curve assumed by a cable of span L and sag h carrying a distributed load $w = w_0 \cos(\pi x/L)$, where x is measured from mid-span. Also determine the maximum and minimum values of the tension in the cable.

PROBLEM 7.124 Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION



$$w(x) = w_0 \cos\frac{\pi x}{L}$$

From Problem 7.124

$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0} = \frac{w_0}{T_0}\cos\frac{\pi x}{L}$$

So

$$\frac{dy}{dx} = \frac{W_0 L}{T_0 \pi}\sin\frac{\pi x}{L} \qquad \left(\text{using } \left.\frac{dy}{dx}\right|_0 = 0\right)$$

$$y = \frac{w_0 L^2}{T_0 \pi^2}\left(1 - \cos\frac{\pi x}{L}\right) \quad [\text{using } y(0) = 0] \blacktriangleleft$$

But

$$y\left(\frac{L}{2}\right) = h = \frac{w_0 L^2}{T_0 \pi^2}\left(1 - \cos\frac{\pi}{2}\right) \quad \text{so} \quad T_0 = \frac{w_0 L^2}{\pi^2 h}$$

And

$$T_0 = T_{\min} \quad \text{so} \qquad T_{\min} = \frac{w_0 L^2}{\pi^2 h} \blacktriangleleft$$

$$T_{\max} = T_A = T_B: \quad \frac{T_{By}}{T_0} = \left.\frac{dy}{dx}\right|_{x=L/2} = \frac{w_0 L}{T_0 \pi}$$

$$T_{By} = \frac{w_0 L}{\pi} \qquad T_B = \sqrt{T_{By}^2 + T_0^2} = \frac{w_0 L}{\pi}\sqrt{1 + \left(\frac{L}{\pi h}\right)^2} \blacktriangleleft$$

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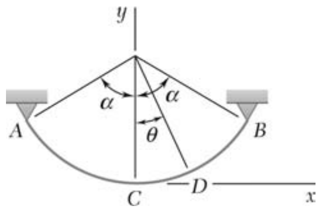
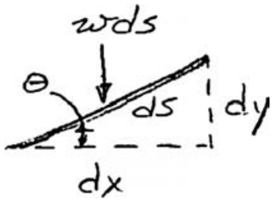


PROBLEM 7.126

If the weight per unit length of the cable *AB* is $w_0/\cos^2 \theta$, prove that the curve formed by the cable is a circular arc. (*Hint:* Use the property indicated in Problem 7.124.)

PROBLEM 7.124 Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION



Elemental Segment:

Load on segment* $\quad w(x)dx = \dfrac{w_0}{\cos^2 \theta} ds$

But $\quad dx = \cos\theta ds, \quad \text{so} \quad w(x) = \dfrac{w_0}{\cos^3 \theta}$

From Problem 7.119 $\quad \dfrac{d^2y}{dx^2} = \dfrac{w(x)}{T_0} = \dfrac{w_0}{T_0 \cos^3 \theta}$

In general $\quad \dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) = \dfrac{d}{dx}(\tan\theta) = \sec^2\theta \dfrac{d\theta}{dx}$

So $\quad \dfrac{d\theta}{dx} = \dfrac{w_0}{T_0 \cos^3\theta \sec^2\theta} = \dfrac{w_0}{T_0 \cos\theta}$

or $\quad \dfrac{T_0}{w_0} \cos\theta d\theta = dx = r d\theta \cos\theta$

Giving $r = \dfrac{T_0}{w_0} = \text{constant}.$ So curve is circular arc $\qquad$ Q.E.D.

*For large sag, it is not appropriate to approximate *ds* by *dx*.

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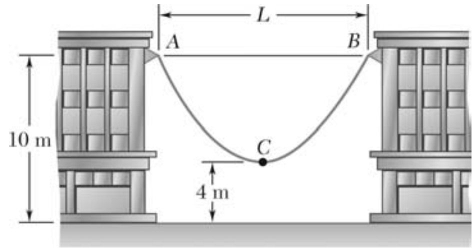
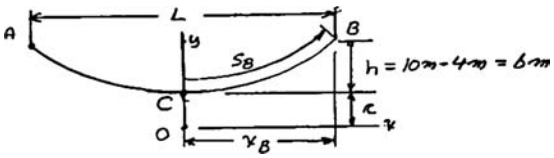


PROBLEM 7.127

A 30-m cable is strung as shown between two buildings. The maximum tension is found to be 500 N, and the lowest point of the cable is observed to be 4 m above the ground. Determine (*a*) the horizontal distance between the buildings, (*b*) the total mass of the cable.

SOLUTION



$$s_B = 15 \text{ m}$$
$$T_m = 500 \text{ N}$$

Eq. 7.17: $y_B^2 - s_B^2 = c^2; \quad (6+c)^2 - 15^2 = c^2$

$$36 + 12c + c^2 - 225 = c^2$$
$$12c = 189 \quad c = 15.75 \text{ m}$$

Eq. 7.15: $s_B = c \sinh \frac{x_B}{c}; \quad 15 = (15.75)\sinh \frac{x_B}{c}$

$$\sinh \frac{x_B}{c} = 0.95238 \quad \frac{x_B}{c} = 0.8473$$

(*a*) $x_B = 0.8473(15.75) = 13.345 \text{ m}; \quad L = 2x_B$ $\qquad L = 26.7 \text{ m}$ ◀

(*b*) Eq. 7.18: $T_m = wy_B; \quad 500 \text{ N} = w(6 + 15.75)$

$$w = 22.99 \text{ N/m}$$

$$W = 2s_B w = (30 \text{ m})(22.99 \text{ N/m})$$
$$= 689.7 \text{ N}$$

$$m = \frac{W}{g} = \frac{689.7 \text{ N}}{9.81 \text{ m/s}^2}$$ $\qquad$ Total mass $= 70.3$ kg ◀

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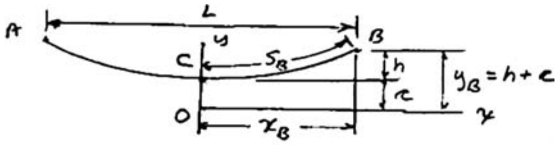
PROBLEM 7.128

A 200-ft steel surveying tape weighs 4 lb. If the tape is stretched between two points at the same elevation and pulled until the tension at each end is 16 lb, determine the horizontal distance between the ends of the tape. Neglect the elongation of the tape due to the tension.

SOLUTION



$$s_B = 100 \text{ ft}$$

$$w = \left(\frac{4 \text{ lb}}{200 \text{ ft}}\right) = 0.02 \text{ lb/ft} \quad T_m = 16 \text{ m}$$

Eq. 7.18: $\quad T_m = wy_B; \quad 16 \text{ lb} = (0.02 \text{ lb/ft})y_B; \quad y_B = 800 \text{ ft}$

Eq. 7.17: $\quad y_B^2 - s_B^2 = c^2; \quad (800)^2 - (100)^2 = c^2; \quad c = 793.73 \text{ ft}$

Eq. 7.15: $\quad s_B = c \sinh \frac{x_B}{c}; \quad 100 = 793.73 \sinh \frac{x_B}{c}$

$$\frac{x_B}{c} = 0.12566; \quad x_B = 99.737 \text{ ft}$$

$$L = 2x_B = 2(99.737 \text{ ft}) \qquad L = 199.5 \text{ ft} \blacktriangleleft$$

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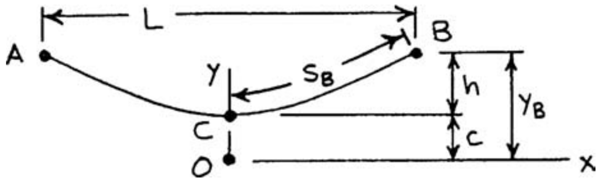
PROBLEM 7.129

A 200-m-long aerial tramway cable having a mass per unit length of 3.5 kg/m is suspended between two points at the same elevation. Knowing that the sag is 50 m, find (*a*) the horizontal distance between the supports, (*b*) the maximum tension in the cable.

SOLUTION



<u>Given</u>:

$$\text{Length} = 200 \text{ m}$$
$$\text{Unit mass} = 3.5 \text{ kg/m}$$
$$h = 50 \text{ m}$$

Then,
$$w = (3.5 \text{ kg/m})(9.81 \text{ m/s}^2) = 34.335 \text{ N/m}$$
$$s_B = 100 \text{ m}$$
$$y_B = h + c = 50 \text{ m} + c$$
$$y_B^2 - s_B^2 = c^2; \quad (50 + c)^2 - (100)^2 = c^2$$
$$50^2 + 100c + c^2 - 100^2 = c^2$$
$$c = 75 \text{ m}$$

$$s_B = c \sinh \frac{x_B}{c}; \quad 100 = 75 \sinh \frac{x_B}{75}$$

$$x_B = 82.396 \text{ m}$$

$$\text{span} = L = 2x_B = 2(82.396 \text{ m}) \qquad L = 164.8 \text{ m} \blacktriangleleft$$

$$T_m = wy_B = (34.335 \text{ N/m})(50 \text{ m} + 75 \text{ m}) \qquad T_m = 4290 \text{ N} \blacktriangleleft$$

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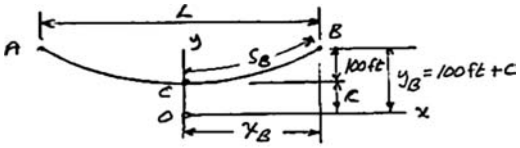
PROBLEM 7.130

An electric transmission cable of length 400 ft weighing 2.5 lb/ft is suspended between two points at the same elevation. Knowing that the sag is 100 ft, determine the horizontal distance between the supports and the maximum tension.

SOLUTION



$$s_B = 200 \text{ ft}$$

Eq. 7.17: $$y_B^2 - s_B^2 = c^2; \quad (100 + c)^2 - 200^2 = c^2$$

$$10000 + 200c + c^2 - 40000 = c^2; \quad c = 150 \text{ ft}$$

Eq. 7.15: $$s_B = c \sinh \frac{x_B}{c}; \quad 200 = 150 \sinh \frac{x_B}{c}$$

$$\sinh \frac{x_B}{c} = \frac{4}{3}; \quad \frac{x_B}{c} = 1.0986$$

$$x_B = (150)(1.0986) = 164.79 \text{ ft}$$

$$L = 2x_B = 2(164.79 \text{ ft}) = 329.58 \text{ ft} \qquad L = 330 \text{ ft} \blacktriangleleft$$

Eq. 7.18: $$T_m = wy_B = (2.5 \text{ lb/ft})(100 \text{ ft} + 150 \text{ ft}) \qquad T_m = 625 \text{ lb} \blacktriangleleft$$

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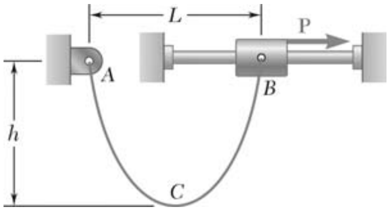
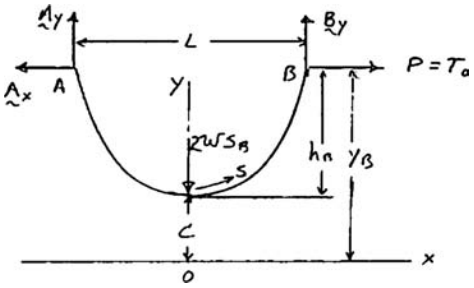


PROBLEM 7.131

A 20-m length of wire having a mass per unit length of 0.2 kg/m is attached to a fixed support at *A* and to a collar at *B*. Neglecting the effect of friction, determine (*a*) the force **P** for which $h = 8$ m, (*b*) the corresponding span *L*.

SOLUTION

FBD Cable:



$$s_T = 20 \text{ m} \quad \left(\text{so } s_B = \frac{20 \text{ m}}{2} = 10 \text{ m}\right)$$

$$w = (0.2 \text{ kg/m})(9.81 \text{ m/s}^2)$$

$$= 1.96200 \text{ N/m}$$

$$h_B = 8 \text{ m}$$

$$y_B^2 = (c + h_B)^2 = c^2 + s_B^2$$

So

$$c = \frac{s_B^2 - h_B^2}{2h_B}$$

$$c = \frac{(10 \text{ m})^2 - (8 \text{ m})^2}{2(8 \text{ m})}$$

$$= 2.250 \text{ m}$$

Now

$$s_B = c \sinh \frac{x_B}{c} \rightarrow x_B = c \sinh^{-1} \frac{s_B}{c}$$

$$= (2.250 \text{ m}) \sinh^{-1}\left(\frac{10 \text{ m}}{2.250 \text{ m}}\right)$$

$$x_B = 4.9438 \text{ m}$$

$$P = T_0 = wc = (1.96200 \text{ N/m})(2.250 \text{ m})$$ (*a*) $\mathbf{P} = 4.41 \text{ N} \longrightarrow$ ◀

$$L = 2x_B = 2(4.9438 \text{ m})$$ (*b*) $L = 9.89 \text{ m}$ ◀

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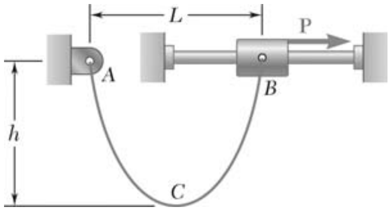
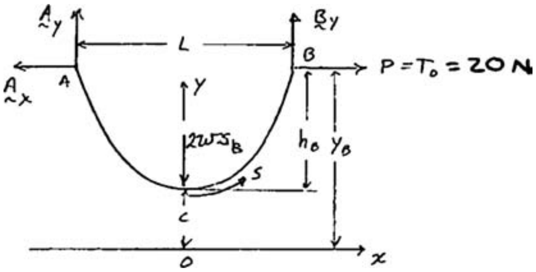


PROBLEM 7.132

A 20-m length of wire having a mass per unit length of 0.2 kg/m is attached to a fixed support at A and to a collar at B. Knowing that the magnitude of the horizontal force applied to the collar is $P = 20$ N, determine (*a*) the sag h, (*b*) the span L.

SOLUTION

FBD Cable:



$$s_T = 20\text{ m},\quad w = (0.2\text{ kg/m})(9.81\text{ m/s}^2) = 1.96200\text{ N/m}$$

$$P = T_0 = wc \quad c = \frac{P}{w}$$

$$c = \frac{20\text{ N}}{1.9620\text{ N/m}} = 10.1937\text{ m}$$

$$y_B^2 = (h_B + c)^2 = c^2 + s_B^2$$

$$h^2 + 2ch - s_B^2 = 0 \quad s_B = \frac{20\text{ m}}{2} = 10\text{ m}$$

$$h^2 + 2(10.1937\text{ m})h - 100\text{ m}^2 = 0$$

$$h = 4.0861\text{ m}$$

(*a*) $h = 4.09\text{ m}$ ◀

$$s_B = c\sinh\frac{x_A}{c} \rightarrow x_B = c\sinh^{-1}\frac{s_B}{c} = (10.1937\text{ m})\sinh^{-1}\left(\frac{10\text{ m}}{10.1937\text{ m}}\right)$$

$$= 8.8468\text{ m}$$

$$L = 2x_B = 2(8.8468\text{ m})$$

(*b*) $L = 17.69\text{ m}$ ◀

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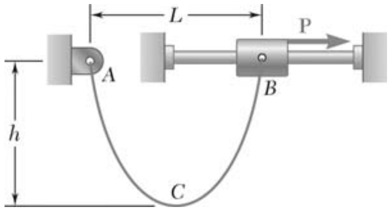
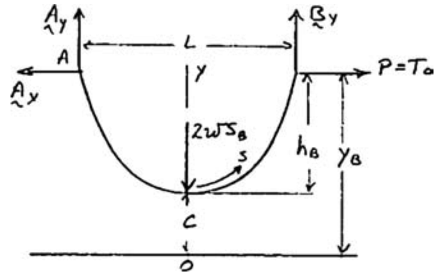


PROBLEM 7.133

A 20-m length of wire having a mass per unit length of 0.2 kg/m is attached to a fixed support at *A* and to a collar at *B*. Neglecting the effect of friction, determine (*a*) the sag *h* for which *L* = 15 m, (*b*) the corresponding force **P**.

SOLUTION

FBD Cable:



$$s_T = 20\text{ m} \rightarrow s_B = \frac{20\text{ m}}{2} = 10\text{ m}$$

$$w = (0.2\text{ kg/m})(9.81\text{ m/s}^2) = 1.96200\text{ N/m}$$

$$L = 15\text{ m}$$

$$s_B = c\sinh\frac{x_B}{c} = c\sinh\frac{\frac{L}{2}}{c}$$

$$10\text{ m} = c\sinh\frac{7.5\text{ m}}{c}$$

Solving numerically: $c = 5.5504\text{ m}$

$$y_B = c\cosh\left(\frac{x_B}{c}\right) = (5.5504)\cosh\left(\frac{7.5}{5.5504}\right)$$

$$y_B = 11.4371\text{ m}$$

$$h_B = y_B - c = 11.4371\text{ m} - 5.5504\text{ m}$$

(*a*) $h_B = 5.89\text{ m}$ ◀

$P = wc = (1.96200\text{ N/m})(5.5504\text{ m})$ (*b*) $\mathbf{P} = 10.89\text{ N} \longrightarrow$ ◀

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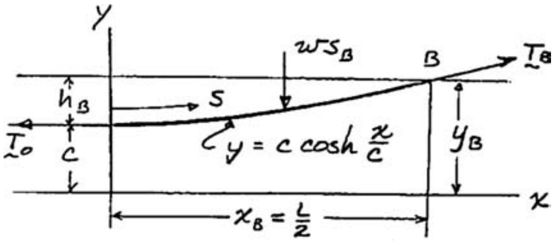
PROBLEM 7.134

Determine the sag of a 30-ft chain that is attached to two points at the same elevation that are 20 ft apart.

SOLUTION



$$s_B = \frac{30 \text{ ft}}{2} = 15 \text{ ft} \qquad L = 20 \text{ ft}$$

$$x_B = \frac{L}{2} = 10 \text{ ft}$$

$$s_B = c \sinh \frac{x_B}{c}$$

$$15 \text{ ft} = c \sinh \frac{10 \text{ ft}}{c}$$

Solving numerically: $c = 6.1647 \text{ ft}$

$$y_B = c \cosh \frac{x_B}{c}$$

$$= (6.1647 \text{ ft}) \cosh \frac{10 \text{ ft}}{6.1647 \text{ ft}}$$

$$= 16.2174 \text{ ft}$$

$$h_B = y_B - c = 16.2174 \text{ ft} - 6.1647 \text{ ft}$$

$h_B = 10.05 \text{ ft}$ ◀

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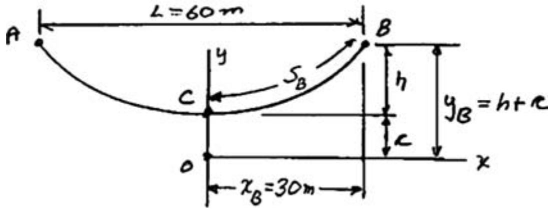
PROBLEM 7.135

A 90-m wire is suspended between two points at the same elevation that are 60 m apart. Knowing that the maximum tension is 300 N, determine (*a*) the sag of the wire, (*b*) the total mass of the wire.

SOLUTION



$$s_B = 45 \text{ m}$$

Eq. 7.17:
$$s_B = c \sinh \frac{x_B}{c}$$

$$45 = c \sinh \frac{30}{c}; \quad c = 18.494 \text{ m}$$

Eq. 7.16:
$$y_B = c \cosh \frac{x_B}{c}$$

$$y_B = (18.494) \cosh \frac{30}{18.494}$$

$$y_B = 48.652 \text{ m}$$

$$y_B = h + c$$

$$48.652 = h + 18.494$$

$$h = 30.158 \text{ m} \qquad\qquad h = 30.2 \text{ m} \blacktriangleleft$$

Eq. 7.18:
$$T_m = w y_B$$

$$300 \text{ N} = w(48.652 \text{ m})$$

$$w = 6.166 \text{ N/m}$$

Total weight of cable
$$W = w(\text{Length})$$

$$= (6.166 \text{ N/m})(90 \text{ m})$$

$$= 554.96 \text{ N}$$

Total mass of cable
$$m = \frac{W}{g} = \frac{554.96 \text{ N}}{9.81 \text{ m/s}} = 56.57 \text{ kg} \qquad\qquad m = 56.6 \text{ kg} \blacktriangleleft$$

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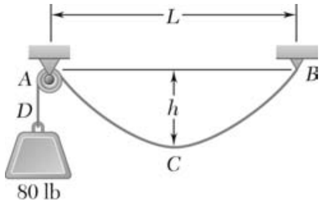
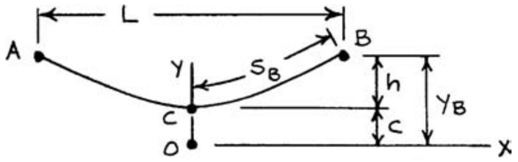


PROBLEM 7.136

A counterweight D is attached to a cable that passes over a small pulley at A and is attached to a support at B. Knowing that $L = 45$ ft and $h = 15$ ft, determine (a) the length of the cable from A to B, (b) the weight per unit length of the cable. Neglect the weight of the cable from A to D.

SOLUTION



Given:

$$L = 45 \text{ ft}$$
$$h = 15 \text{ ft}$$
$$T_A = 80 \text{ lb}$$
$$x_B = 22.5 \text{ ft}$$

By symmetry: $T_B = T_A = T_m = 80$ lb

We have $y_B = c\cosh\frac{x_B}{c} = c\cosh\frac{22.5}{c}$

and $y_B = h + c = 15 + c$

Then $c\cosh\frac{22.5}{c} = 15 + c$

or $\cosh\frac{22.5}{c} = \frac{15}{c} + 1$

Solve by trial for c: $c = 18.9525$ ft

(a)

$$s_B = c\sinh\frac{x_B}{c}$$
$$= (18.9525 \text{ ft})\sinh\frac{22.5}{18.9525}$$
$$= 28.170 \text{ ft}$$

Length $= 2s_B = 2(28.170 \text{ ft}) = 56.3$ ft ◀

(b)

$$T_m = wy_B = w(h + c)$$

$80 \text{ lb} = w(15 \text{ ft} + 18.9525 \text{ ft})$ $w = 2.36$ lb/ft ◀

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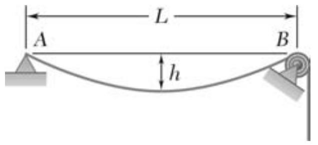
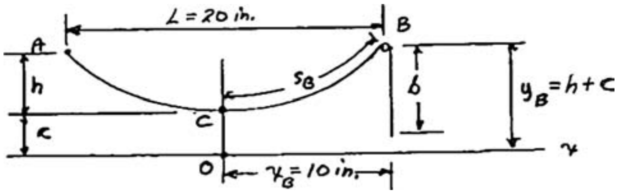


PROBLEM 7.137

A uniform cord 50 in. long passes over a pulley at B and is attached to a pin support at A. Knowing that $L = 20$ in. and neglecting the effect of friction, determine the smaller of the two values of h for which the cord is in equilibrium.

SOLUTION



Length of overhang: $b = 50 \text{ in.} - 2s_B$

Weight of overhang equals max. tension

$$T_m = T_B = wb = w(50 \text{ in.} - 2s_B)$$

Eq. 7.15: $s_B = c \sinh \dfrac{x_B}{c}$

Eq. 7.16: $y_B = c \cosh \dfrac{x_B}{c}$

Eq. 7.18: $T_m = wy_B$

$$w(50 \text{ in.} - 2s_B) = wy_B$$

$$w\left(50 \text{ in.} - 2c \sinh \frac{x_B}{c}\right) = wc \cosh \frac{x_B}{c}$$

$$x_B = 10: \quad 50 - 2c \sinh \frac{10}{c} = c \cosh \frac{10}{c}$$

Solve by trial and error: $c = 5.549$ in. and $c = 27.742$ in.

For $c = 5.549$ in. $y_B = (5.549 \text{ in.}) \cosh \dfrac{10 \text{ in.}}{5.549 \text{ in.}} = 17.277 \text{ in.}$

$$y_B = h + c; \quad 17.277 \text{ in.} = h + 5.549 \text{ in.}$$

$h = 11.728$ in. $\qquad h = 11.73$ in. ◀

For $c = 27.742$ in. $y_B = (27.742 \text{ in.}) \cosh \dfrac{10 \text{ in.}}{27.742 \text{ in.}} = 29.564 \text{ in.}$

$$y_B = h + c; \quad 29.564 \text{ in.} = h + 27.742 \text{ in.}$$

$h = 1.8219$ in. $\qquad h = 1.822$ in. ◀

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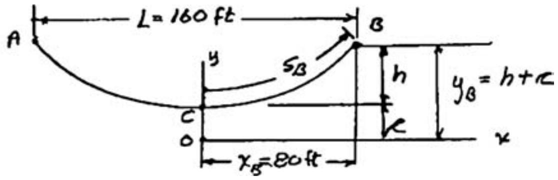
PROBLEM 7.138

A cable weighing 2 lb/ft is suspended between two points at the same elevation that are 160 ft apart. Determine the smallest allowable sag of the cable if the maximum tension is not to exceed 400 lb.

SOLUTION



Eq. 7.18: $T_m = wy_B; \quad 400 \text{ lb} = (2 \text{ lb/ft})y_B; \quad y_B = 200 \text{ ft}$

Eq. 7.16: $y_B = c\cosh\dfrac{x_B}{c}$

$$200 \text{ ft} = c\cosh\frac{80 \text{ ft}}{c}$$

Solve for c: $c = 182.148$ ft and $c = 31.592$ ft

$$y_B = h + c; \quad h = y_B - c$$

For $c = 182.148$ ft; $h = 200 - 182.147 = 17.852$ ft ◁

For $c = 31.592$ ft; $h = 200 - 31.592 = 168.408$ ft ◁

For $T_m \leq 400$ lb: smallest $h = 17.85$ ft ◀

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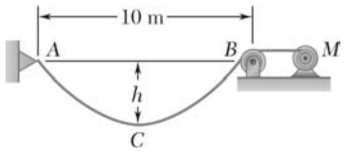
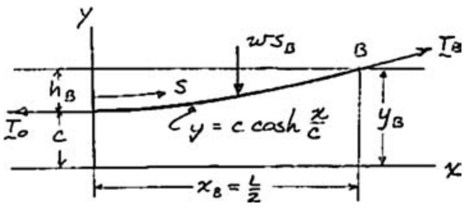


PROBLEM 7.139

A motor M is used to slowly reel in the cable shown. Knowing that the mass per unit length of the cable is 0.4 kg/m, determine the maximum tension in the cable when $h = 5$ m.

SOLUTION



$$w = 0.4 \text{ kg/m} \quad L = 10 \text{ m} \quad h_B = 5 \text{ m}$$

$$y_B = c\cosh\frac{x_B}{c}$$

$$h_B + c = c\cosh\frac{L}{2c}$$

$$5 \text{ m} = c\left(\cosh\frac{5 \text{ m}}{c} - 1\right)$$

Solving numerically:

$$c = 3.0938 \text{ m}$$

$$y_B = h_B + c = 5 \text{ m} + 3.0938 \text{ m}$$

$$= 8.0938 \text{ m}$$

$$T_{\max} = T_B = wy_B$$

$$= (0.4 \text{ kg/m})(9.81 \text{ m/s}^2)(8.0938 \text{ m})$$

$$T_{\max} = 31.8 \text{ N} \blacktriangleleft$$

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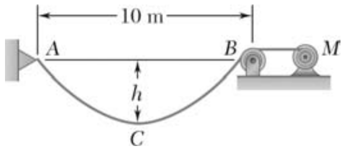
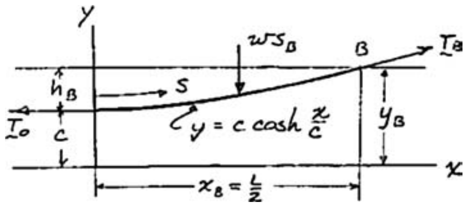


PROBLEM 7.140

A motor M is used to slowly reel in the cable shown. Knowing that the mass per unit length of the cable is 0.4 kg/m, determine the maximum tension in the cable when $h = 3$ m.

SOLUTION



$$w = 0.4 \text{ kg/m}, \quad L = 10 \text{ m}, \quad h_B = 3 \text{ m}$$

$$y_B = h_B + c = c\cosh\frac{x_B}{c} = c\cosh\frac{L}{2c}$$

$$3 \text{ m} = c\left(c\cosh\frac{5 \text{ m}}{c} - 1\right)$$

Solving numerically: $c = 4.5945 \text{ m}$

$$y_B = h_B + c = 3 \text{ m} + 4.5945 \text{ m}$$

$$= 7.5945 \text{ m}$$

$$T_{\max} = T_B = w y_B$$

$$= (0.4 \text{ kg/m})(9.81 \text{ m/s}^2)(7.5945 \text{ m})$$

$T_{\max} = 29.8 \text{ N}$ ◀

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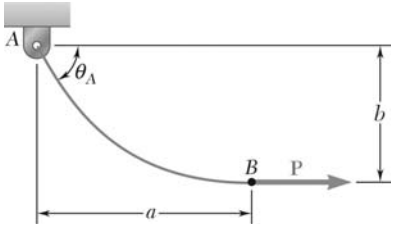
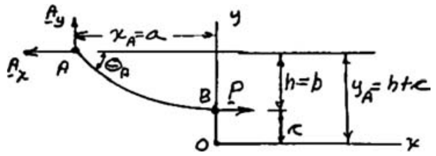


PROBLEM 7.141

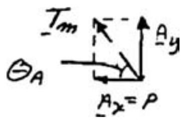
A uniform cable weighing 3 lb/ft is held in the position shown by a horizontal force **P** applied at *B*. Knowing that $P = 180$ lb and $\theta_A = 60°$, determine (*a*) the location of Point *B*, (*b*) the length of the cable.

SOLUTION



Eq. 7.18: $T_0 = P = cw$

$$c = \frac{P}{w} = \frac{180 \text{ lb}}{3 \text{ lb/ft}} \qquad c = 60 \text{ ft}$$



At *A*: $$T_m = \frac{P}{\cos 60°}$$

$$= \frac{cw}{0.5} = 2cw$$

(*a*) Eq. 7.18: $T_m = w(h+c)$

$$2cw = w(h+c)$$

$$2c = h + c \qquad h = b = c \qquad\qquad b = 60.0 \text{ ft} ◀$$

Eq. 7.16: $$y_A = c\cosh\frac{x_A}{c}$$

$$h + c = c\cosh\frac{x_A}{c}$$

$$(60 \text{ ft} + 60 \text{ ft}) = (60 \text{ ft}) \quad \cosh\frac{x_A}{60}$$

$$\cosh\frac{x_A}{60 \text{ m}} = 2 \qquad \frac{x_A}{60 \text{ m}} = 1.3170$$

$$x_A = 79.02 \text{ ft} \qquad\qquad a = 79.0 \text{ ft} ◀$$

(*b*) Eq. 7.15: $$s_A = c\sinh\frac{x_B}{c} = (60 \text{ ft})\sinh\frac{79.02 \text{ ft}}{60 \text{ ft}}$$

$$s_A = 103.92 \text{ ft}$$

$$\text{length} = s_A \qquad\qquad s_A = 103.9 \text{ ft} ◀$$

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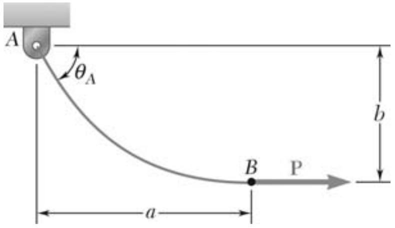
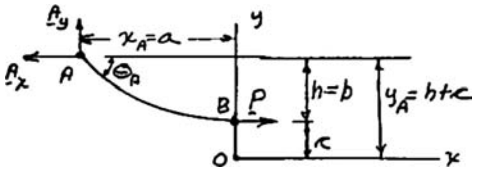


PROBLEM 7.142

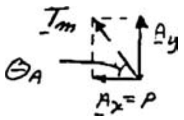
A uniform cable weighing 3 lb/ft is held in the position shown by a horizontal force **P** applied at *B*. Knowing that *P* = 150 lb and $\theta_A = 60°$, determine (*a*) the location of Point *B*, (*b*) the length of the cable.

SOLUTION



Eq. 7.18: $T_0 = P = cw$

$$c = \frac{P}{w} = \frac{150 \text{ lb}}{3 \text{ lb/ft}} = 50 \text{ ft}$$



At *A*: $T_m = \dfrac{P}{\cos 60°}$

$$= \frac{cw}{0.5} = 2\,cw$$

(*a*) Eq. 7.18: $T_m = w(h+c)$

$$2cw = w(h+c)$$

$$2c = h + c \quad h = c = b$$

$b = 50.0 \text{ ft}$ ◀

Eq. 7.16: $y_A = c\cosh\dfrac{x_A}{c}$

$$h + c = c\cosh\frac{x_A}{c}$$

$$(50 \text{ ft} + 50 \text{ ft}) = (50 \text{ ft})\cosh\frac{x_A}{c}$$

$$\cosh\frac{x_A}{c} = 2 \quad \frac{x_A}{c} = 1.3170$$

$$x_A = 1.3170(50 \text{ ft}) = 65.85 \text{ ft}$$

$a = 65.8 \text{ ft}$ ◀

(*b*) Eq. 7.15: $s_A = c\sinh\dfrac{x_A}{c} = (50 \text{ ft})\sinh\dfrac{65.85 \text{ ft}}{50 \text{ ft}}$

$$s_A = 86.6 \text{ ft}$$

$\text{length} = s_A = 86.6 \text{ ft}$ ◀

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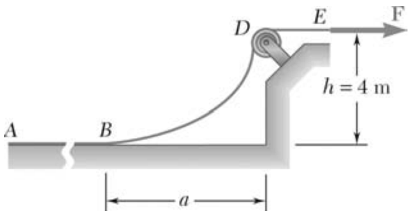
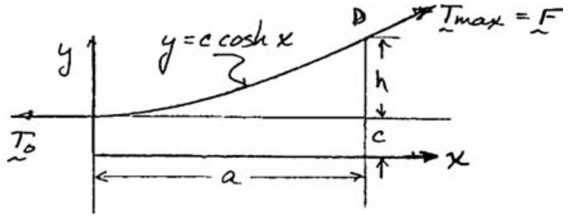


PROBLEM 7.143

To the left of Point B the long cable $ABDE$ rests on the rough horizontal surface shown. Knowing that the mass per unit length of the cable is 2 kg/m, determine the force **F** when $a = 3.6$ m.

SOLUTION



$$x_D = a = 3.6 \text{ m} \quad h = 4 \text{ m}$$

$$y_D = c\cosh\frac{x_D}{c}$$

$$h + c = c\cosh\frac{a}{c}$$

$$4 \text{ m} = c\left(\cosh\frac{3.6 \text{ m}}{c} - 1\right)$$

Solving numerically $\quad c = 2.0712$ m

Then $\quad y_B = h + c = 4 \text{ m} + 2.0712 \text{ m} = 6.0712 \text{ m}$

$$F = T_{\max} = wy_B = (2 \text{ kg/m})(9.81 \text{ m/s}^2)(6.0712 \text{ m})$$

$\mathbf{F} = 119.1 \text{ N} \longrightarrow$ ◀

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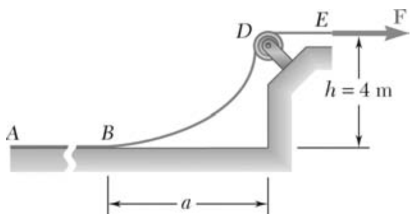
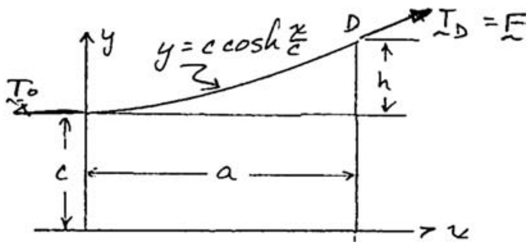


PROBLEM 7.144

To the left of Point B the long cable $ABDE$ rests on the rough horizontal surface shown. Knowing that the mass per unit length of the cable is 2 kg/m, determine the force **F** when $a = 6$ m.

SOLUTION



$$x_D = a = 6\text{ m} \quad h = 4\text{ m}$$

$$y_D = c\cosh\frac{x_D}{c}$$

$$h + c = c\cosh\frac{a}{c}$$

$$4\text{ m} = c\left(\cosh\frac{6\text{ m}}{c} - 1\right)$$

Solving numerically $\quad c = 5.054\text{ m}$

$$y_B = h + c = 4\text{ m} + 5.054\text{ m} = 9.054\text{ m}$$

$$F = T_D = wy_D = (2\text{ kg/m})(9.81\text{ m/s}^2)(9.054\text{ m})$$

F = 177.6 N → ◀

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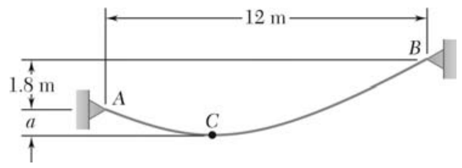
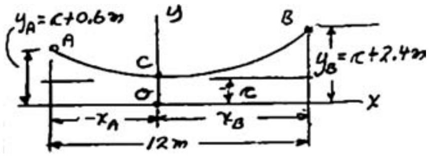


PROBLEM 7.145

The cable *ACB* has a mass per unit length of 0.45 kg/m. Knowing that the lowest point of the cable is located at a distance $a = 0.6$ m below the support *A*, determine (*a*) the location of the lowest Point *C*, (*b*) the maximum tension in the cable.

SOLUTION



Note: $$x_B - x_A = 12 \text{ m}$$

or, $$-x_A = 12 \text{ m} - x_B$$

Point *A*: $$y_A = c\cosh\frac{-x_A}{c}; \quad c + 0.6 = c\cosh\frac{12 - x_B}{c} \tag{1}$$

Point *B*: $$y_B = c\cosh\frac{x_B}{c}; \quad c + 2.4 = c\cosh\frac{x_B}{c} \tag{2}$$

From (1): $$\frac{12}{c} - \frac{x_B}{c} = \cosh^{-1}\left(\frac{c+0.6}{c}\right) \tag{3}$$

From (2): $$\frac{x_B}{c} = \cosh^{-1}\left(\frac{c+2.4}{c}\right) \tag{4}$$

Add (3) + (4): $$\frac{12}{c} = \cosh^{-1}\left(\frac{c+0.6}{c}\right) + \cosh^{-1}\left(\frac{c+2.4}{c}\right)$$

Solve by trial and error: $$c = 13.6214 \text{ m}$$

Eq. (2) $$13.6214 + 2.4 = 13.6214\cosh\frac{x_B}{c}$$

$$\cosh\frac{x_B}{c} = 1.1762; \quad \frac{x_B}{c} = 0.58523$$

$$x_B = 0.58523(13.6214 \text{ m}) = 7.9717 \text{ m}$$

Point *C* is 7.97 m to left of *B* ◀

$$y_B = c + 2.4 = 13.6214 + 2.4 = 16.0214 \text{ m}$$

Eq. 7.18: $$T_m = wy_B = (0.45 \text{ kg/m})(9.81 \text{ m/s}^2)(16.0214 \text{ m})$$

$$T_m = 70.726 \text{ N}$$

$T_m = 70.7$ N ◀

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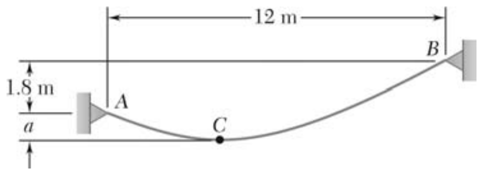
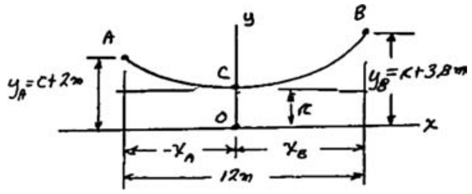


PROBLEM 7.146

The cable *ACB* has a mass per unit length of 0.45 kg/m. Knowing that the lowest point of the cable is located at a distance a = 2 m below the support *A*, determine (*a*) the location of the lowest Point *C*, (*b*) the maximum tension in the cable.

SOLUTION



Note:

$$x_B - x_A = 12 \text{ m}$$

or

$$-x_A = 12 \text{ m} - x_B$$

Point *A*: $$y_A = c\cosh\frac{-x_A}{c}; \quad c + 2 = c\cosh\frac{12 - x_B}{c} \tag{1}$$

Point *B*: $$y_B = c\cosh\frac{x_B}{c}; \quad c + 3.8 = c\cosh\frac{x_B}{c} \tag{2}$$

From (1): $$\frac{12}{c} - \frac{x_B}{c} = \cosh^{-1}\left(\frac{c+2}{c}\right) \tag{3}$$

From (2): $$\frac{x_B}{c} = \cosh^{-1}\left(\frac{c+3.8}{c}\right) \tag{4}$$

Add (3) + (4): $$\frac{12}{c} = \cosh^{-1}\left(\frac{c+2}{c}\right) + \cosh^{-1}\left(\frac{c+3.8}{c}\right)$$

Solve by trial and error: $c = 6.8154 \text{ m}$

Eq. (2): $$6.8154 \text{ m} + 3.8 \text{ m} = (6.8154 \text{ m})\cosh\frac{x_B}{c}$$

$$\cosh\frac{x_B}{c} = 1.5576 \qquad \frac{x_B}{c} = 1.0122$$

$$x_B = 1.0122(6.8154 \text{ m}) = 6.899 \text{ m}$$

Point *C* is 6.90 m to left of *B* ◀

$$y_B = c + 3.8 = 6.8154 + 3.8 = 10.6154 \text{ m}$$

Eq. (7.18): $$T_m = wy_B = (0.45 \text{ kg/m})(9.81 \text{ m/s}^2)(10.6154 \text{ m})$$

$$T_m = 46.86 \text{ N}$$

$T_m = 46.9 \text{ N}$ ◀

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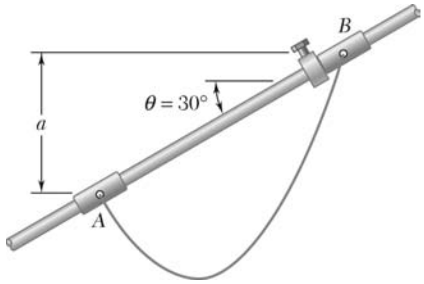
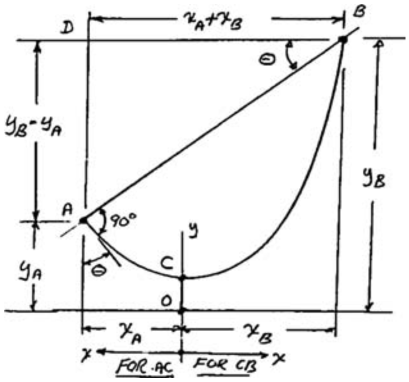


PROBLEM 7.147*

The 10-ft cable AB is attached to two collars as shown. The collar at A can slide freely along the rod; a stop attached to the rod prevents the collar at B from moving on the rod. Neglecting the effect of friction and the weight of the collars, determine the distance a.

SOLUTION

Collar at A: Since $\mu = 0$, cable $\perp$ rod



Point A:

$$y = c\cosh\frac{x}{c}; \quad \frac{dy}{dx} = \sinh\frac{x}{c}$$

$$\tan\theta = \left|\frac{dy}{dx}\right|_A = \sinh\frac{x_A}{c}$$

$$\frac{x_A}{c} = \sinh(\tan(90° - \theta))$$

$$x_A = c\sinh(\tan(90° - \theta)) \quad (1)$$

Length of cable = 10 ft

$$10 \text{ ft} = AC + CB$$

$$10 = c\sinh\frac{x_A}{c} + c\sinh\frac{x_B}{c}$$

$$\sinh\frac{x_B}{c} = \frac{10}{c} - \sinh\frac{x_A}{c}$$

$$x_B = c\sinh^{-1}\left[\frac{10}{c} - \sinh\frac{x_A}{c}\right] \quad (2)$$

$$y_A = c\cosh\frac{x_A}{c} \quad y_B = c\cosh\frac{x_B}{c} \quad (3)$$

In $\Delta\,ABD$:

$$\tan\theta = \frac{y_B - y_A}{x_B + x_A} \quad (4)$$

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PROBLEM 7.147* (Continued)

Method of solution:

For given value of θ, choose trial value of c and calculate:

From Eq. (1): x_A

Using value of x_A and c, calculate:

From Eq. (2): x_B

From Eq. (3): y_A and y_B

Substitute values obtained for x_A, x_B, y_A, y_B into Eq. (4) and calculate θ

Choose new trial value of θ and repeat above procedure until calculated value of θ is equal to given value of θ.

For $\theta = 30°$

Result of trial and error procedure:

$$c = 1.803 \text{ ft}$$
$$x_A = 2.3745 \text{ ft}$$
$$x_B = 3.6937 \text{ ft}$$
$$y_A = 3.606 \text{ ft}$$
$$y_B = 7.109 \text{ ft}$$
$$a = y_B - y_A$$
$$= 7.109 \text{ ft} - 3.606 \text{ ft}$$
$$= 3.503 \text{ ft}$$

$a = 3.50 \text{ ft}$ ◀

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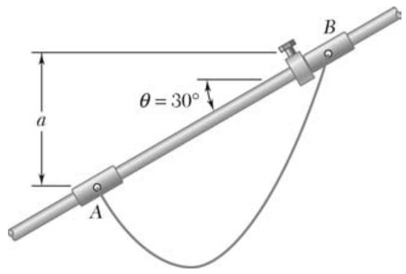


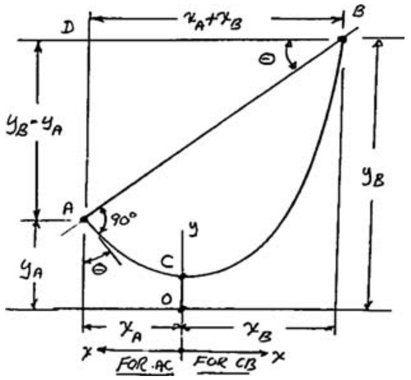
PROBLEM 7.148*

Solve Problem 7.147 assuming that the angle θ formed by the rod and the horizontal is 45°.

PROBLEM 7.147 The 10-ft cable *AB* is attached to two collars as shown. The collar at *A* can slide freely along the rod; a stop attached to the rod prevents the collar at *B* from moving on the rod. Neglecting the effect of friction and the weight of the collars, determine the distance *a*.

SOLUTION

Collar at *A*: Since $\mu = 0,$ cable $\perp$ rod



Point *A*:
$$y = c\cosh\frac{x}{c}; \quad \frac{dy}{dx} = \sinh\frac{x}{c}$$

$$\tan\theta = \left|\frac{dy}{dx}\right|_A = \sinh\frac{x_A}{c}$$

$$\frac{x_A}{c} = \sinh(\tan(90° - \theta))$$

$$x_A = c\sinh(\tan(90° - \theta)) \qquad (1)$$

Length of cable = 10 ft
$$10 \text{ ft} = AC + CB$$

$$10 = c\sinh\frac{x_A}{c} + c\sinh\frac{x_B}{c}$$

$$\sinh\frac{x_B}{c} = \frac{10}{c} - \sinh\frac{x_A}{c}$$

$$x_B = c\sinh^{-1}\left[\frac{10}{c} - \sinh\frac{x_A}{c}\right] \qquad (2)$$

$$y_A = c\cosh\frac{x_A}{c} \quad y_B = c\cosh\frac{x_B}{c} \qquad (3)$$

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PROBLEM 7.148* (Continued)

In $\Delta\, ABD$:

$$\tan\theta = \frac{y_B - y_A}{x_B + x_A} \qquad (4)$$

Method of solution:

For given value of θ, choose trial value of c and calculate:

From Eq. (1): x_A

Using value of x_A and c, calculate:

From Eq. (2): x_B

From Eq. (3): y_A and y_B

Substitute values obtained for x_A, x_B, y_A, y_B into Eq. (4) and calculate θ

Choose new trial value of θ and repeat above procedure until calculated value of θ is equal to given value of θ.

For $\theta = 45°$

Result of trial and error procedure:

$$c = 1.8652 \text{ ft}$$
$$x_A = 1.644 \text{ ft}$$
$$x_B = 4.064 \text{ ft}$$
$$y_A = 2.638 \text{ ft}$$
$$y_B = 8.346 \text{ ft}$$
$$a = y_B - y_A$$
$$= 8.346 \text{ ft} - 2.638 \text{ ft}$$
$$= 5.708 \text{ ft}$$

$a = 5.71$ ft ◀

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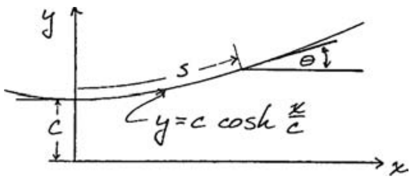
PROBLEM 7.149

Denoting by θ the angle formed by a uniform cable and the horizontal, show that at any point (*a*) $s = c\tan\theta$, (*b*) $y = c\sec\theta$.

SOLUTION



(*a*) $$\tan\theta = \frac{dy}{dx} = \sinh\frac{x}{c}$$

$$s = c\sinh\frac{x}{c} = c\tan\theta \qquad \text{Q.E.D.}$$

(*b*) Also $$y^2 = s^2 + c^2(\cosh^2 x = \sinh^2 x + 1)$$

so $$y^2 = c^2(\tan^2\theta + 1)c^2\sec^2\theta$$

and $$y = c\sec\theta \qquad \text{Q.E.D.}$$

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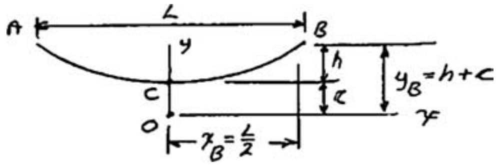
PROBLEM 7.150*

(*a*) Determine the maximum allowable horizontal span for a uniform cable of weight per unit length w if the tension in the cable is not to exceed a given value T_m. (*b*) Using the result of part *a*, determine the maximum span of a steel wire for which $w = 0.25$ lb/ft and $T_m = 8000$ lb.

SOLUTION



(*a*)

$$T_m = wy_B$$

$$= wc\cosh\frac{x_B}{c}$$

$$= wx_B\left(\frac{1}{\frac{x_B}{c}}\right)\cosh\frac{x_B}{c}$$

We shall find ratio $\left(\frac{x_B}{c}\right)$ for when T_m is minimum

$$\frac{dT_m}{d\left(\frac{x_B}{c}\right)} = wx_B\left[\frac{1}{\frac{x_B}{c}}\sinh\frac{x_B}{c} - \left(\frac{1}{\frac{x_B}{c}}\right)^2\cosh\frac{x_B}{c}\right] = 0$$

$$\frac{\sinh\frac{x_B}{c}}{\cosh\frac{x_B}{c}} = \frac{1}{\frac{x_B}{c}}$$

$$\tanh\frac{x_B}{c} = \frac{c}{x_B}$$

Solve by trial and error for: $\qquad \frac{x_B}{c} = 1.200 \qquad (1)$

$s_B = c\sinh\frac{x_B}{c} = c\sinh(1.200)$: $\qquad \frac{s_B}{c} = 1.509$

<u>Eq. 7.17</u>: $\qquad y_B^2 - s_B^2 = c^2$

$$y_B^2 = c^2\left[1 + \left(\frac{s_B}{c}\right)^2\right] = c^2(1 + 1.509^2)$$

$$y_B = 1.810c$$

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PROBLEM 7.150* (Continued)

Eq. 7.18:
$$T_m = wy_B$$
$$= 1.810\,wc$$
$$c = \frac{T_m}{1.810\,w}$$

Eq. (1):
$$x_B = 1.200c = 1.200\frac{T_m}{1.810w} = 0.6630\frac{T_m}{w}$$

Span:
$$L = 2x_B = 2(0.6630)\frac{T_m}{w}$$
$$L = 1.326\frac{T_m}{w} \blacktriangleleft$$

(*b*) For $w = 0.25\,\text{lb/ft}$ and $T_m = 8000\,\text{lb}$

$$L = 1.326\frac{8000\text{ lb}}{0.25\,\text{lb/ft}}$$
$$= 42,432\,\text{ft}$$
$$L = 8.04 \text{ miles} \blacktriangleleft$$

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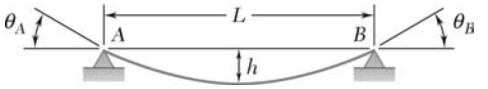
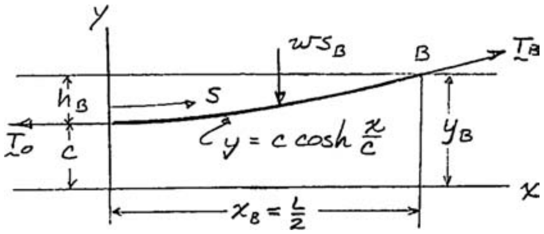


PROBLEM 7.151*

A cable has a mass per unit length of 3 kg/m and is supported as shown. Knowing that the span L is 6 m, determine the *two* values of the sag h for which the maximum tension is 350 N.

SOLUTION



$$y_{max} = c\cosh\frac{L}{2c} = h + c$$

$$w = (3\text{ kg/m})(9.81\text{ m/s}^2) = 29.43\text{ N/m}$$

$$T_{max} = wy_{max}$$

$$y_{max} = \frac{T_{max}}{w}$$

$$y_{max} = \frac{350\text{ N}}{29.43\text{ N/m}} = 11.893\text{ m}$$

$$c\cosh\frac{3\text{ m}}{c} = 11.893\text{ m}$$

Solving numerically

$$c_1 = 0.9241\text{ m}$$

$$c_2 = 11.499\text{ m}$$

$$h = y_{max} - c$$

$$h_1 = 11.893\text{ m} - 0.9241\text{ m}$$

$h_1 = 10.97$ m ◀

$$h_2 = 11.893\text{ m} - 11.499\text{ m}$$

$h_2 = 0.394$ m ◀

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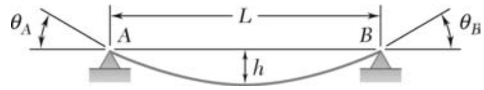
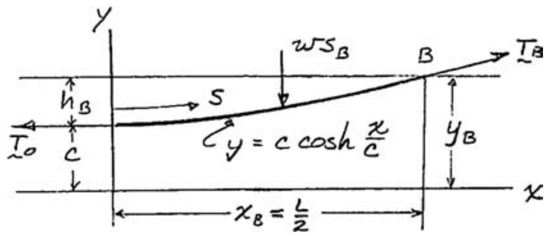


PROBLEM 7.152*

Determine the sag-to-span ratio for which the maximum tension in the cable is equal to the total weight of the entire cable *AB*.

SOLUTION



$$T_{\max} = wy_B = 2ws_B$$

$$y_B = 2s_B$$

$$c\cosh\frac{L}{2c} = 2c\sinh\frac{L}{2c}$$

$$\tanh\frac{L}{2c} = \frac{1}{2}$$

$$\frac{L}{2c} = \tanh^{-1}\frac{1}{2} = 0.549306$$

$$\frac{h_B}{c} = \frac{y_B - c}{c} = \cosh\frac{L}{2c} - 1 = 0.154701$$

$$\frac{h_B}{L} = \frac{\frac{h_B}{c}}{2\left(\frac{L}{2c}\right)} = \frac{0.5(0.154701)}{0.549306} = 0.14081$$

$$\frac{h_B}{L} = 0.1408 \blacktriangleleft$$

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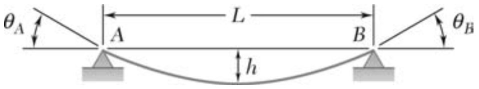
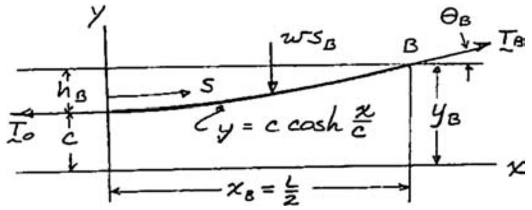


PROBLEM 7.153*

A cable of weight w per unit length is suspended between two points at the same elevation that are a distance L apart. Determine (a) the sag-to-span ratio for which the maximum tension is as small as possible, (b) the corresponding values of θ_B and T_m.

SOLUTION



(a)

$$T_{\max} = wy_B = wc\cosh\frac{L}{2c}$$

$$\frac{dT_{\max}}{dc} = w\left(\cosh\frac{L}{2c} - \frac{L}{2c}\sinh\frac{L}{2c}\right)$$

For $\quad \min T_{\max}, \quad \frac{dT_{\max}}{dc} = 0$

$$\tanh\frac{L}{2c} = \frac{2c}{L} \rightarrow \frac{L}{2c} = 1.1997$$

$$\frac{y_B}{c} = \cosh\frac{L}{2c} = 1.8102$$

$$\frac{h}{c} = \frac{y_B}{c} - 1 = 0.8102$$

$$\frac{h}{L} = \left[\frac{1}{2}\frac{h}{c}\left(\frac{2c}{L}\right)\right] = \frac{0.8102}{2(1.1997)} = 0.3375$$

$\frac{h}{L} = 0.338$ ◀

(b)

$$T_0 = wc \quad T_{\max} = wc\cosh\frac{L}{2c} \quad \frac{T_{\max}}{T_0} = \cosh\frac{L}{2c} = \frac{y_B}{c}$$

But $\quad T_0 = T_{\max}\cos\theta_B \quad \frac{T_{\max}}{T_0} = \sec\theta_B$

So $\quad \theta_B = \sec^{-1}\left(\frac{y_B}{c}\right) = \sec^{-1}(1.8102) = 56.46°$

$\theta_B = 56.5°$ ◀

$$T_{\max} = wy_B = w\frac{y_B}{c}\left(\frac{2c}{L}\right)\left(\frac{L}{2}\right) = w(1.8102)\frac{L}{2(1.1997)}$$

$T_{\max} = 0.755wL$ ◀

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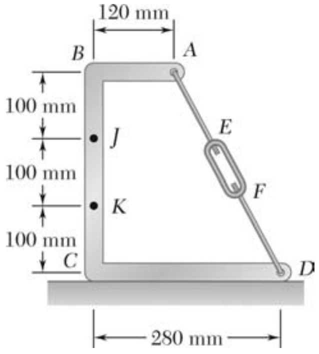
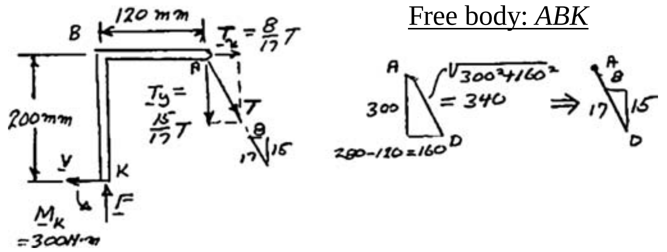


PROBLEM 7.154

It has been experimentally determined that the bending moment at Point K of the frame shown is 300 N · m. Determine (a) the tension in rods AE and FD, (b) the corresponding internal forces at Point J.

SOLUTION

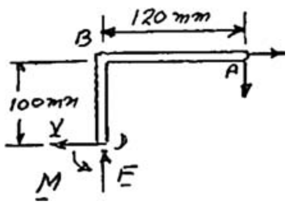
Free body: *ABK*



(a) $\circlearrowleft + \Sigma M_K = 0$: $300\text{ N}\cdot\text{m} - \frac{8}{17}T(0.2\text{ m}) - \frac{15}{17}T(0.12\text{ m}) = 0$ $\qquad T = 1500\text{ N}$ ◀

Free body: *AJ*

$$T_x = \frac{8}{17}T = \frac{8}{17}(1500\text{ N})$$
$$= 705.88\text{ N}$$
$$T_y = \frac{15}{17}T = \frac{15}{17}(1500\text{ N})$$
$$= 1323.53\text{ N}$$



Internal forces on *ABJ*

(b) $\xrightarrow{+} \Sigma F_x = 0$: $705.88\text{ N} - V = 0$

$V = +705.88\text{ N}$ $\qquad \mathbf{V} = 706\text{ N} \longleftarrow$ ◀

$+\uparrow \Sigma F_y = 0$: $F - 1323.53\text{ N} = 0$

$F = +1323.53\text{ N}$ $\qquad \mathbf{F} = 1324\text{ N}\uparrow$ ◀

$\circlearrowleft + \Sigma M_J = 0$: $M - (705.88\text{ N})(0.1\text{ m}) - (1323.53\text{ N})(0.12\text{ m}) = 0$

$M = +229.4\text{ N}\cdot\text{m}$

$\mathbf{M} = 229\text{ N}\cdot\text{m} \circlearrowleft$ ◀

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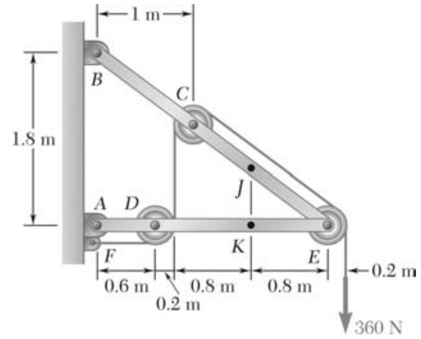
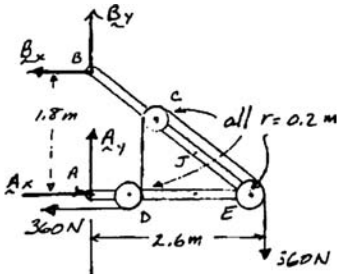


PROBLEM 7.155

Knowing that the radius of each pulley is 200 mm and neglecting friction, determine the internal forces at Point J of the frame shown.

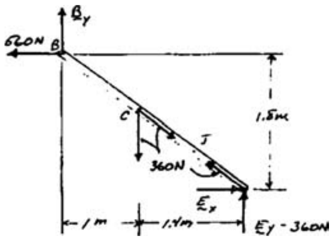
SOLUTION



FBD Frame with pulley and cord:

$$\circlearrowleft \Sigma M_A = 0: \quad (1.8 \text{ m})B_x - (2.6 \text{ m})(360 \text{ N}) - (0.2 \text{ m})(360 \text{ N}) = 0$$

$$\mathbf{B}_x = 560 \text{ N} \longleftarrow$$

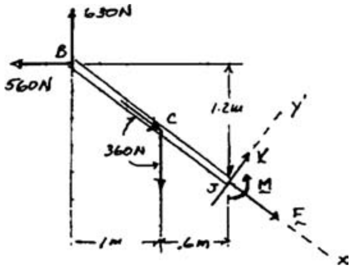


FBD BE:

Note: Cord forces have been moved to pulley hub as per Problem 6.91.

$$\circlearrowleft \Sigma M_E = 0: \quad (1.4 \text{ m})(360 \text{ N}) + (1.8 \text{ m})(560 \text{ N}) - (2.4 \text{ m})B_y = 0$$

$$\mathbf{B}_y = 630 \text{ N} \uparrow$$



FBD BJ:

$$\searrow \Sigma F_{x'} = 0: \quad F + 360 \text{ N} - \frac{3}{5}(630 \text{ N} - 360 \text{ N}) - \frac{4}{5}(560 \text{ N}) = 0$$

$$\mathbf{F} = 250 \text{ N} \nwarrow \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0: \quad V + \frac{4}{5}(630 \text{ N} - 360 \text{ N}) - \frac{3}{5}(560 \text{ N}) = 0$$

$$\mathbf{V} = 120.0 \text{ N} \nearrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad M + (0.6 \text{ m})(360 \text{ N}) + (1.2 \text{ m})(560 \text{ N}) - (1.6 \text{ m})(630 \text{ N}) = 0$$

$$\mathbf{M} = 120.0 \text{ N} \cdot \text{m} \circlearrowright \blacktriangleleft$$

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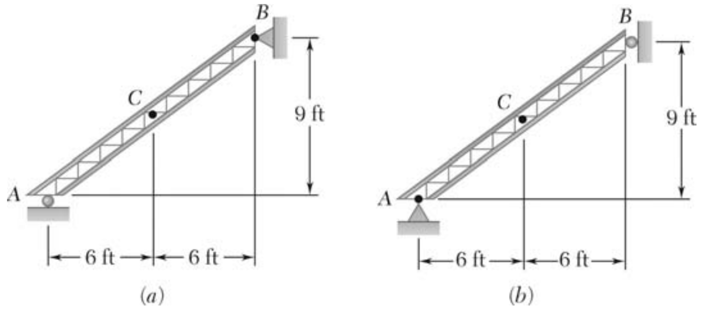
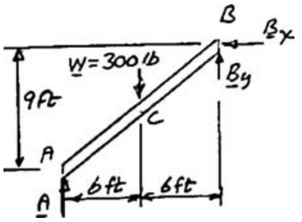
PROBLEM 7.156

A steel channel of weight per unit length $w = 20\text{ lb/ft}$ forms one side of a flight of stairs. Determine the internal forces at the center C of the channel due to its own weight for each of the support conditions shown.



SOLUTION

(*a*) Free body: *AB*



$$AB = \sqrt{9^2 + 12^2} = 15\text{ ft}$$

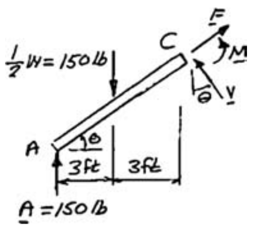
$$W = (20\text{ lb/ft})(15\text{ ft}) = 300\text{ lb}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad A(12\text{ ft}) - (300\text{ lb})(6\text{ ft}) = 0$$

$$A = +150\text{ lb} \qquad \mathbf{A} = 150\text{ lb}\uparrow$$

Free body: *AC*

(150-lb Forces form a couple)



$$\nearrow \Sigma \mathbf{F} = 0 \qquad \mathbf{F} = 0 \blacktriangleleft$$

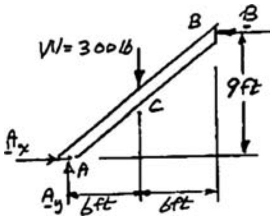
$$\nwarrow \Sigma \mathbf{F} \qquad \mathbf{V} = 0 \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_C = 0: \quad M - (150\text{ lb})(3\text{ ft}) = 0$$

$$M = +450\text{ lb}\cdot\text{ft}$$

$$\mathbf{M} = 450\text{ lb}\cdot\text{ft} \circlearrowleft \blacktriangleleft$$

(*b*) Free body: *AB*



$$+\circlearrowleft \Sigma M_A = 0: \quad B(9\text{ ft}) - (300\text{ lb})(6\text{ ft}) = 0$$

$$B = +200\text{ lb}$$

$$\mathbf{B} = 200\text{ lb} \longleftarrow$$

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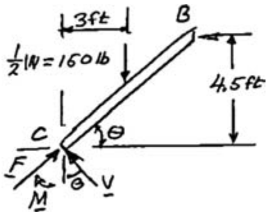
PROBLEM 7.156 (Continued)

Free body: *CB*

$$+\circlearrowleft \Sigma M_C = 0: \quad (200\text{ lb})(4.5\text{ ft}) - (150\text{ lb})(3\text{ ft}) - M = 0$$

$$M = +450\text{ lb}\cdot\text{ft}$$

$$\mathbf{M} = 450\text{ lb}\cdot\text{ft} \circlearrowright$$



$$\tan\theta = \frac{9}{12} = \frac{3}{4}; \quad \sin\theta = \frac{3}{5}; \quad \cos\theta = \frac{4}{5}$$

$$+\nearrow \Sigma F = 0: \quad F - \frac{3}{5}(150\text{ lb}) - \frac{4}{5}(200\text{ lb}) = 0$$

$$F = +250\text{ lb} \quad \mathbf{F} = 250\text{ lb} \nearrow$$

$$+\nwarrow \Sigma F = 0: \quad V - \frac{4}{5}(150\text{ lb}) + \frac{3}{5}(200\text{ lb}) = 0$$

$$V = 0 \quad \mathbf{V} = 0$$

On portion *AC* internal forces are $\mathbf{M} = 450\text{ lb}\cdot\text{ft} \circlearrowleft, \quad \mathbf{F} = 250\text{ lb} \swarrow, \quad \mathbf{V} = 0$ ◀

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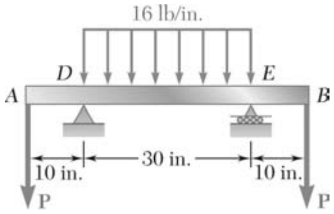
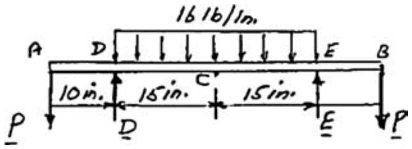


PROBLEM 7.157

For the beam shown, determine (*a*) the magnitude *P* of the two concentrated loads for which the maximum absolute value of the bending moment is as small as possible, (*b*) the corresponding value of $|M|_{max}$.

SOLUTION



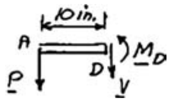
Free body: Entire beam

By symmetry

$$D = E = \frac{1}{2}(16 \text{ lb/in.})(30 \text{ in.}) + P$$

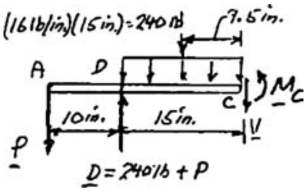
$$D = 240 \text{ lb} + P$$

Free body: Portion *AD*



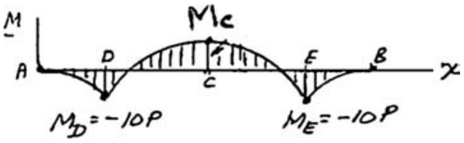
$$+\circlearrowleft \Sigma M_D = 0: \quad M_D = -10P$$

Free body: Portion *ADC*



$$+\circlearrowleft \Sigma M_C = 0: \quad P(25 \text{ in.}) - (240 \text{ lb} + P)(15 \text{ in.}) + (240 \text{ lb})(7.5 \text{ in.}) + M_C = 0$$

$$M_C = 1800 \text{ lb}\cdot\text{in.} - (10 \text{ in.})P$$



(*a*) We equate: $|M_D| = |M_C|$

$$|-10P| = |1800 - 10P|$$

$$10P = 1800 - 10P$$

$P = 90.0 \text{ lb}$ ◀

(*b*) For $P = 90$ lb: $M_D = -10 \text{ lb } (90 \text{ lb})$

$|M|_{max} = 900 \text{ lb}\cdot\text{in.}$ ◀

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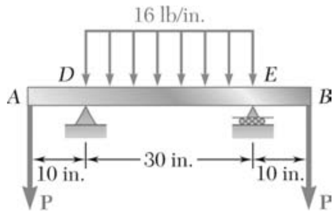


PROBLEM 7.158

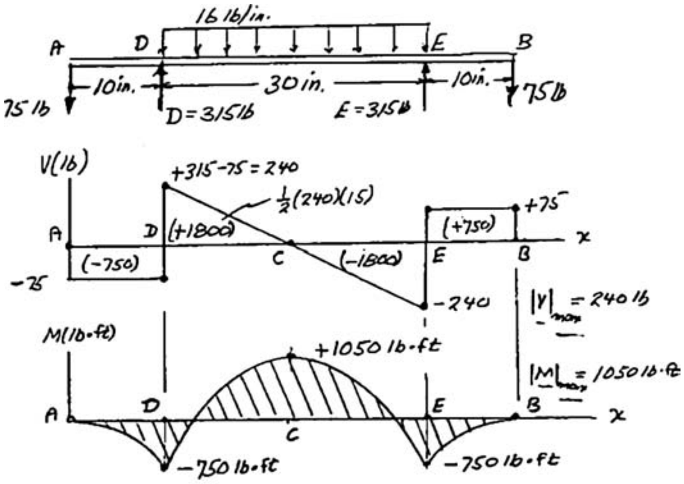
Knowing that the magnitude of the concentrated loads **P** is 75 lb, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

By symmetry: $D = E$

$$= \frac{1}{2}(16 \text{ lb/in.})(30 \text{ in.}) + 75 \text{ lb}$$

$$= 315 \text{ lb}$$



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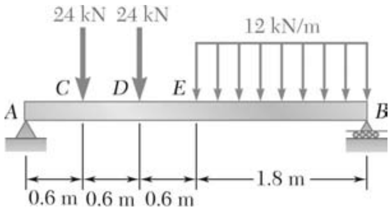
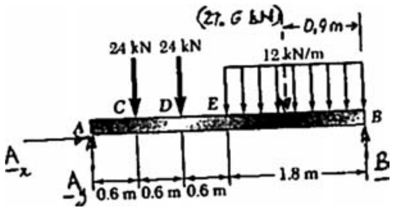


PROBLEM 7.159

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_B = 0: \quad (24 \text{ kN})(3 \text{ m})$$

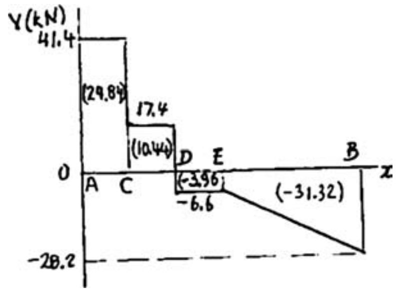
$$+(24 \text{ kN})(2.4 \text{ m}) + (21.6 \text{ kN})(0.9 \text{ m})$$

$$-A_y(3.6 \text{ m}) = 0$$

$$A_y = +91.4 \text{ kN}$$

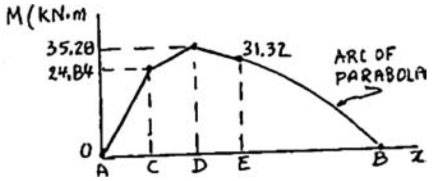
$$\Sigma F_x = 0: \quad A_x = 0$$

Shear diagram



At *A*: $V_A = A_y = +41.4 \text{ kN}$ $\quad |V|_{max} = 41.4 \text{ kN}$ ◀

Bending-moment diagram



At *A*: $M_A = 0$ $\quad |M|_{max} = 35.3 \text{ kN}\cdot\text{m}$ ◀

The slope of the parabola at *E* is the same as that of the segment *DE*

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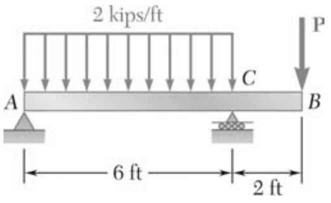
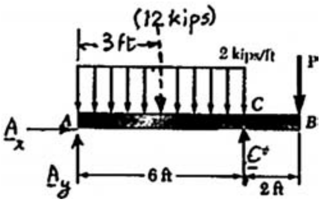


PROBLEM 7.160

For the beam shown, draw the shear and bending-moment diagrams, and determine the magnitude and location of the maximum absolute value of the bending moment, knowing that (*a*) $P = 6$ kips, (*b*) $P = 3$ kips.

SOLUTION

Free body: Beam



$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad C(6 \text{ ft}) - (12 \text{ kips})(3 \text{ ft}) - P(8 \text{ ft}) = 0$$

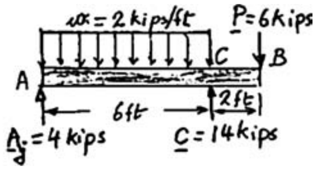
$$C = 6 \text{ kips} + \frac{4}{3}P \qquad (1) \ \triangleleft$$

$$\Sigma F_y = 0: \quad A_y + \left(6 + \frac{4}{3}P\right) - 12 - P = 0$$

$$A_y = 6 \text{ kips} - \frac{1}{3}P \qquad (2) \ \triangleleft$$

(*a*) $P = 6$ kips.

Load diagram

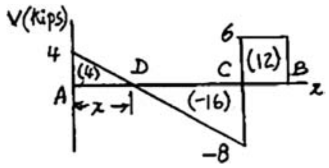


Substituting for P in Eqs. (2) and (1):

$$A_y = 6 - \frac{1}{3}(6) = 4 \text{ kips}$$

$$C = 6 + \frac{4}{3}(6) = 14 \text{ kips}$$

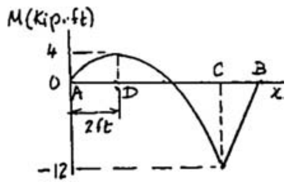
Shear diagram $\qquad V_A = A_y = +4$ kips



To determine Point D where $V = 0$:

$$V_D - V_A = -wx$$

$$0 - 4 \text{ kips} = (2 \text{ kips/ft})x \qquad x = 2 \text{ ft} \ \triangleleft$$



We compute all areas

Bending-moment diagram

At A: $\qquad M_A = 0$

$$|M|_{max} = 12.00 \text{ kip} \cdot \text{ft, at } C \ \blacktriangleleft$$

Parabola from A to C

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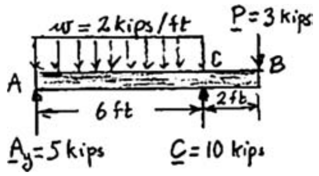
PROBLEM 7.160 (Continued)

(*b*) $P = 3$ kips

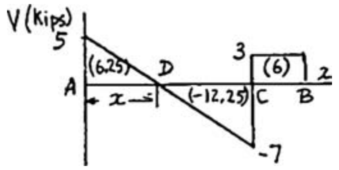


Load diagram

Substituting for P in Eqs. (2) and (1):

$$A = 6 - \frac{1}{3}(3) = 5 \text{ kips}$$

$$C = 6 + \frac{4}{3}(3) = 10 \text{ kips}$$

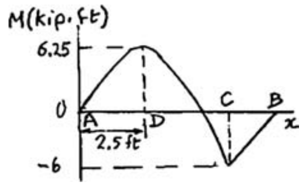


Shear diagram $\qquad V_A = A_y = +5$ kips

To determine D where $V = 0$:

$$V_D - V_A = -wx$$
$$0 - (5 \text{ kips}) = -(2 \text{ kips/ft})\, x$$

$x = 2.5$ ft ◁



We compute all areas

Bending-moment diagram

At A: $\qquad M_A = 0$

$|M|_{max} = 6.25$ kip·ft ◀

2.50 ft from A ◀

Parabola from A to C.

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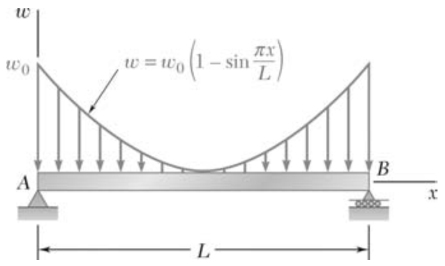

PROBLEM 7.161

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the maximum bending moment.

SOLUTION

(*a*) Reactions at supports: $A = B = \frac{1}{2}W$, where $\frac{W}{L} = $ Total load

$$W = \int_0^L w\,dx = w_0 \int_0^L \left(1 - \sin\frac{\pi x}{L}\right) dx$$

$$= w_0 \left[x + \frac{L}{x}\cos\frac{\pi x}{L} \right]_0^L$$

$$= w_0 L \left(1 - \frac{2}{\pi}\right)$$

Thus $$V_A = A = \frac{1}{2}W = \frac{1}{2} w_0 L \left(1 - \frac{2}{\pi}\right)$$

$$M_A = 0 \qquad (1)$$

Load: $$w(x) = w_0 \left(1 - \sin\frac{\pi x}{L}\right)$$

Shear: From Eq. (7.2):

$$V(x) - V_A = -\int_0^x w(x)\,dx$$

$$= -w_0 \int_0^x \left(1 - \sin\frac{\pi x}{L}\right) dx$$

Integrating and recalling first of Eqs. (1),

$$V(x) - \frac{1}{2} w_0 L \left(1 - \frac{2}{\pi}\right) = -w_0 \left[x + \frac{L}{\pi}\cos\frac{\pi x}{L} \right]_0^x$$

$$V(x) = \frac{1}{2} w_0 L \left(1 - \frac{2}{\pi}\right) - w_0 \left(2 + \frac{L}{\pi}\cos\frac{\pi x}{L}\right) + w_0 \frac{L}{\pi}$$

$$V(x) = w_0 \left(\frac{L}{2} - x - \frac{L}{\pi}\cos\frac{\pi x}{L} \right) \qquad (2) ◀$$

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PROBLEM 7.161 (Continued)

Bending moment: From Eq. (7.4) and recalling that $M_A = 0.$

$$M(x) - M_A = \int_0^x V(x)dx$$

$$= w_0\left[\frac{L}{2}x - \frac{1}{2}x^2 - \left(\frac{L}{\pi}\right)^2 \sin\frac{\pi x}{L}\right]_0^x$$

$$M(x) = \frac{1}{2}w_0\left(Lx - x^2 - \frac{2L^2}{\pi^2}\sin\frac{\pi x}{L}\right) \qquad (3) \blacktriangleleft$$

(*b*) Maximum bending moment

$$\frac{dM}{dx} = V = 0.$$

This occurs at $x = \frac{L}{2}$ as we may check from (2):

$$V\left(\frac{L}{2}\right) = w_0\left(\frac{L}{2} - \frac{L}{2} - \frac{L}{\pi}\cos\frac{\pi}{2}\right) = 0$$

From (3):

$$M\left(\frac{L}{2}\right) = \frac{1}{2}w_0\left(\frac{L^2}{2} - \frac{L^2}{4} - \frac{2L^2}{\pi^2}\sin\frac{\pi}{2}\right)$$

$$= \frac{1}{8}w_0L^2\left(1 - \frac{8}{\pi^2}\right)$$

$$= 0.0237w_0L^2$$

$$M_{\max} = 0.0237w_0L^2, \quad \text{at} \quad x = \frac{L}{2} \qquad \blacktriangleleft$$

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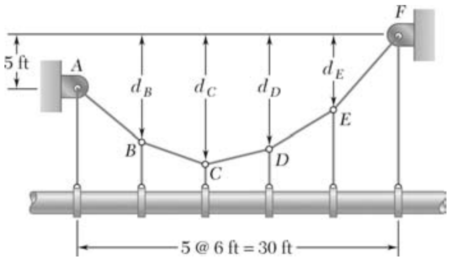
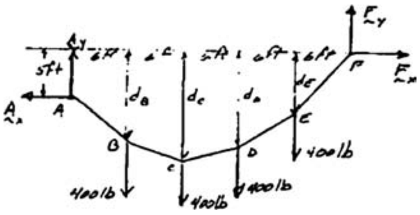


PROBLEM 7.162

An oil pipeline is supported at 6-ft intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents the tension in each hanger is 400 lb. Knowing that $d_C = 12$ ft, determine (*a*) the maximum tension in the cable, (*b*) the distance d_D.

SOLUTION

FBD Cable: Hanger forces at *A* and *F* act on the supports, so $\mathbf{A}_y$ and $\mathbf{F}_y$ act on the cable.



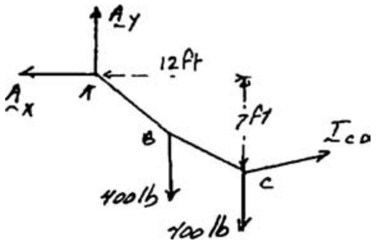
$$\circlearrowleft \Sigma M_F = 0: \quad (6\text{ ft} + 12\text{ ft} + 18\text{ ft} + 24\text{ ft})(400\text{ lb})$$

$$-(30\text{ ft})A_y - (5\text{ ft})A_x = 0$$

$$A_x + 6A_y = 4800\text{ lb} \qquad (1)$$

FBD ABC: $\circlearrowleft \Sigma M_C = 0: \quad (7\text{ ft})A_x - (12\text{ ft})A_y + (6\text{ ft})(400\text{ lb}) = 0 \qquad (2)$

Solving (1) and (2)



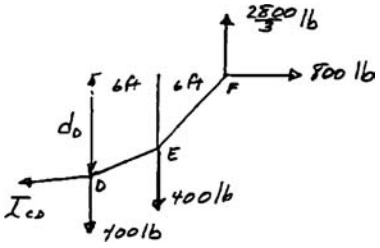
$$\mathbf{A}_x = 800\text{ lb} \longleftarrow$$

$$\mathbf{A}_y = \frac{2000}{3}\text{lb}\uparrow$$

From FBD Cable: $\longrightarrow \Sigma F_x = 0: \quad -800\text{ lb} + F_x = 0$

$$\mathbf{F}_x = 800\text{ lb} \longrightarrow$$

FBD DEF:



$$\uparrow \Sigma F_y = 0: \quad \frac{200}{3}\text{ lb} - 4(400\text{ lb}) + F_y = 0$$

$$\mathbf{F}_y = \frac{2800}{3}\text{ lb}\uparrow$$

Since $A_x = F_x$ and $F_y > A_y$, $\qquad T_{max} = T_{EF} = \sqrt{(800\text{ lb})^2 + \left(\frac{2800}{3}\text{lb}\right)^2}$

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PROBLEM 7.162 (Continued)

(*a*) $T_{\max} = 1229.27$ lb, $T_{\max} = 1229$ lb ◀

$$\circlearrowleft \Sigma M_D = 0: \quad (12 \text{ ft})\left(\frac{2800}{3} \text{lb}\right) - d_D(800 \text{ lb}) - (6 \text{ ft})(400 \text{ lb}) = 0$$

(*b*) $d_D = 11.00$ ft ◀

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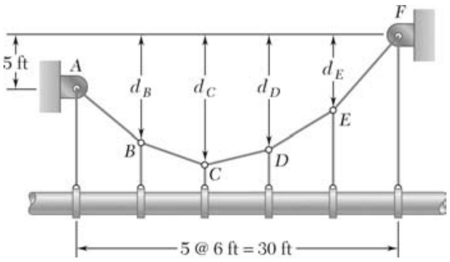


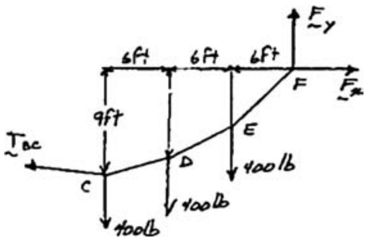
PROBLEM 7.163

Solve Problem 7.162 assuming that $d_C = 9$ ft.

PROBLEM 7.162 An oil pipeline is supported at 6-ft intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents the tension in each hanger is 400 lb. Knowing that $d_C = 12$ ft, determine (*a*) the maximum tension in the cable, (*b*) the distance d_D.

SOLUTION

FBD CDEF:



$$\circlearrowleft \Sigma M_C = 0: \quad (18 \text{ ft})F_y - (9 \text{ ft})F_y - (6 \text{ ft} + 12 \text{ ft})(400 \text{ lb}) = 0$$

$$F_x - 2F_y = -800 \text{ lb} \qquad (1)$$

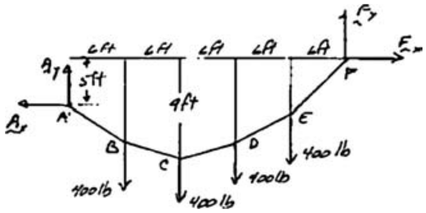
FBD Cable:

$$\circlearrowleft EM_A = 0: \quad (30 \text{ ft})F_y - (5 \text{ ft})F_x$$
$$-(6 \text{ ft})(1+2+3+4)(400 \text{ lb}) = 0$$

$$F_x - 6F_y = -4800 \text{ lb} \qquad (2)$$

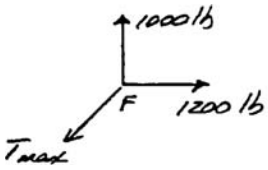
Solving (1) and (2),

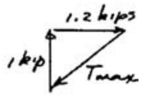
$$\mathbf{F}_x = 1200 \text{ lb} \longrightarrow, \quad \mathbf{F}_y = 1000 \text{ lb} \uparrow$$



$$\longrightarrow \Sigma F_x = 0: \quad -A_x + 1200 \text{ lb} = 0, \quad \mathbf{A}_x = 1200 \text{ lb} \longrightarrow$$

Point F:





$$\uparrow \Sigma F_y = 0: \quad A_y + 1000 \text{ lb} - 4(400 \text{ lb}) = 0, \qquad \mathbf{A}_y = 600 \text{ lb} \uparrow$$

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PROBLEM 7.163 (Continued)

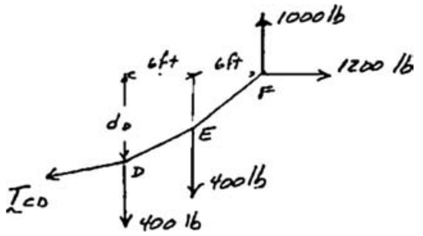
Since

$$A_x = A_y \quad \text{and} \quad F_y > A_y,\ T_{max} = T_{EF}$$

$$T_{max} = \sqrt{(1\text{ kip})^2 + (1.2\text{ kips})^2}$$

(*a*) $T_{max} = 1.562\text{ kips}$ ◀

FBD DEF: $\circlearrowleft \Sigma M_D = 0$: $(12\text{ ft})(1000\text{ lb}) - d_D(1200\text{ lb}) - (6\text{ ft})(400\text{ lb}) = 0$



(*b*) $d_D = 8.00\text{ ft}$ ◀

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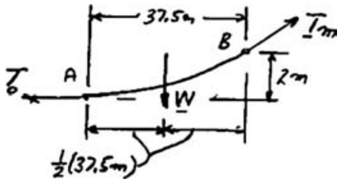
PROBLEM 7.164

A transmission cable having a mass per unit length of 0.8 kg/m is strung between two insulators at the same elevation that are 75 m apart. Knowing that the sag of the cable is 2 m, determine (*a*) the maximum tension in the cable, (*b*) the length of the cable.

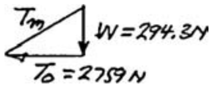
SOLUTION



$$w = (0.8 \text{ kg/m})(9.81 \text{ m/s}^2)$$
$$= 7.848 \text{ N/m}$$
$$W = (7.848 \text{ N/m})(37.5 \text{ m})$$
$$W = 294.3 \text{ N}$$

(*a*) $\quad +\circlearrowleft \Sigma M_B = 0: \quad T_0(2 \text{ m}) - W\left(\frac{1}{2}37.5 \text{ m}\right) = 0$

$$T_0(2 \text{ m}) - (294.3 \text{ N})\frac{1}{2}(37.5 \text{ m}) = 0$$



$$T_0 = 2759 \text{ N}$$
$$T_m^2 = (294.3 \text{ N})^2 + (2759 \text{ N})^2 \qquad T_m = 2770 \text{ N} \blacktriangleleft$$

(*b*)
$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 + \cdots\right]$$
$$= 37.5 \text{ m}\left[1 + \frac{2}{3}\left(\frac{2 \text{ m}}{37.5 \text{ m}}\right)^2 + \cdots\right]$$
$$= 37.57 \text{ m}$$

$$\text{Length} = 2s_B = 2(37.57 \text{ m}) \qquad \text{Length} = 75.14 \text{ m} \blacktriangleleft$$

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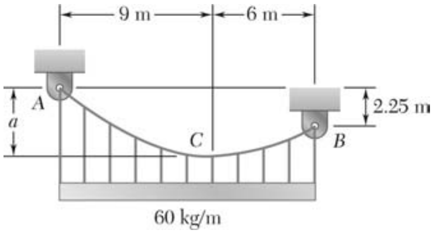
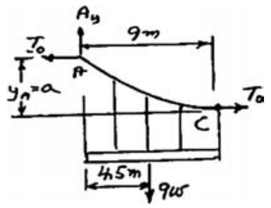


PROBLEM 7.165

Cable *ACB* supports a load uniformly distributed along the horizontal as shown. The lowest Point *C* is located 9 m to the right of *A*. Determine (*a*) the vertical distance *a*, (*b*) the length of the cable, (*c*) the components of the reaction at *A*.

SOLUTION

Free body: Portion *AC*

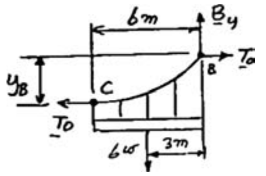


$$+\uparrow \Sigma F_y = 0: \quad A_y - 9w = 0$$

$$\mathbf{A}_y = 9w \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad T_0 a - (9w)(4.5 \text{ m}) = 0 \qquad (1)$$

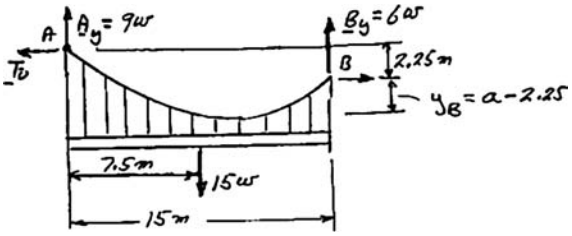
Free body: Portion *CB*



$$+\uparrow \Sigma F_y = 0: \quad B_y - 6w = 0$$

$$\mathbf{B}_y = 6w \uparrow$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad 15w(7.5 \text{ m}) - 6w(15 \text{ m}) - T_0(2.25 \text{ m}) = 0$$

(*a*) $T_0 = 10w$

Eq. (1): $T_0 a - (9w)(4.5 \text{ m}) = 0$

$10wa = (9w)(4.5) = 0$ $\qquad a = 4.05 \text{ m}$ ◀

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PROBLEM 7.165 (Continued)

(b) Length $= AC + CB$

Portion AC:

$$x_A = 9\text{ m},\quad y_A = a = 4.05\text{ m};\quad \frac{y_A}{x_A} = \frac{4.05}{9} = 0.45$$

$$s_{AC} = x_B\left[1 + \frac{2}{3}\left(\frac{y_A}{x_A}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_A}\right)^4 + \cdots\right]$$

$$s_{AC} = 9\text{ m}\left(1 + \frac{2}{3}0.45^2 - \frac{2}{5}0.45^4 + \cdots\right) = 10.067\text{ m}$$

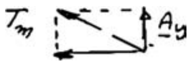
Portion CB:

$$x_B = 6\text{ m},\quad y_B = 4.05 - 2.25 = 1.8\text{ m};\quad \frac{y_B}{x_B} = 0.3$$

$$s_{CB} = 6\text{ m}\left(1 + \frac{2}{3}0.3^2 - \frac{2}{5}0.3^4 + \cdots\right) = 6.341\text{ m}$$

Total length $= 10.067\text{ m} + 6.341\text{ m}$ Total length $= 16.41$ m ◀

(c) Components of reaction at A.



$$A_y = 9w = 9(60\text{ kg/m})(9.81\text{ m/s}^2)$$
$$= 5297.4\text{ N}$$

$$A_x = T_0 = 10w = 10(60\text{ kg/m})(9.81\text{ m/s}^2)$$
$$= 5886\text{ N}$$

$\mathbf{A}_x = 5890$ N ← ◀

$\mathbf{A}_y = 5300$ N ↑ ◀

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CHAPTER 8

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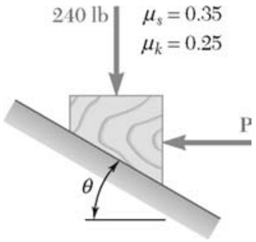
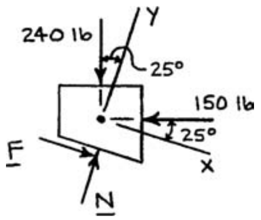


PROBLEM 8.1

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 25°$ and $P = 150$ lb.

SOLUTION



Assume equilibrium:

$$\searrow^{+} \Sigma F_x = 0: \quad F + (240 \text{ lb})\sin 25° - (150 \text{ lb})\cos 25° = 0$$

$$F = +34.518 \text{ lb} \qquad \mathbf{F} = 34.518 \text{ lb} \searrow$$

$$+\nearrow \Sigma F_y = 0: \quad N - (240 \text{ lb})\cos 25° - (150 \text{ lb})\sin 25° = 0$$

$$N = +280.91 \text{ lb} \qquad \mathbf{N} = 280.91 \text{ lb} \nearrow$$

Maximum friction force: $F_m = \mu_s N = 0.35(280.91 \text{ lb}) = 98.319 \text{ lb}$

Block is in equilibrium ◀

Since $F < F_m$, $\mathbf{F} = 34.5 \text{ lb} \searrow$ ◀

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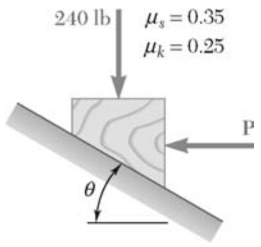
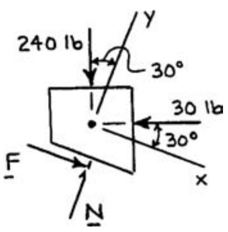


PROBLEM 8.2

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 30°$ and $P = 30$ lb.

SOLUTION

<u>Assume equilibrium</u>:



$\searrow^{+} \Sigma F_x = 0: \quad F + (240 \text{ lb})\sin 30° - (30 \text{ lb})\cos 30° = 0$

$F = -94.019 \text{ lb} \qquad \mathbf{F} = 94.019 \text{ lb} \nwarrow$

$+\nearrow \Sigma F_y = 0: \quad N - (240 \text{ lb})\cos 30° - (30 \text{ lb})\sin 30° = 0$

$N = +222.85 \text{ lb} \qquad \mathbf{N} = 222.85 \text{ lb} \nearrow$

Maximum friction force:

$$F_m = \mu_s N = 0.35(222.85 \text{ lb}) = 77.998 \text{ lb}$$

Since **F** is $\nwarrow$ and $F > F_m$, <u>Block moves down</u> ◀

Actual friction force: $F = F_k = \mu_k N = 0.25(222.85 \text{ lb})$ $\mathbf{F} = 55.7 \text{ lb} \nwarrow$ ◀

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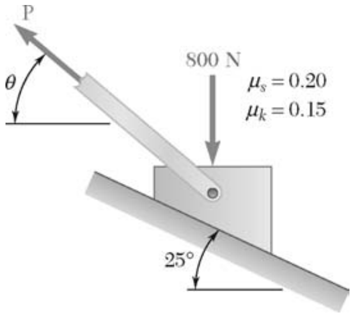
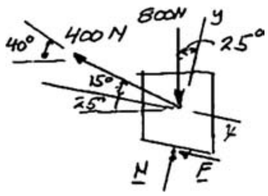


PROBLEM 8.3

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 40°$ and $P = 400$ N.

SOLUTION

Assume equilibrium:



$$+\nearrow \Sigma F_y = 0: \quad N - (800 \text{ N})\cos 25° + (400 \text{ N})\sin 15° = 0$$

$$N = +621.5 \text{ N} \qquad \mathbf{N} = 621.5 \text{ N} \uparrow$$

$$+\searrow \Sigma F_x = 0: \quad -F + (800 \text{ N})\sin 25° - (400 \text{ N})\cos 15° = 0$$

$$F = +48.28 \text{ N} \qquad \mathbf{F} = 48.28 \text{ N} \nwarrow$$

Maximum friction force:

$$F_m = \mu_s N$$
$$= 0.20(621.5 \text{ N})$$
$$= 124.3 \text{ N}$$

Block is in equilibrium ◀

Since $F < F_m$, $\mathbf{F} = 48.3 \text{ N} \nwarrow$ ◀

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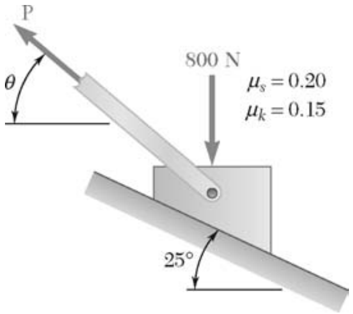
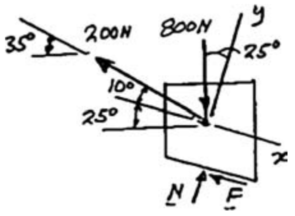


PROBLEM 8.4

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 35°$ and $P = 200$ N.

SOLUTION

Assume equilibrium:



$+\nearrow \Sigma F_y = 0: \quad N - (800 \text{ N})\cos 25° + (200 \text{ N})\sin 10° = 0$

$N = 690.3 \text{ N} \qquad \mathbf{N} = 690.3 \text{ N}$ ↑

$+\searrow \Sigma F_x = 0: \quad -F + (800 \text{ N})\sin 25° - (200 \text{ N})\cos 10° = 0$

$F = 141.13 \text{ N} \qquad \mathbf{F} = 141.13 \text{ N}$ ↖

Maximum friction force:

$$F_m = \mu_s N = (0.20)(690.3 \text{ N}) = 138.06 \text{ N}$$

Since $F > F_m$, Block moves down ↘ ◀

Friction force:

$$F = \mu_k N = (0.15)(690.3 \text{ N}) = 103.547 \text{ N}$$

$\mathbf{F} = 103.5 \text{ N}$ ↖ ◀

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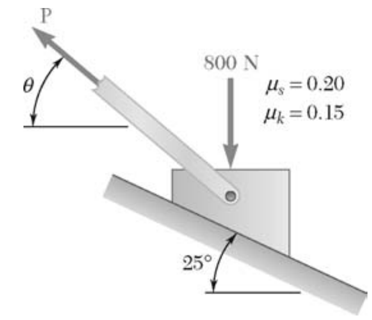


PROBLEM 8.5

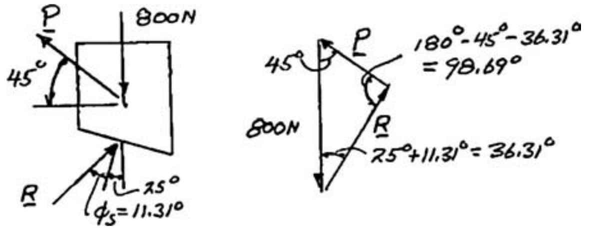
Knowing that $\theta = 45°$, determine the range of values of P for which equilibrium is maintained.

SOLUTION

<u>To start block up the incline</u>:

$$\mu_s = 0.20$$

$$\phi_s = \tan^{-1} 0.20 = 11.31°$$

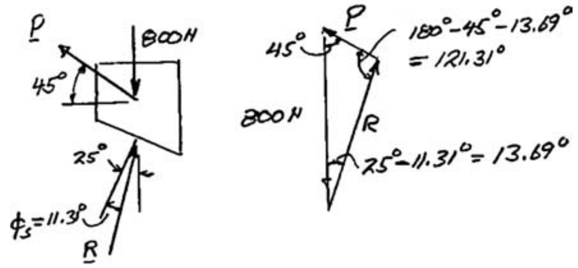


Force triangle:

$$\frac{P}{\sin 36.31°} = \frac{800 \text{ N}}{\sin 98.69°}$$

$P = 479.2$ N ◁

<u>To prevent block from moving down</u>:



Force triangle:

$$\frac{P}{\sin 13.69°} = \frac{800 \text{ N}}{\sin 121.31°}$$

$P = 221.61$ N ◁

Equilibrium is maintained for $222 \text{ N} \le P \le 479 \text{ N}$ ◀

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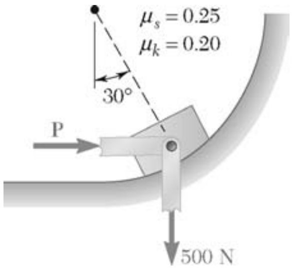


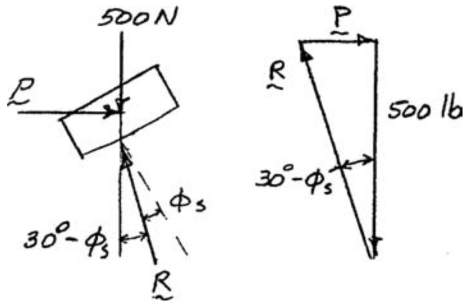
PROBLEM 8.6

Determine the range of values of P for which equilibrium of the block shown is maintained.

SOLUTION

FBD block:

(Impending motion down):

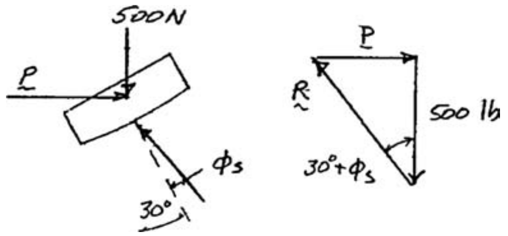


$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.25$$

$$P = (500 \text{ lb}) \tan(30° - \tan^{-1} 0.25)$$

$$= 143.03 \text{ lb}$$

(Impending motion up):



$$P = (500 \text{ lb}) \tan(30° + \tan^{-1} 0.25)$$

$$= 483.46 \text{ lb}$$

Equilibrium is maintained for $143.0 \text{ lb} \le P \le 483 \text{ lb}$ ◀

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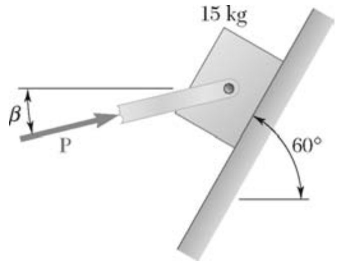
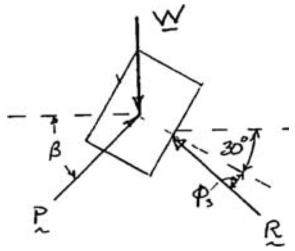


PROBLEM 8.7

Knowing that the coefficient of friction between the 15-kg block and the incline is $\mu_s = 0.25,$ determine (*a*) the smallest value of P required to maintain the block in equilibrium, (*b*) the corresponding value of β.

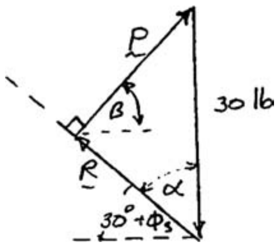
SOLUTION

FBD block (Impending motion downward):



$$W = (15 \text{ kg})(9.81 \text{ m/s}^2)$$
$$= 147.150 \text{ N}$$
$$\phi_s = \tan^{-1} \mu_s$$
$$= \tan^{-1}(0.25)$$
$$= 14.036°$$

(*a*) *Note:* For minimum *P*, $\mathbf{P} \perp \mathbf{R}$



So $\beta = \alpha$

$$= 90° - (30° + 14.036°)$$
$$= 45.964°$$

and $P = (147.150 \text{ N}) \sin \alpha$

$$= (147.150 \text{ N}) \sin(45.964°)$$

$P = 108.8 \text{ N}$ ◀

(*b*) $\beta = 46.0°$ ◀

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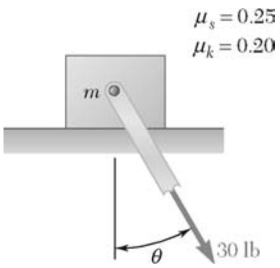
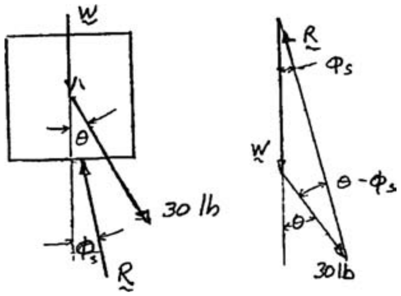


PROBLEM 8.8

Considering only values of θ less than 90°, determine the smallest value of θ required to start the block moving to the right when (*a*) $W = 75$ lb, (*b*) $W = 100$ lb.

SOLUTION

FBD block (Motion impending):



$$\phi_s = \tan^{-1}\mu_s = 14.036°$$

$$\frac{30\text{ lb}}{\sin\phi_s} = \frac{W}{\sin(\theta - \phi_s)}$$

$$\sin(\theta - \phi_s) = \frac{W \sin 14.036°}{30\text{ lb}}$$

or $$\sin(\theta - 14.036°) = \frac{W}{123.695\text{ lb}}$$

(*a*) $W = 75$ lb: $\theta = 14.036° + \sin^{-1}\dfrac{75\text{ lb}}{123.695\text{ lb}}$ $\quad\theta = 51.4°$ ◀

(*b*) $W = 100$ lb: $\theta = 14.036° + \sin^{-1}\dfrac{100\text{ lb}}{123.695\text{ lb}}$ $\quad\theta = 68.0°$ ◀

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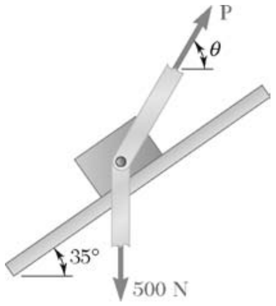


PROBLEM 8.9

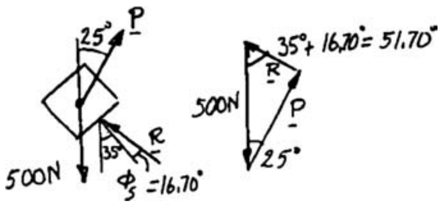
The coefficients of friction between the block and the rail are $\mu_s = 0.30$ and $\mu_k = 0.25.$ Knowing that $\theta = 65°,$ determine the smallest value of P required (*a*) to start the block moving up the rail, (*b*) to keep it from moving down.

SOLUTION

(*a*) To start block up the rail:

$$\mu_s = 0.30$$

$$\phi_s = \tan^{-1} 0.30 = 16.70°$$

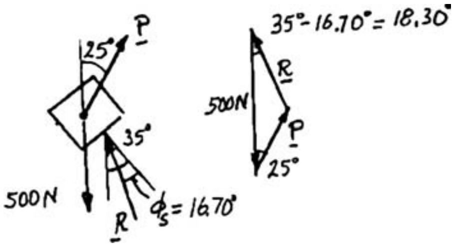


Force triangle:

$$\frac{P}{\sin 51.70°} = \frac{500 \text{ N}}{\sin(180° - 25° - 51.70°)}$$

$P = 403$ N ◀

(*b*) To prevent block from moving down:



Force triangle:

$$\frac{P}{\sin 18.30°} = \frac{500 \text{ N}}{\sin(180° - 25° - 18.30°)}$$

$P = 229$ N ◀

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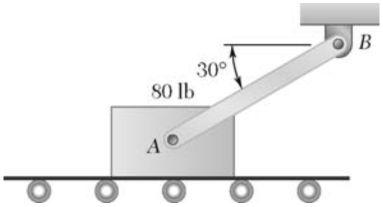


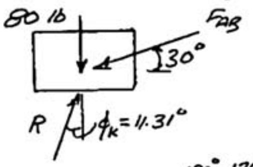
PROBLEM 8.10

The 80-lb block is attached to link *AB* and rests on a moving belt. Knowing that $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the magnitude of the horizontal force **P** that should be applied to the belt to maintain its motion (*a*) to the right, (*b*) to the left.

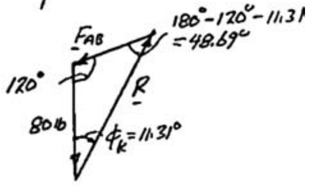
SOLUTION

We note that link *AB* is a two-force member, since there is motion between belt and block $\mu_k = 0.20$ and $\phi_k = \tan^{-1} 0.20 = 11.31°$

(*a*) Belt moves to right



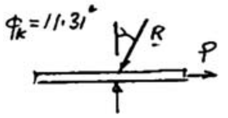
Free body: Block



Force triangle:

$$\frac{R}{\sin 120°} = \frac{80 \text{ lb}}{\sin 48.69°}$$

$$R = 92.23 \text{ lb}$$



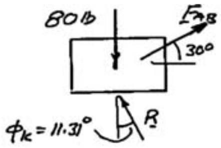
Free body: Belt

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - (92.23 \text{ lb}) \sin 11.31°$$

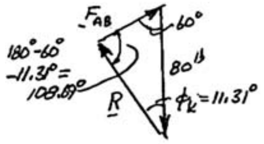
$$P = 18.089 \text{ lb}$$

$\mathbf{P} = 18.09 \text{ lb} \longrightarrow$ ◀

(*b*) Belt moves to left



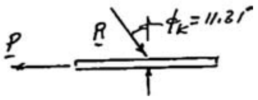
Free body: Block



Force triangle:

$$\frac{R}{\sin 60°} = \frac{80 \text{ lb}}{\sin 108.69°}$$

$$R = 73.139 \text{ lb}$$



Free body: Belt

$$\xrightarrow{+} \Sigma F_x = 0: \quad (73.139 \text{ lb}) \sin 11.31° - P = 0$$

$$P = 14.344 \text{ lb}$$

$\mathbf{P} = 14.34 \text{ lb} \longleftarrow$ ◀

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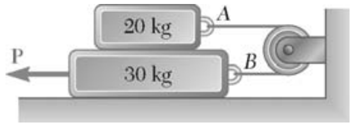
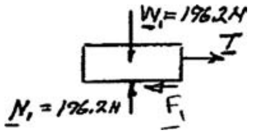


PROBLEM 8.11

The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the smallest force **P** required to start the 30-kg block moving if cable AB (*a*) is attached as shown, (*b*) is removed.

SOLUTION

(*a*) Free body: 20-kg block

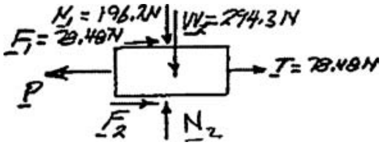


$$W_1 = (20 \text{ kg})(9.81 \text{ m/s}^2) = 196.2 \text{ N}$$

$$F_1 = \mu_s N_1 = 0.4(196.2 \text{ N}) = 78.48 \text{ N}$$

$$\xrightarrow{+} \Sigma F = 0: \quad T - F_1 = 0 \quad T = F_1 = 78.48 \text{ N}$$

Free body: 30-kg block



$$W_2 = (30 \text{ kg})(9.81 \text{ m/s}^2) = 294.3 \text{ N}$$

$$N_2 = 196.2 \text{ N} + 294.3 \text{ N} = 490.5 \text{ N}$$

$$F_2 = \mu_s N_2 = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$

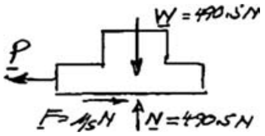
$$\xleftarrow{+} \Sigma F = 0: \quad P - F_1 - F_2 - T = 0$$

$$P = 78.48 \text{ N} + 196.2 \text{ N} + 78.48 \text{ N} = 353.2 \text{ N}$$

$\mathbf{P} = 353 \text{ N} \longleftarrow$ ◀

(*b*) Free body: Both blocks

Blocks move together



$$W = (50 \text{ kg})(9.81 \text{ m/s}^2) = 490.5 \text{ N}$$

$$\xleftarrow{+} \Sigma F = 0: \quad P - F = 0$$

$$P = \mu_s N = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$

$\mathbf{P} = 196.2 \text{ N} \longleftarrow$ ◀

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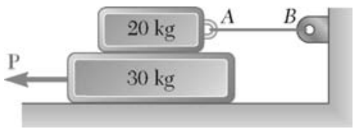


PROBLEM 8.12

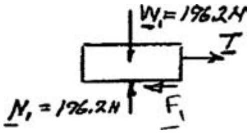
The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the smallest force **P** required to start the 30-kg block moving if cable *AB* (*a*) is attached as shown, (*b*) is removed.

SOLUTION

(*a*) Free body: 20-kg block

$$W_1 = (20 \text{ kg})(9.81 \text{ m/s}^2) = 196.2 \text{ N}$$

$$F_1 = \mu_s N_1 = 0.4(196.2 \text{ N}) = 78.48 \text{ N}$$



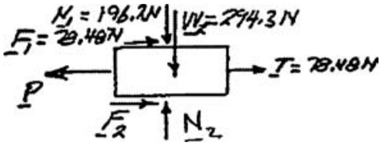
$$\xrightarrow{+} \Sigma F = 0: \quad T - F_1 = 0 \quad T = F_1 = 78.48 \text{ N}$$

Free body: 30-kg block

$$W_2 = (30 \text{ kg})(9.81 \text{ m/s}^2) = 294.3 \text{ N}$$

$$N_2 = 196.2 \text{ N} + 294.3 \text{ N} = 490.5 \text{ N}$$

$$F_2 = \mu_s N_2 = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$



$$\xleftarrow{+} \Sigma F = 0: \quad P - F_1 - F_2 = 0$$

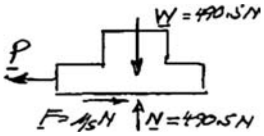
$$P = 78.48 \text{ N} + 196.2 \text{ N} = 274.7 \text{ N}$$

$\mathbf{P} = 275 \text{ N} \longleftarrow$ ◀

(*b*) Free body: Both blocks

Blocks move together

$$W = (50 \text{ kg})(9.81 \text{ m/s}^2) = 490.5 \text{ N}$$



$$\xleftarrow{+} \Sigma F = 0: \quad P - F = 0$$

$$P = \mu_s N = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$

$\mathbf{P} = 196.2 \text{ N} \longleftarrow$ ◀

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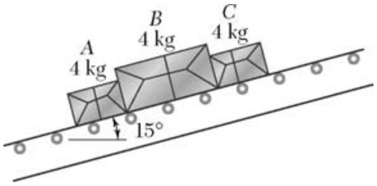
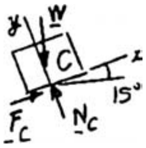


PROBLEM 8.13

Three 4-kg packages A, B, and C are placed on a conveyor belt that is at rest. Between the belt and both packages A and C the coefficients of friction are $\mu_s = 0.30$ and $\mu_k = 0.20$; between package B and the belt the coefficients are $\mu_s = 0.10$ and $\mu_k = 0.08$. The packages are placed on the belt so that they are in contact with each other and at rest. Determine which, if any, of the packages will move and the friction force acting on each package.

SOLUTION

<u>Consider C by itself</u>: Assume equilibrium



$$+\nwarrow \Sigma F_y = 0: \quad N_C - W\cos 15° = 0$$

$$N_C = W\cos 15° = 0.966W$$

$$+\nearrow \Sigma F_x = 0: \quad F_C - W\sin 15° = 0$$

$$F_C = W\sin 15° = 0.259W$$

But
$$F_m = \mu_s N_C = 0.30(0.966W) = 0.290W$$

Thus, $F_C < F_m$ <u>Package C does not move</u> ◀

$$F_C = 0.259W = 0.259(4 \text{ kg})(9.81 \text{ m/s}^2) = 10.16 \text{ N}$$

$$\mathbf{F}_C = 10.16 \text{ N} \nearrow$$ ◀

<u>Consider B by itself</u>: Assume equilibrium. We find,

$$F_B = 0.259W$$
$$N_B = 0.966W$$

But
$$F_m = \mu_s N_B = 0.10(0.966W) = 0.0966W$$

Thus, $F_B > F_m$. <u>Package B would move if alone</u> ◀

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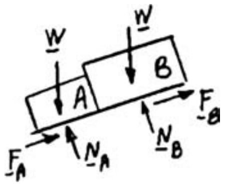
PROBLEM 8.13 (Continued)

Consider A and B together: Assume equilibrium



$$F_A = F_B = 0.259W$$
$$N_A = N_B = 0.966W$$
$$F_A + F_B = 2(0.259W) = 0.518W$$
$$(F_A)_m + (F_B)_m = 0.3N_A + 0.1N_B = 0.386W$$

Thus, $F_A + F_B > (F_A)_m + (F_B)_m$ A and B move ◀

$$F_A = \mu_k N_A = 0.2(0.966)(4)(9.81)$$

$\mathbf{F}_A = 7.58$ N ↗ ◀

$$F_B = \mu_k N_B = 0.08(0.966)(4)(9.81)$$

$\mathbf{F}_B = 3.03$ N ↗ ◀

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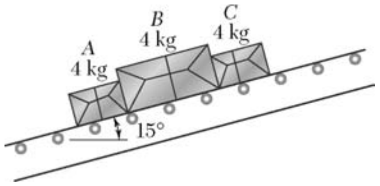


PROBLEM 8.14

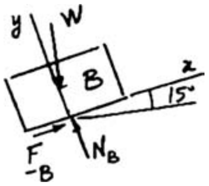
Solve Problem 8.13 assuming that package *B* is placed to the right of both packages *A* and *C*.

PROBLEM 8.13 Three 4-kg packages *A*, *B*, and *C* are placed on a conveyor belt that is at rest. Between the belt and both packages *A* and *C* the coefficients of friction are $\mu_s = 0.30$ and $\mu_k = 0.20$; between package *B* and the belt the coefficients are $\mu_s = 0.10$ and $\mu_k = 0.08$. The packages are placed on the belt so that they are in contact with each other and at rest. Determine which, if any, of the packages will move and the friction force acting on each package.

SOLUTION

Consider package *B* by itself: Assume equilibrium

$$+\nwarrow \Sigma F_y = 0: \quad N_B - W\cos 15° = 0$$



$$N_B = W\cos 15° = 0.966W$$

$$+\nearrow \Sigma F_x = 0: \quad F_B - W\sin 15° = 0$$

$$F_B = W\sin 15° = 0.259W$$

But

$$F_m = \mu_s N_B = 0.10(0.966W) = 0.0966W$$

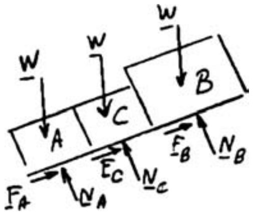
Thus, $F_B > F_m$. Package *B* would move if alone.

Consider all packages together: Assume equilibrium. In a manner similar to above, we find

$$N_A = N_B = N_C = 0.966W$$

$$F_A = F_B = F_C = 0.259W$$

$$F_A + F_B + F_C = 3(0.259W) = 0.777W$$



But

$$(F_A)_m = (F_C)_m = \mu_s N = 0.30(0.966W) = 0.290W$$

and

$$(F_B)_m = 0.10(0.966W) = 0.0966W$$

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PROBLEM 8.14 (Continued)

Thus, $$(F_A)_m + (F_C)_m + (F_B)_m = 2(0.290W) + 0.0966W$$
$$= 0.677W$$

and we note that $$F_A + F_B + F_C > (F_A)_m + (F_C)_m + (F_B)_m$$

All packages move ◀

$$F_A = F_C = \mu_k N$$
$$= 0.20(0.966)(4\text{ kg})(9.81\text{ m/s}^2)$$
$$= 7.58\text{ N}$$
$$F_B = \mu_k N$$
$$= 0.08(0.966)(4\text{ kg})(9.81\text{ m/s}^2)$$
$$= 3.03\text{ N}$$

$\mathbf{F}_A = \mathbf{F}_C = 7.58\text{ N}$ ↗ ; $\mathbf{F}_B = 3.03\text{ N}$ ↗ ◀

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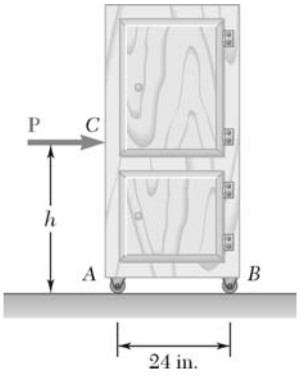
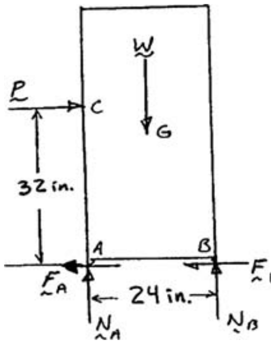


PROBLEM 8.15

A 120-lb cabinet is mounted on casters that can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. If $h = 32$ in., determine the magnitude of the force **P** required to move the cabinet to the right (*a*) if all casters are locked, (*b*) if the casters at *B* are locked and the casters at *A* are free to rotate, (*c*) if the casters at *A* are locked and the casters at *B* are free to rotate.

SOLUTION

FBD cabinet: *Note:* for tipping,



$$N_A = F_A = 0$$

$$\circlearrowleft \Sigma M_B = 0: \quad (12 \text{ in.})W - (32 \text{ in.})P_{\text{tip}} = 0$$

$$P_{\text{tip}} = 2.66667$$

(*a*) All casters locked. Impending slip:

$$F_A = \mu_s N_A$$
$$F_B = \mu_s N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0$$

$$N_A + N_B = W \qquad W = 120 \text{ lb}$$

So $$F_A + F_B = \mu_s W \qquad \mu_s = 0.3$$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_A - F_B = 0$$
$$P = F_A + F_B = \mu_s W$$

$$P = 0.3(120 \text{ lb}) \quad \text{or} \quad \mathbf{P} = 36.0 \text{ lb} \longrightarrow \blacktriangleleft$$

$$(P = 0.3W < P_{\text{tip}} \quad \text{OK})$$

(*b*) Casters at *A* free, so $\quad F_A = 0$

Impending slip: $\quad F_B = \mu_s N_B$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_B = 0$$

$$P = F_B = \mu_s N_B \qquad N_B = \frac{P}{\mu_s}$$

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PROBLEM 8.15 (Continued)

$$\circlearrowleft \Sigma M_A = 0: \quad (32 \text{ in.})P + (12 \text{ in.})W - (24 \text{ in.})N_B = 0$$

$$8P + 3W - 6\frac{P}{0.3} = 0 \quad P = 0.25W$$

$$(P = 0.25W < P_{\text{tip}} \quad \text{OK})$$

$$P = 0.25(120 \text{ lb})$$

or $\mathbf{P} = 30.0 \text{ lb} \longrightarrow$ ◀

(*c*) Casters at *B* free, so $F_B = 0$

Impending slip: $F_A = \mu_s N_A$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_A = 0 \quad P = F_A = \mu_s N_A$$

$$N_A = \frac{P}{\mu_s} = \frac{P}{0.3}$$

$$\circlearrowleft \Sigma M_B = 0: \quad (12 \text{ in.})W - (32 \text{ in.})P - (24 \text{ in.})N_A = 0$$

$$3W - 8P - 6\frac{P}{0.3} = 0$$

$$P = 0.107143W = 12.8572$$

$$(P < P_{\text{tip}} \quad \text{OK})$$

$\mathbf{P} = 12.86 \text{ lb} \longrightarrow$ ◀

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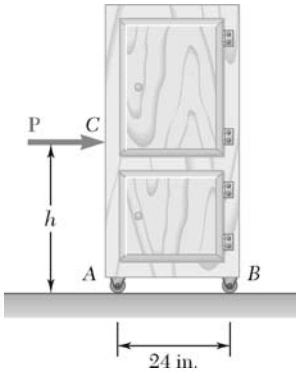
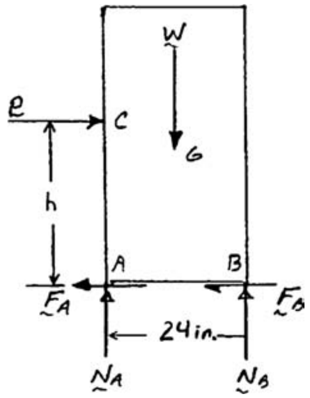


PROBLEM 8.16

A 120-lb cabinet is mounted on casters that can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. Assuming that the casters at both *A* and *B* are locked, determine (*a*) the force **P** required to move the cabinet to the right, (*b*) the largest allowable value of *h* if the cabinet is not to tip over.

SOLUTION

FBD cabinet:



(*a*) $\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0$

$$N_A + N_B = W$$

Impending slip:

$$F_A = \mu_s N_A$$
$$F_B = \mu_s N_B$$

So

$$F_A + F_B = \mu_s W$$

$\longrightarrow \Sigma F_x = 0: \quad P - F_A - F_B = 0$

$$P = F_A + F_B = \mu_s W$$

$W = 120$ lb

$\mu_s = 0.3$

$$P = 0.3(120\text{ lb}) = 141.26\text{ N}$$

$\mathbf{P} = 36.0\text{ lb} \longrightarrow$ ◀

(*b*) For tipping, $N_A = F_A = 0$

$\circlearrowleft \Sigma M_B = 0: \quad hP - (12\text{ in.})W = 0$

$$h_{\max} = (12\text{ in.})\frac{W}{P} = (12\text{ in.})\frac{1}{\mu_s} = \frac{12\text{ in.}}{0.3}$$

$h_{\max} = 40.0$ in. ◀

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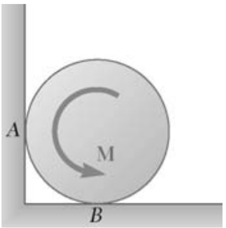
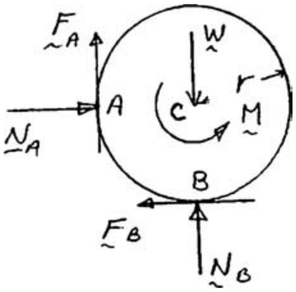


PROBLEM 8.17

The cylinder shown is of weight W and radius r, and the coefficient of static friction μ_s is the same at A and B. Determine the magnitude of the largest couple **M** that can be applied to the cylinder if it is not to rotate.

SOLUTION

FBD cylinder:



For maximum M, motion impends at both A and B

$$F_A = \mu_s N_A$$
$$F_B = \mu_s N_B$$

$$\rightarrow \Sigma F_x = 0: \quad N_A - F_B = 0$$
$$N_A = F_B = \mu_s N_B$$
$$F_A = \mu_s N_A = \mu_s^2 N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B + F_A - W = 0$$
$$N_B + \mu_s^2 N_B = W$$

or
$$N_B = \frac{W}{1+\mu_s^2}$$

and
$$F_B = \frac{\mu_s W}{1+\mu_s^2}$$
$$F_A = \frac{\mu_s^2 W}{1+\mu^2}$$

$$\Sigma M_C = 0: \quad M - r(F_A + F_B) = 0$$
$$M = r\left(\mu_s + \mu_s^2\right)\frac{W}{1+\mu_s^2}$$

$$M_{\max} = Wr\mu_s \frac{1+\mu_s}{1+\mu_s^2} \blacktriangleleft$$

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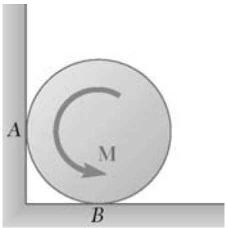
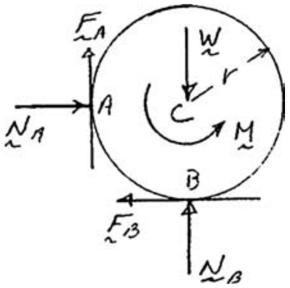


PROBLEM 8.18

The cylinder shown is of weight W and radius r. Express in terms W and r the magnitude of the largest couple **M** that can be applied to the cylinder if it is not to rotate, assuming the coefficient of static friction to be (*a*) zero at A and 0.30 at B, (*b*) 0.25 at A and 0.30 at B.

SOLUTION

FBD cylinder:



For maximum M, motion impends at both A and B

$$F_A = \mu_A N_A$$
$$F_B = \mu_B N_B$$

$$\rightarrow \Sigma F_x = 0: \quad N_A - F_B = 0$$
$$N_A = F_B = \mu_B N_B$$
$$F_A = \mu_A N_A = \mu_A \mu_B N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B + F_A - W = 0$$
$$N_B(1 + \mu_A \mu_B) = W$$

or

$$N_B = \frac{1}{1 + \mu_A \mu_B} W$$

and

$$F_B = \mu_B N_B = \frac{\mu_B}{1 + \mu_A \mu_B} W$$
$$F_A = \mu_A \mu_B N_B = \frac{\mu_A \mu_B}{1 + \mu_A \mu_B} W$$

$$\Sigma M_C = 0: \quad M - r(F_A + F_B) = 0$$
$$M = Wr\mu_B \frac{1 + \mu_A}{1 + \mu_A \mu_B}$$

(*a*) For $\mu_A = 0$ and $\mu_B = 0.30$:

$M = 0.300Wr$ ◀

(*b*) For $\mu_A = 0.25$ and $\mu_B = 0.30$:

$M = 0.349Wr$ ◀

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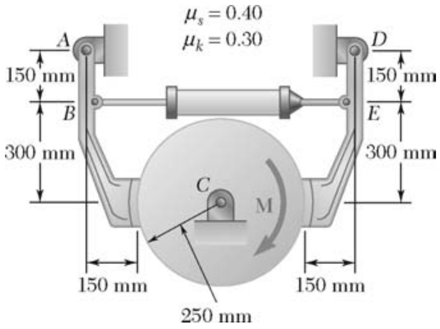
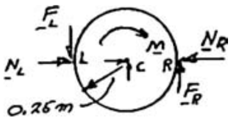


PROBLEM 8.19

The hydraulic cylinder shown exerts a force of 3 kN directed to the right on Point *B* and to the left on Point *E*. Determine the magnitude of the couple **M** required to rotate the drum clockwise at a constant speed.

SOLUTION

<u>Free body: Drum</u>



$$+\circlearrowright \Sigma M_C = 0: \quad M - (0.25\text{ m})(F_L + F_R) = 0$$

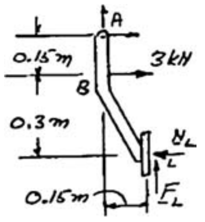
$$M = (0.25\text{ m})(F_L + F_R) \qquad (1)$$

Since drum is rotating

$$F_L = \mu_k N_L = 0.3 N_L$$
$$F_R = \mu_k N_R = 0.3 N_R$$

<u>Free body: Left arm *ABL*</u>

$$+\circlearrowright \Sigma M_A = 0: \quad (3\text{ kN})(0.15\text{ m}) + F_L(0.15\text{ m}) - N_L(0.45\text{ m}) = 0$$

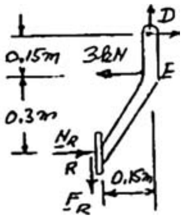


$$0.45\text{ kN}\cdot\text{m} + (0.3N_L)(0.15\text{ m}) - N_L(0.45\text{ m}) = 0$$
$$0.405 N_L = 0.45$$

$$N_L = 1.111\text{ kN}$$
$$F_L = 0.3N_L = 0.3(1.111\text{ kN})$$
$$= 0.3333\text{ kN} \qquad (2)$$

<u>Free body: Right arm *DER*</u>

$$+\circlearrowright \Sigma M_D = 0: \quad (3\text{ kN})(0.15\text{ m}) - F_R(0.15\text{ m}) - N_R(0.45\text{ m}) = 0$$



$$0.45\text{ kN}\cdot\text{m} - (0.3N_R)(0.15\text{ m}) - N_R(0.45\text{ m}) = 0$$

$$0.495 N_R = 0.45$$

$$N_R = 0.9091\text{ kN}$$
$$F_R = \mu_k N_R = 0.3(0.9091\text{ kN})$$
$$= 0.2727\text{ kN} \qquad (3)$$

Substitute for F_L and F_R into (1): $\quad M = (0.25\text{ m})(0.333\text{ kN} + 0.2727\text{ kN})$

$$M = 0.1515\text{ kN}\cdot\text{m}$$

$$M = 151.5\text{ N}\cdot\text{m} \blacktriangleleft$$

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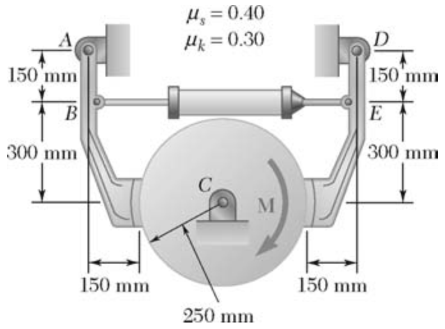
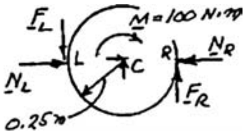


PROBLEM 8.20

A couple **M** of magnitude 100 N · m is applied to the drum as shown. Determine the smallest force that must be exerted by the hydraulic cylinder on joints *B* and *E* if the drum is not to rotate.

SOLUTION

<u>Free body: Drum</u>



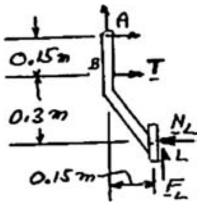
$$+\circlearrowleft \Sigma M_C = 0: \quad 100 \text{ N} \cdot \text{m} - (0.25 \text{ m})(F_L + F_R) = 0$$

$$F_L + F_R = 400 \text{ N} \qquad (1)$$

Since motion impends

$$F_L = \mu_s N_L = 0.4 N_L$$
$$F_R = \mu_s N_R = 0.4 N_R$$

<u>Free body: Left arm *ABL*</u>



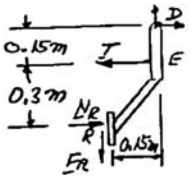
$$+\circlearrowleft \Sigma M_A = 0: \quad T(0.15 \text{ m}) + F_L(0.15 \text{ m}) - N_L(0.45 \text{ m}) = 0$$

$$0.15T + (0.4N_L)(0.15 \text{ m}) - N_L(0.45 \text{ m}) = 0$$

$$0.39N_L = 0.15T; \quad N_L = 0.38462T$$

$$F_L = 0.4N_L = 0.4(0.38462T)$$
$$F_L = 0.15385T \qquad (2)$$

<u>Free body: Right arm *DER*</u>



$$+\circlearrowleft \Sigma M_D = 0: \quad T(0.15 \text{ m}) - F_R(0.15 \text{ m}) - N_R(0.45 \text{ m}) = 0$$

$$0.15T - (0.4N_R)(0.15 \text{ m}) - N_R(0.45 \text{ m}) = 0$$

$$0.51N_R = 0.15T; \quad N_R = 0.29412T$$

$$F_R = 0.4N_R = 0.4(0.29412T)$$
$$F_R = 0.11765T \qquad (3)$$

Substitute for F_L and F_R into Eq. (1):

$$0.15385T + 0.11765T = 400$$
$$T = 1473.3 \text{ N}$$

$$T = 1.473 \text{ kN} \blacktriangleleft$$

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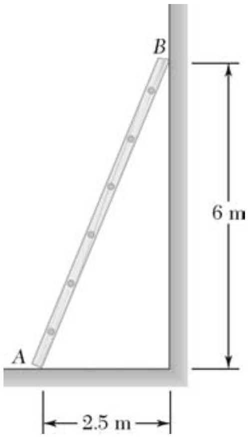


PROBLEM 8.21

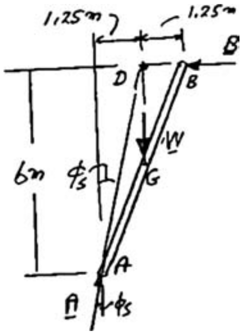
A 6.5-m ladder *AB* leans against a wall as shown. Assuming that the coefficient of static friction μ_s is zero at *B*, determine the smallest value of μ_s at *A* for which equilibrium is maintained.

SOLUTION

Free body: Ladder

Three-force body.

Line of action of **A** must pass through *D*, where **W** and **B** intersect.



At *A*: $$\mu_s = \tan\phi_s = \frac{1.25\text{ m}}{6\text{ m}} = 0.2083$$

$\mu_s = 0.208$ ◀

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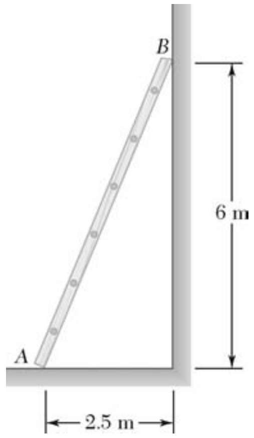


PROBLEM 8.22

A 6.5-m ladder *AB* leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at *A* and *B*, determine the smallest value of μ_s for which equilibrium is maintained.

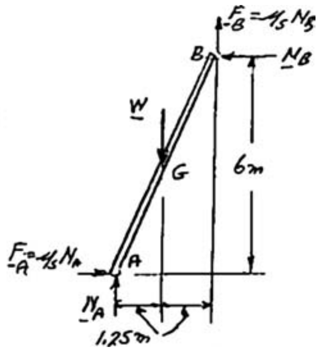
SOLUTION

Free body: Ladder

Motion impending:

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$



$$+\circlearrowleft \Sigma M_A = 0: \quad W(1.25 \text{ m}) - N_B(6 \text{ m}) - \mu_s N_B(2.5 \text{ m}) = 0$$

$$N_B = \frac{1.25W}{6 + 2.5\mu_s} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_A + \mu_s N_B - W = 0$$

$$N_A = W - \mu_s N_B$$

$$N_A = W - \frac{1.25\mu_s W}{6 + 2.5\mu_s} \qquad (2)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad \mu_s N_A - N_B = 0$$

Substitute for N_A and N_B from Eqs. (1) and (2):

$$\mu_s W - \frac{1.25\mu_s^2 W}{6 + 2.5\mu_s} = \frac{1.25W}{6 + 2.5\mu_s}$$

$$6\mu_s + 2.5\mu_s^2 - 1.25\mu_s^2 = 1.25$$

$$1.25\mu_s^2 + 6\mu_s - 1.25 = 0$$

$$\mu_s = 0.2$$

and $\mu_s = -5$ (Discard)

$$\mu_s = 0.200 \blacktriangleleft$$

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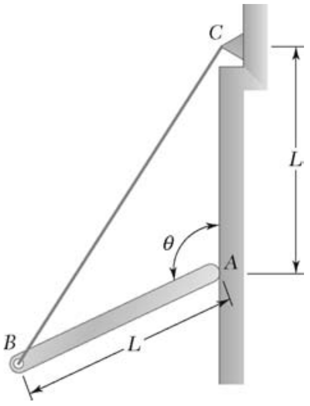
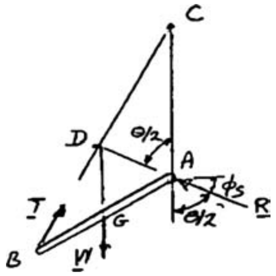


PROBLEM 8.23

End A of a slender, uniform rod of length L and weight W bears on a surface as shown, while end B is supported by a cord BC. Knowing that the coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine (a) the largest value of θ for which motion is impending, (b) the corresponding value of the tension in the cord.

SOLUTION



Free-body diagram

Three-force body. Line of action of **R** must pass through D, where **T** and **R** intersect.

Motion impends:

$$\tan\phi_s = 0.4$$

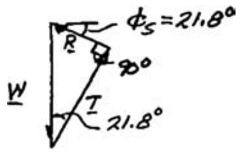
$$\phi_s = 21.80°$$

(a) Since $BG = GA$, it follows that $BD = DC$ and AD bisects $\angle BAC$

$$\frac{\theta}{2} + \phi_s = 90°$$

$$\frac{\theta}{2} + 21.8° = 90°$$

$$\theta = 136.4° \blacktriangleleft$$



(b) Force triangle (right triangle):

$$T = W\cos 21.8°$$

$$T = 0.928W \blacktriangleleft$$

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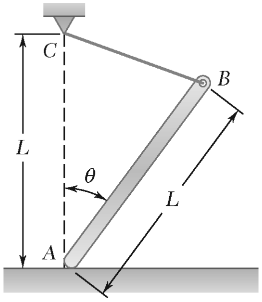
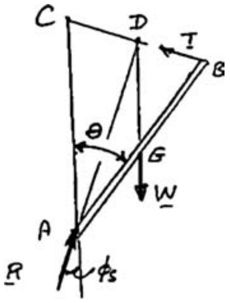


PROBLEM 8.24

End A of a slender, uniform rod of length L and weight W bears on a surface as shown, while end B is supported by a cord BC. Knowing that the coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine (*a*) the largest value of θ for which motion is impending, (*b*) the corresponding value of the tension in the cord.

SOLUTION



Free-body diagram

Rod *AB* is a three-force body. Thus, line of action of **R** must pass through *D*, where **W** and **T** intersect.

Since $AG = GB$, $CD = DB$ and the median *AD* of the isosceles triangle *ABC* bisects the angle θ.

(*a*) Thus,

$$\phi_s = \frac{1}{2}\theta$$

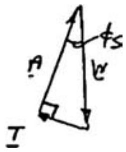
Since motion impends,

$$\phi_s = \tan^{-1} 0.40 = 21.80°$$
$$\theta = 2\phi_s = 2(21.8°)$$

$\theta = 43.6°$ ◀

(*b*) Force triangle:

This is a right triangle.



$$T = W \sin\phi_s$$
$$= W \sin 21.8°$$

$T = 0.371W$ ◀

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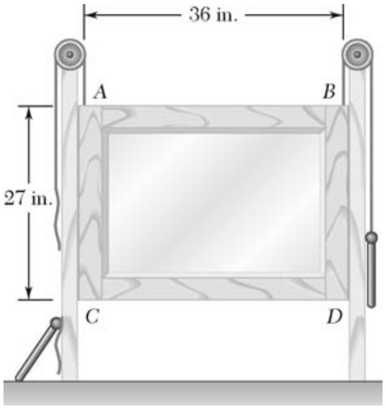
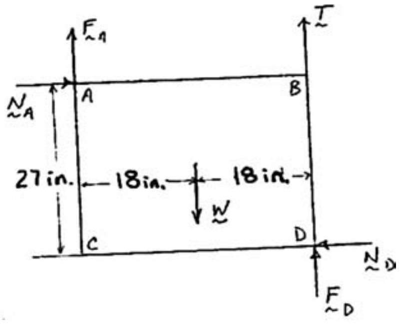


PROBLEM 8.25

A window sash weighing 10 lb is normally supported by two 5-lb sash weights. Knowing that the window remains open after one sash cord has broken, determine the smallest possible value of the coefficient of static friction. (Assume that the sash is slightly smaller than the frame and will bind only at Points *A* and *D*.)

SOLUTION

FBD window: $T = 5\text{ lb}$



$$\xrightarrow{} \Sigma F_x = 0: \quad N_A - N_D = 0$$

$$N_A = N_D$$

Impending motion:

$$F_A = \mu_s N_A$$

$$F_D = \mu_s N_D$$

$$\circlearrowleft \Sigma M_D = 0: \quad (18\text{ in.})W - (27\text{ in.})N_A - (36\text{ in.})F_A = 0 \qquad W = 10\text{ lb}$$

$$W = \frac{3}{2}N_A + 2\mu_s N_A$$

$$N_A = \frac{2W}{3 + 4\mu_s}$$

$$\uparrow \Sigma F_y = 0: \quad F_A - W + T + F_D = 0$$

$$F_A + F_D = W - T = \frac{W}{2}$$

Now

$$F_A + F_D = \mu_s (N_A + N_D)$$

$$= 2\mu_s N_A$$

Then

$$\frac{W}{2} = 2\mu_s \frac{2W}{3 + 4\mu_s}$$

or $\mu_s = 0.750$ ◀

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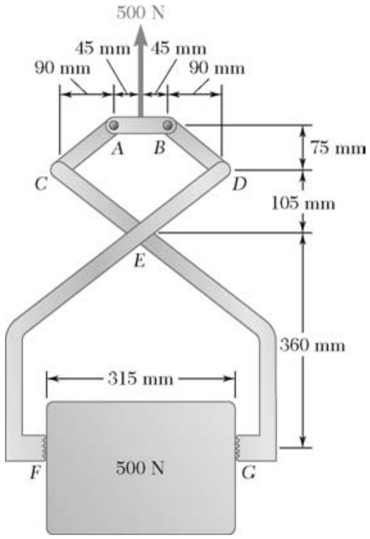
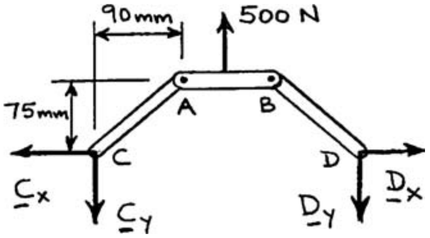


PROBLEM 8.26

A 500-N concrete block is to be lifted by the pair of tongs shown. Determine the smallest allowable value of the coefficient of static friction between the block and the tongs at *F* and *G*.

SOLUTION



Free body: Members *CA*, *AB*, *BD*

By symmetry: $$C_y = D_y = \frac{1}{2}(500) = 250 \text{ N}$$

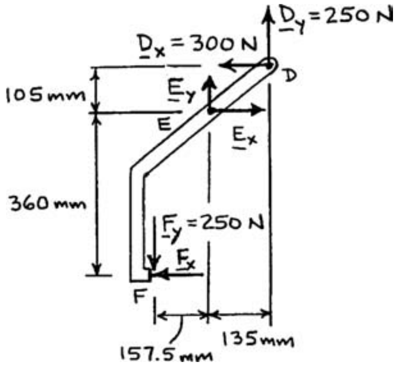
Since *CA* is a two-force member,

$$\frac{C_x}{90 \text{ mm}} = \frac{C_y}{75 \text{ mm}} = \frac{250 \text{ N}}{75 \text{ mm}}$$

$$C_x = 300 \text{ N}$$

$$\Sigma F_x = 0: \quad D_x = C_x$$

$$D_x = 300 \text{ N}$$



Free body: Tong *DEF*

$$+\circlearrowleft \Sigma M_E = 0: \quad (300 \text{ N})(105 \text{ mm}) + (250 \text{ N})(135 \text{ mm}) + (250 \text{ N})(157.5 \text{ mm}) - F_x(360 \text{ mm}) = 0$$

$$F_x = +290.625 \text{ N}$$

Minimum value of μ_s: $$\mu_s = \frac{F_y}{F_x} = \frac{250 \text{ N}}{290.625 \text{ N}}$$

$\mu_s = 0.860$ ◀

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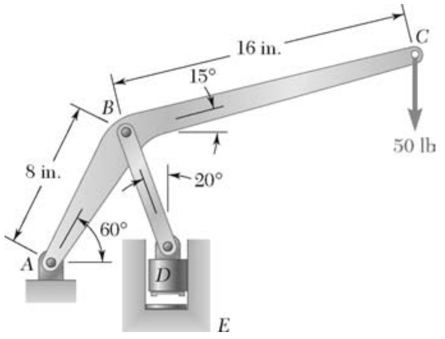
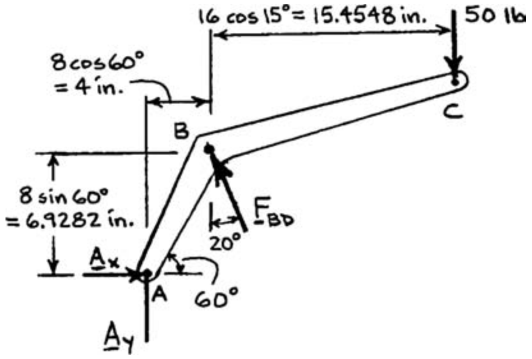


PROBLEM 8.27

The press shown is used to emboss a small seal at E. Knowing that the coefficient of static friction between the vertical guide and the embossing die D is 0.30, determine the force exerted by the die on the seal.

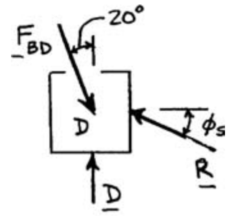
SOLUTION

Free body: Member ABC



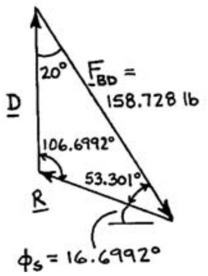
$$+\circlearrowleft \Sigma M_A = 0: \quad F_{BD} \cos 20°(4) + F_{BD} \sin 20°(6.9282)$$
$$-(50 \text{ lb})(4 + 15.4548) = 0$$
$$F_{BD} = 158.728 \text{ lb}$$

Free body: Die D



$$\phi_s = \tan^{-1} \mu_s$$
$$= \tan^{-1} 0.3$$
$$= 16.6992°$$

Force triangle:



$$\frac{D}{\sin 53.301°} = \frac{158.728 \text{ lb}}{\sin 106.6992°}$$
$$D = 132.869 \text{ lb}$$

On seal: $\mathbf{D} = 132.9 \text{ lb} \downarrow$ ◀

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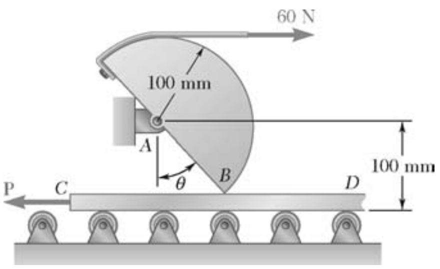
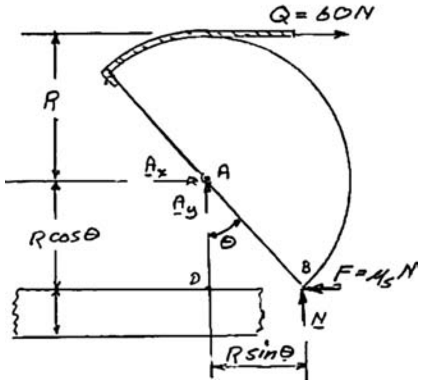


PROBLEM 8.28

The 100-mm-radius cam shown is used to control the motion of the plate *CD*. Knowing that the coefficient of static friction between the cam and the plate is 0.45 and neglecting friction at the roller supports, determine (*a*) the force **P** required to maintain the motion of the plate, knowing that the plate is 20 mm thick, (*b*) the largest thickness of the plate for which the mechanism is self locking (i.e., for which the plate cannot be moved however large the force **P** may be).

SOLUTION

Free body: Cam



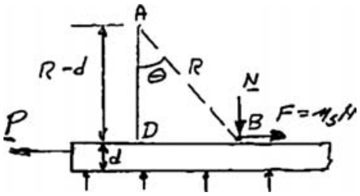
Impending motion: $F = \mu_s N$

$$+\circlearrowleft \Sigma M_A = 0: \quad QR - NR\sin\theta + (\mu_s N)R\cos\theta = 0$$

$$N = \frac{Q}{\sin\theta - \mu_s\cos\theta} \tag{1}$$

Free body: Plate $\quad \Sigma F_x = 0 \quad P = \mu_s N \quad (2)$

Geometry in ΔABD with $R = 100$ mm and $d = 20$ mm



$$\cos\theta = \frac{R-d}{R} = \frac{80\text{ mm}}{100\text{ mm}} = 0.8$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = 0.6$$

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PROBLEM 8.28 (Continued)

(*a*) Eq. (1) using $Q = 60 \text{ N} \quad \text{and} \quad \mu_s = 0.45$

$$N = \frac{60 \text{ N}}{0.6 - (0.45)(0.8)}$$

$$= \frac{60}{0.24} = 250 \text{ N}$$

Eq. (2) $P = \mu_s N = (0.45)(250 \text{ N})$

$P = 112.5 \text{ N}$ ◀

(*b*) <u>For $P = \infty$,</u> $N = \infty$. Denominator is zero in Eq. (1).

$$\sin\theta - \mu_s \cos\theta = 0$$

$$\tan\theta = \mu_s = 0.45$$

$$\theta = 24.23°$$

$$\cos\theta = \frac{R - d}{R}$$

$$\cos 24.23 = \frac{100 - d}{100}$$

$d = 8.81 \text{ mm}$ ◀

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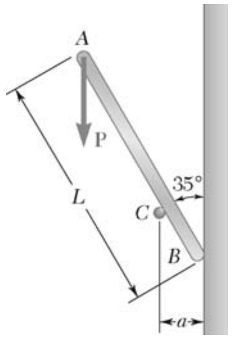
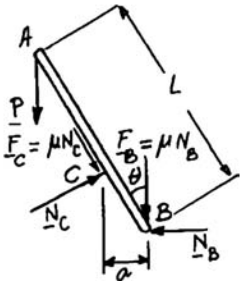


PROBLEM 8.29

A slender rod of length *L* is lodged between peg *C* and the vertical wall and supports a load **P** at end *A*. Knowing that the coefficient of static friction is 0.20 at both *B* and *C*, find the range of values of the ratio *L*/*a* for which equilibrium is maintained.

SOLUTION

We shall first assume that the motion of end *B* is *impending upward*. The friction forces at *B* and *C* will have the values and directions indicated in the *FB* diagram.



$$+\circlearrowleft \Sigma M_B = 0: \quad PL\sin\theta - N_C\left(\frac{a}{\sin\theta}\right) = 0$$

$$N_C = \frac{PL}{a}\sin^2\theta \qquad (1)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_C\cos\theta + \mu N_C\sin\theta - N_B = 0 \qquad (2)$$

$$+\uparrow \Sigma F_y = 0: \quad N_C\sin\theta - \mu N_C\cos\theta - \mu N_B - P = 0 \qquad (3)$$

Multiply Eq. (2) by μ and subtract from Eq. (3):

$$N_C(\sin\theta - \mu\cos\theta) - \mu N_C(\cos\theta + \mu\sin\theta) - P = 0$$

$$P = N_C[\sin\theta(1-\mu^2) - 2\mu\cos\theta]$$

Substitute for N_C from Eq. (1) and solve for *a*/*L*:

$$\frac{a}{L} = \sin^2\theta[(1-\mu^2)\sin\theta - 2\mu\cos\theta] \qquad (4)$$

Making $\theta = 35°$ and $\mu = 0.20$ in Eq. (4):

$$\frac{a}{L} = \sin^2 35°[(1-0.04)\sin 35° - 2(0.20)\cos 35°]$$

$$= 0.07336$$

$$\frac{L}{a} = 13.63 \quad \triangleleft$$

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PROBLEM 8.29 (Continued)

Assuming now that the motion at B is *impending downward,* we should reverse the direction of $\mathbf{F}_B$ and $\mathbf{F}_C$ in. the *FB* diagram. The same result may be obtained by making $\theta = 35°$ and $\mu = -0.20$ in Eq. (4):

$$\frac{a}{L} = \sin^2 35°[(1 - 0.04)\sin 35° - 2(-0.20)\cos 35°]$$
$$= 0.2889$$

$$\frac{L}{a} = 3.461 \quad \triangleleft$$

Thus, the range of values of L/a for which equilibrium is maintained is $\quad 3.46 \le \frac{L}{a} \le 13.63 \quad \blacktriangleleft$

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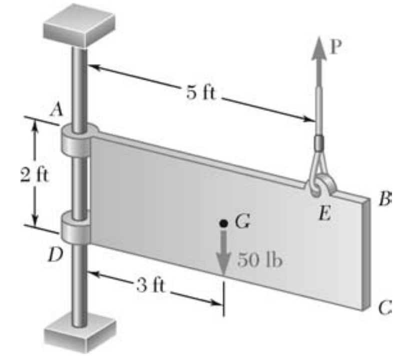
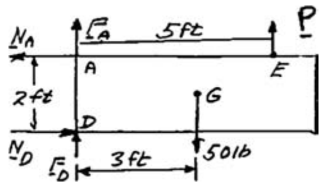


PROBLEM 8.30

The 50-lb plate *ABCD* is attached at *A* and *D* to collars that can slide on the vertical rod. Knowing that the coefficient of static friction is 0.40 between both collars and the rod, determine whether the plate is in equilibrium in the position shown when the magnitude of the vertical force applied at *E* is (*a*) $P = 0$, (*b*) $P = 20$ lb.

SOLUTION



(*a*) $\underline{P = 0}$

$$+\circlearrowleft \Sigma M_D = 0: \quad N_A(2\text{ ft}) - (50\text{ lb})(3\text{ ft}) = 0$$

$$N_A = 75\text{ lb}$$

$$\Sigma F_x = 0: \quad N_D = N_A = 75\text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + F_D - 50\text{ lb} = 0$$

$$F_A + F_D = 50\text{ lb}$$

But:
$$(F_A)_m = \mu_s N_A = 0.40(75\text{ lb}) = 30\text{ lb}$$
$$(F_D)_m = \mu_s N_D = 0.40(75\text{ lb}) = 30\text{ lb}$$

Thus:
$$(F_A)_m + (F_D)_m = 60\text{ lb}$$

and
$$(F_A)_m + (F_D)_m > F_A + F_D$$

Plate is in equilibrium ◀

(*b*) $\underline{P = 20\text{ lb}}$

$$+\circlearrowleft \Sigma M_D = 0: \quad N_A(2\text{ ft}) - (50\text{ lb})(3\text{ ft}) + (20\text{ lb})(5\text{ ft}) = 0$$

$$N_A = 25\text{ lb}$$

$$\Sigma F_x = 0: \quad N_D = N_A = 25\text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + F_D - 50\text{ lb} + 20\text{ lb} = 0$$

$$F_A + F_D = 30\text{ lb}$$

But:
$$(F_A)_m = \mu_s N_A = 0.4(25\text{ lb}) = 10\text{ lb}$$
$$(F_D)_m = \mu_s N_D = 0.4(25\text{ lb}) = 10\text{ lb}$$

Thus:
$$(F_A)_m + (F_D)_m = 20\text{ lb}$$

and
$$F_A + F_D > (F_A)_m + (F_D)_m$$

Plate moves downward ◀

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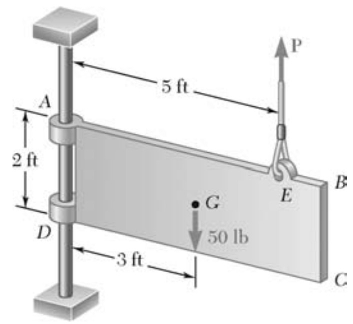
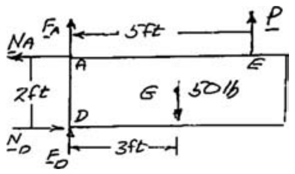


PROBLEM 8.31

In Problem 8.30, determine the range of values of the magnitude P of the vertical force applied at E for which the plate will move downward.

PROBLEM 8.30 The 50-lb plate $ABCD$ is attached at A and D to collars that can slide on the vertical rod. Knowing that the coefficient of static friction is 0.40 between both collars and the rod, determine whether the plate is in equilibrium in the position shown when the magnitude of the vertical force applied at E is (*a*) $P = 0$, (*b*) $P = 20$ lb.

SOLUTION



We shall consider the following two cases:

(1) $0 < P < 30$ lb

$$+\circlearrowleft \Sigma M_D = 0: \quad N_A(2\text{ ft}) - (50\text{ lb})(3\text{ ft}) + P(5\text{ ft}) = 0$$

$$N_A = 75\text{ lb} - 2.5P$$

(*Note:* $N_A \geq 0$ and directed $\longleftarrow$ for $P \leq 30$ lb as assumed here)

$$\Sigma F_x = 0: \quad N_A = N_D$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + F_D + P - 50 = 0$$

$$F_A + F_D = 50 - P$$

But:

$$\begin{aligned}(F_A)_m = (F_0)_m &= \mu_s N_A \\ &= 0.40(75 - 2.5P) \\ &= 30 - P\end{aligned}$$

Plate moves $\downarrow$ if: $\quad F_A + F_D > (F_A)_m + (F_D)_m$

or $\quad 50 - P > (30 - P) + (30 - P) \qquad P > 10\text{ lb} \;\triangleleft$

(2) $30\text{ lb} < P < 50$ lb

$$+\circlearrowleft \Sigma M_D = 0: \quad -N_A(2\text{ ft}) - (50\text{ lb})(3\text{ ft}) + P(5\text{ ft}) = 0$$

$$N_A = 2.5P - 75$$

(*Note:* $N_A >$ and directed $\longrightarrow$ for $P > 30$ lb as assumed)

$$\Sigma F_x = 0: \quad N_A = N_D$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + F_D + P - 50 = 0$$

$$F_A + F_D = 50 - P$$

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PROBLEM 8.31 (Continued)

But: $$(F_A)_m = (F_D)_m = \mu_s N_A$$
$$= 0.40(2.5P - 75)$$
$$= P - 30 \text{ lb}$$

Plate moves ↓ if: $$F_A + F_D > (F_A)_m + (F_D)_m$$

$$50 - P > (P - 30) + (P - 30) \qquad P < \frac{110}{3} = 36.7 \text{ lb} \;\triangleleft$$

Thus, plate moves downward for: $10.00 \text{ lb} < P < 36.7 \text{ lb}$ ◀

(*Note:* For $P > 50$ lb, plate is in equilibrium)

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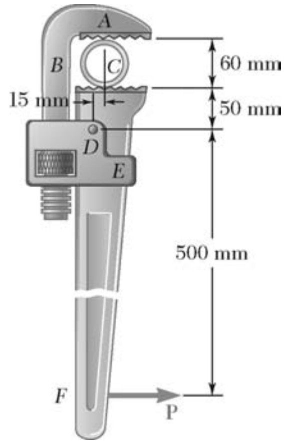
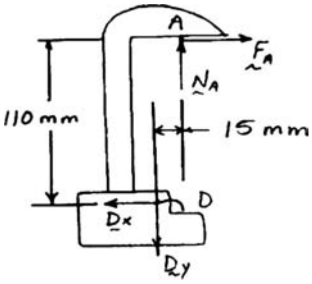


PROBLEM 8.32

A pipe of diameter 60 mm is gripped by the stillson wrench shown. Portions *AB* and *DE* of the wrench are rigidly attached to each other, and portion *CF* is connected by a pin at *D*. If the wrench is to grip the pipe and be self-locking, determine the required minimum coefficients of friction at *A* and *C*.

SOLUTION

FBD *ABD*:



$$\circlearrowleft \Sigma M_D = 0: \quad (15 \text{ mm})N_A - (110 \text{ mm})F_A = 0$$

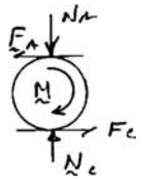
Impending motion: $F_A = \mu_A N_A$

Then $15 - 110\mu_A = 0$

or $\mu_A = 0.13636$ $\qquad \mu_A = 0.1364$ ◀

$$\rightarrow \Sigma F_x = 0: \quad F_A - D_x = 0 \quad D_x = F_A$$

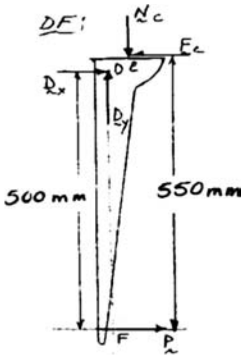
FBD pipe:



$$\uparrow \Sigma F_y = 0: \quad N_C - N_A = 0$$

$$N_C = N_A$$

FBD *DF*:



$$\circlearrowleft \Sigma M_F = 0: \quad (550 \text{ mm})F_C - (15 \text{ mm})N_C - (500 \text{ mm})D_x = 0$$

Impending motion: $F_C = \mu_C N_C$

Then $550\mu_C - 15 = 500\dfrac{F_A}{N_C}$

But $N_C = N_A$ and $\dfrac{F_A}{N_A} = \mu_A = 0.13636$

So $550\mu_C = 15 + 500(0.13636)$ $\qquad \mu_C = 0.1512$ ◀

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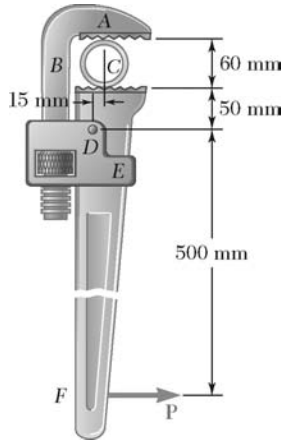


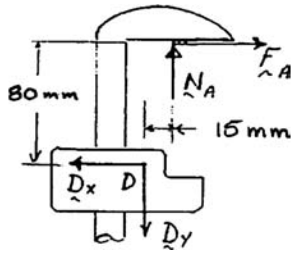
PROBLEM 8.33

Solve Problem 8.32 assuming that the diameter of the pipe is 30 mm.

PROBLEM 8.32 A pipe of diameter 60 mm is gripped by the stillson wrench shown. Portions *AB* and *DE* of the wrench are rigidly attached to each other, and portion *CF* is connected by a pin at *D*. If the wrench is to grip the pipe and be self-locking, determine the required minimum coefficients of friction at *A* and *C*.

SOLUTION

FBD *ABD*:



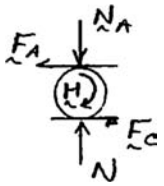
$$\circlearrowleft \Sigma M_D = 0: \quad (15 \text{ mm})N_A - (80 \text{ mm})F_A = 0$$

Impending motion: $F_A = \mu_A N_A$

Then $15 \text{ mm} - (80 \text{ mm})\mu_A = 0$ $\mu_A = 0.1875$ ◀

$$\rightarrow \Sigma F_x = 0: \quad F_A - D_x = 0$$

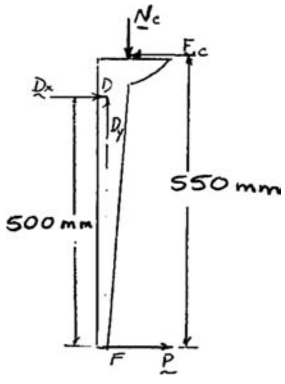
So that $D_x = F_A = 0.1875 N_A$



FBD pipe: $\uparrow \Sigma F_y = 0: \quad N_C - N_A = 0$

$$N_C = N_A$$

FBD *DF*:



$$\circlearrowleft \Sigma M_F = 0: \quad (550 \text{ mm})F_C - (15 \text{ mm})N_C - (500 \text{ mm})D_x = 0$$

Impending motion: $F_C = \mu_C N_C$

$$550\mu_C - 15 = 500(0.1875)\frac{N_A}{N_C}$$

But $N_A = N_C$ (from pipe *FBD*) so $\dfrac{N_A}{N_C} = 1$

and $\mu_C = 0.1977$ ◀

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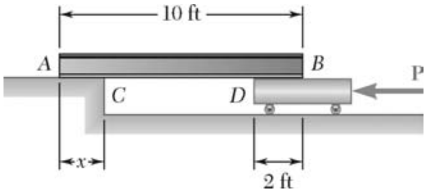
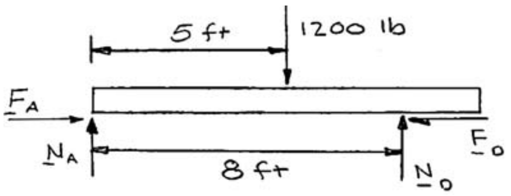


PROBLEM 8.34

A 10-ft beam, weighing 1200 lb, is to be moved to the left onto the platform. A horizontal force **P** is applied to the dolly, which is mounted on frictionless wheels. The coefficients of friction between all surfaces are $\mu_s = 0.30$ and $\mu_k = 0.25$, and initially $x = 2$ ft. Knowing that the top surface of the dolly is slightly higher than the platform, determine the force **P** required to start moving the beam. (*Hint:* The beam is supported at *A* and *D*.)

SOLUTION

FBD beam:



$$+\circlearrowleft \Sigma M_A = 0: \quad N_D(8\,\text{ft}) - (1200\text{ lb})(5\,\text{ft}) = 0$$

$$\mathbf{N}_D = 750 \text{ lb} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad N_A - 1200 + 750 = 0$$

$$\mathbf{N}_A = 450 \text{ lb} \uparrow$$

$$(F_A)_m = \mu_s N_A = 0.3(450) = 135.0 \text{ lb}$$

$$(F_D)_m = \mu_s N_D = 0.3(750) = 225 \text{ lb}$$

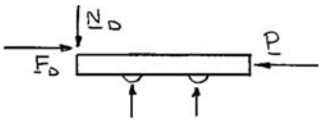
Since $(F_A)_m < (F_D)_m$, sliding first impends at *A* with

$$F_A = (F_A)_m = 135 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_A - F_D = 0$$

$$F_D = F_A = 135.0 \text{ lb}$$

FBD dolly:



From *FBD* of dolly:

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_D - P = 0$$

$$P = F_D = 135.0 \text{ lb}$$

$P = 135.0$ lb ◀

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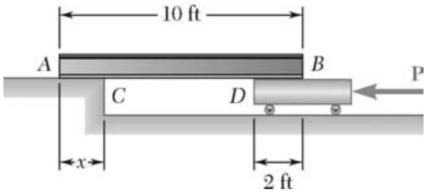


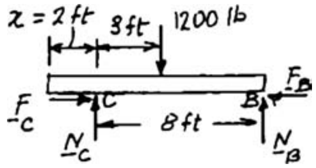
PROBLEM 8.35

(*a*) Show that the beam of Problem 8.34 *cannot* be moved if the top surface of the dolly is slightly *lower* than the platform. (*b*) Show that the beam *can* be moved if two 175-lb workers stand on the beam at *B* and determine how far to the left the beam can be moved.

PROBLEM 8.34 A 10-ft beam, weighing 1200 lb, is to be moved to the left onto the platform. A horizontal force **P** is applied to the dolly, which is mounted on frictionless wheels. The coefficients of friction between all surfaces are $\mu_s = 0.30$ and $\mu_k = 0.25$, and initially $x = 2$ ft. Knowing that the top surface of the dolly is slightly higher than the platform, determine the force **P** required to start moving the beam. (*Hint:* The beam is supported at *A* and *D*.)

SOLUTION

(*a*) <u>Beam alone</u>



$$+\circlearrowleft \Sigma M_C = 0: \quad N_B(8\text{ ft}) - (1200\text{ lb})(3\text{ ft}) = 0$$

$$\mathbf{N}_B = 450\text{ lb}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad N_C + 450 - 1200 = 0$$

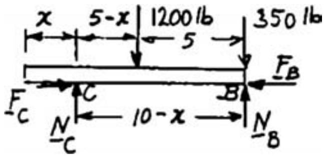
$$\mathbf{N}_C = 750\text{ lb}\uparrow$$

$$(F_C)_m = \mu_s N_C = 0.3(750) = 225\text{ lb}$$

$$(F_B)_m = \mu_s N_B = 0.3(450) = 135\text{ lb}$$

Since $(F_B)_m < (F_C)_m$, sliding first impends at *B*, and <u>Beam cannot be moved</u> ◀

(*b*) <u>Beam with workers standing at *B*</u>



$$+\circlearrowleft \Sigma M_C = 0: \quad N_B(10 - x) - (1200)(5 - x) - 350(10 - x) = 0$$

$$N_B = \frac{9500 - 1550x}{10 - x}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (1200)(5) - N_C(10 - x) = 0$$

$$N_C = \frac{6000}{10 - x}$$

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PROBLEM 8.35 (Continued)

Check that beam starts moving for $x = 2$ ft:

For $x = 2$ ft:

$$N_B = \frac{9500 - 1550(2)}{10 - 2} = 800 \text{ lb}$$

$$N_C = \frac{6000}{10 - 2} = 750 \text{ lb}$$

$$(F_C)_m = \mu_s N_C = 0.3(750) = 225 \text{ lb}$$

$$(F_B)_m = \mu_s N_B = 0.3(800) = 240 \text{ lb}$$

Since $(F_C)_m < (F_B)_m$, sliding first impends at C, Beam moves ◀

How far does beam move?

Beam will stop moving when

$$F_C = (F_B)_m$$

But

$$F_C = \mu_k N_C = 0.25 \frac{6000}{10 - x} = \frac{1500}{10 - x}$$

and

$$(F_B)_m = \mu_s N_B = 0.30 \frac{9500 - 1550x}{10 - x} = \frac{2850 - 465x}{10 - x}$$

Setting $F_C = (F_B)_m$: $1500 = 2850 - 465x$ $x = 2.90 \text{ ft}$ ◀

(*Note:* We have assumed that, once started, motion is continuous and uniform (no acceleration).)

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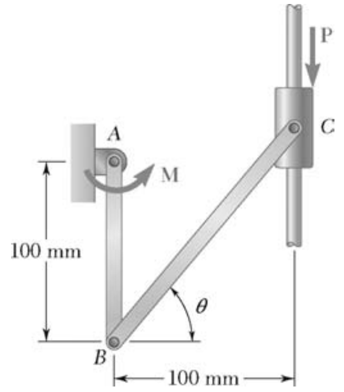
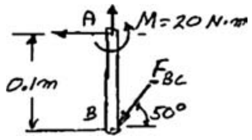


PROBLEM 8.36

Knowing that the coefficient of static friction between the collar and the rod is 0.35, determine the range of values of P for which equilibrium is maintained when $\theta = 50°$ and $M = 20\ \text{N}\cdot\text{m}$.

SOLUTION

Free body member *AB*:

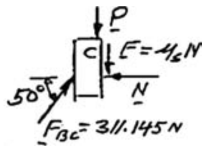


BC is a two-force member.

$$+\circlearrowleft \Sigma M_A = 0: \quad 20\ \text{N}\cdot\text{m} - F_{BC}\cos 50°(0.1\ \text{m}) = 0$$

$$F_{BC} = 311.145\ \text{N}$$

<u>Motion of *C* impending upward</u>:



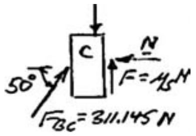
$$\xrightarrow{+} \Sigma F_x = 0: \quad (311.145\ \text{N})\cos 50° - N = 0$$

$$N = 200\ \text{N}$$

$$+\uparrow \Sigma F_y = 0: \quad (311.145\ \text{N})\sin 50° - P - (0.35)(200\ \text{N}) = 0$$

$$P = 168.351\ \text{N} \ \triangleleft$$

<u>Motion of *C* impending downward</u>:



$$\xrightarrow{+} \Sigma F_x = 0: \quad (311.145\ \text{N})\cos 50° - N = 0$$

$$N = 200\ \text{N}$$

$$+\uparrow \Sigma F_y = 0: \quad (311.145\ \text{N})\sin 50° - P + (0.35)(200\ \text{N}) = 0$$

$$P = 308.35\ \text{N} \ \triangleleft$$

<u>Range of *P*</u>: $168.4\ \text{N} \le P \le 308\ \text{N}$ ◀

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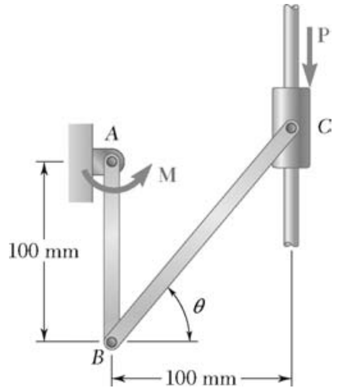
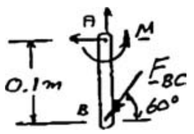


PROBLEM 8.37

Knowing that the coefficient of static friction between the collar and the rod is 0.40, determine the range of values of M for which equilibrium is maintained when $\theta = 60°$ and $P = 200$ N.

SOLUTION

Free body member *AB*:

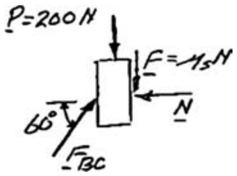


BC is a two-force member.

$$+\circlearrowleft \Sigma M_A = 0: \quad M - F_{BC} \cos 60°(0.1 \text{ m}) = 0$$

$$M = 0.05F_{BC} \qquad (1)$$

<u>Motion of *C* impending upward</u>:



$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BC} \cos 60° - N = 0$$

$$N = 0.5F_{BC}$$

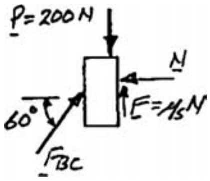
$$+\uparrow \Sigma F_y = 0: \quad F_{BC} \sin 60° - 200\,\text{N} - (0.40)(0.5F_{BC}) = 0$$

$$F_{BC} = 300.29\,\text{N}$$

Eq. (1): $\quad M = 0.05(300.29)$

$$M = 15.014 \text{ N}\cdot\text{m} \quad \triangleleft$$

<u>Motion of *C* impending downward</u>:



$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BC} \cos 60° - N = 0$$

$$N = 0.5F_{BC}$$

$$+\uparrow \Sigma F_y = 0: \quad F_{BC} \sin 60° - 200 \text{ N} + (0.40)(0.5\,F_{BC}) = 0$$

$$F_{BC} = 187.613 \text{ N}$$

Eq. (1): $\quad M = 0.05(187.613)$

$$M = 9.381 \text{ N}\cdot\text{m} \quad \triangleleft$$

Range of M: $\quad 9.38 \text{ N}\cdot\text{m} \le M \le 15.01 \text{ N}\cdot\text{m} \quad \blacktriangleleft$

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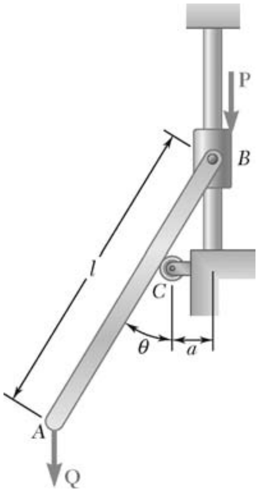


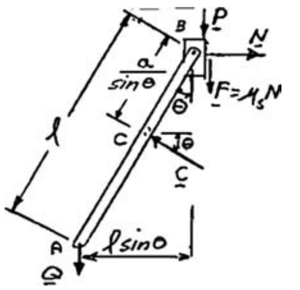
PROBLEM 8.38

The slender rod AB of length $l = 600$ mm is attached to a collar at B and rests on a small wheel located at a horizontal distance $a = 80$ mm from the vertical rod on which the collar slides. Knowing that the coefficient of static friction between the collar and the vertical rod is 0.25 and neglecting the radius of the wheel, determine the range of values of P for which equilibrium is maintained when $Q = 100$ N and $\theta = 30°$.

SOLUTION

For motion of collar at B impending upward:

$$\mathbf{F} = \mu_s N \downarrow$$



$$+\circlearrowleft \Sigma M_B = 0: \quad Ql\sin\theta - \frac{Ca}{\sin\theta} = 0$$

$$C = Q\left(\frac{l}{a}\right)\sin^2\theta$$

$$\Sigma F_x = 0: \quad N = C\cos\theta = Q\left(\frac{l}{a}\right)\sin^2\theta\cos\theta$$

$$+\downarrow \Sigma F_y = 0: \quad P + Q - C\sin\theta - \mu_s N = 0$$

$$P + Q - Q\left(\frac{l}{a}\right)\sin^3\theta - \mu_s Q\left(\frac{l}{a}\right)\sin^2\theta\cos\theta = 0$$

$$P = Q\left[\frac{l}{a}\sin^2\theta(\sin\theta - \mu_s\cos\theta) - 1\right] \qquad (1)$$

Substitute data:

$$P = (100\text{ N})\left[\frac{600\text{ mm}}{80\text{ mm}}\sin^2 30°(\sin 30° - 0.25\cos 30°) - 1\right]$$

$$P = -46.84\text{ N } (P \text{ is directed } \uparrow)$$

$$P = -46.8\text{ N} \quad \triangleleft$$

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PROBLEM 8.38 (Continued)

For motion of collar, impending downward:

$$\mathbf{F} = \mu_s N \uparrow$$

In Eq. (1) we substitute $-\mu_s$ for μ_s.

$$P = Q\left[\frac{l}{a}\sin^2\theta(\sin\theta + \mu_s\cos\theta) - 1\right]$$

$$P = (100\text{ N})\left[\frac{600\text{ mm}}{80\text{ mm}}\sin^2 30°(\sin 30° + 0.25\cos\theta) - 1\right]$$

$$P = +34.34\text{ N} \quad \triangleleft$$

For equilibrium: $-46.8\text{ N} \leq P \leq 34.3\text{ N}$ ◀

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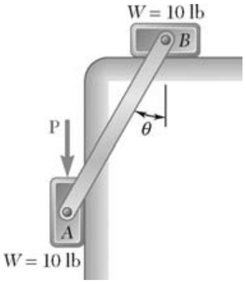


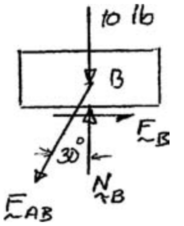
PROBLEM 8.39

Two 10-lb blocks A and B are connected by a slender rod of negligible weight. The coefficient of static friction is 0.30 between all surfaces of contact, and the rod forms an angle $\theta = 30°$. with the vertical. (a) Show that the system is in equilibrium when $P = 0$. (b) Determine the largest value of P for which equilibrium is maintained.

SOLUTION

FBD block B:

(a) Since $P = 2.69$ lb to initiate motion, equilibrium exists with $P = 0$ ◀



(b) For P_{max}, motion impends at both surfaces:

Block B: $\uparrow \Sigma F_y = 0: \quad N_B - 10\,\text{lb} - F_{AB}\cos 30° = 0$

$$N_B = 10\text{ lb} + \frac{\sqrt{3}}{2}F_{AB} \qquad (1)$$

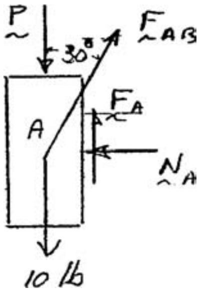
Impending motion: $F_B = \mu_s N_B = 0.3N_B$

$\longrightarrow \Sigma F_x = 0: \quad F_B - F_{AB}\sin 30° = 0$

$$F_{AB} = 2F_B = 0.6N_B \qquad (2)$$

Solving Eqs. (1) and (2): $N_B = 10\text{ lb} + \frac{\sqrt{3}}{2}(0.6N_B) = 20.8166\text{ lb}$

FBD block A:



Then $F_{AB} = 0.6N_B = 12.4900\text{ lb}$

Block A: $\longrightarrow \Sigma F_x = 0: \quad F_{AB}\sin 30° - N_A = 0$

$$N_A = \frac{1}{2}F_{AB} = \frac{1}{2}(12.4900\text{ lb}) = 6.2450\text{ lb}$$

Impending motion: $F_A = \mu_s N_A = 0.3(6.2450\text{ lb}) = 1.8735\text{ lb}$

$\uparrow \Sigma F_y = 0: \quad F_A + F_{AB}\cos 30° - P - 10\text{ lb} = 0$

$$P = F_A + \frac{\sqrt{3}}{2}F_{AB} - 10\text{ lb}$$

$$= 1.8735\text{ lb} + \frac{\sqrt{3}}{2}(12.4900\text{ lb}) - 10\text{ lb}$$

$$= 2.69\text{ lb}$$

$P = 2.69$ lb ◀

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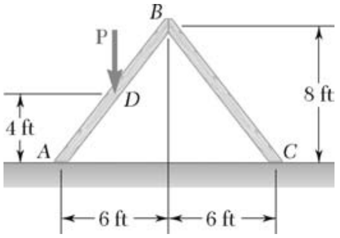
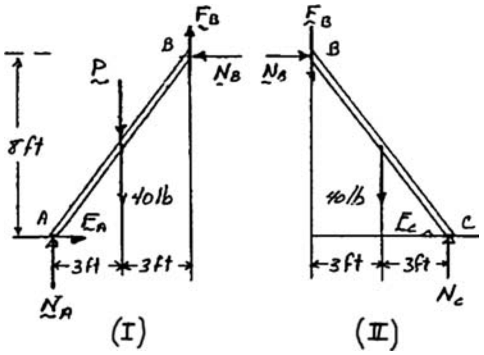


PROBLEM 8.40

Two identical uniform boards, each of weight 40 lb, are temporarily leaned against each other as shown. Knowing that the coefficient of static friction between all surfaces is 0.40, determine (*a*) the largest magnitude of the force **P** for which equilibrium will be maintained, (*b*) the surface at which motion will impend.

SOLUTION

Board FBDs:



Assume impending motion at *C*, so

$$F_C = \mu_s N_C = 0.4 N_C$$

FBD II: $\circlearrowleft \Sigma M_B = 0$: $(6\text{ ft})N_C - (8\text{ ft})F_C - (3\text{ ft})(40\text{ lb}) = 0$

$$[6\text{ ft} - 0.4(8\text{ ft})]N_C = (3\text{ ft})(40\text{ lb})$$

or

$$N_C = 42.857\text{ lb}$$

and

$$F_C = 0.4N_C = 17.143\text{ lb}$$

$\rightarrow \Sigma F_x = 0$: $N_B - F_C = 0$

$$N_B = F_C = 17.143\text{ lb}$$

$\uparrow \Sigma M_y = 0$: $-F_B - 40\text{ lb} + N_C = 0$

$$F_B = N_C - 40\text{ lb} = 2.857\text{ lb}$$

Check for motion at *B*: $\dfrac{F_B}{N_B} = \dfrac{2.857\text{ lb}}{17.143\text{ lb}} = 0.167 < \mu_s$, OK, no motion.

FBD I: $\circlearrowleft \Sigma M_A = 0$: $(8\text{ ft})N_B + (6\text{ ft})F_B - (3\text{ ft})(P + 40\text{ lb}) = 0$

$$P = \frac{(8\text{ ft})(17.143\text{ lb}) + (6\text{ ft})(2.857\text{ lb})}{3\text{ ft}} - 40\text{ lb}$$

$$= 11.429\text{ lb}$$

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PROBLEM 8.40 (Continued)

Check for slip at A (unlikely because of P):

$$\rightarrow \Sigma F_x = 0: \quad F_A - N_B = 0 \quad \text{or} \quad F_A = N_B = 17.143 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad N_A - P - 40 \text{ lb} + F_B = 0$$

or

$$N_A = 11.429 \text{ lb} + 40 \text{ lb} - 2.857 \text{ lb}$$
$$= 48.572 \text{ lb}$$

Then

$$\frac{F_A}{N_A} = \frac{17.143 \text{ lb}}{48.572 \text{ lb}} = 0.353 < \mu_s$$

OK, no slip $\Rightarrow$ assumption is correct.

Therefore

(a) $P_{\max} = 11.43$ lb ◀

(b) Motion impends at C ◀

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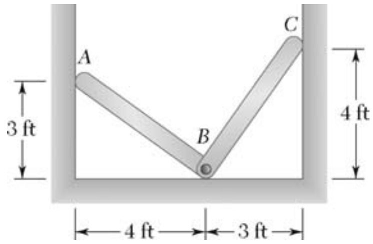
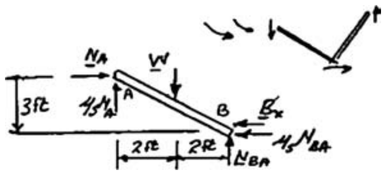


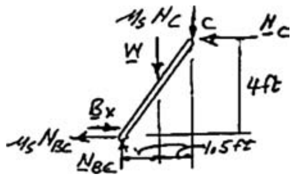
PROBLEM 8.41

Two identical 5-ft-long rods connected by a pin at B are placed between two walls and a horizontal surface as shown. Denoting by μ_s the coefficient of static friction at A, B, and C, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

Sense of impending motion:





$$+\circlearrowleft \Sigma M_B = 0: \quad 2W - 3N_A - 4\mu_s N_A = 0$$

$$N_A = \frac{2W}{(3+4\mu_s)} \qquad (1)$$

$$\Sigma F_y: \quad N_{AB} = W - \mu_s N_A \qquad (3)$$

$$\overset{+}{\longleftarrow} \Sigma F_x = 0: \quad B_x + \mu_s N_{BA} - N_A = 0$$

$$B_x = N_A - \mu_s N_{BA} \qquad (5)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 1.5W - 4N_C + 3\mu_s N_C = 0$$

$$N_C = \frac{1.5W}{(4-3\mu_s)} \qquad (2)$$

$$\Sigma F_y: \quad N_{BC} = W + \mu_s N_C \qquad (4)$$

$$\overset{+}{\longrightarrow} \Sigma F_x = 0: \quad B_x - N_C - \mu_s N_{BC} = 0$$

$$B_x = N_C + \mu_s N_{BC} \qquad (6)$$

Equate (5) and (6):
$$N_A - \mu_s N_{BA} = N_C + \mu_s N_{BC}$$

Substitute from Eqs. (3) and (4):
$$N_A - \mu_s (W - \mu_s N_A) = N_C + \mu_s (W + \mu_s N_C)$$
$$N_A (1+\mu_s^2) - \mu_s W = N_C (1+\mu_s^2) + \mu_s W$$

Substitute from Eqs. (1) and (2):
$$\frac{2W}{3+4\mu_s}(1+\mu_s^2) - \mu_s W = \frac{1.5W}{4-3\mu_s}(1+\mu_s^2) + \mu_s W$$
$$\frac{2}{3+4\mu_s} - \frac{1.5}{4-3\mu_s} = \frac{2\mu_s}{1+\mu_s^2}$$

Solve for μ_s: $\mu_s = 0.0949$ ◀

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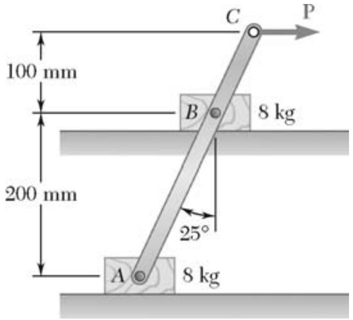
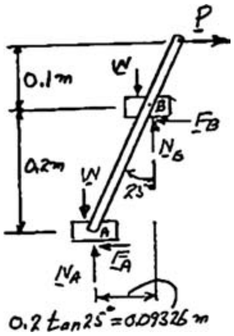


PROBLEM 8.42

Two 8-kg blocks A and B resting on shelves are connected by a rod of negligible mass. Knowing that the magnitude of a horizontal force **P** applied at C is slowly increased from zero, determine the value of P for which motion occurs, and what that motion is, when the coefficient of static friction between all surfaces is (a) $\mu_s = 0.40$, (b) $\mu_s = 0.50$.

SOLUTION

(a) $\mu_s = 0.40$: Assume blocks slide to right.



$$W = mg = (8\text{ kg})(9.81\text{ m/s}^2) = 78.48\text{ N}$$

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

$$+\uparrow \Sigma F_y = 0: \quad N_A + N_B - 2W = 0$$

$$N_A + N_B = 2W$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - F_A - F_B = 0$$

$$P = F_A + F_B = \mu_s(N_A + N_B) = \mu_s(2W) \qquad (1)$$

$$P = 0.40(2)(78.48\text{ N}) = 62.78\text{ N}$$

$$+\circlearrowright \Sigma M_B = 0: \quad P(0.1\text{ m}) - (N_A - W)(0.09326\text{ m}) + F_A(0.2\text{ m}) = 0$$

$$(62.78)(0.1) - (N_A - 78.48)(0.09326) + (0.4)(N_B)(0.2) = 0$$

$$0.17326 N_A = 1.041$$

$$N_A = 6.01\text{ N} > 0 \quad \text{OK}$$

System slides: $P = 62.8$ N ◀

(b) $\mu_s = 0.50$: See part a.

Eq. (1): $P = 0.5(2)(78.48\text{ N}) = 78.48\text{ N}$

$$+\circlearrowright \Sigma M_B = 0: \quad P(0.1\text{ m}) + (N_A - W)(0.09326\text{ m}) + F_A(0.2\text{ m}) = 0$$

$$(78.48)(0.1) + (N_A - 78.48)(0.09326) + (0.5)N_A(0.2) = 0$$

$$0.19326 N_A = -0.529$$

$$N_A = -2.73\text{ N} < 0 \text{ uplift, rotation about } B$$

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PROBLEM 8.42 (Continued)

For $N_A = 0$: $\circlearrowright \Sigma M_B = 0$: $P(0.1\text{ m}) - W(0.09326\text{ m}) = 0$

$$P = (78.48\text{ N})(0.09326\text{ m})/(0.1) = 73.19$$

<u>System rotates about *B*</u>: $P = 73.2\text{ N}$ ◀

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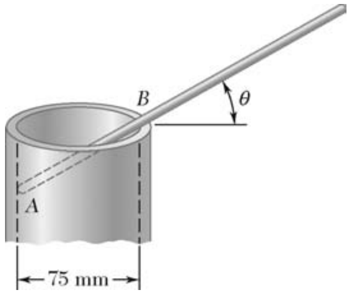
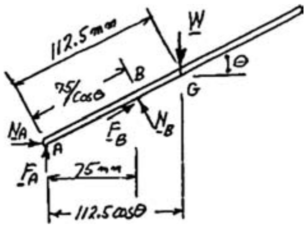


PROBLEM 8.43

A slender steel rod of length 225 mm is placed inside a pipe as shown. Knowing that the coefficient of static friction between the rod and the pipe is 0.20, determine the largest value of θ for which the rod will not fall into the pipe.

SOLUTION

Motion of rod impends down at *A* and to left at *B*.



$$F_A = \mu_s N_A \quad F_B = \mu_s N_B$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_A - N_B \sin\theta + F_B \cos\theta = 0$$

$$N_A - N_B \sin\theta + \mu_s N_B \cos\theta = 0$$

$$N_A = N_B(\sin\theta - \mu_s \cos\theta) \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + N_B \cos\theta + F_B \sin\theta - W = 0$$

$$\mu_s N_A + N_B \cos\theta + \mu_s N_B \sin\theta - W = 0 \qquad (2)$$

Substitute for N_A from Eq. (1) into Eq. (2):

$$\mu_s N_B(\sin\theta - \mu_s \cos\theta) + N_B \cos\theta + \mu_s N_B \sin\theta - W = 0$$

$$N_B = \frac{W}{(1-\mu_s^2)\cos\theta + 2\mu_s \sin\theta} = \frac{W}{(1-0.2^2)\cos\theta + 2(0.2)\sin\theta} \qquad (3)$$

$$+\circlearrowleft \Sigma M_A = 0: \quad N_B\left(\frac{75}{\cos\theta}\right) - W(112.5\cos\theta) = 0$$

Substitute for N_B from Eq. (3), cancel *W*, and simplify to find

$$9.6\cos^3\theta + 4\sin\theta\cos^2\theta - 6.6667 = 0$$

$$\cos^3\theta(2.4 + \tan\theta) = 1.6667$$

Solve by trial & error: $\theta = 35.8° \blacktriangleleft$

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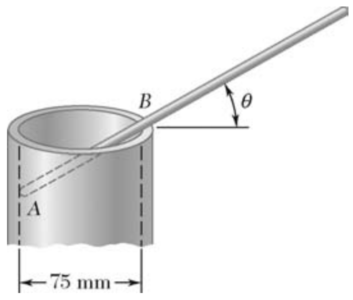


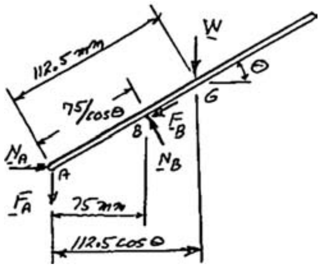
PROBLEM 8.44

In Problem 8.43, determine the smallest value of θ for which the rod will not fall out the pipe.

PROBLEM 8.43 A slender steel rod of length 225 mm is placed inside a pipe as shown. Knowing that the coefficient of static friction between the rod and the pipe is 0.20, determine the largest value of θ for which the rod will not fall into the pipe.

SOLUTION

Motion of rod impends up at A and right at B.



$$F_A = \mu_s N_A \quad F_B = \mu_s N_B$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_A - N_B \sin\theta - F_B \cos\theta = 0$$

$$N_A - N_B \sin\theta - \mu_s N_B \cos\theta = 0$$

$$N_A = N_B(\sin\theta + \mu_s \cos\theta) \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad -F_A + N_B \cos\theta - F_B \sin\theta - W = 0$$

$$-\mu_s N_A + N_B \cos\theta - \mu_s N_B \sin\theta - W = 0 \tag{2}$$

Substitute for N_A from Eq. (1) into Eq. (2):

$$-\mu_s N_B(\sin\theta + \mu_s \cos\theta) + N_B \cos\theta - \mu_s N_B \sin\theta - W = 0$$

$$N_B = \frac{W}{(1-\mu_s^2)\cos\theta - 2\mu_s \sin\theta} = \frac{W}{(1-0.2^2)\cos\theta - 2(0.2)\sin\theta} \tag{3}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad N_B\left(\frac{75}{\cos\theta}\right) - W(112.5\cos\theta) = 0$$

Substitute for N_B from Eq. (3), cancel W, and simplify to find

$$9.6\cos^3\theta - 4\sin\theta\cos^2\theta - 6.6667 = 0$$

$$\cos^3\theta(2.4 - \tan\theta) = 1.6667$$

Solve by trial + error: $\theta = 20.5°$ ◀

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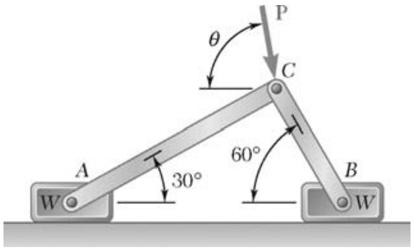
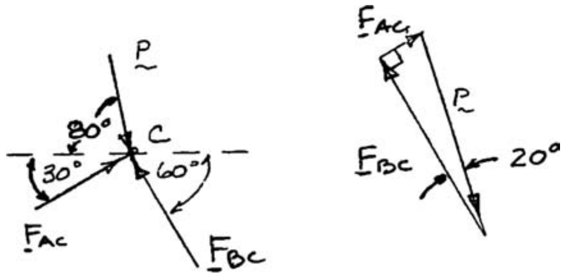


PROBLEM 8.45

Two slender rods of negligible weight are pin-connected at C and attached to blocks A and B, each of weight W. Knowing that $\theta = 80°$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the largest value of P for which equilibrium is maintained.

SOLUTION

FBD pin C:

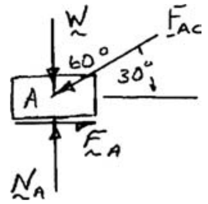


$$F_{AC} = P\sin 20° = 0.34202P$$
$$F_{BC} = P\cos 20° = 0.93969P$$

$\uparrow \Sigma F_y = 0$: $\quad N_A - W - F_{AC}\sin 30° = 0$

or $\quad N_A = W + 0.34202P\sin 30° = W + 0.171010P$

FBD block A: $\quad \rightarrow \Sigma F_x = 0$: $\quad F_A - F_{AC}\cos 30° = 0$



or $\quad F_A = 0.34202P\cos 30° = 0.29620P$

For impending motion at A: $\quad F_A = \mu_s N_A$

Then $\quad N_A = \dfrac{F_A}{\mu_s}$: $\quad W + 0.171010P = \dfrac{0.29620}{0.3}P$

or $\quad P = 1.22500W$

$\uparrow \Sigma F_y = 0$: $\quad N_B - W - F_{BC}\cos 30° = 0$

$$N_B = W + 0.93969P\cos 30° = W + 0.81380P$$

$\rightarrow \Sigma F_x = 0$: $\quad F_{BC}\sin 30° - F_B = 0$

$$F_B = 0.93969P\sin 30° = 0.46985P$$

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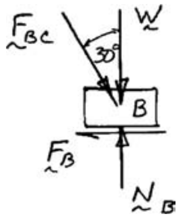
PROBLEM 8.45 (Continued)

FBD block *B*:



For impending motion at *B*: $F_B = \mu_s N_B$

Then $N_B = \dfrac{F_B}{\mu_s}: \quad W + 0.81380P = \dfrac{0.46985P}{0.3}$

or $P = 1.32914W$

Thus, maximum *P* for equilibrium $P_{max} = 1.225W$ ◀

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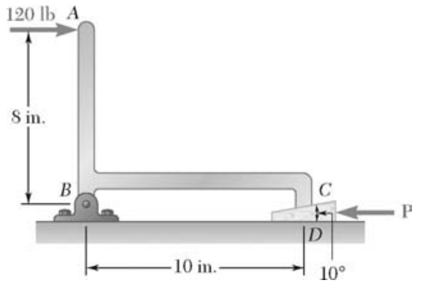


PROBLEM 8.46

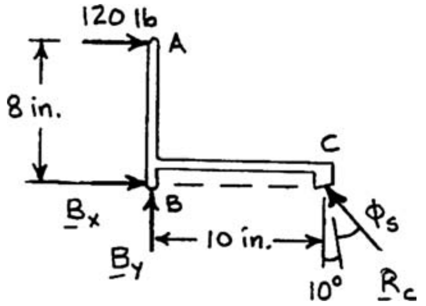
The machine part *ABC* is supported by a frictionless hinge at *B* and a 10° wedge at *C*. Knowing that the coefficient of static friction is 0.20 at both surfaces of the wedge, determine (*a*) the force **P** required to move the wedge to the left, (*b*) the components of the corresponding reaction at *B*.

SOLUTION

$$\mu_s = 0.20$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.20 = 11.3099°$$

Free body: *ABC*

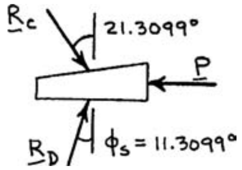


$$10° + 11.3099° = 21.3099°$$

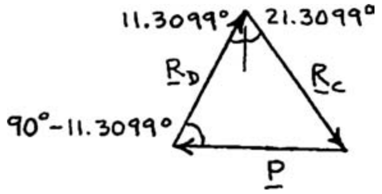
$$+\circlearrowleft \Sigma M_B = 0: \quad (R_C \cos 21.3099°)(10) - (120 \text{ lb})(8) = 0$$

$$R_C = 103.045 \text{ lb}$$

Free body: Wedge



Force triangle:



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PROBLEM 8.46 (Continued)

Law of sines:

$$\frac{P}{\sin 32.6198°} = \frac{(R_C = 103.045 \text{ lb})}{\sin 78.690°}$$

(*a*) $\mathbf{P} = 56.6 \text{ lb} \longleftarrow$ ◀

(*b*) Returning to free body of *ABC*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 120 - (103.045)\sin 21.3099° = 0$$

$$B_x = -82.552 \text{ lb}$$

$\mathbf{B}_x = 82.6 \text{ lb} \longleftarrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad B_y + (103.045)\cos 21.3099° = 0$$

$$B_y = -96.000 \text{ lb}$$

$\mathbf{B}_y = 96.0 \text{ lb} \downarrow$ ◀

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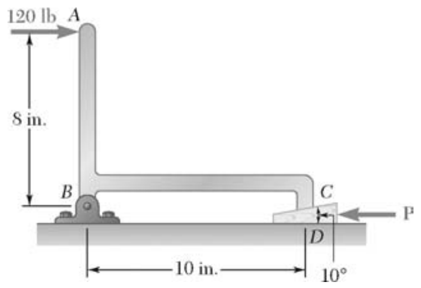


PROBLEM 8.47

Solve Problem 8.46 assuming that the wedge is to be moved to the right.

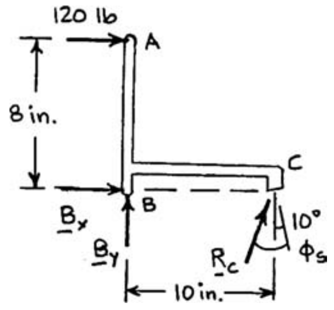
PROBLEM 8.46 The machine part *ABC* is supported by a frictionless hinge at *B* and a 10° wedge at *C*. Knowing that the coefficient of static friction is 0.20 at both surfaces of the wedge, determine (*a*) the force **P** required to move the wedge to the left, (*b*) the components of the corresponding reaction at *B*.

SOLUTION

$$\mu_s = 0.20$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.20 = 11.30993°$$

Free body: *ABC*

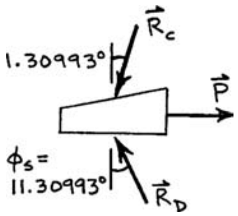


$$11.30993° - 10° = 1.30993°$$

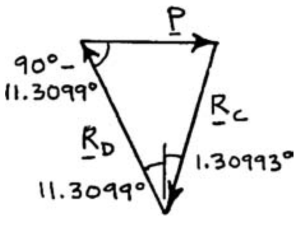
$$+\circlearrowleft \Sigma M_B = 0: \quad (R_C \cos 1.30993°)(10) - (120 \text{ lb})(8) = 0$$

$$R_C = 96.025 \text{ lb}$$

Free body: Wedge



Force triangle:



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PROBLEM 8.47 (Continued)

Law of sines:

$$\frac{P}{\sin 12.6198°} = \frac{(R_C = 96.025 \text{ lb})}{\sin 78.690°}$$

(*a*) $\mathbf{P} = 21.4 \text{ lb} \longrightarrow$ ◀

(*b*) Returning to free body of *ABC*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 120 + (96.025)\sin 1.30993° = 0$$

$$B_x = -122.195 \text{ lb}$$

$\mathbf{B}_x = 122.2 \text{ lb} \longleftarrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad B_y + (96.025)\cos 1.30993° = 0$$

$$B_y = -96.000 \text{ lb}$$

$\mathbf{B}_y = 96.0 \text{ lb} \downarrow$ ◀

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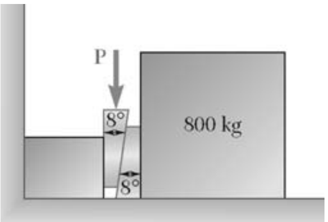


PROBLEM 8.48

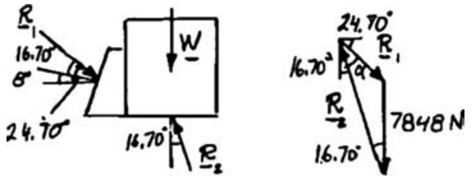
Two 8° wedges of negligible weight are used to move and position the 800-kg block. Knowing that the coefficient of static friction is 0.30 at all surfaces of contact, determine the smallest force **P** that should be applied as shown to one of the wedges.

SOLUTION

$$\mu_s = 0.30 \quad \phi_s = \tan^{-1} 0.30 = 16.70°$$

<u>Free body</u>: <u>800-kg block and right-hand wedge</u>

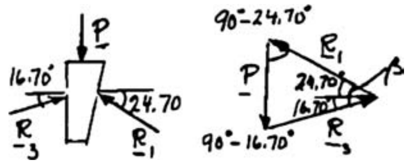
$$W = (800 \text{ kg})(9.81 \text{ m/s}^2) = 7848 \text{ N}$$



Force triangle:
$$\alpha = 90° - 16.70° - 24.70°$$
$$= 48.60°$$

Law of sines:
$$\frac{R_1}{\sin 16.70°} = \frac{7848 \text{ N}}{\sin 48.60°}$$
$$R_1 = 3006.5 \text{ N}$$

<u>Free body</u>: <u>Left-hand wedge</u>



Force triangle:
$$\beta = 16.70° + 24.70°$$
$$= 41.40°$$

Law of sines:
$$\frac{P}{\sin 41.40°} = \frac{R_1}{\sin(90° - 16.70°)}$$

$$\frac{P}{\sin 41.40°} = \frac{3006.5 \text{ N}}{\cos 16.70°}$$

P = 2080 N ↓ ◀

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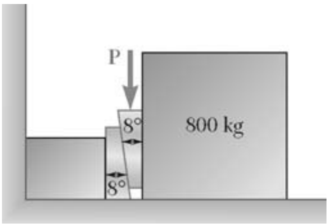


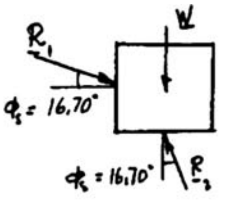
PROBLEM 8.49

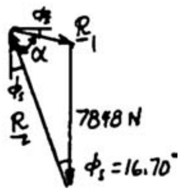
Two 8° wedges of negligible weight are used to move and position the 800-kg block. Knowing that the coefficient of static friction is 0.30 at all surfaces of contact, determine the smallest force **P** that should be applied as shown to one of the wedges.

SOLUTION

$$\mu_s = 0.30 \quad \phi_s = \tan^{-1} 0.30 = 16.70°$$

Free body: 800-kg block





Force triangle:

$$W = (800 \text{ kg})(9.81 \text{ m/s}^2) = 7848 \text{ N}$$

$$\alpha = 90° - 2\phi_s = 90° - 2(16.70°)$$

$$= 56.60°$$

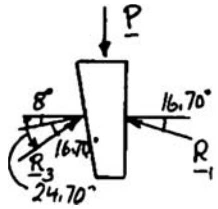
Law of sines:

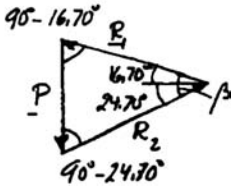
$$\frac{R_1}{\sin 16.70°} = \frac{7848 \text{ N}}{\sin 56.60°}$$

$$R_1 = 2701 \text{ N}$$

Free body: Right-hand wedge

Force triangle:





$$\beta = 16.70° + 24.70°$$

$$= 41.40°$$

Law of sines:

$$\frac{P}{\sin 41.40°} = \frac{R_1}{\sin(90° - 24.70°)}$$

$$\frac{P}{\sin 41.40°} = \frac{2701 \text{ N}}{\cos 24.70°}$$

$\mathbf{P} = 1966 \text{ N} \downarrow$ ◀

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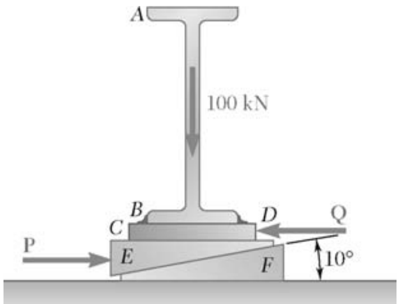
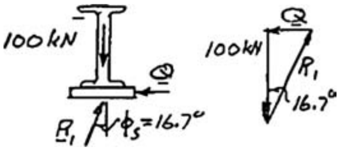


PROBLEM 8.50

The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 100 kN. The coefficient of static friction is 0.30 between two steel surfaces and 0.60 between steel and concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

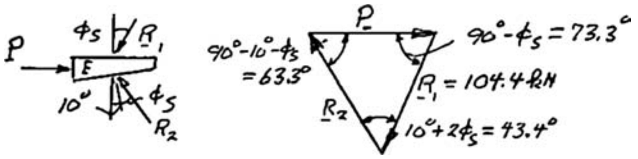
SOLUTION

<u>Free body: Beam and plate *CD*</u>



$$R_1 = \frac{(100 \text{ kN})}{\cos 16.7°}$$

$$R_1 = 104.4 \text{ kN}$$



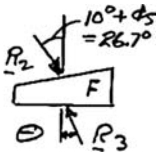
(*a*) $$\frac{P}{\sin 43.4°} = \frac{104.4 \text{ kN}}{\sin 63.3°}$$ $\mathbf{P} = 80.3 \text{ kN} \longrightarrow$ ◀

(*b*) $$\phi_s = \tan^{-1} 0.3 = 16.7°$$

$$Q = (100 \text{ kN}) \tan 16.7°$$ $\mathbf{Q} = 30 \text{ kN} \longleftarrow$ ◀

<u>Free body: Wedge *F*</u>

(To check that it does not move.)



Since wedge *F* is a two-force body, R_2 and R_3 are colinear

Thus $\theta = 26.7°$

But $\phi_{\text{concrete}} = \tan^{-1} 0.6 = 31.0° > \theta$ OK

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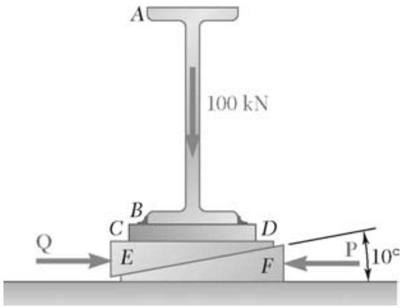
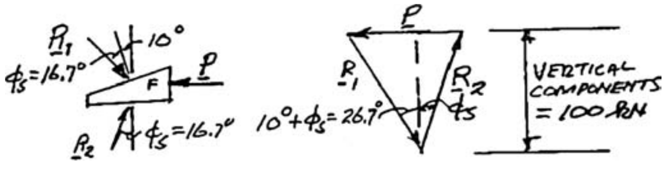


PROBLEM 8.51

The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 100 kN. The coefficient of static friction is 0.30 between two steel surfaces and 0.60 between steel and concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

SOLUTION

Free body: Wedge *F* $\qquad \phi_s = \tan^{-1} 0.30 = 16.7°$



(*a*)

$$P = (100 \text{ kN}) \tan 26.7° + (100 \text{ kN}) \tan \phi_s$$

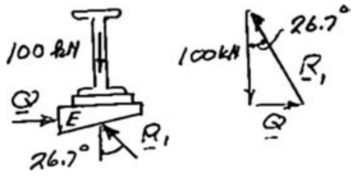
$$P = 50.29 \text{ kN} + 30 \text{ kN}$$

$$P = 80.29 \text{ kN}$$

$\mathbf{P} = 80.3 \text{ kN} \longleftarrow$ ◀

$$R_1 = \frac{(100 \text{ kN})}{\cos 26.7°} = 111.94 \text{ kN}$$

Free body: Beam, plate, and wedge *E*



(*b*)

$$Q = W \tan 26.7° = (100 \text{ kN}) \tan 26.7°$$

$$Q = 50.29 \text{ kN}$$

$\mathbf{Q} = 50.3 \text{ kN} \longrightarrow$ ◀

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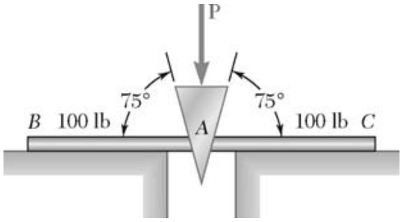


PROBLEM 8.52

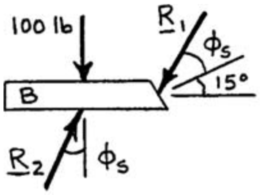
A wedge *A* of negligible weight is to be driven between two 100-lb plates *B* and *C*. The coefficient of static friction between all surfaces of contact is 0.35. Determine the magnitude of the force **P** required to start moving the wedge (*a*) if the plates are equally free to move, (*b*) if plate *C* is securely bolted to the surface.

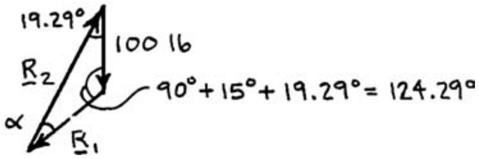
SOLUTION

(*a*) <u>With plates equally free to move</u>

<u>Free body: Plate *B*</u> $\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.35 = 19.2900°$

Force triangle:



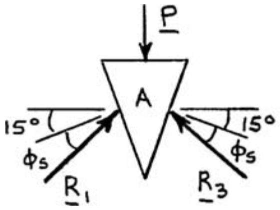


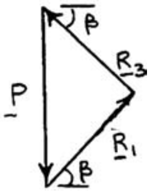
$$\alpha = 180° - 124.29° - 19.29° = 36.42°$$

Law of sines:
$$\frac{R_1}{\sin 19.29°} = \frac{100\text{ lb}}{\sin 36.42°}$$
$$R_1 = 55.643\text{ lb}$$

<u>Free body: Wedge *A*</u>

Force triangle:





By symmetry, $R_3 = R_1 = 55.643\text{ lb}$

$$\beta = 19.29° + 15° = 34.29°$$

Then $P = 2R_1 \sin\beta$

or $P = 2(55.643)\sin 34.29°$ $P = 62.7\text{ lb}$ ◀

(*b*) <u>With plate *C* bolted</u>

The free body diagrams of plate *B* and wedge *A* (the only members to move) are same as above. Answer is thus the same.

$P = 62.7\text{ lb}$ ◀

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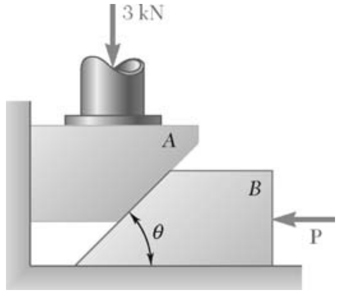


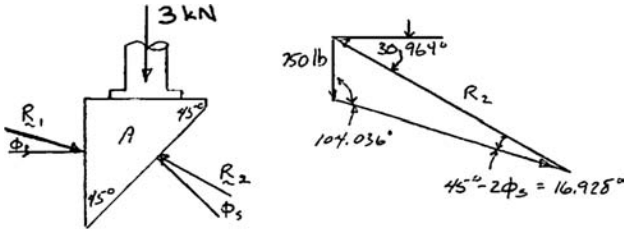
PROBLEM 8.53

Block *A* supports a pipe column and rests as shown on wedge *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force **P** required to raise block *A*.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.25 = 14.036°$$

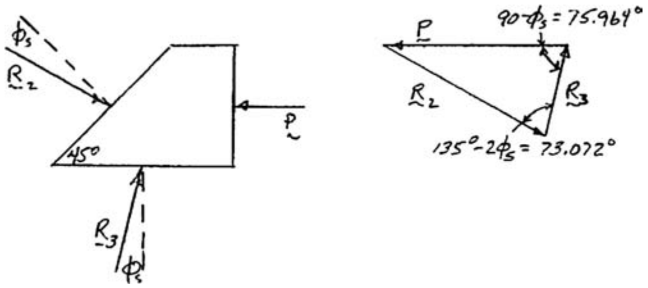
FBD block *A*:



$$\frac{R_2}{\sin 104.036°} = \frac{3\text{ kN}}{\sin 16.928°}$$

$$R_2 = 10.0000\text{ kN}$$

FBD wedge *B*:



$$\frac{P}{\sin 73.072°} = \frac{10.0000\text{ kN}}{\sin 75.964°}$$

$$P = 9.8611\text{ kN}$$

$\mathbf{P} = 9.86\text{ kN} \longleftarrow$ ◀

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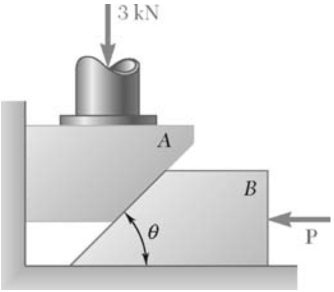


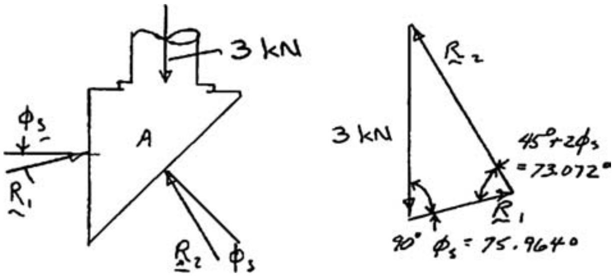
PROBLEM 8.54

Block A supports a pipe column and rests as shown on wedge B. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force **P** for which equilibrium is maintained.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.25 = 14.036°$$

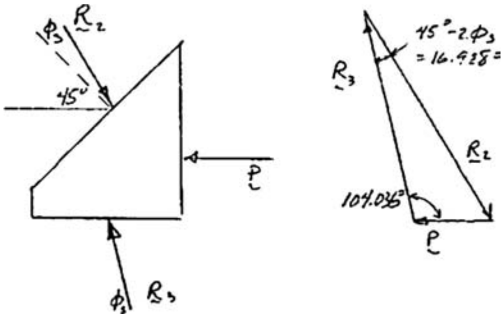
FBD block A:



$$\frac{R_2}{\sin(75.964°)} = \frac{3\text{ kN}}{\sin(73.072°)}$$

$$R_2 = 3.0420\text{ kN}$$

FBD wedge B:



$$\frac{P}{\sin 16.928°} = \frac{3.0420\text{ kN}}{\sin 104.036°}$$

$$P = 0.91300\text{ kN}$$

$\mathbf{P} = 913\text{ N} \longleftarrow$ ◀

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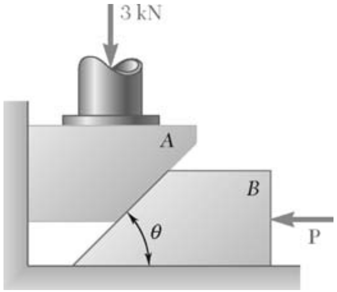


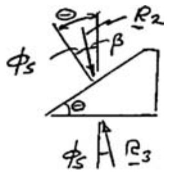
PROBLEM 8.55

Block A supports a pipe column and rests as shown on wedge B. The coefficient of static friction at all surfaces of contact is 0.25. If $\mathbf{P} = 0$, determine (a) the angle θ for which sliding is impending, (b) the corresponding force exerted on the block by the vertical wall.

SOLUTION

Free body: Wedge B $\qquad \phi_s = \tan^{-1} 0.25 = 14.04°$

(a) Since wedge is a two-force body, $\mathbf{R}_2$ and $\mathbf{R}_3$ must be equal and opposite. Therefore, they form equal angles with vertical



$$\beta = \phi_s$$

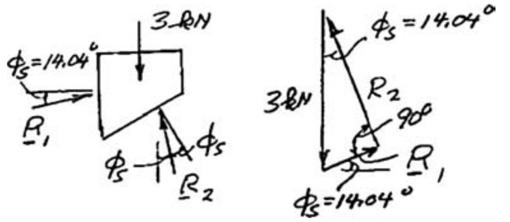
and

$$\theta - \phi_s = \phi_s$$

$$\theta = 2\phi_s = 2(14.04°)$$

$\theta = 28.1°$ ◀

Free body: Block A



$$R_1 = (3\text{ kN})\sin 14.04° = 0.7278\text{ kN}$$

(b) Force exerted by wall: $\qquad \mathbf{R}_1 = 728\text{ N} \measuredangle\ 14.04°$ ◀

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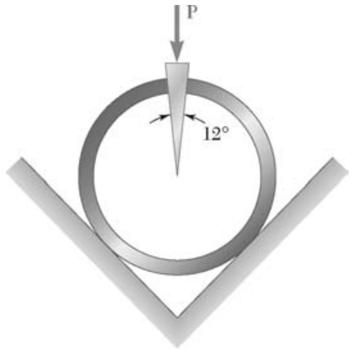


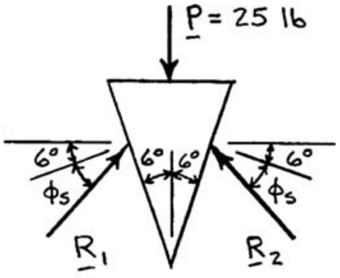
PROBLEM 8.56

A 12° wedge is used to spread a split ring. The coefficient of static friction between the wedge and the ring is 0.30. Knowing that a force **P** of magnitude 25 lb was required to insert the wedge, determine the magnitude of the forces exerted on the ring by the wedge after insertion.

SOLUTION

Free body: Wedge

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.30 = 16.6992°$$

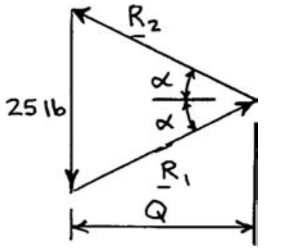


Force triangle:

$$\alpha = 6° + \phi_s$$
$$= 6° + 16.6992°$$
$$= 22.6992°$$

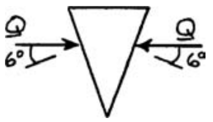
$Q =$ Horizontal component of R

$$Q = \frac{\frac{1}{2}(25\text{ lb})}{\tan 22.6992°}$$
$$= 29.9\text{ lb}$$



Free body: After wedge has been inserted

Wedge is now a two-force body with forces shown.



$Q = 29.9$ lb ◀

(*Note:* Since angles between force **Q** and normal to wedge is $6° < \phi_s$, wedge stays in place.)

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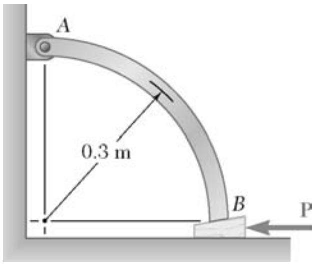
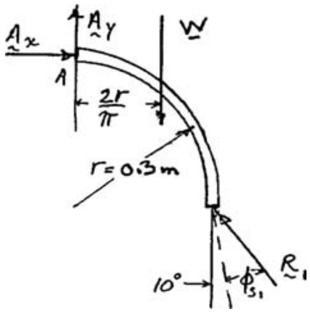


PROBLEM 8.57

A 10° wedge is to be forced under end B of the 5-kg rod AB. Knowing that the coefficient of static friction is 0.40 between the wedge and the rod and 0.20 between the wedge and the floor, determine the smallest force **P** required to raise end B of the rod.

SOLUTION

FBD *AB*:



$$W = mg$$

$$W = (5\text{ kg})(9.81\text{ m/s}^2)$$

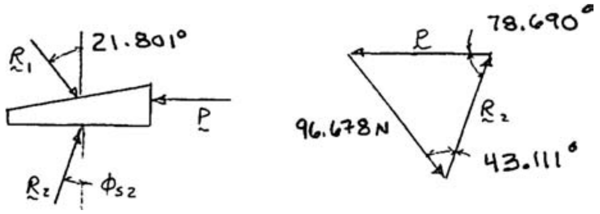
$$W = 49.050\text{ N}$$

$$\phi_{s1} = \tan^{-1}(\mu_s)_1 = \tan^{-1} 0.40 = 21.801°$$

$$\circlearrowleft \Sigma M_A = 0: \quad rR_1 \cos(10° + 21.801°) - rR_1 \sin(10° + 21.801°) - \frac{2r}{\pi}(49.050\text{ N}) = 0$$

$$R_1 = 96.678\text{ N}$$

FBD wedge:



$$\phi_{s2} = \tan^{-1}(\mu_s)_2 = \tan^{-1} 0.20 = 11.3099$$

$$\frac{P}{\sin(43.111°)} = \frac{96.678\text{ N}}{\sin 78.690°}$$

P = 67.4 N ⟵ ◀

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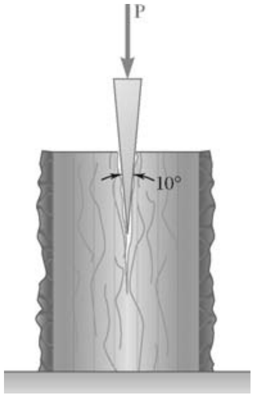
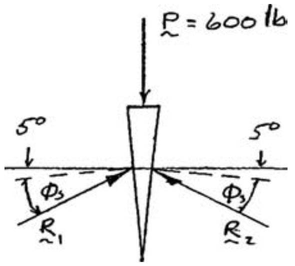


PROBLEM 8.58

A 10° wedge is used to split a section of a log. The coefficient of static friction between the wedge and the log is 0.35. Knowing that a force **P** of magnitude 600 lb was required to insert the wedge, determine the magnitude of the forces exerted on the wood by the wedge after insertion.

SOLUTION

FBD wedge (impending motion ↓):



$$\phi_s = \tan^{-1}\mu_s$$
$$= \tan^{-1} 0.35$$
$$= 19.29°$$

By symmetry: $R_1 = R_2$

$$+\uparrow \Sigma F_y = 0: \quad 2R_1 \sin(5° + \phi_s) - 600 \text{ lb} = 0$$

or $$R_1 = R_2 = \frac{300 \text{ lb}}{\sin(5°+19.29°)} = 729.30 \text{ lb}$$

When P is removed, the vertical components of R_1 and R_2 vanish, leaving the horizontal components

$$R_{1x} = R_{2x}$$
$$= R_1 \cos(5° + \phi_s)$$
$$= (729.30 \text{ lb})\cos(5° + 19.29°)$$

$R_{1x} = R_{2x} = 665 \text{ lb}$ ◀

(Note that $\phi_s > 5°$, so wedge is self-locking.)

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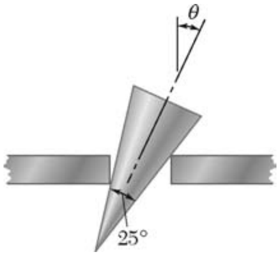
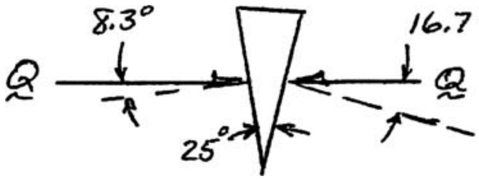


PROBLEM 8.59

A conical wedge is placed between two horizontal plates that are then slowly moved toward each other. Indicate what will happen to the wedge (*a*) if $\mu_s = 0.20$, (*b*) if $\mu_s = 0.30$.

SOLUTION



As the plates are moved, the angle θ will decrease.

(*a*) $\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.2 = 11.31°.$

As θ decrease, the minimum angle at the contact approaches

$12.5° > \phi_s = 11.31°$, so the wedge will slide up and out from the slot. ◀

(*b*) $\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.3 = 16.70°.$

As θ decreases, the angle at one contact reaches 16.7°. (At this time the angle at the other contact is $25° - 16.7° = 8.3° < \phi_s$). The wedge binds in the slot. ◀

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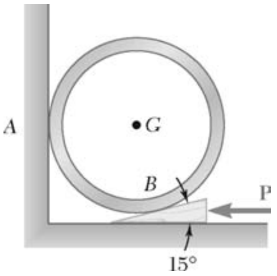


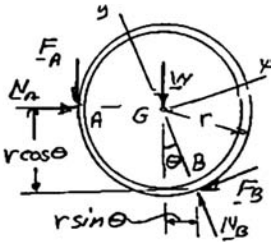
PROBLEM 8.60

A 15° wedge is forced under a 50-kg pipe as shown. The coefficient of static friction at all surfaces is 0.20. (*a*) Show that slipping will occur between the pipe and the vertical wall. (*b*) Determine the force **P** required to move the wedge.

SOLUTION

Free body: Pipe

$$+\circlearrowleft \Sigma M_B = 0: \quad Wr\sin\theta + F_A r(1+\sin\theta) - N_A r\cos\theta = 0$$



Assume slipping at A:

$$F_A = \mu_s N_A$$

$$N_A\cos\theta - \mu_s N_A(1+\sin\theta) = W\sin\theta$$

$$N_A = \frac{W\sin\theta}{\cos\theta - \mu_s(1+\sin\theta)}$$

$$N_A = \frac{W\sin 15^\circ}{\cos 15^\circ - (0.20)(1+\sin 15^\circ)}$$

$$= 0.36241W$$

$$+\nearrow \Sigma F_x = 0: \quad -F_B - W\sin\theta - F_A\sin\theta + N_A\cos\theta = 0$$

$$F_B = N_A\cos\theta - \mu_s N_A\sin\theta - W\sin\theta$$

$$F_B = 0.36241W\cos 15^\circ - 0.20(0.36241W)\sin 15^\circ - W\sin 15^\circ$$

$$F_B = 0.072482W$$

$$+\nwarrow \Sigma F_y = 0: \quad N_B - W\cos\theta - F_A\cos\theta - N_A\sin\theta = 0$$

$$N_B = N_A\sin\theta + \mu_s N_A\cos\theta + W\cos\theta$$

$$N_B = (0.36241W)\sin 15^\circ + 0.20(0.36241W)\cos 15^\circ + W\cos 15^\circ$$

$$N_B = 1.12974W$$

Maximum available: $\quad F_B = \mu_s N_B = 0.22595W$

(*a*) We note that $F_B < F_{max}$ $\quad$ No slip at B ◀

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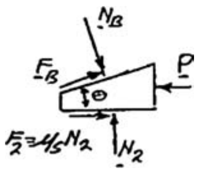
PROBLEM 8.60 (Continued)

(*b*) Free body: Wedge



$+\uparrow \Sigma F_y = 0: \quad N_2 - N_B \cos\theta + F_B \sin\theta = 0$

$$N_2 = N_B \cos\theta - F_B \sin\theta$$
$$N_2 = (1.12974W)\cos 15° - (0.07248W)\sin 15°$$
$$N_2 = 1.07249W$$

$\xrightarrow{+} \Sigma F_x = 0: \quad F_B \cos\theta + N_B \sin\theta + \mu_s N_2 - P = 0$

$$P = F_B \cos\theta + N_B \sin\theta + \mu_s N_2$$
$$P = (0.07248W)\cos 15° + (1.12974W)\sin 15° + 0.2(1.07249W)$$
$$P = 0.5769W$$

$W = mg: \quad P = 0.5769(50 \text{ kg})(9.81 \text{ m/s}^2)$ **P** = 283 N ⟵ ◀

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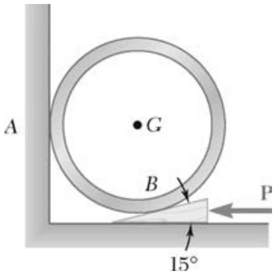
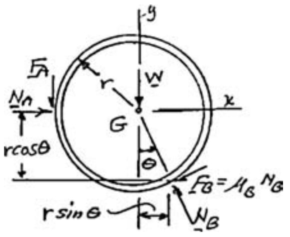


PROBLEM 8.61

A 15° wedge is forced under a 50-kg pipe as shown. Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20, determine the largest coefficient of static friction between the pipe and the vertical wall for which slipping will occur at A.

SOLUTION

<u>Free body: Pipe</u>



$$+\circlearrowleft \Sigma M_A = 0: \quad N_B r\cos\theta - \mu_B N_B r - (\mu_B N_B \sin\theta)r - Wr = 0$$

$$N_B = \frac{W}{\cos\theta - \mu_B(1+\sin\theta)}$$

$$N_B = \frac{W}{\cos 15° - 0.2(1+\sin 15°)}$$

$$N_B = 1.4002W$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_A - N_B \sin\theta - \mu_B N_B \cos\theta = 0$$

$$N_A = N_B(\sin\theta + \mu_B \cos\theta)$$

$$= (1.4002W)(\sin 15° + 0.2\times\cos 15°)$$

$$N_A = 0.63293W$$

$$+\uparrow \Sigma F_y = 0: \quad -F_A - W + N_B\cos\theta - \mu_B N_B \sin\theta = 0$$

$$F_A = N_B(\cos\theta - \mu_B\sin\theta) - W$$

$$F_A = (1.4002W)(\cos 15° - 0.2\times\sin\theta) - W$$

$$F_A = 0.28001W$$

For slipping at A: $\quad F_A = \mu_A N_A$

$$\mu_A = \frac{F_A}{N_A} = \frac{0.28001W}{0.63293W}$$

$\mu_A = 0.442$ ◀

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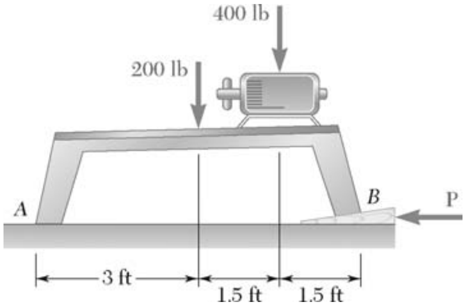
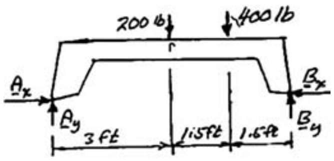


PROBLEM 8.62

An 8° wedge is to be forced under a machine base at *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.15, (*a*) determine the force **P** required to move the wedge, (*b*) indicate whether the machine base will slide on the floor.

SOLUTION

<u>Free body: Machine base</u>



$$+\circlearrowleft \Sigma M_B = 0: \quad (200 \text{ lb})(3 \text{ ft}) + (400 \text{ lb})(1.5 \text{ ft}) - A_y(6 \text{ ft}) = 0$$

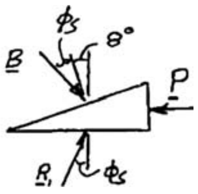
$$A_y = 200 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + B_y - 200 \text{ lb} - 400 \text{ lb} = 0$$

$$200 \text{ lb} + B_y - 200 \text{ lb} - 400 \text{ lb} = 0$$

$$B_y = 400 \text{ lb}$$

<u>Free body: Wedge</u>

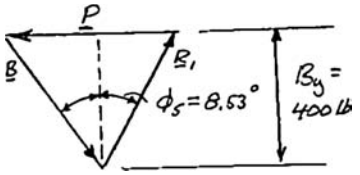


(Assume machine base will not move)

$$\mu_s = 0.15, \quad \phi_s = \tan^{-1} 0.15 = 8.53°$$

We know that $B_y = 400 \text{ lb}$

<u>Force triangle</u>: $8° + \phi_s = 8° + 8.53° = 16.53°$



$$P = (400 \text{ lb}) \tan 16.53° + (400 \text{ lb}) \tan 8.53°$$

$$P = 178.7 \text{ lb}$$

Total maximum friction force at *A* and *B*:

$$F_m = \mu_s W = 0.15(200 \text{ lb} + 400 \text{ lb}) = 90 \text{ lb}$$

If machine moves with wedge: $P = F_m = 90 \text{ lb}$

Using smaller *P*, we have

(*a*) $P = 90.0 \text{ lb}$ ◀

(*b*) Machine base moves ◀

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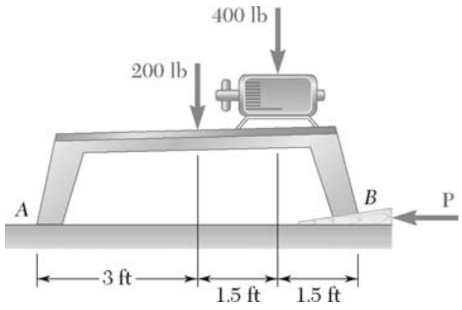


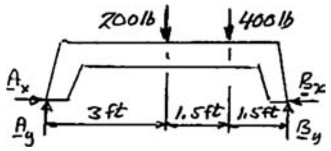
PROBLEM 8.63

Solve Problem 8.62 assuming that the wedge is to be forced under the machine base at *A* instead of *B*.

PROBLEM 8.62 An 8° wedge is to be forced under a machine base at *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.15, (*a*) determine the force **P** required to move the wedge, (*b*) indicate whether the machine base will slide on the floor.

SOLUTION

FBD: Machine base



$$+\circlearrowleft \Sigma M_B = 0: \quad (200 \text{ lb})(3 \text{ ft}) + (400 \text{ lb})(1.5 \text{ ft}) - A_y(6 \text{ ft}) = 0$$

$$\mathbf{A}_y = 200 \text{ lb} \uparrow$$

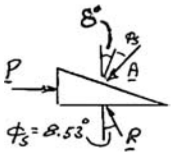
$$+\uparrow \Sigma F_y = 0: \quad A_y + B_y - 200 \text{ lb} - 400 \text{ lb} = 0$$

$$\mathbf{B}_y = 400 \text{ lb} \uparrow$$

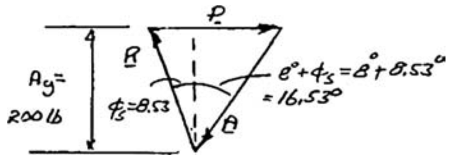
$$\phi_s = \tan^{-1} 0.15 = 8.53°$$

We known that $A_y = 200$ lb

FBD: Wedge



Force triangle:



(*a*) $P = (200 \text{ lb}) \tan 8.53° + (200 \text{ lb}) \tan 16.53°$ $P = 89.4$ lb ◀

(*b*) Total maximum friction force at *A* and *B*:

$$F_m = \mu_s(W) = 0.15(200 \text{ lb} + 400 \text{ lb}) = 90 \text{ lb}$$

Since $P < F_m$, machine base will not move ◀

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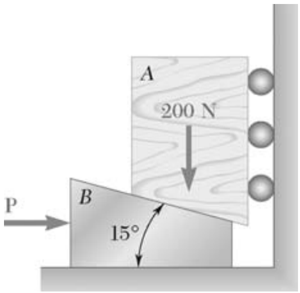


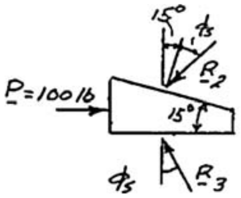
PROBLEM 8.64*

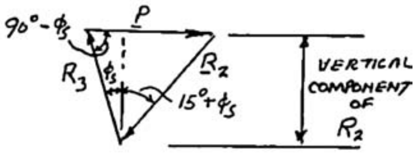
A 200-N block rests as shown on a wedge of negligible weight. The coefficient of static friction μ_s is the same at both surfaces of the wedge, and friction between the block and the vertical wall may be neglected. For $P = 100$ N, determine the value of μ_s for which motion is impending. (*Hint:* Solve the equation obtained by trial and error.)

SOLUTION

Free body: Wedge

Force triangle:



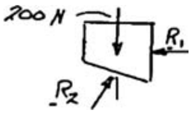


Law of sines:

$$\frac{R_2}{\sin(90° - \phi_s)} = \frac{P}{\sin(15° + 2\phi_s)}$$

$$R_2 = P\frac{\sin(90° - \phi_s)}{\sin(15° + 2\phi_s)} \qquad (1)$$

Free body: Block



$$\sum F_y = 0$$

Vertical component of R_2 is 200 N

Return to force triangle of wedge. Note $P = 100$ N

$$100\,\text{N} = (200\,\text{N})\tan\phi + (200\,\text{N})\tan(15° + \phi_s)$$
$$0.5 = \tan\phi + \tan(15° + \phi_s)$$

Solve by trial and error $\qquad \phi_s = 6.292$

$$\mu_s = \tan\phi_s = \tan 6.292° \qquad \mu_s = 0.1103 \blacktriangleleft$$

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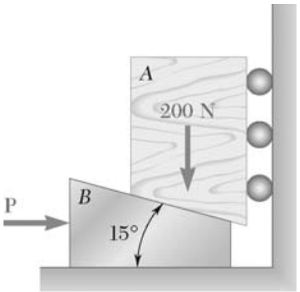


PROBLEM 8.65*

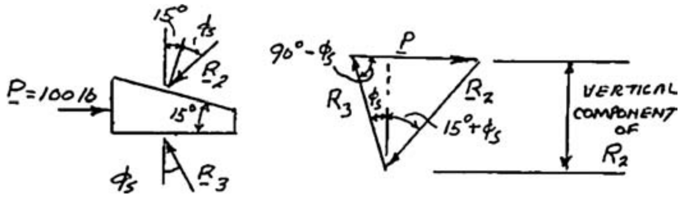
Solve Problem 8.64 assuming that the rollers are removed and that μ_s is the coefficient of friction at all surfaces of contact.

PROBLEM 8.64* A 200-N block rests as shown on a wedge of negligible weight. The coefficient of static friction μ_s is the same at both surfaces of the wedge, and friction between the block and the vertical wall may be neglected. For P = 100 N, determine the value of μ_s for which motion is impending. (*Hint:* Solve the equation obtained by trial and error.)

SOLUTION

Free body: Wedge

Force triangle:



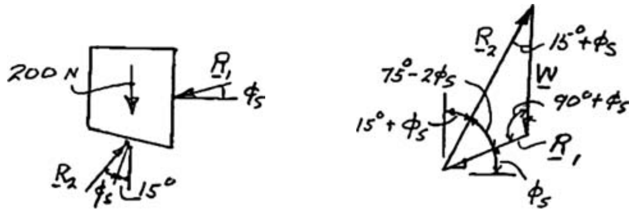
Law of sines:

$$\frac{R_2}{\sin(90^\circ-\phi_s)}=\frac{P}{\sin(15^\circ+2\phi_s)}$$

$$R_2=P\frac{\sin(90^\circ-\phi_s)}{\sin(15^\circ+2\phi_s)} \tag{1}$$

Free body: Block (Rollers removed)

Force triangle:



Law of sines:

$$\frac{R_2}{\sin(90^\circ+\phi_s)}=\frac{W}{\sin(75^\circ-2\phi_s)}$$

$$R_2=W\frac{\sin(90^\circ+\phi_s)}{\sin(75^\circ-2\theta_s)} \tag{2}$$

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PROBLEM 8.65* (Continued)

Equate R_2 from Eq. (1) and Eq. (2):

$$P\frac{\sin(90°-\phi_s)}{\sin(15°+2\phi_s)} = W\frac{\sin(90°+\phi_s)}{\sin(75°-2\phi_s)}$$

$$P = 100 \text{ lb}$$

$$W = 200 \text{ N}$$

$$0.5 = \frac{\sin(90°+\phi_s)\sin(15°+2\phi_s)}{\sin(75°-2\phi_s)\sin(90°-\phi_s)}$$

Solve by trial and error: $\phi_s = 5.784°$

$\mu_s = \tan\phi_s = \tan 5.784°$ $\qquad$ $\mu_s = 0.1013$ ◀

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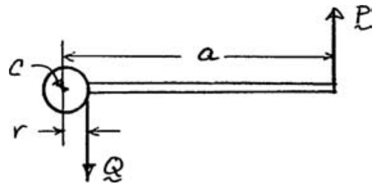
PROBLEM 8.66

Derive the following formulas relating the load **W** and the force **P** exerted on the handle of the jack discussed in Section 8.6. (*a*) $P = (Wr/a)\tan(\theta + \phi_s)$, to raise the load; (*b*) $P = (Wr/a)\tan(\phi_s - \theta)$, to lower the load if the screw is self-locking; (*c*) $P = (Wr/a)\tan(\theta - \phi_s)$, to hold the load if the screw is not self-locking.

SOLUTION

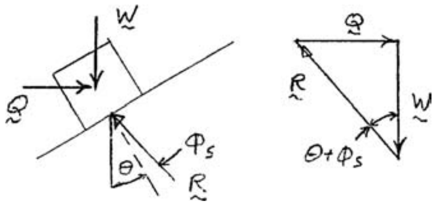
FBD jack handle:



See Section 8.6. $\circlearrowleft \Sigma M_C = 0{:}\quad aP - rQ = 0 \quad \text{or} \quad P = \frac{r}{a}Q$

FBD block on incline:

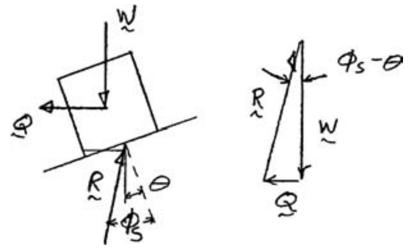
(*a*) Raising load



$$Q = W\tan(\theta + \phi_s)$$

$$P = \frac{r}{a}W\tan(\theta + \phi_s) \blacktriangleleft$$

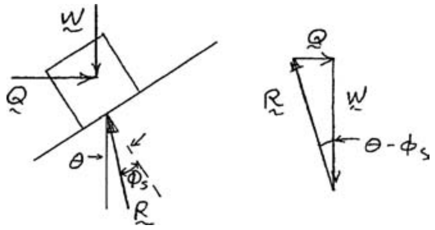
(*b*) Lowering load if screw is self-locking (i.e.,: if $\phi_s > \theta$)



$$Q = W\tan(\phi_s - \theta)$$

$$P = \frac{r}{a}W\tan(\phi_s - \theta) \blacktriangleleft$$

(*c*) Holding load is screw is not self-locking (i.e., if $\phi_s < \theta$)



$$Q = W\tan(\theta - \phi_s)$$

$$P = \frac{r}{a}W\tan(\theta - \phi_s) \blacktriangleleft$$

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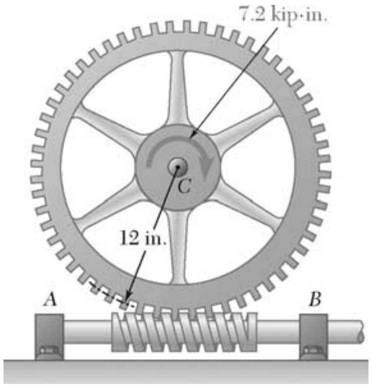


PROBLEM 8.67

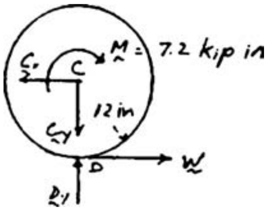
The square-threaded worm gear shown has a mean radius of 1.5 in. and a lead of 0.375 in. The large gear is subjected to a constant clockwise couple of 7.2 kip · in. Knowing that the coefficient of static friction between the two gears is 0.12, determine the couple that must be applied to shaft *AB* in order to rotate the large gear counterclockwise. Neglect friction in the bearings at *A*, *B*, and *C*.

SOLUTION

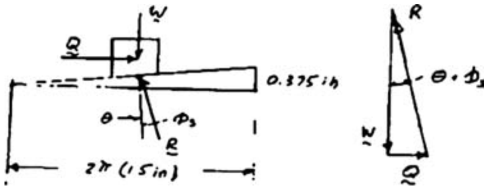
FBD large gear:

$$\circlearrowleft \Sigma M_C = 0: \quad (12\text{ in.})W - 7.2\text{ kip}\cdot\text{in.} = 0$$

$$W = 0.600\text{ kips}$$
$$= 600\text{ lb}$$



Block on incline:

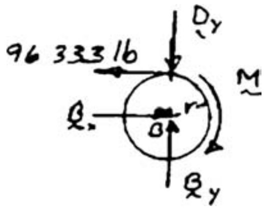


$$\theta = \tan^{-1}\frac{0.375\text{ in.}}{2\pi(1.5\text{ in.})} = 2.2785°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.12$$
$$= 6.8428°$$

$$Q = W\tan(\theta + \phi_s)$$
$$= (600\text{ lb})\tan 9.1213°$$
$$= 96.333\text{lb}$$

FBD worm gear:



$$r = 1.5\text{ in.}$$

$$\circlearrowleft \Sigma M_B = 0: \quad (1.5\text{in.})(96.333\text{lb}) - M = 0$$

$$M = 144.5\text{ lb}\cdot\text{in.} \blacktriangleleft$$

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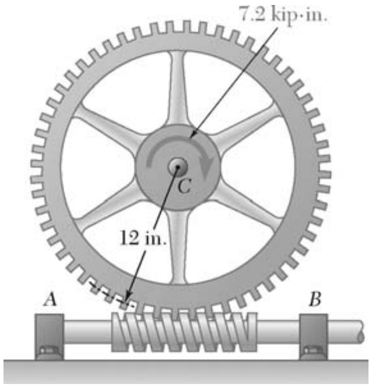


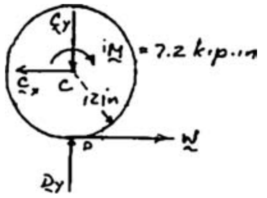
PROBLEM 8.68

In Problem 8.67, determine the couple that must be applied to shaft *AB* in order to rotate the large gear clockwise.

PROBLEM 8.67 The square-threaded worm gear shown has a mean radius of 1.5 in. and a lead of 0.375 in. The large gear is subjected to a constant clockwise couple of 7.2 kip · in. Knowing that the coefficient of static friction between the two gears is 0.12, determine the couple that must be applied to shaft *AB* in order to rotate the large gear counterclockwise. Neglect friction in the bearings at *A*, *B*, and *C*.

SOLUTION

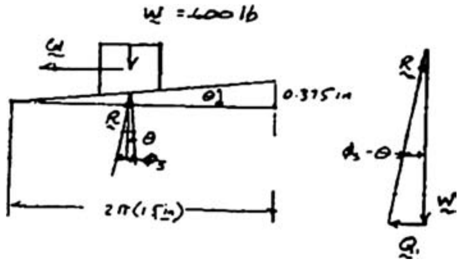
FBD large gear:



$$\circlearrowleft \Sigma M_C = 0: \quad (12\text{ in.})W - 7.2\text{ kip}\cdot\text{in.} = 0$$

$$W = 0.600\text{ kips} = 600\text{ lb}$$

Block on incline:



$$\theta = \tan^{-1}\frac{0.375\text{ in.}}{2\pi(1.5\text{ in.})}$$

$$= 2.2785°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.12$$

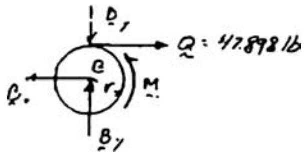
$$= 6.8428°$$

$$Q = W\tan(\phi_s - \theta)$$

$$= (600\text{ lb})\tan 4.5643°$$

$$= 47.898\text{ lb}$$

FBD worm gear:



$$r = 1.5\text{ in.}$$

$$\circlearrowleft \Sigma M_B = 0: \quad M - (1.5\text{ in.})(47.898\text{ lb}) = 0$$

$$M = 71.8\text{ lb}\cdot\text{in.} \blacktriangleleft$$

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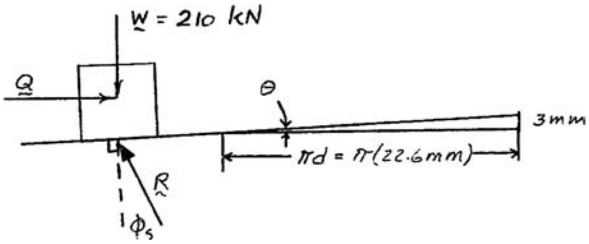
PROBLEM 8.69

High-strength bolts are used in the construction of many steel structures. For a 24-mm-nominal-diameter bolt the required minimum bolt tension is 210 kN. Assuming the coefficient of friction to be 0.40, determine the required couple that should be applied to the bolt and nut. The mean diameter of the thread is 22.6 mm, and the lead is 3 mm. Neglect friction between the nut and washer, and assume the bolt to be square-threaded.

SOLUTION

FBD block on incline:



$$\theta = \tan^{-1}\frac{3\text{ mm}}{(22.6\text{ mm})\pi}$$

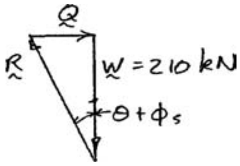
$$= 2.4195°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.40$$

$$\phi_s = 21.801°$$

$$Q = (210\text{ kN})\tan(21.801° + 2.4195°)$$

$$Q = 94.468\text{ kN}$$



$$\text{Torque} = \frac{d}{2}Q$$

$$= \frac{22.6\text{ mm}}{2}(94.468\text{ kN})$$

$$= 1067.49\text{ N}\cdot\text{m}$$

$\text{Torque} = 1068\text{ N}\cdot\text{m}$ ◀

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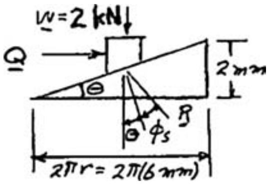
PROBLEM 8.70

The ends of two fixed rods *A* and *B* are each made in the form of a single-threaded screw of mean radius 6 mm and pitch 2 mm. Rod *A* has a right-handed thread and rod *B* has a left-handed thread. The coefficient of static friction between the rods and the threaded sleeve is 0.12. Determine the magnitude of the couple that must be applied to the sleeve in order to draw the rods closer together.

SOLUTION



To draw rods together:

Screw at *A*

$$\tan\theta = \frac{2\text{ mm}}{2\pi(6\text{ mm})}$$

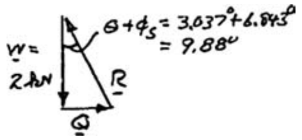
$$\theta = 3.037°$$

$$\phi_s = \tan^{-1} 0.12$$

$$= 6.843°$$

$$Q = (2\text{ kN})\tan 9.88°$$

$$= 348.3\text{ N}$$



$$\text{Torque at } A = Qr$$

$$= (348.3\text{ N})(6\text{ mm})$$

$$= 2.09\text{ N}\cdot\text{m}$$

Same torque required at *B*

$$\text{Total torque} = 4.18\text{ N}\cdot\text{m} \blacktriangleleft$$

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PROBLEM 8.71

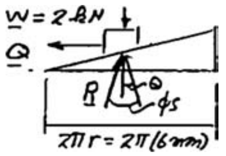
Assuming that in Problem 8.70 a right-handed thread is used on *both* rods A and B, determine the magnitude of the couple that must be applied to the sleeve in order to rotate it.

PROBLEM 8.70 The ends of two fixed rods A and B are each made in the form of a single-threaded screw of mean radius 6 mm and pitch 2 mm. Rod A has a right-handed thread and rod B has a left-handed thread. The coefficient of static friction between the rods and the threaded sleeve is 0.12. Determine the magnitude of the couple that must be applied to the sleeve in order to draw the rods closer together.

SOLUTION

From the solution to Problem 8.70,

$$\text{Torque at } A = 2.09\ \text{N}\cdot\text{m}$$



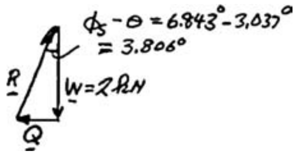
Screw at B: Loosening

$$\theta = 3.037°$$

$$\phi_s = 6.843°$$

$$Q = (2\ \text{kN})\tan 3.806°$$

$$= 133.1\ \text{N}$$



$$\text{Torque at } B = Qr$$

$$= (133.1\ \text{N})(6\ \text{mm})$$

$$= 0.798\ \text{N}\cdot\text{m}$$

$$\text{Total torque} = 2.09\ \text{N}\cdot\text{m} + 0.798\ \text{N}\cdot\text{m}$$

Total torque $= 2.89\ \text{N}\cdot\text{m}$ ◀

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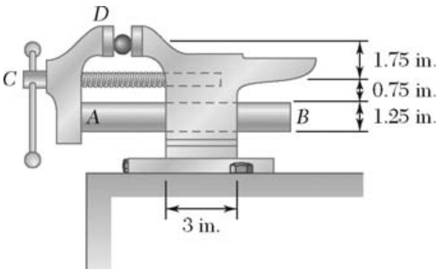
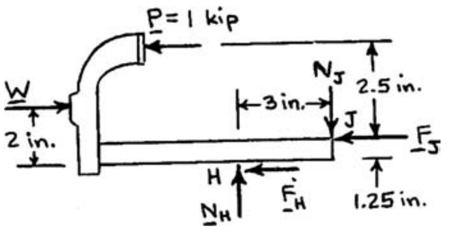


PROBLEM 8.72

In the machinist's vise shown, the movable jaw *D* is rigidly attached to the tongue *AB* that fits loosely into the fixed body of the vise. The screw is single-threaded into the fixed base and has a mean diameter of 0.75 in. and a pitch of 0.25 in. The coefficient of static friction is 0.25 between the threads and also between the tongue and the body. Neglecting bearing friction between the screw and the movable head, determine the couple that must be applied to the handle in order to produce a clamping force of 1 kip.

SOLUTION

<u>Free body: Jaw *D* and tongue *AB*</u>



P is due to elastic forces in clamped object.

W is force exerted by screw.

$$+\uparrow \Sigma F_y = 0: \quad N_H - N_J = 0 \quad N_J = N_H = N$$

For final tightening,

$$F_H = F_J = \mu_s N = 0.25\text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad W - P - 2(0.25\text{ N}) = 0$$

$$N = 2(W - P) \qquad (1)$$

$$+\circlearrowleft \Sigma M_H = 0: \quad P(3.75) - W(2) - N(3) + (0.25\text{ N})(1.25) = 0$$

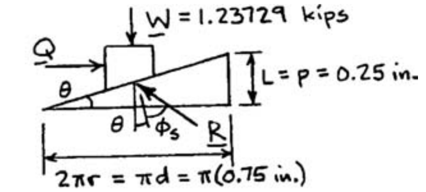
$$3.75P - 2W - 2.6875\text{ N} = 0 \qquad (2)$$

Substitute Eq. (1) into Eq. (2): $\quad 3.75P - 2W - 2.6875[2(W - P)] = 0$

$$7.375W = 9.125P = 9.125(1\text{ kip})$$

$$W = 1.23729\text{ kips}$$

<u>Block-and-incline analysis of screw</u>:



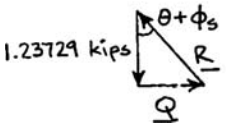
$$\tan\phi_s = \mu_s = 0.25$$

$$\phi_s = 14.0362°$$

$$\tan\theta = \frac{0.25\text{ in.}}{\pi(0.75\text{ in.})}$$

$$\theta = 6.0566°$$

$$\theta + \phi_s = 20.093°$$



$$Q = (1.23729\text{ kips})\tan 20.093°$$

$$= 0.45261\text{ kip}$$

$$T = Qr = (452.61\text{ lb})\left(\frac{0.75\text{ in.}}{2}\right)$$

$T = 169.7\text{ lb}\cdot\text{in.}$ ◀

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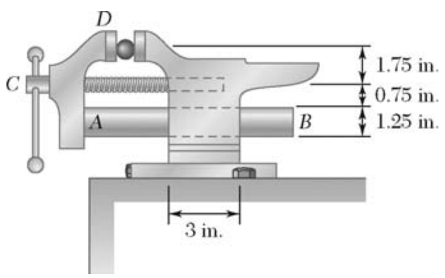
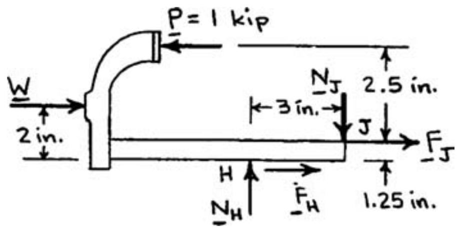
PROBLEM 8.73



In Problem 8.72, a clamping force of 1 kip was obtained by tightening the vise. Determine the couple that must be applied to the screw to loosen the vise.

PROBLEM 8.72 In the machinist's vise shown, the movable jaw *D* is rigidly attached to the tongue *AB* that fits loosely into the fixed body of the vise. The screw is single-threaded into the fixed base and has a mean diameter of 0.75 in. and a pitch of 0.25 in. The coefficient of static friction is 0.25 between the threads and also between the tongue and the body. Neglecting bearing friction between the screw and the movable head, determine the couple that must be applied to the handle in order to produce a clamping force of 1 kip.

SOLUTION



Free body: Jaw *D* and tongue *AB*

P is due to elastic forces in clamped object.

W is force exerted by screw.

$$+\uparrow \Sigma F_y = 0: \quad N_H - N_J = 0 \quad N_J = N_H = N$$

As vise is just about to loosen, $\quad F_H = F_J = \mu_s N = 0.25\text{ N}$

$$\xrightarrow{+} \Sigma F_x = 0: \quad W - P + 2(0.25\text{ N}) = 0$$

$$N = 2(P - W) \qquad (1)$$

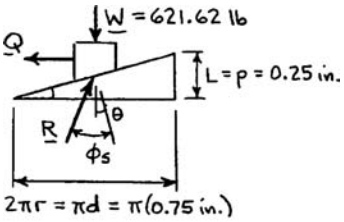
$$+\circlearrowleft \Sigma M_H = 0: \quad P(3.75) - W(2) - N(3) - (0.25\text{ N})(1.25) = 0$$

$$3.75P - 2W - 3.3125\text{ N} = 0 \qquad (2)$$

Substitute Eq. (1) into Eq. (2): $\quad 3.75P - 2W - 3.3125[2(P - W)] = 0$

$$4.625W = 2.875P = 2.875(1\text{ kip})$$

$$W = 0.62162\text{ kip}$$

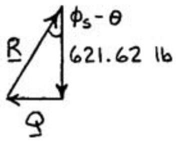


Block-and-incline analysis of screw:

$$\tan\phi_s = \mu_s = 0.25 \qquad \phi_s = 14.0362°$$

$$\tan\theta = \frac{0.25\text{ in.}}{\pi(0.75\text{ in.})} \qquad \theta = 6.0566°$$

$$\phi_s - \theta = 7.9796°$$



$$Q = (621.62\text{ lb})\tan 7.9796° = 87.137\text{ lb}$$

$$M = Qr = (87.137\text{ lb})\left(\frac{0.75\text{ in.}}{2}\right)$$

$M = 32.7\text{ lb}\cdot\text{in.}$ ◀

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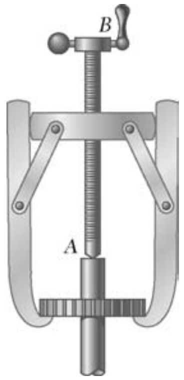
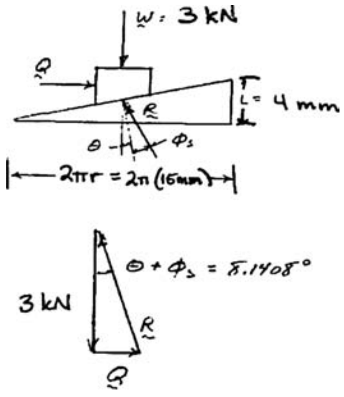


PROBLEM 8.74

In the gear-pulling assembly shown the square-threaded screw *AB* has a mean radius of 15 mm and a lead of 4 mm. Knowing that the coefficient of static friction is 0.10, determine the couple that must be applied to the screw in order to produce a force of 3 kN on the gear. Neglect friction at end *A* of the screw.

SOLUTION

Block/Incline:



$$\theta = \tan^{-1}\frac{0.25\text{ in.}}{1.875\pi\text{ in.}}$$
$$= 2.4302°$$

$$\phi_s = \tan^{-1}\mu_s$$
$$= \tan^{-1}(0.10)$$
$$= 5.7106°$$

$$Q = (3000\text{ N})\tan(8.1408°)$$
$$= 429.14\text{ N}$$

$$\text{Couple} = rQ$$
$$= (0.015\text{ m})(429.14\text{ N})$$
$$= 6.4371\text{ N}\cdot\text{m}$$

$M = 6.44\text{ N}\cdot\text{m}$ ◀

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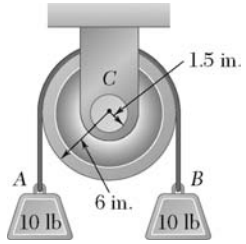
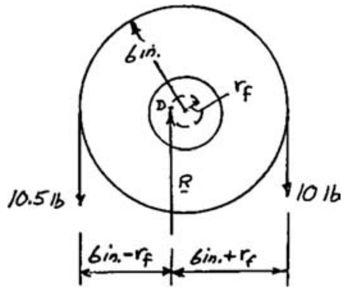


PROBLEM 8.75

A 6-in.-radius pulley of weight 5 lb is attached to a 1.5-in.-radius shaft that fits loosely in a fixed bearing. It is observed that the pulley will just start rotating if a 0.5-lb weight is added to block *A*. Determine the coefficient of static friction between the shaft and the bearing.

SOLUTION



$$+\circlearrowright \Sigma M_D = 0: \quad (10.5 \text{ lb})(6 \text{ in.} - r_f) - (10 \text{ lb})(6 \text{ in.} + r_f) = 0$$

$$(0.5 \text{ lb})(6 \text{ in.}) = (20.5 \text{ lb})r_f$$

$$r_f = 0.146341 \text{ in.}$$

$$r_f = r \sin \phi_s$$

$$\sin \phi_s = \frac{0.146341 \text{ in.}}{1.5 \text{ in.}}$$

$$= 0.097561$$

$$\phi_s = 5.5987°$$

$$\mu_s = \tan \phi_s$$

$$= \tan 5.5987°$$

$$\mu_s = 0.0980 \blacktriangleleft$$

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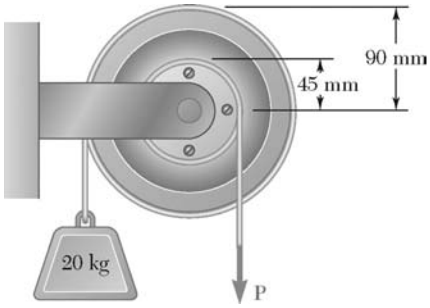
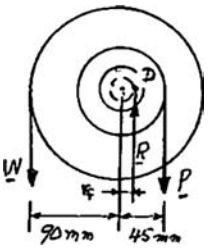


PROBLEM 8.76

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to start raising the load.

SOLUTION



$$+\circlearrowleft \Sigma M_D = 0: \quad P(45 - r_f) - W(90 + r_f) = 0$$

$$P = W\frac{90 + r_f}{45 - r_f}$$

$$= (196.2\text{ N})\frac{90\text{ mm} + 4\text{ mm}}{45\text{ mm} - 4\text{ mm}}$$

$$P = 449.82\text{ N}$$

P = 450 N ↓ ◀

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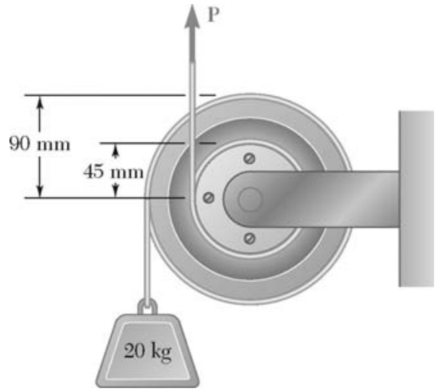
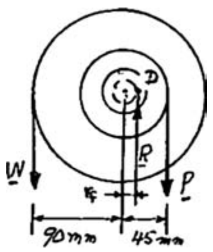


PROBLEM 8.77

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to start raising the load.

SOLUTION

Find **P** required to start raising load



$$+\circlearrowleft \Sigma M_D = 0: \quad P(45 - r_f) - W(90 - r_f) = 0$$

$$P = W\frac{90 - r_f}{45 - r_f}$$

$$= (196.2 \text{ N})\frac{90 \text{ mm} - 4 \text{ mm}}{45 \text{ mm} - 4 \text{ mm}}$$

$$P = 411.54 \text{ N}$$

P = 412 N ↑ ◀

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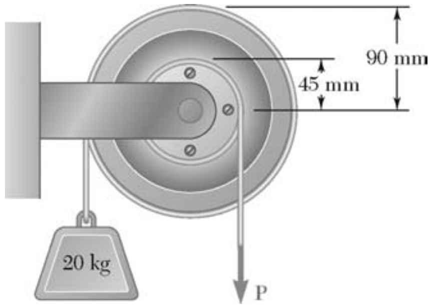
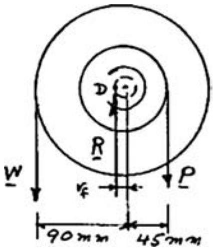


PROBLEM 8.78

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the smallest force **P** required to maintain equilibrium.

SOLUTION

Find smallest **P** to maintain equilibrium



$$+\circlearrowleft \Sigma M_D = 0: \quad P(45 + r_f) - W(90 - r_f) = 0$$

$$P = W\frac{90 - r_f}{45 + r_f}$$

$$= (196.2\text{ N})\frac{90\text{ mm} - 4\text{ mm}}{45\text{ mm} + 4\text{ mm}}$$

$$P = 344.35\text{ N}$$

$\mathbf{P} = 344\text{ N}\downarrow$ ◀

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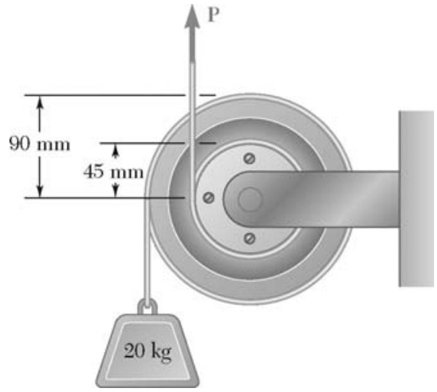
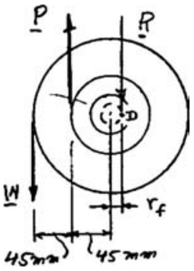


PROBLEM 8.79

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the smallest force **P** required to maintain equilibrium.

SOLUTION

Find smallest **P** to maintain equilibrium



$$+\circlearrowleft \Sigma M_D = 0: \quad P(45 + r_f) - W(90 + r_f) = 0$$

$$P = W\frac{90 + r_f}{45 + r_f}$$

$$= (196.2 \text{ N})\frac{90 \text{ mm} + 4 \text{ mm}}{45 \text{ mm} + 4 \text{ mm}}$$

$$P = 376.38 \text{ N}$$

P $= 376$ N ↑ ◀

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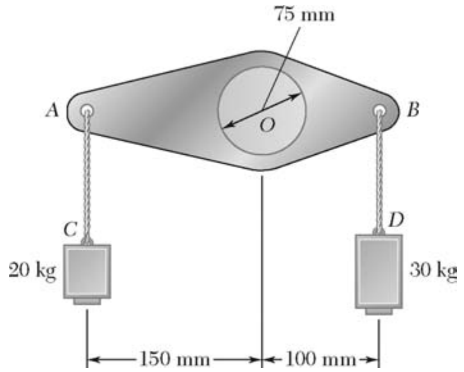
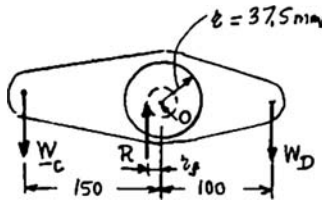


PROBLEM 8.80

A lever of negligible weight is loosely fitted onto a 75-mm-diameter fixed shaft. It is observed that the lever will just start rotating if a 3-kg mass is added at *C*. Determine the coefficient of static friction between the shaft and the lever.

SOLUTION

$$+\circlearrowleft \Sigma M_O = 0: \quad W_C(150) - W_D(100) - Rr_f = 0$$



But
$$W_C = (23\text{ kg})(9.81\text{ m/s}^2)$$
$$W_D = (30\text{ kg})(9.81\text{ m/s}^2)$$
$$R = W_C + W_D = (53\text{ kg})(9.81)$$

Thus, after dividing by 9.81,

$$23(150) - 30(100) - 53\,r_f = 0$$
$$r_f = 8.49\text{ mm}$$

But
$$\mu_s \approx \frac{r_f}{r} = \frac{8.49\text{ mm}}{37.5\text{ mm}}$$

$\mu_s \approx 0.226$ ◀

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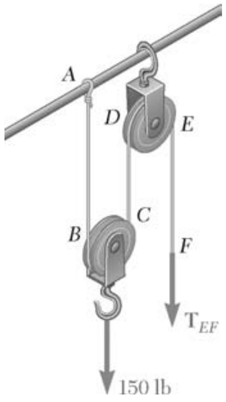


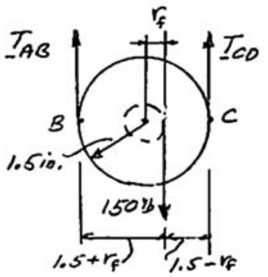
PROBLEM 8.81

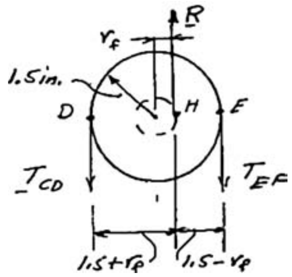
The block and tackle shown are used to raise a 150-lb load. Each of the 3-in.-diameter pulleys rotates on a 0.5-in.-diameter axle. Knowing that the coefficient of static friction is 0.20, determine the tension in each portion of the rope as the load is slowly raised.

SOLUTION

<u>For each pulley</u>: Axle diameter $= 0.5$ in.

$$r_f = r\sin\phi_s \approx \mu_s r = 0.20\left(\frac{0.5\text{ in.}}{2}\right) = 0.05\text{ in.}$$





<u>Pulley *BC*</u>:

$$+\circlearrowleft \Sigma M_B = 0: \quad T_{CD}(3\text{ in.}) - (150\text{ lb})(1.5\text{ in.} + r_f) = 0$$

$$T_{CD} = \frac{1}{3}(150\text{ lb})(1.5\text{ in.} + 0.05\text{ in.}) \qquad T_{CD} = 77.5\text{ lb} ◀$$

$$+\uparrow \Sigma F_y = 0: \quad T_{AB} + 77.5\text{ lb} - 150\text{ lb} = 0 \qquad T_{AB} = 72.5\text{ lb} ◀$$

<u>Pulley *DE*</u>:

$$+\circlearrowleft \Sigma M_B = 0: \quad T_{CD}(1.5 + r_f) - T_{EF}(1.5 - r_f) = 0$$

$$T_{EF} = T_{CD}\frac{1.5 + r_f}{1.5 - r_f}$$

$$= (77.5\text{ lb})\frac{1.5\text{ in.} + 0.05\text{ in.}}{1.5\text{ in.} - 0.05\text{ in.}} \qquad T_{EF} = 82.8\text{ lb} ◀$$

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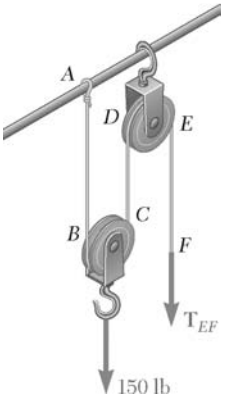


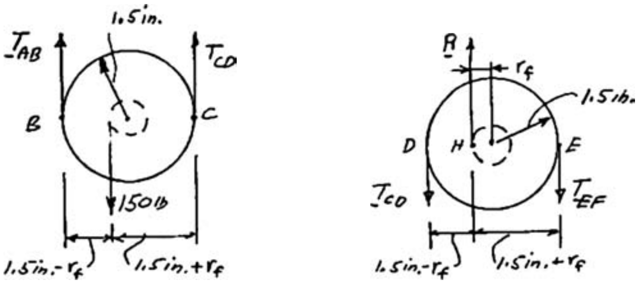
PROBLEM 8.82

The block and tackle shown are used to lower a 150-lb load. Each of the 3-in.-diameter pulleys rotates on a 0.5-in.-diameter axle. Knowing that the coefficient of static friction is 0.20, determine the tension in each portion of the rope as the load is slowly lowered.

SOLUTION

For each pulley:

$$r_f = r\mu_s = \left(\frac{0.5\text{ in.}}{2}\right)0.2 = 0.05\text{ in.}$$



Pulley *BC*:

$$+\circlearrowleft \Sigma M_B = 0: \quad T_{CD}(3\text{ in.}) - (150\text{ lb})(1.5\text{ in.} - r_f) = 0$$

$$T_{CD} = \frac{(150\text{ lb})(1.5\text{ in.} - 0.05\text{ in.})}{3\text{ in.}} \qquad T_{CD} = 72.5\text{ lb} \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad T_{AB} + 72.5\text{ lb} - 150\text{ lb} = 0 \qquad T_{AB} = 77.5\text{ lb} \blacktriangleleft$$

Pulley *DE*:

$$T_{CD}(1.5\text{ in.} - r_f) - T_{EF}(1.5\text{ in.} + r_f) = 0$$

$$T_{EF} = T_{CD}\frac{1.5\text{ in.} - r_f}{1.5\text{ in.} + r_f}$$

$$= (72.5\text{ lb})\frac{1.5\text{ in.} - 0.05\text{ in.}}{1.5\text{ in.} + 0.05\text{ in.}} \qquad T_{EF} = 67.8\text{ lb} \blacktriangleleft$$

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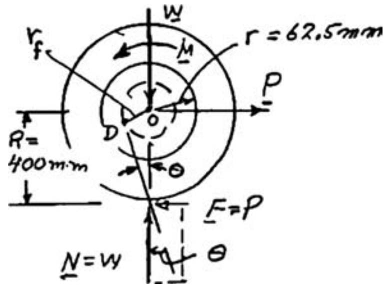
PROBLEM 8.83

A loaded railroad car has a mass of 30 Mg and is supported by eight 800-mm-diameter wheels with 125-mm-diameter axles. Knowing that the coefficients of friction are $\mu_s = 0.020$ and $\mu_k = 0.015$, determine the horizontal force required (*a*) to start the car moving, (*b*) to keep the car moving at a constant speed. Neglect rolling resistance between the wheels and the track.

SOLUTION



$$r_f = \mu r; \quad R = 400 \text{ mm}$$

$$\sin\theta = \tan\theta = \frac{r_f}{R} = \frac{\mu r}{R}$$

$$P = W \tan\theta = W \frac{\mu r}{R}$$

$$P = W \mu \frac{62.5 \text{ mm}}{400 \text{ mm}}$$

$$= 0.15625 W\mu$$

<u>For one wheel</u>:

$$W = \frac{1}{8}(30 \text{ mg})(9.81 \text{ m/s}^2)$$

$$= \frac{1}{8}(294.3 \text{ kN})$$

<u>For eight wheels of rail road car</u>:

$$\Sigma P = 8(0.15625)\frac{1}{8}(294.3 \text{ kN})\mu$$

$$= (45.984\mu) \text{ kN}$$

(*a*) To start motion: $\mu_s = 0.020$

$$\Sigma P = (45.984)(0.020)$$

$$= 0.9197 \text{ kN}$$

$\Sigma P = 920 \text{ N}$ ◀

(*b*) To maintain motion: $\mu_k = 0.015$

$$\Sigma P = (45.984)(0.015)$$

$$= 0.6897 \text{ kN}$$

$\Sigma P = 690 \text{ N}$ ◀

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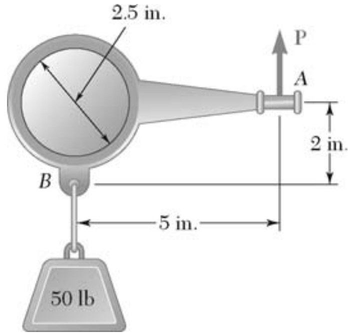
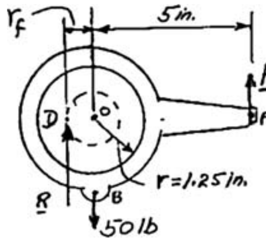


PROBLEM 8.84

A lever *AB* of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating counterclockwise.

SOLUTION



$$r_f = \mu_s r$$
$$= 0.15(1.25 \text{ in.})$$
$$= 0.1875 \text{ in.}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad P(5 \text{ in.} + r_f) - (50 \text{ lb})r_f = 0$$

$$P = \frac{50(0.1875)}{5.1875}$$
$$= 1.807 \text{ lb}$$

$\mathbf{P} = 1.807 \text{ lb} \uparrow$ ◀

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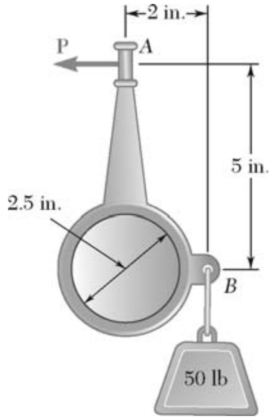
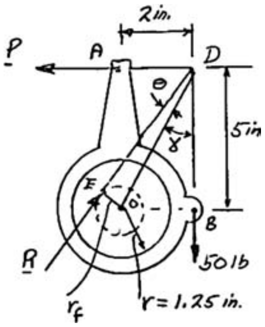


PROBLEM 8.85

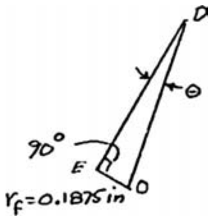
A lever AB of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating counterclockwise.

SOLUTION



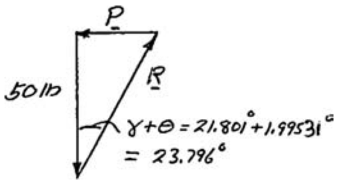
$$r_f = \mu_s r$$
$$= 0.15(1.25\text{ in.})$$
$$r_f = 0.1875\text{ in.}$$
$$\tan\gamma = \frac{2\text{ in.}}{5\text{ in.}}$$
$$\gamma = 21.801°$$

In ΔEOD:



$$OD = \sqrt{(2\text{ in.})^2 + (5\text{ in.})^2}$$
$$= 5.3852\text{ in.}$$
$$\sin\theta = \frac{OE}{OD} = \frac{r_f}{OD}$$
$$= \frac{0.1875\text{ in.}}{5.3852\text{ in.}}$$
$$\theta = 1.99531°$$

Force triangle:



$$P = (50\text{ lb})\tan(\gamma + \theta)$$
$$= (50\text{ lb})\tan 23.796°$$
$$= 22.049\text{ lb}$$

P = 22.0 lb ← ◀

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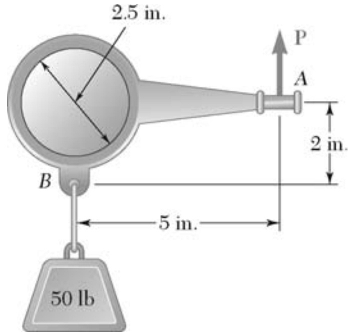
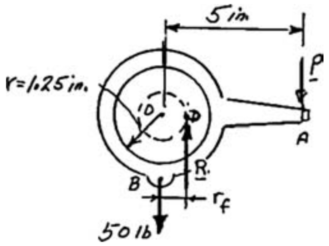


PROBLEM 8.86

A lever *AB* of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating clockwise.

SOLUTION



$$r_f = \mu_s r$$
$$= 0.15(1.25\text{ in.})$$
$$r_f = 0.1875\text{ in.}$$

$$+\circlearrowright \Sigma M_D = 0: \quad P(5\text{ in.} - r_f) - (50\text{ lb})r_f = 0$$

$$P = \frac{50(0.1875)}{5 - 0.1875}$$
$$= 1.948\text{ lb}$$

$\mathbf{P} = 1.948\text{ lb}\downarrow$ ◀

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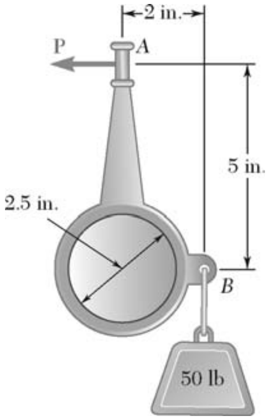
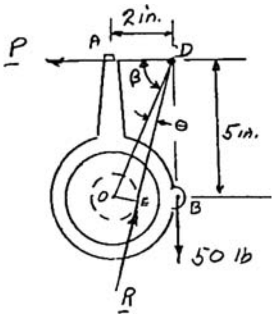


PROBLEM 8.87

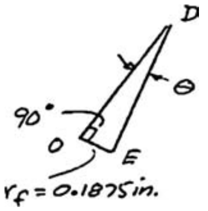
A lever *AB* of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating clockwise.

SOLUTION



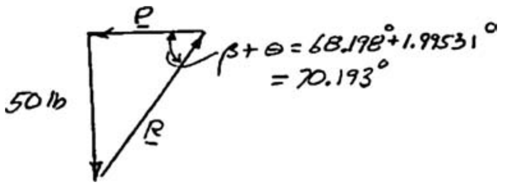
$$r_f = \mu_s r$$
$$= 0.15(1.25\text{ in.})$$
$$= 0.1875\text{ in.}$$
$$\tan\beta = \frac{5\text{ in.}}{2\text{ in.}}$$
$$\beta = 68.198°$$

In ΔEOD:



$$OD = \sqrt{(2\text{ in.})^2 + (5\text{ in.})^2}$$
$$OD = 5.3852\text{ in.}$$
$$\sin\theta = \frac{OE}{OD} = \frac{0.1875\text{ in.}}{5.3852\text{ in.}}$$
$$\theta = 1.99531°$$

Force triangle:



$$P = \frac{50}{\tan(\beta+\theta)} = \frac{50\text{ lb}}{\tan 70.193°}$$

P = 18.01 lb ⟵ ◀

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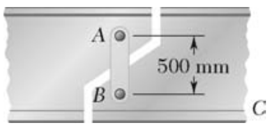
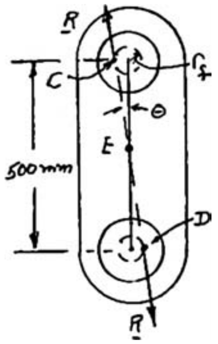
PROBLEM 8.88



The link arrangement shown is frequently used in highway bridge construction to allow for expansion due to changes in temperature. At each of the 60-mm-diameter pins *A* and *B* the coefficient of static friction is 0.20. Knowing that the vertical component of the force exerted by *BC* on the link is 200 kN, determine (*a*) the horizontal force that should be exerted on beam *BC* to just move the link, (*b*) the angle that the resulting force exerted by beam *BC* on the link will form with the vertical.

SOLUTION



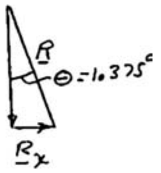
Bearing:

$$r = 30 \text{ mm}$$

$$\begin{aligned} r_f &= \mu_s r \\ &= 0.20(30 \text{ mm}) \\ &= 6 \text{ mm} \end{aligned}$$

Resultant forces **R** must be tangent to friction circles at Points *C* and *D*.

(*a*)

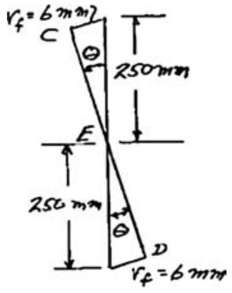


$$R_y = \text{Vertical component} = 200 \text{ kN}$$

$$\begin{aligned} R_x &= R_y \tan\theta \\ &= (200 \text{ kN}) \tan 1.375° \\ &= 4.80 \text{ kN} \end{aligned}$$

Horizontal force = 4.80 kN ◀

(*b*)



$$\sin\theta = \frac{6 \text{ mm}}{250 \text{ mm}}$$

$$\sin\theta = 0.024$$

$\theta = 1.375°$ ◀

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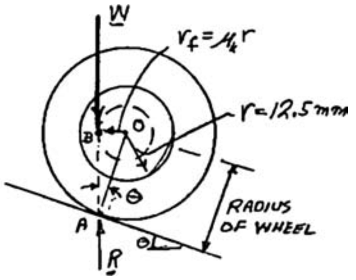
PROBLEM 8.89

A scooter is to be designed to roll down a 2 percent slope at a constant speed. Assuming that the coefficient of kinetic friction between the 25-mm-diameter axles and the bearings is 0.10, determine the required diameter of the wheels. Neglect the rolling resistance between the wheels and the ground.

SOLUTION



$$\tan\theta = \frac{2}{100} = 0.02$$

Since a scooter rolls at constant speed, each wheel is in equilibrium. Thus, **W** and **R** must have a common line of action tangent to the friction circle.

$$r_f = \mu_k r = (0.10)(12.5 \text{ mm})$$
$$= 1.25 \text{ mm}$$

$$OA = \frac{OB}{\tan\theta} = \frac{r_f}{\tan\theta} = \frac{1.25 \text{ mm}}{0.02}$$
$$= 62.5 \text{ mm}$$

Diameter of wheel $= 2(OA) = 125.0$ mm ◀

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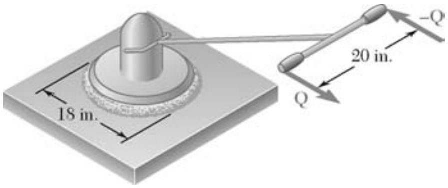

PROBLEM 8.90

A 50-lb electric floor polisher is operated on a surface for which the coefficient of kinetic friction is 0.25. Assuming that the normal force per unit area between the disk and the floor is uniformly distributed, determine the magnitude Q of the horizontal forces required to prevent motion of the machine.

SOLUTION

See Figure 8.12 and Eq. (8.9).

Using:

$$R = 9 \text{ in.}$$
$$P = 50 \text{ lb}$$

and

$$\mu_k = 0.25$$

$$M = \frac{2}{3}\mu_k PR = \frac{2}{3}(0.25)(50 \text{ lb})(9 \text{ in.})$$
$$= 75 \text{ lb} \cdot \text{in.}$$

$\Sigma M_y = 0$ yields:

$$M = Q(20 \text{ in.})$$
$$75 \text{ lb} \cdot \text{in.} = Q(20 \text{ in.})$$

$$Q = 3.75 \text{ lb} \blacktriangleleft$$

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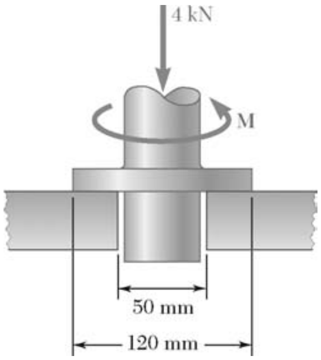

PROBLEM 8.91

Knowing that a couple of magnitude $30\ \text{N}\cdot\text{m}$ is required to start the vertical shaft rotating, determine the coefficient of static friction between the annular surfaces of contact.

SOLUTION

For annular contact regions, use Equation 8.8 with impending slipping:

$$M = \frac{2}{3}\mu_s N \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

So,

$$30\ \text{N}\cdot\text{m} = \frac{2}{3}\mu_s (4000\ \text{N}) \frac{(0.06\ \text{m})^3 - (0.025\ \text{m})^3}{(0.06\ \text{m})^2 - (0.025\ \text{m})^2}$$

$$\mu_s = 0.1670 \blacktriangleleft$$

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PROBLEM 8.92*

The frictional resistance of a thrust bearing decreases as the shaft and bearing surfaces wear out. It is generally assumed that the wear is directly proportional to the distance traveled by any given point of the shaft and thus to the distance r from the point to the axis of the shaft. Assuming, then, that the normal force per unit area is inversely proportional to r, show that the magnitude M of the couple required to overcome the frictional resistance of a worn-out end bearing (with contact over the full circular area) is equal to 75 percent of the value given by Eq. (8.9) for a new bearing.

SOLUTION

Using Figure 8.12, we assume

$$\Delta N = \frac{k}{r}\Delta A: \quad \Delta A = r\Delta\theta\,\Delta r$$

$$\Delta N = \frac{k}{r} r\Delta\theta\Delta r = k\Delta\theta\Delta r$$

We write $\quad P = \Sigma\Delta N \quad$ or $\quad P = \int dN$

$$P = \int_0^{2\pi}\int_0^R k\Delta\theta\Delta r = 2\pi Rk; \quad k = \frac{P}{2\pi R}$$

$$\Delta N = \frac{P\Delta\theta\Delta r}{2\pi R}$$

$$\Delta M = r\Delta F = r\mu_k\Delta N = r\mu_k\frac{P\Delta\theta\Delta r}{2\pi R}$$

$$M = \int_0^{2\pi}\int_0^R \frac{\mu_k P}{2\pi R} rdrd\theta = \frac{2\pi\mu_k P}{2\pi R}\cdot\frac{R^2}{2} = \frac{1}{2}\mu_k PR$$

From Eq. (8.9) for a new bearing,

$$M_{\text{New}} = \frac{2}{3}\mu_k PR$$

Thus, $\quad \dfrac{M}{M_{\text{New}}} = \dfrac{\frac{1}{2}}{\frac{2}{3}} = \dfrac{3}{4}$

$$M = 0.75M_{\text{New}} \blacktriangleleft$$

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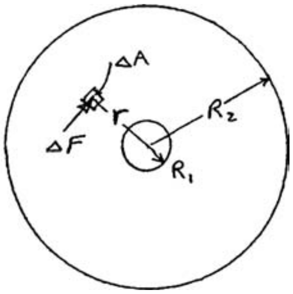
PROBLEM 8.93*

Assuming that bearings wear out as indicated in Problem 8.92, show that the magnitude M of the couple required to overcome the frictional resistance of a worn-out collar bearing is

$$M = \frac{1}{2}\mu_k P(R_1 + R_2)$$

where P = magnitude of the total axial force
R_1, R_2 = inner and outer radii of collar

SOLUTION



Let normal force on ΔA be $\Delta N,$ and $\dfrac{\Delta N}{\Delta A} = \dfrac{k}{r}.$

As in the text, $\qquad \Delta F = \mu\Delta N \qquad \Delta M = r\Delta F$

The total normal force P is

$$P = \lim_{\Delta A \to 0} \Sigma\Delta N = \int_0^{2\pi}\left(\int_{R_1}^{R_2} \frac{k}{r}\dot{r}dr\right)d\theta$$

$$P = 2\pi\int_{R_1}^{R_2} kdr = 2\pi k(R_2 - R_1) \quad \text{or} \quad k = \frac{P}{2\pi(R_2 - R_1)}$$

Total couple: $\qquad M_{\text{worn}} = \lim_{\Delta A \to 0} \Sigma\Delta M = \int_0^{2\pi}\left(\int_{R_1}^{R_2} r\mu\frac{k}{r}rdr\right)d\theta$

$$M_{\text{worn}} = 2\pi\mu k\int_{R_1}^{R_2}(rdr) = \pi\mu k\left(R_2^2 - R_1^2\right) = \frac{\pi\mu P\left(R_2^2 - R_1^2\right)}{2\pi(R_2 - R_1)}$$

$$M_{\text{worn}} = \frac{1}{2}\mu P(R_2 + R_1) \blacktriangleleft$$

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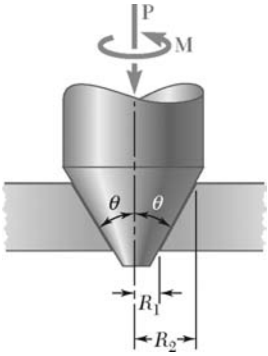


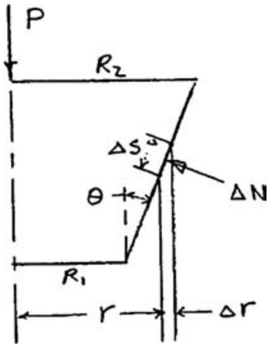
PROBLEM 8.94*

Assuming that the pressure between the surfaces of contact is uniform, show that the magnitude M of the couple required to overcome frictional resistance for the conical bearing shown is

$$M = \frac{2}{3}\frac{\mu_k P}{\sin\theta}\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

SOLUTION

Let normal force on ΔA be ΔN and $\dfrac{\Delta N}{\Delta A} = k.$



So $\qquad \Delta N = k\Delta A \qquad \Delta A = r\Delta s\Delta\phi \qquad \Delta s = \dfrac{\Delta r}{\sin\theta}$

where ϕ is the azimuthal angle around the symmetry axis of rotation.

$$\Delta F_y = \Delta N \sin\theta = kr\Delta r\Delta\phi$$

Total vertical force: $\qquad P = \lim_{\Delta A \to 0} \Sigma \Delta F_y$

$$P = \int_0^{2\pi}\left(\int_{R_1}^{R_2} krdr\right)d\phi = 2\pi k\int_{R_1}^{R_2} rdr$$

$$P = \pi k\left(R_2^2 - R_1^2\right) \quad \text{or} \quad k = \frac{P}{\pi\left(R_2^2 - R_1^2\right)}$$

Friction force: $\qquad \Delta F = \mu\Delta N = \mu k\Delta A$

Moment: $\qquad \Delta M = r\Delta F = r\mu kr\dfrac{\Delta r}{\sin\theta}\Delta\phi$

Total couple: $\qquad M = \lim_{\Delta A\to 0}\Sigma\Delta M = \displaystyle\int_0^{2\pi}\left(\int_{R_1}^{R_2}\frac{\mu k}{\sin\theta}r^2dr\right)d\phi$

$$M = 2\pi\frac{\mu k}{\sin\theta}\int_{R_1}^{R_2} r^2dr = \frac{2}{3}\frac{\pi\mu}{\sin\theta}\frac{P}{\pi\left(R_2^2 - R_3^2\right)}\left(R_2^3 - R_3^3\right)$$

$$M = \frac{2}{3}\frac{\mu P}{\sin\theta}\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \blacktriangleleft$$

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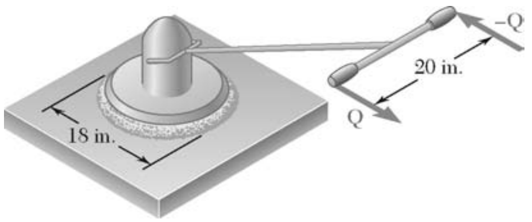


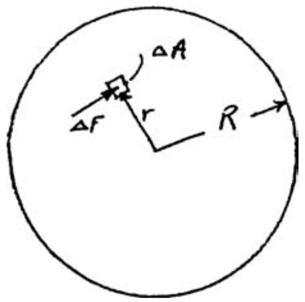
PROBLEM 8.95

Solve Problem 8.90 assuming that the normal force per unit area between the disk and the floor varies linearly from a maximum at the center to zero at the circumference of the disk.

PROBLEM 8.90 A 50-lb electric floor polisher is operated on a surface for which the coefficient of kinetic friction is 0.25. Assuming that the normal force per unit area between the disk and the floor is uniformly distributed, determine the magnitude Q of the horizontal forces required to prevent motion of the machine.

SOLUTION

Let normal force on ΔA be ΔN and $\dfrac{\Delta N}{\Delta A} = k\left(1-\dfrac{r}{R}\right)$.



$$\Delta F = \mu\Delta N = \mu k\left(1-\frac{r}{R}\right)\Delta A = \mu k\left(1-\frac{r}{R}\right) r\Delta r\Delta\theta$$

$$P = \lim_{\Delta A\to 0}\Sigma\Delta N = \int_0^{2\pi}\left[\int_0^R k\left(1-\frac{r}{R}\right) rdr\right] d\theta$$

$$P = 2\pi k\int_0^R\left(1-\frac{r}{R}\right) rdr = 2\pi k\left(\frac{R^2}{2}-\frac{R^3}{3R}\right)$$

$$P = \frac{1}{3}\pi kR^2 \quad \text{or} \quad k = \frac{3P}{\pi R^2}$$

$$M = \lim_{\Delta A\to 0}\Sigma r\Delta F = \int_0^{2\pi}\left[\int_0^R r\mu k\left(1-\frac{r}{R}\right) rdr\right] d\theta$$

$$= 2\pi\mu k\int_0^R\left(r^2-\frac{r^3}{R}\right) dr$$

$$= 2\pi\mu k\left(\frac{R^3}{3}-\frac{R^4}{4R}\right) = \frac{1}{6}\pi\mu kR^3$$

$$= \frac{\pi\mu}{6}\frac{3P}{\pi R^2}R^3 = \frac{1}{2}\mu PR$$

where $\mu = \mu_k = 0.25 \quad R = 9\text{ in.}$

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PROBLEM 8.95 (Continued)

$$P = W = 50 \text{ lb}$$

Then $$M = \frac{1}{2}(0.25)(50 \text{ lb})(9 \text{ in.})$$

$$= 56.250 \text{ lb} \cdot \text{in.}$$

Finally $$Q = \frac{M}{d} = \frac{56.250 \text{ lb} \cdot \text{in.}}{20 \text{ in.}}$$

$Q = 2.81 \text{ lb}$ ◀

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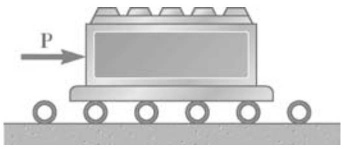
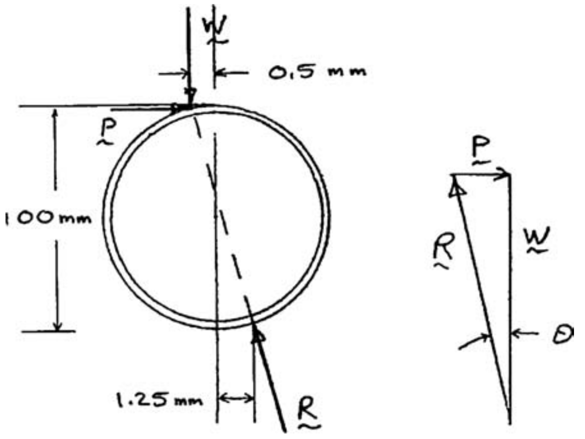


PROBLEM 8.96

A 900-kg machine base is rolled along a concrete floor using a series of steel pipes with outside diameters of 100 mm. Knowing that the coefficient of rolling resistance is 0.5 mm between the pipes and the base and 1.25 mm between the pipes and the concrete floor, determine the magnitude of the force **P** required to slowly move the base along the floor.

SOLUTION

FBD pipe:



$$W = mg$$
$$= (900 \text{ kg})(9.81 \text{ m/s}^2)$$
$$= 8829.0 \text{ N}$$
$$\theta = \sin^{-1} \frac{.5 \text{ mm} + 1.25 \text{ mm}}{100 \text{ mm}}$$
$$= 1.00273°$$

$P = W \tan\theta$ for each pipe, so also for total

$$P = (8829.0 \text{ N}) \tan(1.00273°)$$

$\mathbf{P} = 154.4$ N ◀

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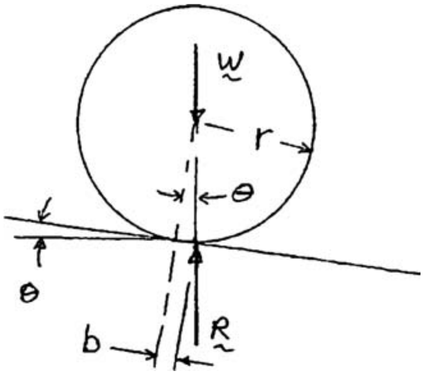
PROBLEM 8.97

Knowing that a 6-in.-diameter disk rolls at a constant velocity down a 2 percent incline, determine the coefficient of rolling resistance between the disk and the incline.

SOLUTION

FBD disk:



$$\tan\theta = \text{slope} = 0.02$$

$$b = r\tan\theta = (3\text{ in.})(0.02) \qquad b = 0.0600\text{ in.} \blacktriangleleft$$

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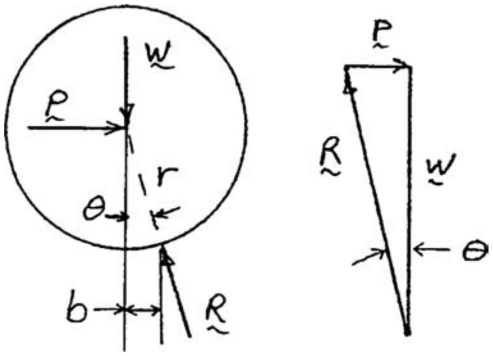
PROBLEM 8.98

Determine the horizontal force required to move a 2500-lb automobile with 23-in.-diameter tires along a horizontal road at a constant speed. Neglect all forms of friction except rolling resistance, and assume the coefficient of rolling resistance to be 0.05 in.

SOLUTION

FBD wheel:



$$r = 11.5 \text{ in.}$$

$$b = 0.05 \text{ in.}$$

$$\theta = \sin^{-1}\frac{b}{r}$$

$P = W\tan\theta = W\tan\left(\sin^{-1}\frac{b}{r}\right)$ for each wheel, so for total

$$P = 2500 \text{ lb} \tan\left(\sin^{-1}\frac{0.05}{11.5}\right)$$

$P = 10.87$ lb ◀

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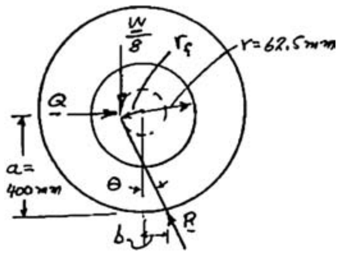
PROBLEM 8.99

Solve Problem 8.83 including the effect of a coefficient of rolling resistance of 0.5 mm.

PROBLEM 8.83 A loaded railroad car has a mass of 30 Mg and is supported by eight 800-mm-diameter wheels with 125-mm-diameter axles. Knowing that the coefficients of friction are $\mu_s = 0.020$ and $\mu_k = 0.015$, determine the horizontal force required (*a*) to start the car moving, (*b*) to keep the car moving at a constant speed. Neglect rolling resistance between the wheels and the track.

SOLUTION

For one wheel:



$$r_f = \mu r$$

$$\tan\theta \approx \sin\theta \approx \frac{r_f + b}{a}$$

$$\tan\theta = \frac{\mu r + b}{a}$$

$$Q = \frac{W}{8}\tan\theta = \frac{W}{8}\frac{\mu r + b}{a}$$

For eight wheels of car:

$$P = W\frac{\mu r + b}{a}$$

$$W = mg = (30 \text{ Mg})(9.81 \text{ m/s}^2) = 294.3 \text{ kN}$$

$$a = 400 \text{ mm}, \quad r = 62.5 \text{ mm}, \quad b = 0.5 \text{ mm}$$

(*a*) To start motion:

$$\mu = \mu_s = 0.02$$

$$P = (294.3 \text{ kN})\frac{(0.020)(62.5 \text{ mm}) + 0.5 \text{ mm}}{400 \text{ mm}}$$

$$P = 1.288 \text{ kN} \blacktriangleleft$$

(*b*) To maintain constant speed

$$\mu = \mu_k = 0.015$$

$$P = (294.3 \text{ kN})\frac{(0.015)(62.5 \text{ mm}) + 0.5 \text{ mm}}{400 \text{ mm}}$$

$$P = 1.058 \text{ kN} \blacktriangleleft$$

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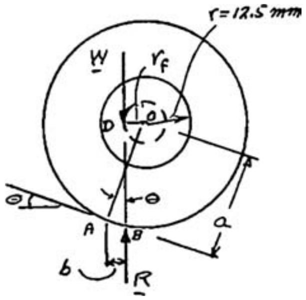
PROBLEM 8.100

Solve Problem 8.89 including the effect of a coefficient of rolling resistance of 1.75 mm.

PROBLEM 8.89 A scooter is to be designed to roll down a 2 percent slope at a constant speed. Assuming that the coefficient of kinetic friction between the 25-mm-diameter axles and the bearings is 0.10, determine the required diameter of the wheels. Neglect the rolling resistance between the wheels and the ground.

SOLUTION

Since the scooter rolls at a constant speed, each wheel is in equilibrium. Thus, **W** and **R** must have a common line of action tangent to the friction circle.



$$a = \text{Radius of wheel}$$

$$\tan\theta = \frac{2}{100} = 0.02$$

Since b and r_f are small compared to a,

$$\tan\theta \approx \frac{r_f + b}{a} = \frac{\mu_k r + b}{a} = 0.02$$

Data: $\mu_k = 0.10, \quad b = 1.75 \text{ mm}, \quad r = 12.5 \text{ mm}$

$$\frac{(0.10)(12.5 \text{ mm}) + 1.75 \text{ mm}}{a} = 0.02$$

$$a = 150 \text{ mm} \qquad \text{Diameter} = 2a = 300 \text{ mm} \blacktriangleleft$$

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PROBLEM 8.101

A hawser is wrapped two full turns around a bollard. By exerting an 80-lb force on the free end of the hawser, a dockworker can resist a force of 5000 lb on the other end of the hawser. Determine (*a*) the coefficient of static friction between the hawser and the bollard, (*b*) the number of times the hawser should be wrapped around the bollard if a 20,000-lb force is to be resisted by the same 80-lb force.

SOLUTION

(*a*)

$$\beta = 2 \text{ turns} = 2(2\pi) = 4\pi$$

$$T_1 = 80 \text{ lb}, \qquad T_2 = 5000 \text{ lb}$$

$$\ln\frac{T_2}{T_1} = \mu_s\beta$$

$$\mu_s = \frac{1}{\beta}\ln\frac{T_2}{T_1} = \frac{1}{4\pi}\ln\frac{5000 \text{ lb}}{80 \text{ lb}}$$

$$\mu_s = \frac{1}{4\pi}\ln 62.5 = \frac{4.1351}{4\pi}$$

$\mu_s = 0.329$ ◀

(*b*)

$$T_1 = 80 \text{ lb}, \quad T_2 = 20{,}000 \text{ lb}, \quad \mu_s = 0.329$$

$$\ln\frac{T_2}{T_1} = \mu_s\beta$$

$$\beta = \frac{1}{\mu}\ln\frac{T_2}{T_1} = \frac{1}{0.329}\ln\frac{20{,}000 \text{ lb}}{80 \text{ lb}}$$

$$\beta = \frac{1}{0.329}\ln(250) = \frac{5.5215}{0.329} = 16.783$$

$$\text{Number of turns} = \frac{16.783}{2\pi}$$

Number of turns $= 2.67$ ◀

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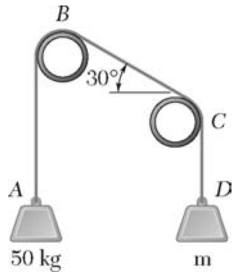


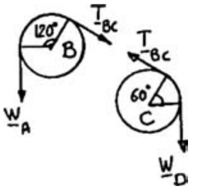
PROBLEM 8.102

A rope *ABCD* is looped over two pipes as shown. Knowing that the coefficient of static friction is 0.25, determine (*a*) the smallest value of the mass *m* for which equilibrium is possible, (*b*) the corresponding tension in portion *BC* of the rope.

SOLUTION

We apply Eq. (8.14) to pipe *B* and pipe *C*.

$$\frac{T_2}{T_1} = e^{\mu_s \beta} \qquad (8.14)$$



Pipe *B*:

$$T_2 = W_A, \quad T_1 = T_{BC}$$

$$\mu_s = 0.25, \quad \beta = \frac{2\pi}{3}$$

$$\frac{W_A}{T_{BC}} = e^{0.25(2\pi/3)} = e^{\pi/6} \qquad (1)$$

Pipe *C*:

$$T_2 = T_{BC}, \quad T_1 = W_D, \quad \mu_s = 0.25, \quad \beta = \frac{\pi}{3}$$

$$\frac{T_{BC}}{W_D} = e^{0.025(\pi/3)} = e^{\pi/12} \qquad (2)$$

(*a*) Multiplying Eq. (1) by Eq. (2):

$$\frac{W_A}{W_D} = e^{\pi/6} \cdot e^{\pi/12} = e^{\pi/6 + \pi/12} = e^{\pi/4} = 2.193$$

$$W_D = \frac{W_A}{2.193} \quad m = \frac{W_D}{g} = \frac{\frac{W_A}{g}}{2.193} = \frac{m_A}{2.193} = \frac{50 \text{ kg}}{2.193}$$

$$m = 22.8 \text{ kg} \blacktriangleleft$$

(*b*) From Eq. (1):

$$T_{BC} = \frac{W_A}{e^{\pi/6}} = \frac{(50 \text{ kg})(9.81 \text{ m/s}^2)}{1.688} = 291 \text{ N} \blacktriangleleft$$

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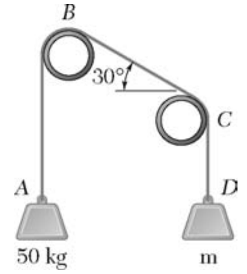

PROBLEM 8.103

A rope *ABCD* is looped over two pipes as shown. Knowing that the coefficient of static friction is 0.25, determine (*a*) the largest value of the mass *m* for which equilibrium is possible, (*b*) the corresponding tension in portion *BC* of the rope.

SOLUTION

See *FB* diagrams of Problem 8.102. We apply Eq. (8.14) to pipes *B* and *C*.

Pipe *B*: $T_1 = W_A, \quad T_2 = T_{BC}, \quad \mu_s = 0.25, \quad \beta = \dfrac{2\pi}{3}$

$$\frac{T_2}{T_1} = e^{\mu_s \beta}: \quad \frac{T_{BC}}{W_A} = e^{0.25(2\pi/3)} = e^{\pi/6} \qquad (1)$$

Pipe *C*: $T_1 = T_{BC}, \quad T_2 = W_D, \quad \mu_s = 0.25, \quad \beta = \dfrac{\pi}{3}$

$$\frac{T_2}{T_1} = e^{\mu_s \beta}: \quad \frac{W_D}{T_{BC}} = e^{0.25(\pi/3)} = e^{\pi/12} \qquad (2)$$

(*a*) Multiply Eq. (1) by Eq. (2):

$$\frac{W_D}{W_A} = e^{\pi/6} \cdot e^{\pi/12} = e^{\pi/6 + \pi/12} = e^{\pi/4} = 2.193$$

$$W_0 = 2.193 W_A \quad m = 2.193 m_A = 2.193(50 \text{ kg}) \qquad m = 109.7 \text{ kg} \blacktriangleleft$$

(*b*) From Eq. (1): $T_{BC} = W_A e^{\pi/6} = (50 \text{ kg})(9.81 \text{ m/s}^2)(1.688) = 828 \text{ N}$ ◀

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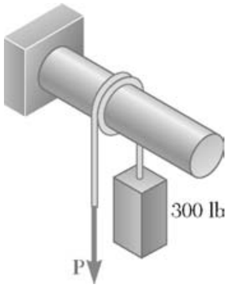
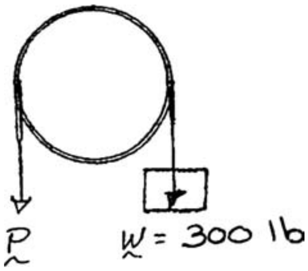


PROBLEM 8.104

A 300-lb block is supported by a rope that is wrapped $1\frac{1}{2}$ times around a horizontal rod. Knowing that the coefficient of static friction between the rope and the rod is 0.15, determine the range of values of P for which equilibrium is maintained.

SOLUTION



$$\beta = 1.5 \text{ turns} = 3\pi \text{ rad}$$

For impending motion of W up,

$$P = We^{\mu_s \beta} = (300 \text{ lb})e^{(0.15)3\pi}$$
$$= 1233.36 \text{ lb}$$

For impending motion of W down,

$$P = We^{-\mu_s \beta} = (300 \text{ lb})e^{-(0.15)3\pi}$$
$$= 72.971 \text{ lb}$$

For equilibrium, $73.0 \text{ lb} \le P \le 1233 \text{ lb}$ ◀

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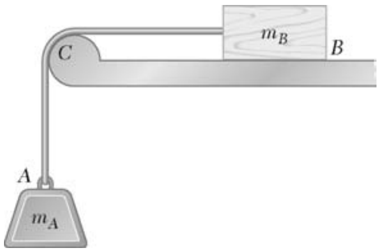
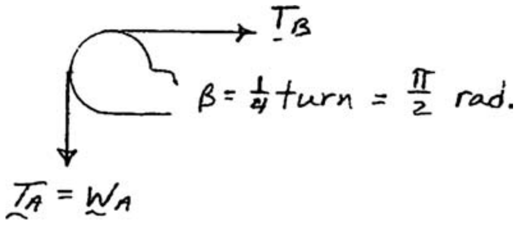


PROBLEM 8.105

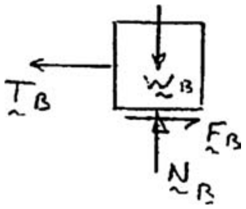
The coefficient of static friction between block *B* and the horizontal surface and between the rope and support *C* is 0.40. Knowing that $m_A = 12$ kg, determine the smallest mass of block *B* for which equilibrium is maintained.

SOLUTION

Support at C:



FBD block B:



$$W_A = mg = (12\text{ kg})g$$

$$+\uparrow \Sigma F_y = 0: \quad N_B - W_B = 0 \quad \text{or} \quad N_B = W_B$$

Impending motion: $\quad F_B = \mu_s N_B = 0.4 N_B = 0.4 W_B$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_B - T_B = 0 \quad \text{or} \quad T_B = F_B = 0.4 W_B$$

At support, for impending motion of W_A down:

$$W_A = T_B e^{\mu_s \beta}$$

so $\quad T_B = W_A e^{-\mu_s \beta} = (12\text{ kg})g^{-(0.4)\pi/2} = (6.4019\text{ kg})g$

Now $\quad W_B = \dfrac{T_B}{0.4} = \left(\dfrac{6.4019\text{ kg}}{0.4}\right) g = 16.0048 g$

so that $\quad m_B = \dfrac{W_B}{g} = \dfrac{16.0048 g}{g}$ $\quad m_B = 16.00\text{ kg}$ ◀

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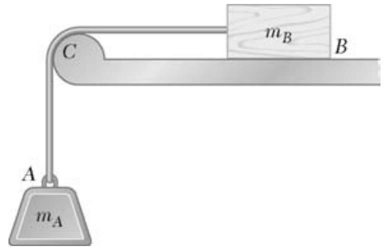
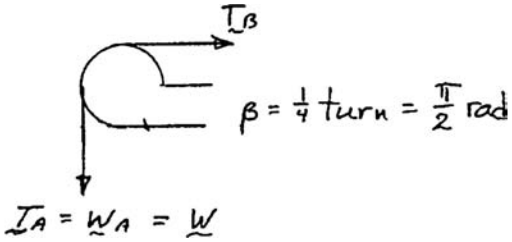


PROBLEM 8.106

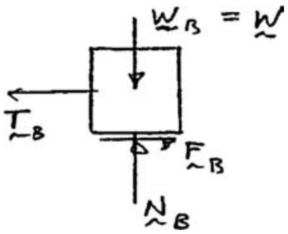
The coefficient of static friction μ_s is the same between block B and the horizontal surface and between the rope and support C. Knowing that $m_A = m_B$, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

Support at C:



FBD B:



$$\uparrow \Sigma F_y = 0: \quad N_B - W = 0 \quad \text{or} \quad N_B = W$$

Impending motion: $\quad F_B = \mu_s N_B = \mu_s W$

$$\longrightarrow \Sigma F_x = 0: \quad F_B - T_B = 0 \quad \text{or} \quad T_B = F_B = \mu_s W$$

Impending motion of rope on support: $\quad W = T_B e^{\mu_s \beta} = \mu_s W e^{\mu_s \beta}$

or $\quad 1 = \mu_s e^{\mu_s \beta}$

or $\quad \mu_s e^{\pi/2 \mu_s} = 1$

Solving numerically: $\quad \mu_s = 0.475$ ◀

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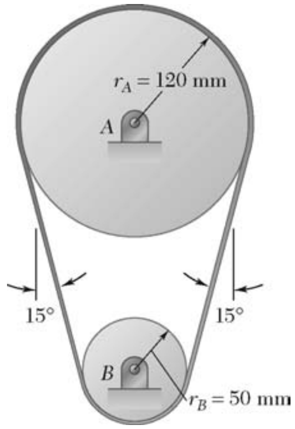
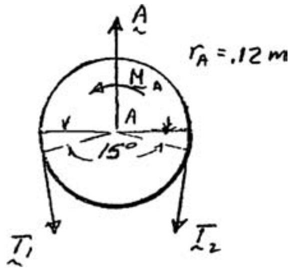


PROBLEM 8.107

A flat belt is used to transmit a couple from drum *B* to drum *A*. Knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is 450 N, determine the largest couple that can be exerted on drum *A*.

SOLUTION

FBD's drums:



$$\beta_A = 180° + 30° = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

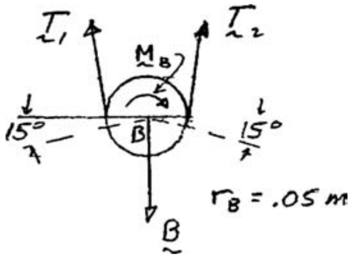
$$\beta_B = 180° - 30° = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Since $\beta_B < \beta_A$, slipping will impend first on *B* (friction coefficients being equal)

So

$$T_2 = T_{\max} = T_1 e^{\mu_s \beta_B}$$

$$450 \text{ N} = T_1 e^{(0.4)5\pi/6} \quad \text{or} \quad T_1 = 157.914 \text{ N}$$



$$\circlearrowleft \Sigma M_A = 0: \quad M_A + (0.12 \text{ m})(T_1 - T_2) = 0$$

$$M_A = (0.12 \text{ m})(450 \text{ N} - 157.914 \text{ N}) = 35.05 \text{ N} \cdot \text{m}$$

$$M_A = 35.1 \text{ N} \cdot \text{m} \blacktriangleleft$$

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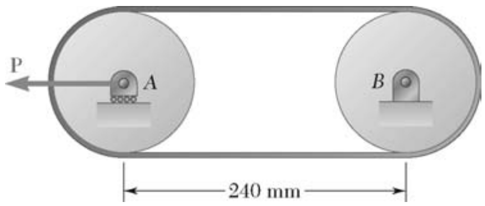


PROBLEM 8.108

A flat belt is used to transmit a couple from pulley A to pulley B. The radius of each pulley is 60 mm, and a force of magnitude $P = 900$ N is applied as shown to the axle of pulley A. Knowing that the coefficient of static friction is 0.35, determine (a) the largest couple that can be transmitted, (b) the corresponding maximum value of the tension in the belt.

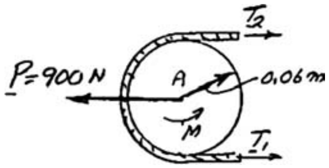
SOLUTION

Drum A:

$$\frac{T_2}{T_1} = e^{\mu_s \pi} = e^{(0.35)\pi}$$

$$T_2 = 3.0028 T_1$$

$$\beta = 180° = \pi \text{ radians}$$



(a) Torque: $+\circlearrowleft \Sigma M_A = 0: \quad M - (675.15\text{ N})(0.06\text{ m}) + (224.84\text{ N})(0.06\text{ m})$

$$M = 27.0\text{ N}\cdot\text{m} \blacktriangleleft$$

(b) $\xrightarrow{+} \Sigma F_x = 0: \quad T_1 + T_2 - 900\text{ N} = 0$

$$T_1 + 3.0028 T_1 - 900\text{ N} = 0$$

$$4.00282 T_1 = 900$$

$$T_1 = 224.841\text{ N}$$

$$T_2 = 3.0028(224.841\text{ N}) = 675.15\text{ N}$$

$$T_{max} = 675\text{ N} \blacktriangleleft$$

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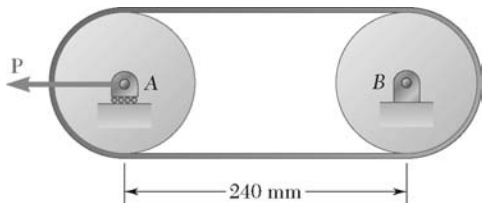


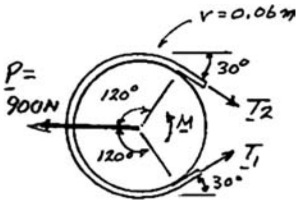
PROBLEM 8.109

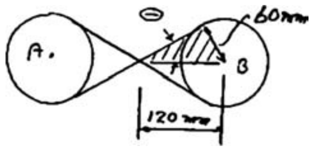
Solve Problem 8.108 assuming that the belt is looped around the pulleys in a figure eight.

PROBLEM 8.108 A flat belt is used to transmit a couple from pulley *A* to pulley *B*. The radius of each pulley is 60 mm, and a force of magnitude $P = 900$ N is applied as shown to the axle of pulley *A*. Knowing that the coefficient of static friction is 0.35, determine (*a*) the largest couple that can be transmitted, (*b*) the corresponding maximum value of the tension in the belt.

SOLUTION

Drum *A*:





$$\beta = 240° = 240° \frac{\pi}{180°} = \frac{4}{3}\pi$$

$$\sin\theta = \frac{60}{120} = \frac{1}{2}$$

$$\theta = 30°$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.35(4/3\pi)}$$

$$T_2 = 4.3322 T_1$$

(*a*) Torque: $+\circlearrowleft \Sigma M_B = 0: \quad M - (844.3\text{ N})(0.06\text{ m}) + (194.9\text{ N})(0.06\text{ m}) = 0$

$$M = 39.0\text{ N}\cdot\text{m} \blacktriangleleft$$

(*b*) $\xrightarrow{+} \Sigma F_x = 0: \quad (T_1 + T_2)\cos 30° - 900\text{ N}$

$$(T_1 + 4.3322 T_1)\cos 30° = 900$$

$$T_1 = 194.90\text{ N}$$

$$T_2 = 4.3322(194.90\text{ N}) = 844.3\text{ N}$$

$$T_{max} = 844\text{ N} \blacktriangleleft$$

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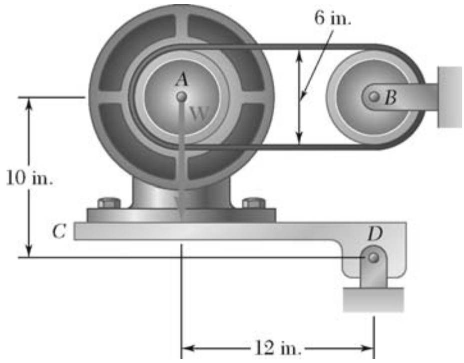
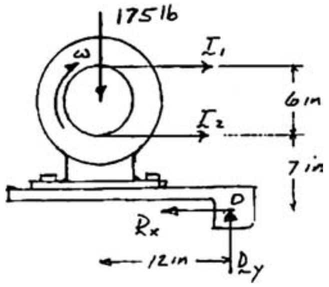


PROBLEM 8.110

In the pivoted motor mount shown the weight **W** of the 175-lb motor is used to maintain tension in the drive belt. Knowing that the coefficient of static friction between the flat belt and drums *A* and *B* is 0.40, and neglecting the weight of platform *CD*, determine the largest couple that can be transmitted to drum *B* when the drive drum *A* is rotating clockwise.

SOLUTION

FBD motor and mount:



Impending belt slip: cw rotation

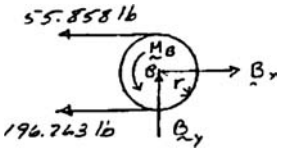
$$T_2 = T_1 e^{\mu_s \beta} = T_1 e^{0.40\pi} = 3.5136 T_1$$

$$\circlearrowleft \Sigma M_D = 0: \quad (12 \text{ in.})(175 \text{ lb}) - (7 \text{ in.})T_2 - (13 \text{ in.})T_1 = 0$$

$$2100 \text{ lb} = [(7 \text{ in.})(3.5136) + 13 \text{ in.}]T_1$$

$$T_1 = 55.858 \text{ lb}, \quad T_2 = 3.5136 T_1 = 196.263 \text{ lb}$$

FBD drum at *B*:



$$\circlearrowleft \Sigma M_B = 0: \quad M_B - (3 \text{ in.})(196.263 \text{ lb} - 55.858 \text{ lb}) = 0 \qquad r = 3 \text{ in.}$$

$$M_B = 421 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

(Compare to $857 \text{ lb} \cdot \text{in.}$ using V-belt, Problem 8.130)

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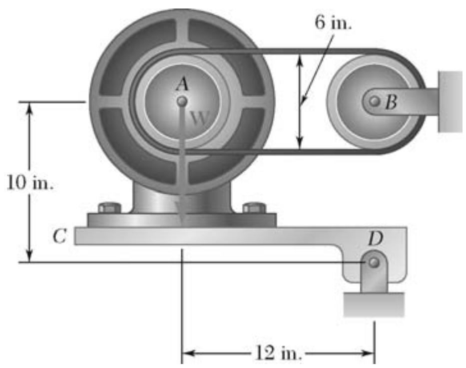


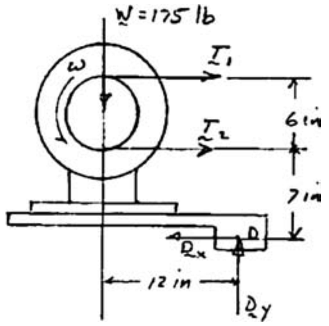
PROBLEM 8.111

Solve Problem 8.110 assuming that the drive drum *A* is rotating counterclockwise.

PROBLEM 8.110 In the pivoted motor mount shown the weight **W** of the 175-lb motor is used to maintain tension in the drive belt. Knowing that the coefficient of static friction between the flat belt and drums *A* and *B* is 0.40, and neglecting the weight of platform *CD*, determine the largest couple that can be transmitted to drum *B* when the drive drum *A* is rotating clockwise.

SOLUTION

FBD motor and mount:



Impending belt slip: ccw rotation

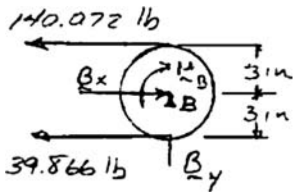
$$T_1 = T_2 e^{\mu_s \beta} = T_2 e^{0.40\pi} = 3.5136 T_2$$

$$\circlearrowleft \Sigma M_D = 0: \quad (12 \text{ in.})(175 \text{ lb}) - (13 \text{ in.})T_1 - (7 \text{ in.})T_2 = 0$$

$$2100 \text{ lb} = [(13 \text{ in.})(3.5136) + 7 \text{ in.}]T_2 = 0$$

$$T_2 = 39.866 \text{ lb}, \quad T_1 = 3.5136 T_2 = 140.072 \text{ lb}$$

FBD drum at B:



$$\circlearrowleft \Sigma M_B = 0: \quad (3 \text{ in.})(140.072 \text{ lb} - 39.866 \text{ lb}) - M_B = 0$$

$$M_B = 301 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

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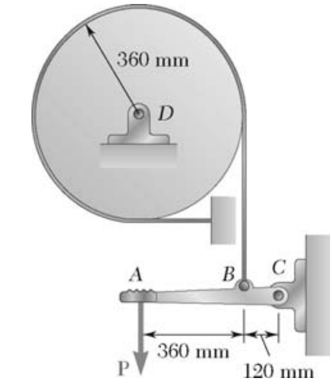
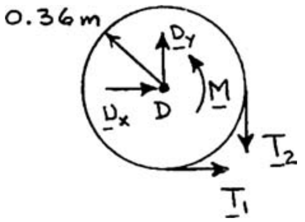


PROBLEM 8.112

A band brake is used to control the speed of a flywheel as shown. The coefficients of friction are $\mu_s = 0.30$ and $\mu_k = 0.25$. Determine the magnitude of the couple being applied to the flywheel, knowing that $P = 45$ N and that the flywheel is rotating counterclockwise at a constant speed.

SOLUTION

<u>Free body: Cylinder</u>



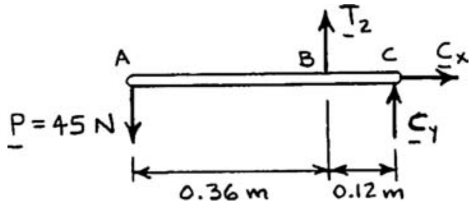
Since slipping of band relative to cylinder is clockwise, $\mathbf{T}_1$ and $\mathbf{T}_2$ are located as shown.

From free body: Lever *ABC*

$$+\circlearrowleft \Sigma M_C = 0: \quad (45\text{ N})(0.48\text{ m}) - T_2(0.12\text{ m}) = 0$$

$$T_2 = 180\text{ N}$$

<u>Free body: Lever *ABC*</u>



From free body: Cylinder

Using Eq. (8.14) with $\mu_k = 0.25$ and $\beta = 270° = \dfrac{3\pi}{2}$ rad:

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{(0.25)(3\pi/2)} = e^{3\pi/8}$$

$$T_1 = \frac{T_2}{e^{3\pi/8}} = \frac{180\text{ N}}{3.2482} = 55.415\text{ N}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (55.415\text{ N})(0.36\text{ m}) - (180\text{ N})(0.36\text{ m}) + M = 0 \qquad \mathbf{M} = 44.9\text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

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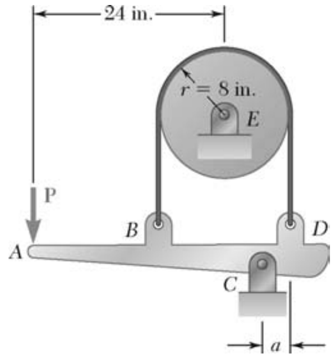


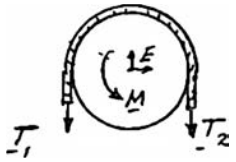
PROBLEM 8.113

The speed of the brake drum shown is controlled by a belt attached to the control bar *AD*. A force **P** of magnitude 25 lb is applied to the control bar at *A*. Determine the magnitude of the couple being applied to the drum, knowing that the coefficient of kinetic friction between the belt and the drum is 0.25, that $a = 4$ in., and that the drum is rotating at a constant speed (*a*) counterclockwise, (*b*) clockwise.

SOLUTION

(*a*) Counterclockwise rotation

Free body: Drum

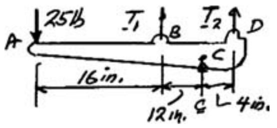


$$r = 8 \text{ in.} \quad \beta = 180° = \pi \text{ radians}$$

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933 T_1$$

Free body: Control bar



$$+\circlearrowleft \Sigma M_C = 0: \quad T_1(12 \text{ in.}) - T_2(4 \text{ in.}) - (25 \text{ lb})(28 \text{ in.}) = 0$$

$$T_1(12) - 2.1933T_1(4) - 700 = 0$$

$$T_1 = 216.93 \text{ lb}$$

$$T_2 = 2.1933(216.93 \text{ lb}) = 475.80 \text{ lb}$$

Return to free body of drum

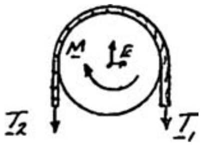
$$+\circlearrowleft \Sigma M_E = 0: \quad M + T_1(8 \text{ in.}) - T_2(8 \text{ in.}) = 0$$

$$M + (216.96 \text{ lb})(8 \text{ in.}) - (475.80 \text{ lb})(8 \text{ in.}) = 0$$

$$M = 2070.9 \text{ lb} \cdot \text{in.}$$

$M = 2070 \text{ lb} \cdot \text{in.}$ ◀

(*b*) Clockwise rotation



$$r = 8 \text{ in.} \quad \beta = \pi \text{ rad}$$

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933 T_1$$

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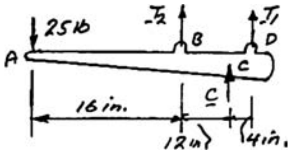
PROBLEM 8.113 (Continued)

Free body: Control rod



$$+\circlearrowright \Sigma M_C = 0: \quad T_2(12\text{ in.}) - T_1(4\text{ in.}) - (25\text{ lb})(28\text{ in.}) = 0$$

$$2.1933T_1(12) - T_1(4) - 700 = 0$$

$$T_1 = 31.363\text{ lb}$$

$$T_2 = 2.1933(31.363\text{ lb})$$

$$T_2 = 68.788\text{ lb}$$

Return to free body of drum

$$+\circlearrowright \Sigma M_E = 0: \quad M + T_1(8\text{ in.}) - T_2(8\text{ in.}) = 0$$

$$M + (31.363\text{ lb})(8\text{ in.}) - (68.788\text{ lb})(8\text{ in.}) = 0$$

$$M = 299.4\text{ lb}\cdot\text{in.}$$

$$M = 299\text{ lb}\cdot\text{in.} \blacktriangleleft$$

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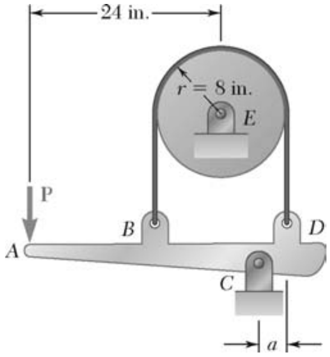
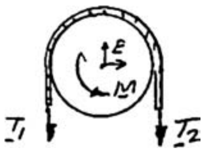


PROBLEM 8.114

Knowing that $a = 4$ in., determine the maximum value of the coefficient of static friction for which the brake is not self-locking when the drum rotates counterclockwise.

SOLUTION

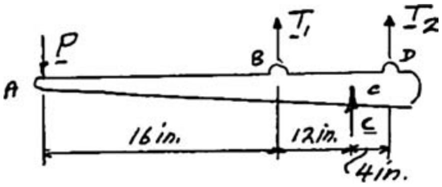


$$r = 8 \text{ in.}, \quad \beta = 180° = \pi \text{ radians}$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{\mu_s \pi}$$

$$T_2 = e^{\mu_s \pi} T_1$$

<u>Free body: Control rod</u>



$$+\circlearrowleft \Sigma M_C = 0: \quad P(28 \text{ in.}) - T_1(12 \text{ in.}) + T_2(4 \text{ in.}) = 0$$

$$28P - 12T_1 + e^{\mu\pi} T_1(4) = 0$$

For self-locking brake: $P = 0$

$$12T_1 = 4T_1 e^{\mu_s \pi}$$

$$e^{\mu_s \pi} = 3$$

$$\mu_s \pi = \ln 3 = 1.0986$$

$$\mu_s = \frac{1.0986}{\pi} = 0.3497$$

$\mu_s = 0.350$ ◀

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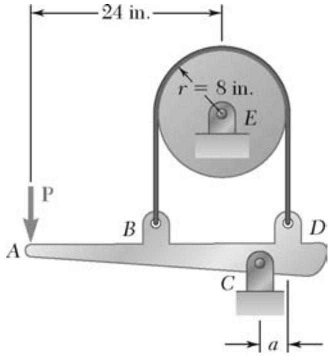
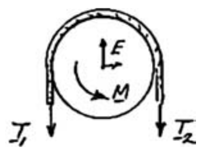


PROBLEM 8.115

Knowing that the coefficient of static friction is 0.30 and that the brake drum is rotating counterclockwise, determine the minimum value of a for which the brake is not self-locking.

SOLUTION

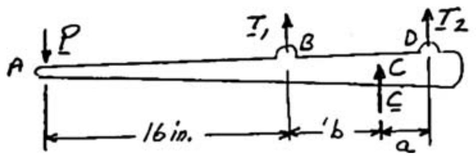


$$r = 8\text{ in.},\quad \beta = \pi \text{ radians}$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.30\pi} = 2.5663$$

$$T_2 = 2.5663 T_1$$

Free body: Control rod



$$b = 16\text{ in.} - a$$

$$+\circlearrowright \Sigma M_C = 0:\quad P(16\text{ in.} + b) - T_1 b + T_2 a = 0$$

For brake to be self-locking, $P = 0$

$$T_2 a = T_1 b;\quad 2.5663 T_1 a = T_1(16 - a)$$

$$2.5663a = 16 - a$$

$$3.5663a = 16$$

$a = 4.49$ in. ◀

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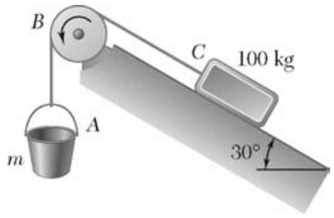
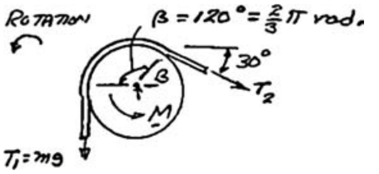


PROBLEM 8.116

Bucket *A* and block *C* are connected by a cable that passes over drum *B*. Knowing that drum *B* rotates slowly counterclockwise and that the coefficients of friction at all surfaces are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the smallest combined mass *m* of the bucket and its contents for which block *C* will (*a*) remain at rest, (*b*) start moving up the incline, (*c*) continue moving up the incline at a constant speed.

SOLUTION

Free body: Drum



$$\frac{T_2}{mg} = e^{\mu \frac{2}{3}\pi}$$

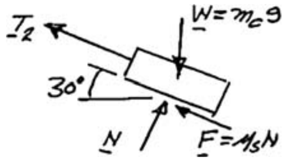
$$T_2 = mge^{2\mu\pi/3} \qquad (1)$$

(*a*) Smallest *m* for block *C* to remain at rest

Cable slips on drum.

Eq. (1) with $\mu_k = 0.25$; $T_2 = mge^{2(0.25)\pi/3} = 1.6881mg$

Block *C*: At rest, motion impending ↘



$$+\nearrow \Sigma F = 0: \quad N - m_C g\cos 30°$$

$$N = m_C g\cos 30°$$

$$F = \mu_s N = 0.35 m_C g\cos 30°$$

$$m_C = 100 \text{ kg}$$

$$+\nwarrow \Sigma F = 0: \quad T_2 + F - m_C g\sin 30° = C$$

$$1.6881mg + 0.35m_C g\cos 30° - m_C g\sin 30° = 0$$

$$1.6881m = 0.19689m_C$$

$$m = 0.11663m_C = 0.11663(100 \text{ kg}); \qquad m = 11.66 \text{ kg} \blacktriangleleft$$

(*b*) Smallest *m* to start block moving up

No slipping at both drum and block: $\mu_s = 0.35$

Eq. (1): $T_2 = mge^{2(0.35)\pi/3} = 2.0814mg$

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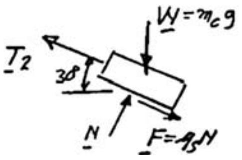
PROBLEM 8.116 (Continued)

Block C:

Motion impending $\nwarrow$



$$m_C = 100\,\text{kg}$$

$$+\nearrow \Sigma F = 0: \quad N - mg\cos 30°$$

$$N = m_C g\cos 30°$$
$$F = \mu_s N = 0.35 m_C g\cos 30°$$

$$+\nwarrow \Sigma F = 0: \quad T_2 - F - m_C g\sin 30° = 0$$

$$2.0814mg - 0.35m_C g\cos 30° - m_C g\sin 30° = 0$$
$$2.0814m = 0.80311m_C$$
$$m = 0.38585m_C = 0.38585(100\text{ kg})$$

$m = 38.6$ kg ◀

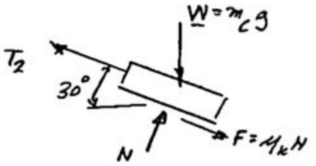
(c) Smallest m to keep block moving up drum: No slipping: $\mu_s = 0.35$

Eq. (1) with $\mu_s = 0.35$

$$T_2 = mg^{2\mu_s\pi/3} = mge^{2(0.35)\pi/3}$$
$$T_2 = 2.0814mg$$

Block C: Moving up plane, thus $\mu_k = 0.25$

Motion up $\nwarrow$



$$+\nearrow \Sigma F = 0: \quad N - m_C g\cos 30° = 0$$

$$N = m_C g\cos 30°$$
$$F = \mu_k N = 0.25 m_C g\cos 30°$$

$$+\nwarrow \Sigma F = 0: \quad T_2 - F - m_C g\sin 30° = 0$$

$$2.0814mg - 0.25m_C g\cos 30° - m_C g\sin 30° = 0$$
$$2.0814m = 0.71651m_C$$
$$m = 0.34424m_C = 0.34424(100\text{ kg})$$

$m = 34.4$ kg ◀

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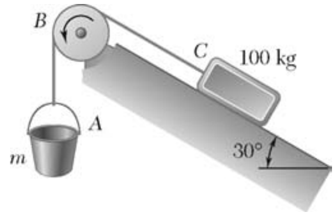


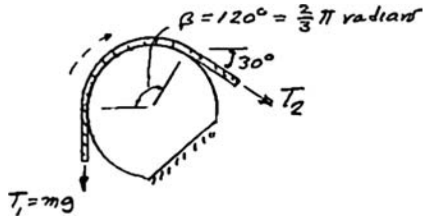
PROBLEM 8.117

Solve Problem 8.116 assuming that drum *B* is frozen and cannot rotate.

PROBLEM 8.116 Bucket *A* and block *C* are connected by a cable that passes over drum *B*. Knowing that drum *B* rotates slowly counterclockwise and that the coefficients of friction at all surfaces are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the smallest combined mass *m* of the bucket and its contents for which block *C* will (*a*) remain at rest, (*b*) start moving up the incline, (*c*) continue moving up the incline at a constant speed.

SOLUTION

(*a*) Block *C* remains at rest: Motion impends ↘

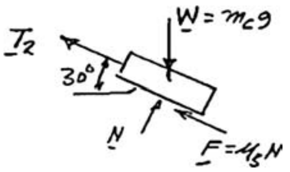


Drum:

$$\frac{T_2}{mg} = e^{\mu_k \beta} = e^{0.35(2\pi/3)}$$

$$T_2 = 2.0814mg$$

Block *C*: Motion impends ↘



$$\nearrow \Sigma F = 0: \quad N - m_C g \cos 30° = 0$$

$$N = m_C g \cos 30°$$

$$F = \mu_s N = 0.35 m_C g \cos 30°$$

$$+\nwarrow \Sigma F = 0: \quad T_2 + F - m_C g \sin 30° = 0$$

$$2.0814mg + 0.35 m_C g \cos 30° - m_C g \sin 30° = 0$$

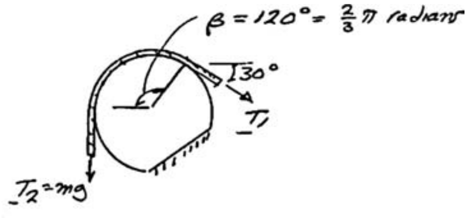
$$2.0814mg = 0.19689 m_C$$

$$m = 0.09459 m_C = 0.09459(100 \text{ kg})$$

$m = 9.46$ kg ◀

(*b*) Block *C*: Starts moving up $\quad \mu_s = 0.35$

Drum: Impending motion of cable ↺



$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

$$\frac{mg}{T_1} = e^{0.35(2/3\pi)}$$

$$T_1 = \frac{mg}{2.0814}$$

$$= 0.48045mg$$

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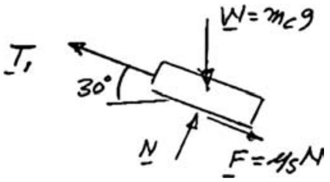
PROBLEM 8.117 (Continued)

Block C: Motion impends ↖



$$+\nearrow \Sigma F = 0: \quad N - m_C g \cos 30°$$

$$N = m_C g \cos 30°$$

$$F = \mu_s N = 0.35 m_C g \cos 30°$$

$$+\nwarrow \Sigma F = 0: \quad T_1 - F - m_C g \sin 30° = 0$$

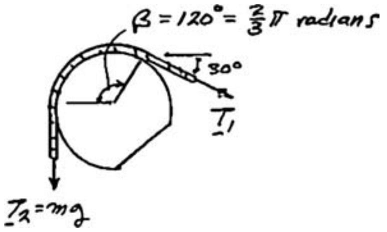
$$0.48045 mg - 0.35 m_C g \cos 30° - 0.5 m_C g = 0$$

$$0.48045 m = 0.80311 m_C$$

$$m = 1.67158 m_C = 1.67158(100 \text{ kg}) \qquad m = 167.2 \text{ kg} \blacktriangleleft$$

(c) Smallest m to keep block moving ↖

Drum: Motion of cable ↺



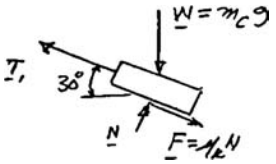
$$\mu_k = 0.25$$

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25(2/3\pi)}$$

$$\frac{mg}{T_1} = 1.6881$$

$$T_1 = \frac{mg}{1.6881} = 0.59238 mg$$

Block C: Block moves ↖



$$+\nearrow \Sigma F = 0: \quad N - m_C g \cos 30° = 0$$

$$N = m_C g \cos 30°$$

$$F = \mu_k N = 0.25 m_C g \cos 30°$$

$$+\nwarrow \Sigma F = 0: \quad T_1 - F - m_C g \sin 30° = 0$$

$$0.59238 mg - 0.25 m_C g \cos 30° - 0.5 m_C g = 0$$

$$0.59238 m = 0.71651 m_C$$

$$m = 1.20954 m_C = 1.20954(100 \text{ kg}) \qquad m = 121.0 \text{ kg} \blacktriangleleft$$

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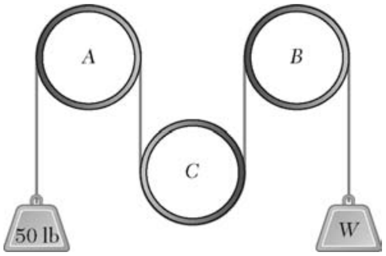
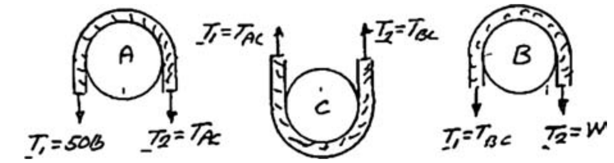


PROBLEM 8.118

A cable is placed around three parallel pipes. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine (*a*) the smallest weight *W* for which equilibrium is maintained, (*b*) the largest weight *W* that can be raised if pipe *B* is slowly rotated counterclockwise while pipes *A* and *C* remain fixed.

SOLUTION

(*a*) <u>Smallest *W* for equilibrium</u> $\quad B = \pi, \quad \mu = \mu_s$



$$\frac{T_{AC}}{50 \text{ lb}} = e^{0.25\pi} \qquad \frac{T_{BC}}{T_{AC}} = e^{0.25\pi} \qquad \frac{W}{T_{BC}} = e^{0.25\pi}$$

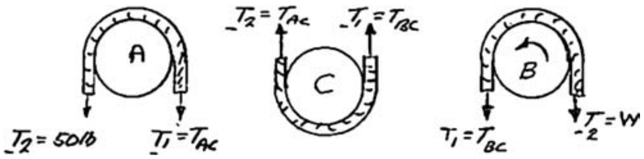
$$\frac{T_{AC}}{50 \text{ lb}} \cdot \frac{T_{BC}}{T_{AC}} \cdot \frac{W}{T_{BC}} = e^{\pi/4} \cdot e^{\pi/4} \cdot e^{\pi/4} = e^{3\pi/4} = 10.551$$

$$\frac{W}{50 \text{ lb}} = 10.551; \qquad W = 4.739 \text{ lb}$$

$W = 4.74$ lb ◀

(*b*) <u>Largest *W* which can be raised by pipe *B* rotated</u> ↺

$$\beta = \pi, \mu = \mu_k \qquad \beta = \pi, \mu = \mu_k \qquad \beta = \pi, \mu = \mu_s$$



$$\frac{50 \text{ lb}}{T_{AC}} = e^{0.2\pi} \qquad \frac{T_{AC}}{T_{BC}} = e^{0.2\pi} \qquad \frac{W}{T_{BC}} = e^{0.25\pi}$$

$$\frac{50 \text{ lb}}{T_{AC}} \cdot \frac{T_{AC}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/5} \cdot e^{\pi/5} \cdot e^{-\pi/4} = e^{\pi(1/5+1/5-1/4)}$$

$$= e^{3\pi/20} = 1.602$$

$$\frac{50 \text{ lb}}{W} = 1.602; \qquad W = \frac{50 \text{ lb}}{1.602} = 31.21 \text{ lb}$$

$W = 31.2$ lb ◀

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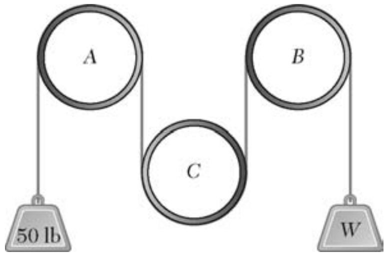


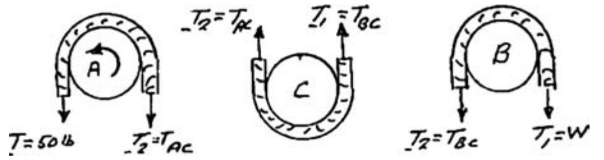
PROBLEM 8.119

A cable is placed around three parallel pipes. Two of the pipes are fixed and do not rotate; the third pipe is slowly rotated. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the largest weight W that can be raised (*a*) if only pipe *A* is rotated counterclockwise, (*b*) if only pipe *C* is rotated clockwise.

SOLUTION

(*a*) Pipe a rotates ↺

$\beta = \pi, \mu = \mu_s \qquad \beta = \pi, \mu = \mu_k \qquad \beta = \pi, \mu = \mu_k$



$$\frac{T_{AC}}{50\text{ lb}} = e^{0.25\pi} \qquad \frac{T_{AC}}{T_{BC}} = e^{0.2\pi} \qquad \frac{T_{BC}}{W} = e^{0.2\pi}$$

$$\frac{T_{AC}}{50\text{ lb}} \cdot \frac{T_{BC}}{T_{AC}} \cdot \frac{W}{T_{BC}} = e^{\pi/4} \cdot e^{-\pi/5} \cdot e^{-\pi/5}$$

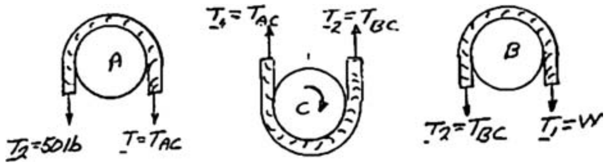
$$= e^{\pi(1/4-1/5-1/5)} = e^{-3\pi/20} = 0.62423$$

$$\frac{W}{50\text{ lb}} = 0.62423; \quad W = 31.21\text{ lb}$$

$W = 31.2$ lb ◀

(*b*) Pipe *C* rotates ↻

$\beta = \pi, \mu = \mu_k \qquad \beta = \pi, \mu = \mu_s \qquad \beta = \pi, \mu = \mu_k$



$$\frac{50\text{ lb}}{T_{AC}} = e^{0.2\pi} \qquad \frac{T_{BC}}{T_{AC}} = e^{0.25\pi} \qquad \frac{T_{BC}}{W} = e^{0.2\pi}$$

$$\frac{50\text{ lb}}{T_{AC}} \cdot \frac{T_{AC}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/5} \cdot e^{-\pi/4} \cdot e^{\pi/5} = e^{\pi(1/5-1/4+1/5)} = e^{3\pi/20}$$

$$\frac{50\text{ lb}}{W} = e^{3\pi/20} = 1.602$$

$$W = \frac{50\text{ lb}}{1.602} = 31.21\text{ lb}$$

$W = 31.2$ lb ◀

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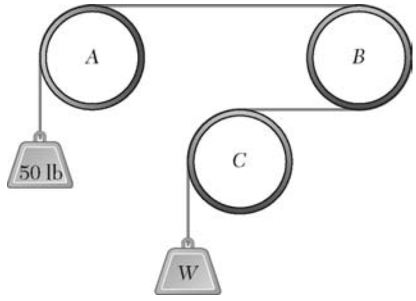
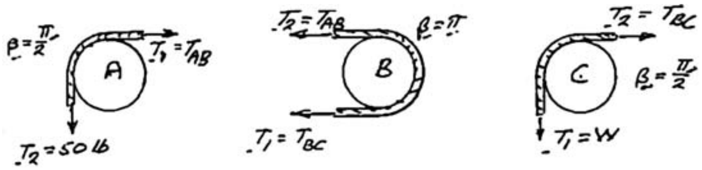


PROBLEM 8.120

A cable is placed around three parallel pipes. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine (*a*) the smallest weight *W* for which equilibrium is maintained, (*b*) the largest weight *W* that can be raised if pipe *B* is slowly rotated counterclockwise while pipes *A* and *C* remain fixed.

SOLUTION

(*a*) $\mu = \mu_s = 0.25$ at all pipes.



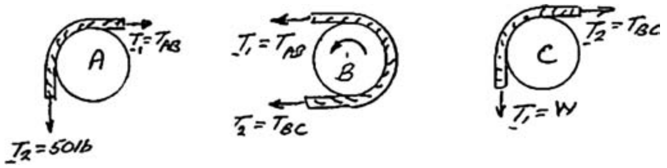
$$\frac{50\text{ lb}}{T_{AB}} = e^{0.25\pi/2} \qquad \frac{T_{AC}}{T_{BC}} = e^{0.25\pi} \qquad \frac{T_{BC}}{W} = e^{0.25\pi/2}$$

$$\frac{50\text{ lb}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/8} \cdot e^{\pi/4} \cdot e^{\pi/8} = e^{\pi/8+\pi/4+\pi/8} = e^{\pi/2} = 4.8105$$

$$\frac{50\text{ lb}}{W} = 4.8015; \quad W = 10.394\text{ lb}$$

$W = 10.39$ lb ◀

(*b*) Pipe *B* rotated ↺ $\qquad \beta = \frac{\pi}{2};\ \mu = \mu_k \qquad \beta = \pi;\ \mu = \mu_s \qquad \beta = \frac{\pi}{2};\ \mu = \mu_k$



$$\frac{50\text{ lb}}{T_{AB}} = e^{0.2\pi/2} \qquad \frac{T_{BC}}{T_{AB}} = e^{0.25\pi} \qquad \frac{T_{BC}}{W} = e^{0.2\pi/2}$$

$$\frac{50\text{ lb}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/10} \cdot e^{-\pi/4} \cdot e^{\pi/10}$$

$$= e^{\pi/10-\pi/4+\pi/10} = e^{-\pi/20} = 0.85464$$

$$\frac{50\text{ lb}}{W} = 0.85464$$

$$W = \frac{50\text{ lb}}{0.85464} = 58.504\text{ lb}$$

$W = 58.5$ lb ◀

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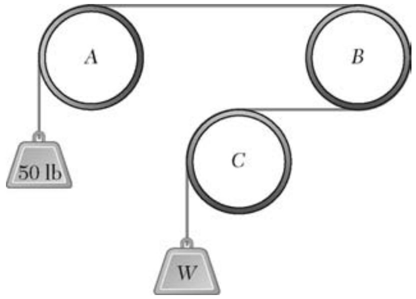
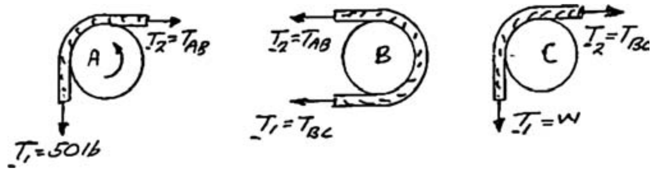


PROBLEM 8.121

A cable is placed around three parallel pipes. Two of the pipes are fixed and do not rotate; the third pipe is slowly rotated. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the largest weight W that can be raised (*a*) if only pipe *A* is rotated counterclockwise, (*b*) if only pipe *C* is rotated clockwise.

SOLUTION

(*a*) Pipe a rotates ↺ $\quad \beta = \frac{\pi}{2};\ \mu = \mu_s \qquad \beta = \pi,\ \mu = \mu_k \qquad \beta = \frac{\pi}{2};\ \mu = \mu_k$



$$\frac{T_{AB}}{50\text{ lb}} = e^{0.25\pi/2} \qquad \frac{T_{AB}}{T_{BC}} = e^{0.2\pi} \qquad \frac{T_{BC}}{W} = e^{0.2\pi/2}$$

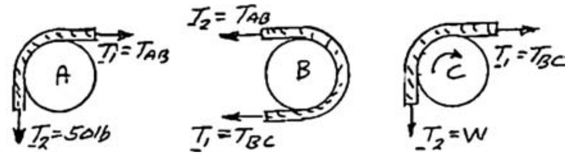
$$\frac{T_{AB}}{50\text{ lb}} \cdot \frac{T_{BC}}{T_{AB}} \cdot \frac{W}{T_{BC}} = e^{\pi/8} \cdot e^{-\pi/5} \cdot e^{-\pi/10}$$

$$= e^{\pi(1/8-1/5-1/10)} = e^{-7\pi/40} = 0.57708$$

$$\frac{W}{50\text{ lb}} = 0.57708; \quad W = 28.854\text{ lb}$$

$W = 28.9\text{ lb}$ ◀

(*b*) Pipe *C* rotates ↻ $\quad \beta = \frac{\pi}{2};\ \mu = \mu_k \quad \beta = \pi;\ \mu = \mu_k \quad \beta = \frac{\pi}{2},\ \mu = \mu_s$



$$\frac{50\text{ lb}}{T_{AB}} = e^{0.2\pi/2} \qquad \frac{T_{AB}}{T_{BC}} = e^{0.2\pi} \qquad \frac{W}{T_{BC}} = e^{0.25\pi/2}$$

$$\frac{50\text{ lb}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/10} \cdot e^{\pi/5} \cdot e^{-\pi/8} = e^{7\pi/40} = 0.57708$$

$$\frac{50\text{ lb}}{W} = 0.57708$$

$$W = 28.854\text{ lb}$$

$W = 28.9\text{ lb}$ ◀

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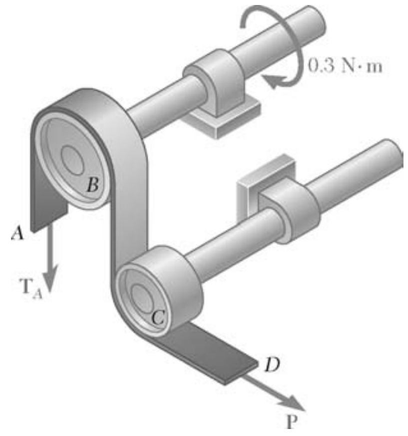
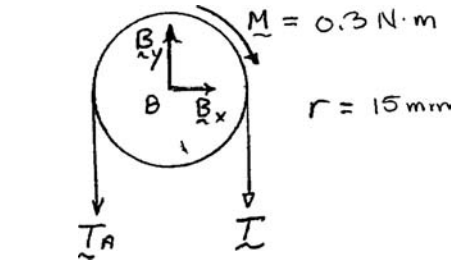


PROBLEM 8.122

A recording tape passes over the 20-mm-radius drive drum B and under the idler drum C. Knowing that the coefficients of friction between the tape and the drums are $\mu_s = 0.40$ and $\mu_k = 0.30$ and that drum C is free to rotate, determine the smallest allowable value of P if slipping of the tape on drum B is not to occur.

SOLUTION

FBD drive drum:



$$\circlearrowleft \Sigma M_B = 0: \quad r(T_A - T) - M = 0$$

$$T_A - T = \frac{M}{r} = \frac{300 \text{ N}\cdot\text{mm}}{20 \text{ mm}} = 15.0000 \text{ N}$$

Impending slipping: $\quad T_A = Te^{\mu_s\beta} = Te^{0.4\pi}$

So $\quad T(e^{0.4\pi} - 1) = 15.0000 \text{ N}$

or $\quad T = 5.9676 \text{ N}$

If C is free to rotate, $P = T$ $\quad P = 5.97 \text{ N}$ ◀

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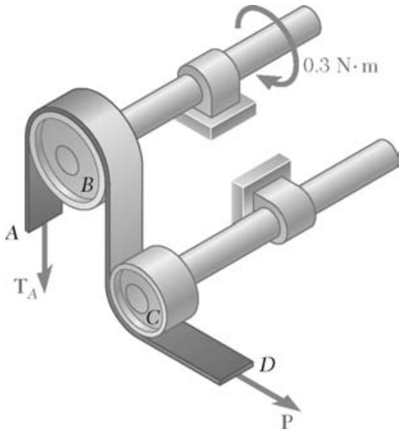


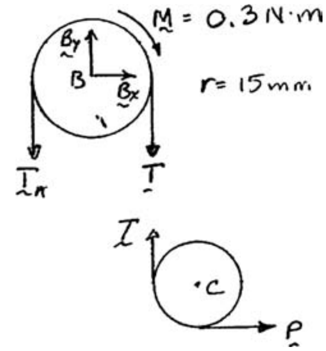
PROBLEM 8.123

Solve Problem 8.122 assuming that the idler drum C is frozen and cannot rotate.

PROBLEM 8.122 A recording tape passes over the 20-mm-radius drive drum B and under the idler drum C. Knowing that the coefficients of friction between the tape and the drums are $\mu_s = 0.40$ and $\mu_k = 0.30$ and that drum C is free to rotate, determine the smallest allowable value of P if slipping of the tape on drum B is not to occur.

SOLUTION

FBD drive drum:



$$\circlearrowleft \Sigma M_B = 0: \quad r(T_A - T) - M = 0$$

$$T_A - T = \frac{M}{r} = 300 \text{ N}\cdot\text{mm} = 15.0000 \text{ N}$$

Impending slipping: $T_A = Te^{\mu_s \beta} = Te^{0.4\pi}$

So $(e^{0.4\pi} - 1)T = 15.000 \text{ N}$

or $T = 5.9676 \text{ N}$

If C is fixed, the tape must slip $\circlearrowleft$

So $P = Te^{\mu_k \beta_C} = (5.9676 \text{ N})e^{0.3\pi/2} = 9.5600 \text{ N}$ $\qquad P = 9.56 \text{ N}$ ◀

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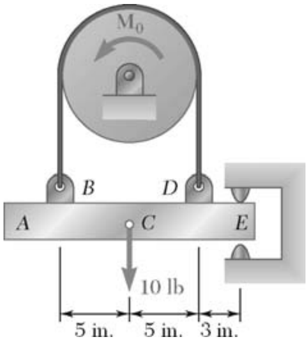
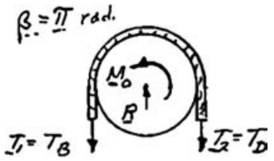


PROBLEM 8.124

The 10-lb bar *AE* is suspended by a cable that passes over a 5-in.-radius drum. Vertical motion of end *E* of the bar is prevented by the two stops shown. Knowing that $\mu_s = 0.30$ between the cable and the drum, determine (*a*) the largest counterclockwise couple $\mathbf{M}_0$ that can be applied to the drum if slipping is not to occur, (*b*) the corresponding force exerted on end *E* of the bar.

SOLUTION

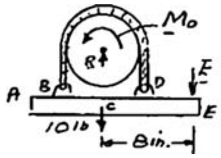


<u>Drum</u>: Slipping impends $\qquad \mu_s = 0.30$

$$\frac{T_2}{T_1} = e^{\mu\beta}: \quad \frac{T_D}{T_B} = e^{0.30\pi} = 2.5663$$

$$T_D = 2.5663 T_B$$

(*a*) <u>Free-body: Drum and bar</u>



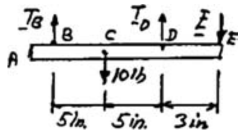
$$+\circlearrowleft \Sigma M_C = 0: \quad M_0 - E(8\text{ in.}) = 0$$

$$M_0 = (3.78649\text{ lb})(8\text{ in.})$$

$$= 30.27\text{ lb}\cdot\text{in.}$$

$\mathbf{M}_0 = 30.3\text{ lb}\cdot\text{in.} \circlearrowleft$ ◀

(*b*) <u>Bar *AE*</u>:



$$+\uparrow \Sigma F_y = 0: \quad T_B + T_D - E - 10\text{ lb} = 0$$

$$T_B + 2.5663 T_B - E - 10\text{ lb} = 0$$

$$3.5663 T_B - E - 10\text{ lb} = 0$$

$$E = 3.5663 T_B - 10\text{ lb} \qquad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: \quad E(3\text{ in.}) - (10\text{ lb})(5\text{ in.}) + T_B(10\text{ in.}) = 0$$

$$(3.5663 T_B - 10\text{ lb})(3\text{ in.}) - 50\text{ lb}\cdot\text{in.} + T_B(10\text{ in.}) = 0$$

$$20.699 T_B = 80 \quad T_B = 3.8649\text{ lb}$$

Eq. (1): $\qquad E = 3.5663(3.8649\text{ lb}) - 10\text{ lb}$

$$E = +3.78347\text{ lb}$$

$\mathbf{E} = 3.78\text{ lb} \downarrow$ ◀

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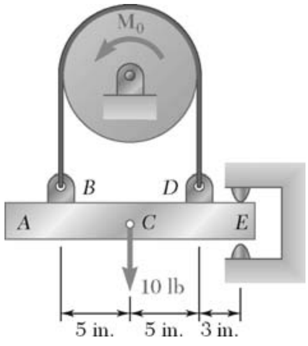


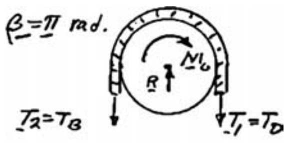
PROBLEM 8.125

Solve Problem 8.124 assuming that a clockwise couple $\mathbf{M}_0$ is applied to the drum.

PROBLEM 8.124 The 10-lb bar *AE* is suspended by a cable that passes over a 5-in.-radius drum. Vertical motion of end *E* of the bar is prevented by the two stops shown. Knowing that $\mu_s = 0.30$ between the cable and the drum, determine (*a*) the largest counterclockwise couple $\mathbf{M}_0$ that can be applied to the drum if slipping is not to occur, (*b*) the corresponding force exerted on end *E* of the bar.

SOLUTION

<u>Drum</u>: Slipping impends



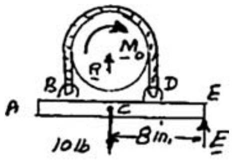
$$\mu_s = 0.30$$

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$\frac{T_B}{T_D} = e^{0.30\pi} = 2.5663$$

$$T_B = 2.5663T_D$$

(*a*) <u>Free body: Drum and bar</u>

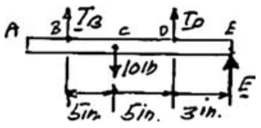


$$+\circlearrowright \Sigma M_C = 0: \quad M_0 - E(8\text{ in.}) = 0$$

$$M_0 = (2.1538\text{ lb})(8\text{ in.})$$

$$\mathbf{M}_0 = 17.23\text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

(*b*) <u>Bar *AE*</u>:



$$+\uparrow \Sigma F_y = 0: \quad T_B + T_D + E - 10\text{ lb} = 0$$

$$= 2.5663T_D + T_D + E - 10\text{ lb}$$

$$E = -3.5663T_D + 10\text{ lb} \qquad (1)$$

$$+\circlearrowright \Sigma M_B = 0: \quad T_D(10\text{ in.}) - (10\text{ lb})(5\text{ in.}) + E(13\text{ in.}) = 0$$

$$T_D(10\text{ in.}) - 50\text{ lb}\cdot\text{in.} + (-3.5663T_D + 10\text{ lb})(13\text{ in.}) = 0$$

$$-36.362T_D + 80\text{ lb}\cdot\text{in.} = 0; \quad T_D = 2.200\text{ lb}$$

Eq. (1):

$$E = -3.5633(2.200\text{ lb}) + 10\text{ lb}$$

$$E = +2.1538\text{ lb} \qquad \mathbf{E} = 2.15\text{ lb}\uparrow \blacktriangleleft$$

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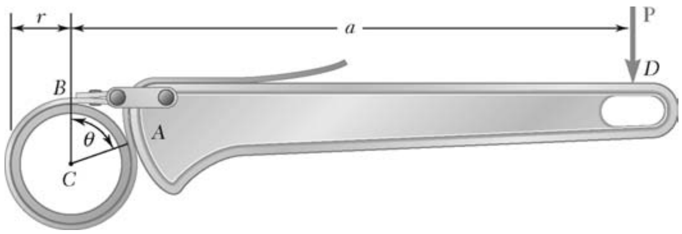
PROBLEM 8.126

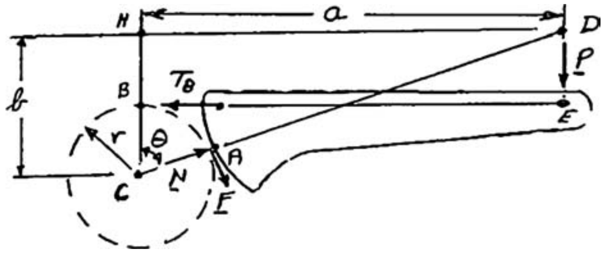
The strap wrench shown is used to grip the pipe firmly without marring the external surface of the pipe. Knowing that the coefficient of static friction is the same for all surfaces of contact, determine the smallest value of μ_s for which the wrench will be self-locking when $a = 200$ mm, $r = 30$ mm, and $\theta = 65°$.



SOLUTION

For wrench to be self-locking $(P = 0)$, the value of μ_s must prevent slipping of strap which is in contact with the pipe from Point A to Point B and must be large enough so that at Point A the strap tension can increase from zero to the minimum tension required to develop "belt friction" between strap and pipe.

Free body: Wrench handle



Geometry   In ΔCDH:

$$CH = \frac{a}{\tan\theta}$$

$$CD = \frac{a}{\sin\theta}$$

$$DE = BH = CH - BC$$

$$DE = \frac{a}{\tan\theta} - r$$

$$AD = CD - CA = \frac{a}{\sin\theta} - r$$

On wrench handle

$$+\circlearrowleft \Sigma M_D = 0: \quad T_B(DE) - F(AD) = 0$$

$$\frac{T_B}{F} = \frac{AD}{DE} = \frac{\dfrac{a}{\sin\theta} - r}{\dfrac{a}{\tan\theta} - r} \qquad (1)$$

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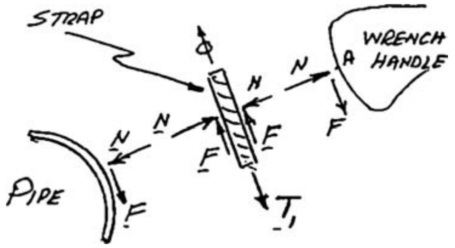
PROBLEM 8.126 (Continued)

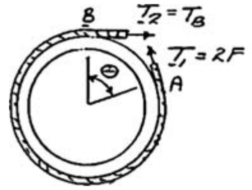
Free body: Strap at Point A



$$+\searrow \Sigma F = 0: \quad T_1 - 2F = 0$$

$$T_1 = 2F \qquad (2)$$

Pipe and strap



$$\beta = (2\pi - \theta) \text{ radians}$$

Eq. (8.13):
$$\mu_s \beta = \ln \frac{T_2}{T_1}$$

$$\mu_s = \frac{1}{\beta} \ln \frac{T_B}{2F} \qquad (3)$$

Return to free body of wrench handle

$$\xrightarrow{+} \Sigma F_x = 0: \quad N \sin\theta + F\cos\theta - T_B = 0$$

$$\frac{N}{F}\sin\theta = \frac{T_B}{F} - \cos\theta$$

Since $F = \mu_s N$, we have
$$\frac{1}{\mu_s}\sin\theta = \frac{T_B}{F} - \cos\theta$$

or
$$\mu_s = \frac{\sin\theta}{\frac{T_B}{F} - \cos\theta} \qquad (4)$$

(*Note:* For a given set of data, we seek the larger of the values of μ_s from Eqs. (3) and (4).)

For
$$a = 200 \text{ mm}, \quad r = 30 \text{ mm}, \quad \theta = 65°$$

Eq. (1):
$$\frac{T_B}{F} = \frac{\frac{200 \text{ mm}}{\sin 65°} - 30 \text{ mm}}{\frac{200 \text{ mm}}{\tan 65°} - 30 \text{ mm}}$$

$$= \frac{190.676 \text{ mm}}{63.262 \text{ mm}} = 3.0141$$

$$\beta = 2\pi - \theta = 2\pi - 65°\frac{\pi}{180°} = 5.1487 \text{ radians}$$

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PROBLEM 8.126 (Continued)

Eq. (3):
$$\mu_s = \frac{1}{5.1487 \text{ rad}} \ln \frac{3.0141}{2}$$
$$= \frac{0.41015}{5.1487}$$
$$= 0.0797 \quad \triangleleft$$

Eq. (4):
$$\mu_s = \frac{\sin 65^\circ}{3.0141 - \cos 65^\circ}$$
$$= \frac{0.90631}{2.1595}$$
$$= 0.3497 \quad \triangleleft$$

We choose the larger value: $\mu_s = 0.350$ ◀

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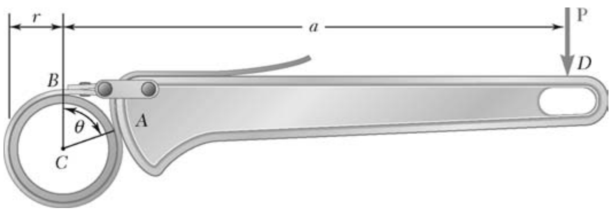
PROBLEM 8.127

Solve Problem 8.126 assuming that $\theta = 75°$.

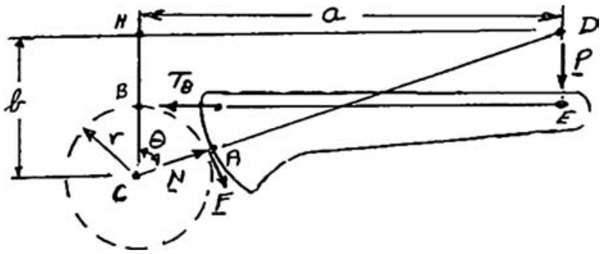
PROBLEM 8.126 The strap wrench shown is used to grip the pipe firmly without marring the external surface of the pipe. Knowing that the coefficient of static friction is the same for all surfaces of contact, determine the smallest value of μ_s for which the wrench will be self-locking when $a = 200$ mm, $r = 30$ mm, and $\theta = 65°$.



SOLUTION

For wrench to be self-locking $(P = 0)$, the value of μ_s must prevent slipping of strap which is in contact with the pipe from Point A to Point B <u>and</u> must be large enough so that at Point A the strap tension can increase from zero to the minimum tension required to develop "belt friction" between strap and pipe.

<u>Free body: Wrench handle</u>



<u>Geometry</u> In ΔCDH:

$$CH = \frac{a}{\tan\theta}$$

$$CD = \frac{a}{\sin\theta}$$

$$DE = BH = CH - BC$$

$$DE = \frac{a}{\tan\theta} - r$$

$$AD = CD - CA = \frac{a}{\sin\theta} - r$$

<u>On wrench handle</u> $+\circlearrowleft \Sigma M_D = 0: \quad T_B(DE) - F(AD) = 0$

$$\frac{T_B}{F} = \frac{AD}{DE} = \frac{\dfrac{a}{\sin\theta} - r}{\dfrac{a}{\tan\theta} - r} \qquad (1)$$

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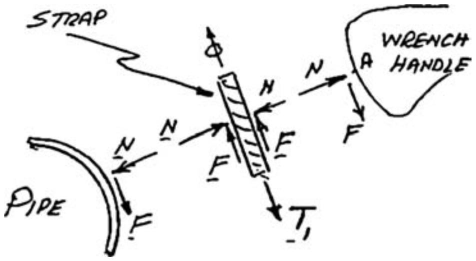
PROBLEM 8.127 (Continued)

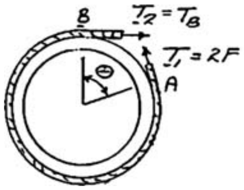
Free body: Strap at Point A



$$+\searrow \Sigma F = 0: \quad T_1 - 2F = 0$$

$$T_1 = 2F \tag{2}$$

Pipe and strap



$$\beta = (2\pi - \theta) \text{ radians}$$

Eq. (8.13): $$\mu_s \beta = \ln \frac{T_2}{T_1}$$

$$\mu_s = \frac{1}{\beta} \ln \frac{T_B}{2F} \tag{3}$$

Return to free body of wrench handle

$$\xrightarrow{+} \Sigma F_x = 0: \quad N \sin\theta + F \cos\theta - T_B = 0$$

$$\frac{N}{F} \sin\theta = \frac{T_B}{F} - \cos\theta$$

Since $F = \mu_s N$, we have $$\frac{1}{\mu_s} \sin\theta = \frac{T_B}{F} - \cos\theta$$

or $$\mu_s = \frac{\sin\theta}{\frac{T_B}{F} - \cos\theta} \tag{4}$$

(*Note:* For a given set of data, we seek the larger of the values of μ_s from Eqs. (3) and (4).)

For $$a = 200 \text{ mm}, \quad r = 30 \text{ mm}, \quad \theta = 75°$$

Eq. (1): $$\frac{T_B}{F} = \frac{\frac{200 \text{ mm}}{\sin 75°} - 30 \text{ mm}}{\frac{200 \text{ mm}}{\tan 75°} - 30 \text{ mm}}$$

$$= \frac{177.055 \text{ mm}}{23.590 \text{ mm}} = 7.5056$$

$$\beta = 2\pi - \theta = 2\pi - 75° \frac{\pi}{180°} = 4.9742$$

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PROBLEM 8.127 (Continued)

Eq. (3):
$$\mu_s = \frac{1}{4.9742 \text{ rad}} \ln \frac{7.5056}{2}$$
$$= \frac{1.3225}{4.9742}$$
$$= 0.2659 \quad \triangleleft$$

Eq. (4):
$$\mu_s = \frac{\sin 75°}{7.5056 - \cos 75°}$$
$$= \frac{0.96953}{7.2468}$$
$$= 0.1333 \quad \triangleleft$$

We choose the larger value: $\mu_s = 0.266$ ◀

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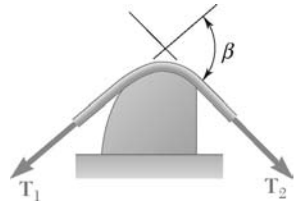
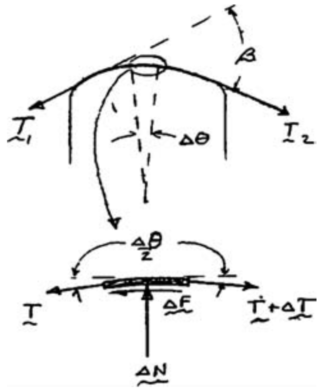


PROBLEM 8.128

Prove that Eqs. (8.13) and (8.14) are valid for any shape of surface provided that the coefficient of friction is the same at all points of contact.

SOLUTION



$$\uparrow \Sigma F_n = 0: \quad \Delta N - [T + (T + \Delta T)]\sin\frac{\Delta\theta}{2} = 0$$

or
$$\Delta N = (2T + \Delta T)\sin\frac{\Delta\theta}{2}$$

$$\longrightarrow \Sigma F_t = 0: \quad [(T + \Delta T) - T]\cos\frac{\Delta\theta}{2} - \Delta F = 0$$

or
$$\Delta F = \Delta T \cos\frac{\Delta\theta}{2}$$

Impending slipping:
$$\Delta F = \mu_s \Delta N$$

So
$$\Delta T \cos\frac{\Delta\theta}{2} = \mu_s 2T \sin\frac{\Delta\theta}{2} + \mu_s \Delta T \frac{\sin\Delta\theta}{2}$$

In limit as
$$\Delta\theta \longrightarrow 0: \quad dT = \mu_s T d\theta \quad \text{or} \quad \frac{dT}{T} = \mu_s d\theta$$

So
$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\beta} \mu_s d\theta$$

and
$$\ln\frac{T_2}{T_1} = \mu_s \beta$$

or $T_2 = T_1 e^{\mu_s \beta}$ ◀

(*Note:* Nothing above depends on the shape of the surface, except it is assumed to be a smooth curve.)

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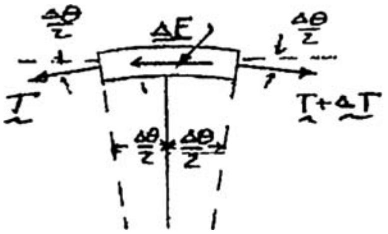
PROBLEM 8.129

Complete the derivation of Eq. (8.15), which relates the tension in both parts of a V belt.

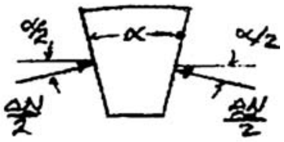
SOLUTION

Small belt section:

Side view:



End view:



$$\uparrow \Sigma F_y = 0: \quad 2\frac{\Delta N}{2}\sin\frac{\alpha}{2} - [T + (T + \Delta T)]\sin\frac{\Delta\theta}{2} = 0$$

$$\longrightarrow \Sigma F_x = 0: \quad [(T + \Delta T) - T]\cos\frac{\Delta\theta}{2} - \Delta F = 0$$

Impending slipping: $$\Delta F = \mu_s \Delta N \Rightarrow \Delta T\cos\frac{\Delta\theta}{2} = \mu_s \frac{2T + \Delta T}{\sin\frac{\alpha}{2}}\sin\frac{\Delta\theta}{2}$$

In limit as $$\Delta\theta \longrightarrow 0: \quad dT = \frac{\mu_s T d\theta}{\sin\frac{\alpha}{2}} \quad \text{or} \quad \frac{dT}{T} = \frac{\mu_s}{\sin\frac{\alpha}{2}} d\theta$$

so $$\int_{T_1}^{T_2} \frac{dT}{T} = \frac{\mu_s}{\sin\frac{\alpha}{2}} \int_0^{\beta} d\theta$$

or $$\ln\frac{T_2}{T_1} = \frac{\mu_s \beta}{\sin\frac{\alpha}{2}}$$

or $$T_2 = T_1 e^{\mu_s \beta / \sin\frac{\alpha}{2}} \blacktriangleleft$$

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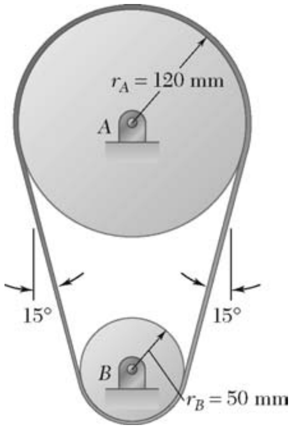


PROBLEM 8.130

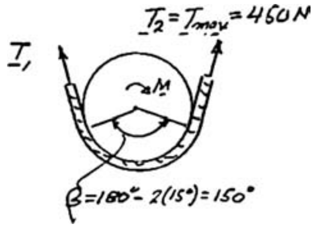
Solve Problem 8.107 assuming that the flat belt and drums are replaced by a V belt and V pulleys with $\alpha = 36°$. (The angle α is as shown in Figure 8.15*a*.)

PROBLEM 8.107 A flat belt is used to transmit a couple from drum *B* to drum *A*. Knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is 450 N, determine the largest couple that can be exerted on drum *A*.

SOLUTION

Since β is smaller for pulley *B*. The belt will slip first at *B*.

$$\beta = 150°\left(\frac{\pi \text{ rad}}{180°}\right) = \frac{5}{6}\pi \text{ rad}$$

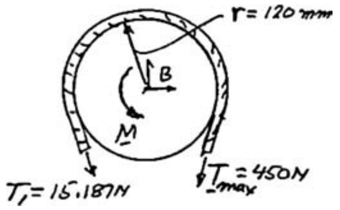


$$\frac{T_2}{T_1} = e^{\mu_s \beta / \sin\frac{\alpha}{2}}$$

$$\frac{450 \text{ N}}{T_1} = e^{(0.4)\frac{5}{6}\pi/\sin 18°} = e^{3.389}$$

$$\frac{450 \text{ N}}{T_1} = 29.63; \quad T_1 = 15.187 \text{ N}$$

<u>Torque on pulley *A*</u>:



$$+\circlearrowleft \Sigma M_B = 0: \quad M - (T_{\text{max}} - T_1)(0.12 \text{ m}) = 0$$

$$M - (450 \text{ N} - 15.187 \text{ N})(0.12 \text{ m}) = 0$$

$$M = 52.18 \text{ N}\cdot\text{m} \qquad M = 52.2 \text{ N}\cdot\text{m} \blacktriangleleft$$

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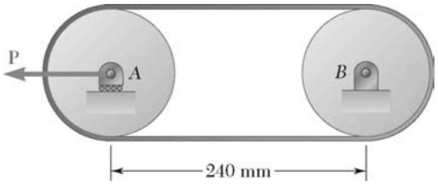
PROBLEM 8.131

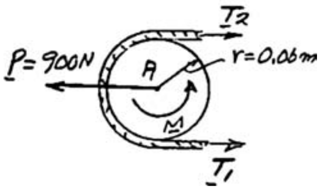


Solve Problem 8.108 assuming that the flat belt and pulleys are replaced by a V belt and V pulleys with $\alpha = 36°$. (The angle α is as shown in Figure 8.15*a*.)

PROBLEM 8.108 A flat belt is used to transmit a couple from pulley *A* to pulley *B*. The radius of each pulley is 60 mm, and a force of magnitude $P = 900$ N is applied as shown to the axle of pulley *A*. Knowing that the coefficient of static friction is 0.35, determine (*a*) the largest couple that can be transmitted, (*b*) the corresponding maximum value of the tension in the belt.

SOLUTION

Pulley *A*: $\beta = \pi$ rad



$$\frac{T_2}{T_1} = e^{\mu_s \beta / \sin\frac{\alpha}{2}}$$

$$\frac{T_2}{T_1} = e^{0.35\pi/\sin 18°}$$

$$\frac{T_2}{T_1} = e^{3.558} = 35.1$$

$$T_2 = 35.1 T_1$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad T_1 + T_2 - 900 \text{ N} = 0$$

$$T_1 + 35.1T_1 - 900 \text{ N} = 0$$

$$T_1 = 24.93 \text{ N}$$

$$T_2 = 35.1(24.93 \text{ N}) = 875.03 \text{ N}$$

$$+\circlearrowright \Sigma M_A = 0: \quad M - T_2(0.06 \text{ m}) + T_1(0.06 \text{ m}) = 0$$

$$M - (875.03 \text{ N})(0.06 \text{ m}) + (24.93 \text{ N})(0.06 \text{ m}) = 0$$

$$M = 51.0 \text{ N}\cdot\text{m} \blacktriangleleft$$

$$T_{max} = T_2 \qquad T_{max} = 875 \text{ N} \blacktriangleleft$$

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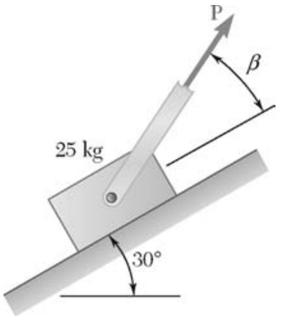
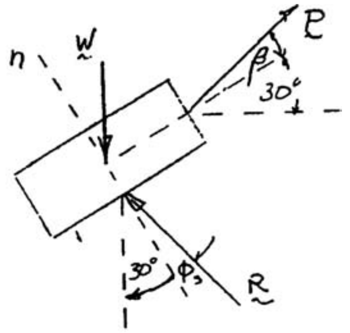


PROBLEM 8.132

Knowing that the coefficient of friction between the 25-kg block and the incline is $\mu_s = 0.25,$ determine (*a*) the smallest value of *P* required to start the block moving up the incline, (*b*) the corresponding value of β.

SOLUTION

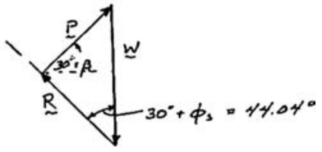
FBD block (Impending motion up)



$$W = mg$$
$$= (25\text{ kg})(9.81\text{ m/s}^2)$$
$$= 245.25\text{ N}$$

$$\phi_s = \tan^{-1}\mu_s$$
$$= \tan^{-1}(0.25)$$
$$= 14.04°$$

(*a*) (*Note:* For minimum *P*, $\mathbf{P} \perp \mathbf{R}$ so $\beta = \phi_s$.)



Then

$$P = W\sin(30° + \phi_s)$$
$$= (245.25\text{ N})\sin 44.04°$$

$P_{\min} = 170.5\text{ N}$ ◀

(*b*) We have $\beta = \phi_s$ $\beta = 14.04°$ ◀

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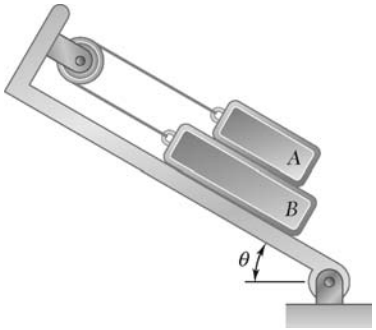


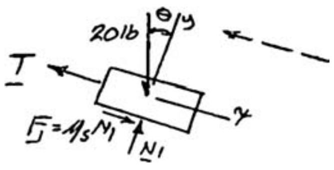
PROBLEM 8.133

The 20-lb block A and the 30-lb block B are supported by an incline that is held in the position shown. Knowing that the coefficient of static friction is 0.15 between all surfaces of contact, determine the value of θ for which motion is impending.

SOLUTION

Since motion impends, $F = \mu_s N$ at all surfaces.

Free body: Block A



Impending motion:

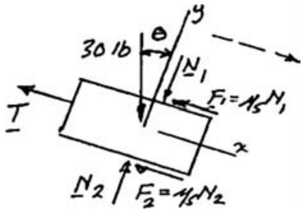
$$\Sigma F_y = 0: \quad N_1 = 20\cos\theta$$

$$\nwarrow^{+} \Sigma F_x = 0: \quad T - 20\sin\theta - \mu_s N_1 = 0$$

$$T = 20\sin\theta + 0.15(20\cos\theta)$$

$$T = 20\sin\theta + 3\cos\theta \qquad (1)$$

Free body: Block B



Impending motion:

$$+\nearrow \Sigma F_y = 0: \quad N_2 - 30\cos\theta - N_1 = 0$$

$$N_2 = 30\cos\theta + 20\cos\theta = 50\cos\theta$$

$$F_2 = \mu_s N_2 = 0.15(50\cos\theta) = 6\cos\theta$$

$$\nwarrow^{+} \Sigma F_x = 0: \quad T - 30\sin\theta + \mu_s N_1 + \mu_s N_2 = 0$$

$$T = 30\sin\theta - 0.15(20\cos\theta) - 0.15(50\cos\theta)$$

$$T = 30\sin\theta - 3\cos\theta - 7.5\cos\theta \qquad (2)$$

Eq. (1) subtracted by Eq. (2): $20\sin\theta + 3\cos\theta - 30\sin\theta + 3\cos\theta + 7.5\cos\theta = 0$

$$13.5\cos\theta = 10\sin\theta, \quad \tan\theta = \frac{13.5°}{10}$$

$\theta = 53.5°$ ◀

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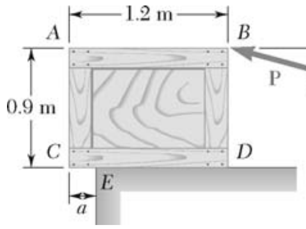


PROBLEM 8.134

A worker slowly moves a 50-kg crate to the left along a loading dock by applying a force **P** at corner *B* as shown. Knowing that the crate starts to tip about the edge *E* of the loading dock when *a* = 200 mm, determine (*a*) the coefficient of kinetic friction between the crate and the loading dock, (*b*) the corresponding magnitude *P* of the force.

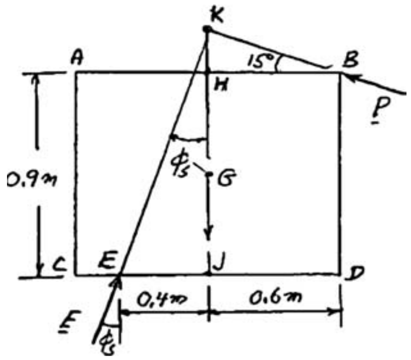
SOLUTION

Free body: Crate Three-force body.

Reaction *E* must pass through *K* where **P** and **W** intersect.

Geometry:

(*a*) $HK = (0.6\text{ m})\tan 15° = 0.16077\text{ m}$

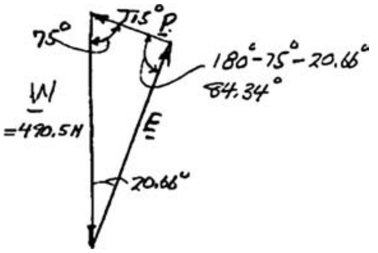


$$JK = 0.9\text{ m} + HK = 1.06077\text{ m}$$

$$\tan\phi_s = \frac{0.4\text{ m}}{1.06077\text{ m}} = 0.37708 \qquad \mu_s = \tan\phi_s = 0.377 \blacktriangleleft$$

$$\phi_s = 20.66°$$

Force triangle:



(*b*) $W = (50\text{ kg})(9.81\text{ m/s})$
$= 490.5\text{ N}$

Law of sines:

$$\frac{P}{\sin 20.66°} = \frac{490.5\text{ N}}{\sin 84.34°}$$

$$P = 173.91\text{ N} \qquad P = 173.9\text{ N} \blacktriangleleft$$

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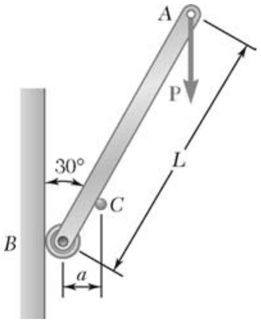


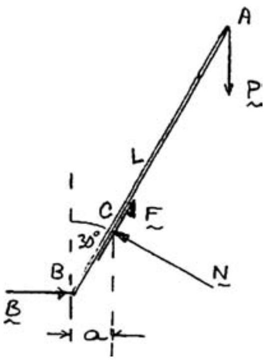
PROBLEM 8.135

A slender rod of length L is lodged between peg C and the vertical wall and supports a load **P** at end A. Knowing that the coefficient of static friction between the peg and the rod is 0.15 and neglecting friction at the roller, determine the range of values of the ratio L/a for which equilibrium is maintained.

SOLUTION

FBD rod:

<u>Free-body diagram</u>: <u>For motion of *B* impending upward</u>:



$$+\circlearrowleft \Sigma M_B = 0: \quad PL\sin\theta - N_C\left(\frac{a}{\sin\theta}\right) = 0$$

$$N_C = \frac{PL}{a}\sin^2\theta \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_C\sin\theta - \mu_s N_C\cos\theta - P = 0$$

$$N_C(\sin\theta - \mu\cos\theta) = P$$

Substitute for N_C from Eq. (1), and solve for a/L.

$$\frac{a}{L} = \sin^2\theta(\sin\theta - \mu_s\cos\theta) \qquad (2)$$

For $\theta = 30°$ and $\mu_s = 0.15$:

$$\frac{a}{L} = \sin^2 30°(\sin 30° - 0.15\cos 30°)$$

$$\frac{a}{L} = 0.092524 \qquad \frac{L}{a} = 10.808$$

<u>For motion of *B* impending downward</u>, reverse sense of friction force F_C. To do this we make $\mu_s = -0.15$ in. Eq. (2).

Eq. (2):

$$\frac{a}{L} = \sin^2 30°(\sin 30° - (-0.15)\cos 30°)$$

$$\frac{a}{L} = 0.15748 \qquad \frac{L}{a} = 6.350$$

Range of values of L/a for equilibrium: $6.35 \le \frac{L}{a} \le 10.81$ ◀

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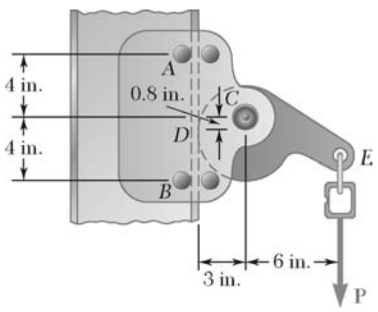
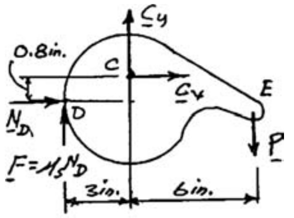


PROBLEM 8.136

A safety device used by workers climbing ladders fixed to high structures consists of a rail attached to the ladder and a sleeve that can slide on the flange of the rail. A chain connects the worker's belt to the end of an eccentric cam that can be rotated about an axle attached to the sleeve at *C*. Determine the smallest allowable common value of the coefficient of static friction between the flange of the rail, the pins at *A* and *B*, and the eccentric cam if the sleeve is not to slide down when the chain is pulled vertically downward.

SOLUTION

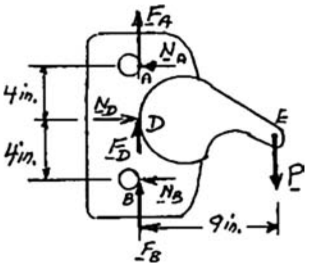
Free body: Cam



$$+\circlearrowleft \Sigma M_C = 0: \quad N_D(0.8 \text{ in.}) - \mu_s N_D(3\text{in.}) - P(6 \text{ in.}) = 0$$

$$N_D = \frac{6P}{0.8 - 3\mu_s} \qquad (1)$$

Free body: Sleeve and cam



$$\xrightarrow{+} \Sigma F_x = 0: \quad N_D - N_A - N_B = 0$$

$$N_A + N_B = N_D \qquad (2)$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + F_B + F_D - P = 0$$

or

$$\mu_s(N_A + N_B + N_D) = P \qquad (3)$$

Substitute from Eq. (2) into Eq. (3): $\mu_s(2N_D) = P \qquad N_D = \dfrac{P}{2\mu_s}$ (4)

Equate expressions for N_D from Eq. (1) and Eq. (4):

$$\frac{P}{2\mu_s} = \frac{6P}{0.8 - 3\mu_s}; \quad 0.8 - 3\mu_s = 12\mu_s$$

$$\mu_s = \frac{0.8}{15} \qquad \mu_s = 0.0533 \blacktriangleleft$$

(*Note:* To verify that contact at pins *A* and *B* takes places as assumed, we shall check that $N_A > 0$ and $N_B = 0$.)

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PROBLEM 8.136 (Continued)

From Eq. (4):
$$N_D = \frac{P}{2\mu_s} = \frac{P}{2(0.0533)} = 9.375P$$

From free body of cam and sleeve:
$$+\circlearrowleft \Sigma M_B = 0: \quad N_A(8 \text{ in.}) - N_D(4 \text{ in.}) - P(9 \text{ in.}) = 0$$
$$8N_A = (9.375P)(4) + 9P$$
$$N_A = 5.8125P > 0 \quad \text{OK}$$

From Eq. (2):
$$N_A + N_B = N_D$$
$$5.8125P + N_B = 9.375P$$
$$N_B = 3.5625P > 0 \quad \text{OK}$$

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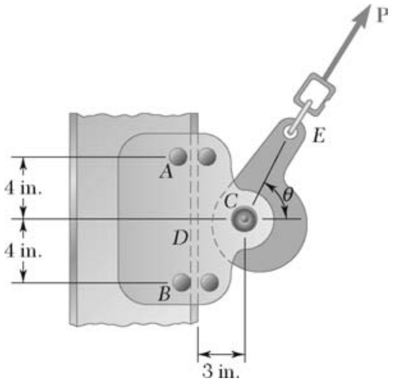


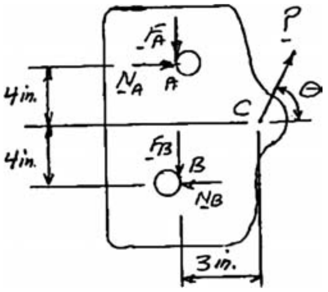
PROBLEM 8.137

To be of practical use, the safety sleeve described in the preceding problem must be free to slide along the rail when pulled upward. Determine the largest allowable value of the coefficient of static friction between the flange of the rail and the pins at A and B if the sleeve is to be free to slide when pulled as shown in the figure, assuming (*a*) $\theta = 60°$, (*b*) $\theta = 50°$, (*c*) $\theta = 40°$.

SOLUTION

Note the cam is a two-force member.

Free body: Sleeve



We assume contact between rail and pins as shown.

$$+\circlearrowright \Sigma M_C = 0: \quad F_A(3\text{ in.}) + F_B(3\text{ in.}) - N_A(4\text{ in.}) - N_B(4\text{ in.}) = 0$$

But $\quad F_A = \mu_s N_A$

$$F_B = \mu_s N_B$$

We find $\quad 3\mu_s(N_A + N_B) - 4(N_A + N_B) = 0$

$$\mu_s = \frac{4}{3} = 1.33333$$

We now verify that our assumption was correct.

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_A - N_B + P\cos\theta = 0$$

$$N_B - N_A = P\cos\theta \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad -F_A - F_B + P\sin\theta = 0$$

$$\mu_s N_A + \mu_s N_B = P\sin\theta$$

$$N_A + N_B = \frac{P\sin\theta}{\mu_s} \tag{2}$$

Add Eqs. (1) and (2): $\quad 2N_B = P\left(\cos\theta + \frac{\sin\theta}{\mu_s}\right) > 0 \quad \text{OK}$

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PROBLEM 8.137 (Continued)

Subtract Eq. (1) from Eq. (2): $$2N_A = P\left(\frac{\sin\theta}{\mu_s} - \cos\theta\right)$$

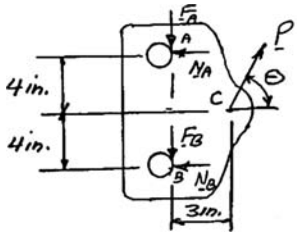
$N_A > 0$ only if $$\frac{\sin\theta}{\mu_s} - \cos\theta > 0$$

$$\tan\theta > \mu_s = 1.33333$$
$$\theta = 53.130°$$

(*a*) For case (*a*): Condition is satisfied, contact takes place as shown. Answer is correct.

$\mu_s = 1.333$ ◀

But for (*b*) and (*c*): $\theta < 53.130°$ and our assumption is wrong, N_A is directed to left.



$$\xrightarrow{+} \Sigma F_x = 0: \quad -N_A - N_B + P\cos\theta = 0$$

$$N_A + N_B = P\cos\theta \qquad (3)$$

$$+\uparrow \Sigma F_y = 0: \quad -F_A - F_A + P\sin\theta = 0$$

$$\mu_s(N_A + N_B) = P\sin\theta \qquad (4)$$

Divide Eq. (4) by Eq. (3):

$$\mu_s = \tan\theta \qquad (5)$$

(*b*) We make $\theta = 50°$ in Eq. (5):

$$\mu_s = \tan 50°$$ $\mu_s = 1.192$ ◀

(*c*) We make $\theta = 40°$ in Eq. (5):

$$\mu_s = \tan 40°$$ $\mu_s = 0.839$ ◀

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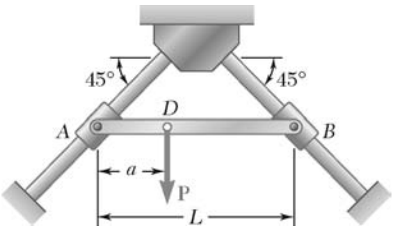
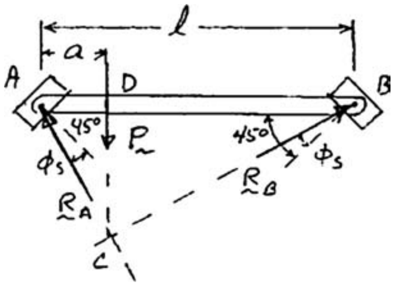


PROBLEM 8.138

Bar *AB* is attached to collars that can slide on the inclined rods shown. *A* force **P** is applied at Point *D* located at a distance *a* from end *A*. Knowing that the coefficient of static friction μ_s between each collar and the rod upon which it slides is 0.30 and neglecting the weights of the bar and of the collars, determine the smallest value of the ratio *a*/*L* for which equilibrium is maintained.

SOLUTION

FBD bar and collars:



Impending motion:

$$\phi_s = \tan^{-1}\mu_s$$
$$= \tan^{-1} 0.3$$
$$= 16.6992°$$

Neglect weights: 3-force *FBD* and $\measuredangle\, ACB = 90°$

so

$$AC = \frac{a}{\cos(45° + \phi_s)}$$
$$= l\sin(45° - \phi_s)$$

$$\frac{a}{l} = \sin(45° - 16.6992°)\cos(45° + 16.6992°)$$

$$\frac{a}{l} = 0.225 \blacktriangleleft$$

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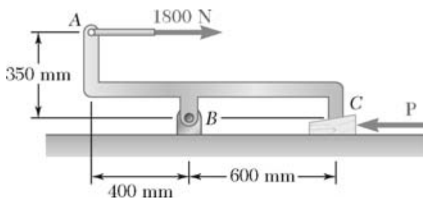


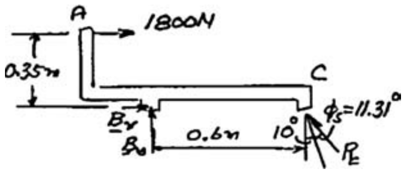
PROBLEM 8.139

The machine part *ABC* is supported by a frictionless hinge at *B* and a 10° wedge at *C*. Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20, determine (*a*) the force **P** required to move the wedge, (*b*) the components of the corresponding reaction at *B*.

SOLUTION

$$\phi_s = \tan^{-1} 0.20 = 11.31°$$

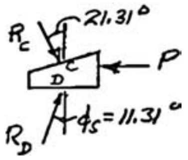
Free body: Part *ABC*



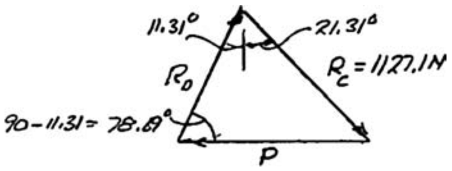
$$+\circlearrowright \Sigma M_B = 0 \quad (1800\text{ N})(0.35\text{ m}) - R\cos 21.31°(0.6\text{ m}) = 0$$

$$R_C = 1127.1\text{ N}$$

Free body: Wedge



Force triangle:



(*a*) Law of sines:

$$\frac{P}{\sin(11.31° + 21.31°)} = \frac{1127.1\text{ N}}{\sin 78.69°}$$

$$P = 619.6\text{ N}$$

$\mathbf{P} = 620\text{ N} \longleftarrow$ ◀

(*b*) Return to part *ABC*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 1800\text{ N} - R_C \sin 21.31° = 0$$

$$B_x + 1800\text{ N} - (1127.1\text{ N})\sin 21.31°$$

$$B_x = -1390.4\text{ N}$$

$\mathbf{B}_x = 1390\text{ N} \longleftarrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad B_y + R_C \cos 21.31° = 0$$

$$B_y + (1127.1\text{ N})\cos 21.31° = 0$$

$$B_y = -1050\text{ N}$$

$\mathbf{B}_y = 1050\text{ N} \downarrow$ ◀

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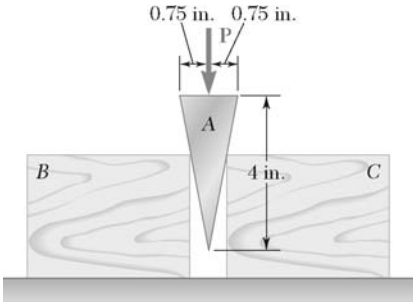
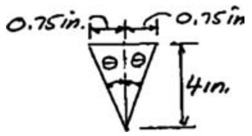


PROBLEM 8.140

A wedge *A* of negligible weight is to be driven between two 100-lb blocks *B* and *C* resting on a horizontal surface. Knowing that the coefficient of static friction at all surfaces of contact is 0.35, determine the smallest force **P** required to start moving the wedge (*a*) if the blocks are equally free to move, (*b*) if block *C* is securely bolted to the horizontal surface.

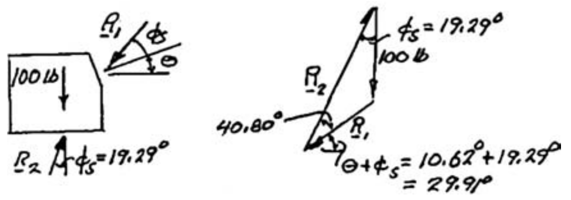
SOLUTION



Wedge angle θ: $\theta = \tan^{-1}\dfrac{0.75\text{ in.}}{4\text{ in.}}$

$\theta = 10.62°$

(*a*) Free body: Block *B*



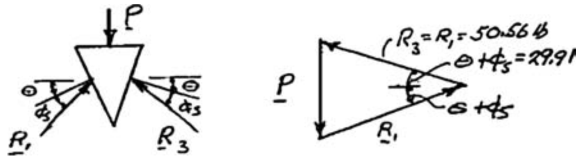
$$\phi_s = \tan^{-1} 0.35 = 19.29°$$

$$\frac{R_1}{\sin 19.29°} = \frac{100\text{ lb}}{\sin 40.80°}$$

$$R_1 = 50.56\text{ lb}$$

Free body: Wedge

By symmetry: $R_3 = R_1$



$$P = 2R_1 \sin(\theta + \phi_s) = 2(50.56)\sin 29.91°$$

$P = 50.42\text{ lb}$ **P** $= 50.4$ lb ↓ ◀

(*b*) Free bodies unchanged: Same result. **P** $= 50.4$ lb ↓ ◀

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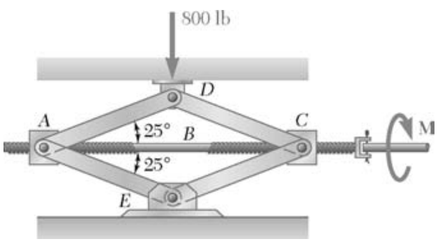
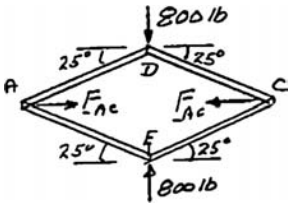


PROBLEM 8.141

The position of the automobile jack shown is controlled by a screw *ABC* that is single-threaded at each end (right-handed thread at *A*, left-handed thread at *C*). Each thread has a pitch of 0.1 in. and a mean diameter of 0.375 in. If the coefficient of static friction is 0.15, determine the magnitude of the couple **M** that must be applied to raise the automobile.

SOLUTION

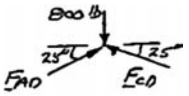


Free body: Parts *A*, *D*, *C*, *E*

Two-force members

Joint *D*:

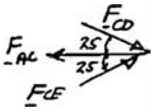
Symmetry: $F_{AD} = F_{CD}$



$$+\uparrow \Sigma F_y = 0: \quad 2F_{CD}\sin 25° - 800 \text{ lb} = 0$$

$$F_{CD} = 946.5 \text{ lb}$$

Joint *C*:



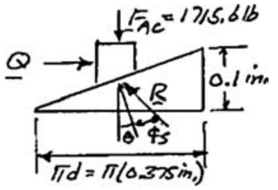
Symmetry: $F_{CE} = F_{CD}$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 2F_{CD}\cos 25° - F_{AC} = 0$$

$$F_{AC} = 2(946.5 \text{ lb})\cos 25°$$

$$F_{AC} = 1715.6 \text{ lb}$$

Block-and-incline analysis of *one* screw:

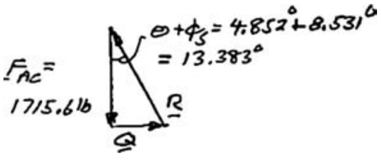


$$\tan\theta = \frac{0.1 \text{ m}}{\pi(0.375 \text{ in.})}$$

$$\theta = 4.852°$$

$$\phi_s = \tan^{-1} 0.15$$

$$= 8.531°$$



$$Q = (1715.6 \text{ lb})\tan 13.383°$$

$$Q = 408.2 \text{ lb}$$

But, we have *two* screws: $\text{Torque} = 2Qr = 2(408.2 \text{ lb})\left(\dfrac{0.375 \text{ in.}}{2}\right)$ $\text{Torque} = 153.1 \text{ lb}\cdot\text{in.}$ ◀

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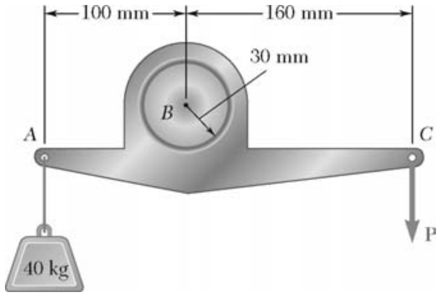
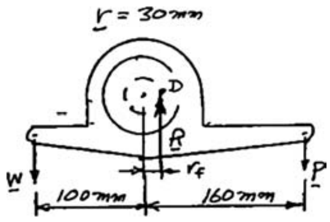


PROBLEM 8.142

A lever of negligible weight is loosely fitted onto a 30-mm-radius fixed shaft as shown. Knowing that a force **P** of magnitude 275 N will just start the lever rotating clockwise, determine (*a*) the coefficient of static friction between the shaft and the lever, (*b*) the smallest force **P** for which the lever does not start rotating counterclockwise.

SOLUTION

(*a*) Impending motion ↻



$$W = (40 \text{ kg})(9.81 \text{ m/s}^2) = 392.4 \text{ N}$$

$$+\circlearrowright \Sigma M_D = 0: \quad P(160 - r_f) - W(100 + r_f) = 0$$

$$r_f = \frac{160P - 100W}{P + W}$$

$$r_f = \frac{(160 \text{ mm})(275 \text{ N}) - (100 \text{ mm})(392.4 \text{ N})}{275 \text{ N} + 392.4 \text{ N}}$$

$$r_f = 7.132 \text{ mm}$$

$$r_f = r \sin\phi_s = r\mu_s$$

$$\mu_s = \frac{r_f}{r} = \frac{7.132 \text{ mm}}{30 \text{ mm}} = 0.2377$$

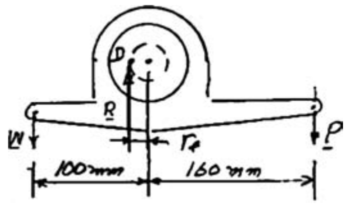
$\mu_s = 0.238$ ◀

(*b*) Impending motion ↺

$$r_f = r \sin\phi_s = r\mu_s$$

$$= (30 \text{ mm})(0.2377)$$

$$r_f = 7.132 \text{ mm}$$



$$+\circlearrowright \Sigma M_D = 0: \quad P(160 + r_f) - W(100 - r_f) = 0$$

$$P = W\frac{100 - r_f}{160 + r_f}$$

$$P = (392.4 \text{ N})\frac{100 \text{ mm} - 7.132 \text{ mm}}{160 \text{ mm} + 7.132 \text{ mm}}$$

$$P = 218.04 \text{ N}$$

P = 218 N ↓ ◀

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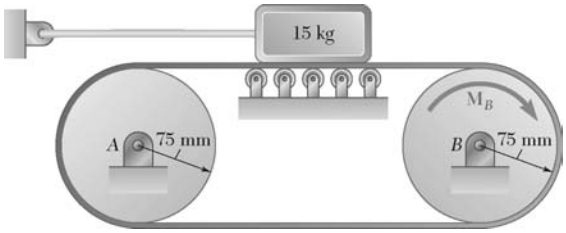
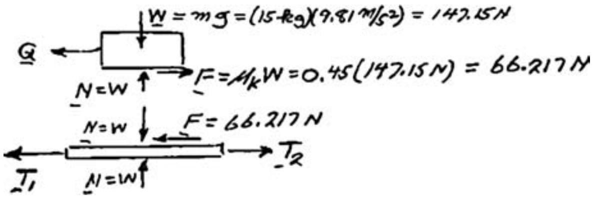


PROBLEM 8.143

A couple $\mathbf{M}_B$ is applied to the drive drum B to maintain a constant speed in the polishing belt shown. Knowing that $\mu_k = 0.45$ between the belt and the 15-kg block being polished and $\mu_s = 0.30$ between the belt and the drive drum B, determine (*a*) the couple $\mathbf{M}_B$, (*b*) the minimum tension in the lower portion of the belt if no slipping is to occur between the belt and the drive drum.

SOLUTION

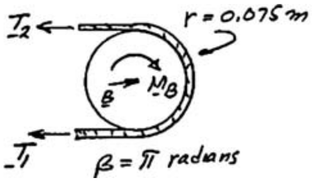
Block:



Portion of belt located under block:

$$\xrightarrow{+} \Sigma F_x = 0: \quad T_2 - T_1 - 66.217 \text{ N} = 0 \qquad (1)$$

Drum B:



$$\frac{T_2}{T_1} = e^{\mu_s \pi} = e^{0.3\pi} = 2.5663$$

$$T_2 = 2.5663T_1 \qquad (2)$$

Eq. (1):

$$2.5663T_1 - T_1 - 66.217 \text{ N} = 0$$

$$1.5663T_1 = 66.217 \text{ N}$$

$$T_1 = 42.276 \text{ N} \qquad T_{\min} = 42.3 \text{ N} \blacktriangleleft$$

Eq. (2):

$$T_2 = 2.5663(42.276 \text{ N}) = 108.493 \text{ N}$$

$$+\circlearrowright \Sigma M_B = 0: \quad M_B - (108.493 \text{ N})(0.075 \text{ m}) + (42.276 \text{ N})(0.075 \text{ m}) = 0$$

$$M_B = 4.966 \text{ N}\cdot\text{m} \qquad \mathbf{M}_B = 4.97 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

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CHAPTER 9

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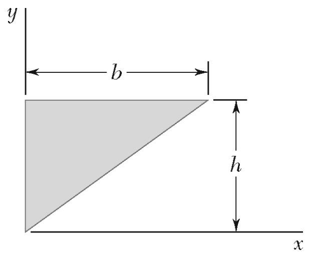
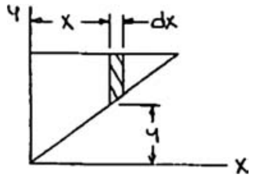


PROBLEM 9.1

Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION



By observation $$y = \frac{h}{b}x$$

Now $$dI_y = x^2 \quad dA = x^2[(h - y)dx]$$

$$= hx^2\left(1 - \frac{x}{b}\right)dx$$

Then $$I_y = \int dI_y = \int_0^b hx^2\left(1 - \frac{x}{b}\right)dx$$

$$= h\left[\frac{1}{3}x^3 - \frac{x^4}{4b}\right]_0^b$$

or $I_y = \frac{1}{12}b^3h$ ◀

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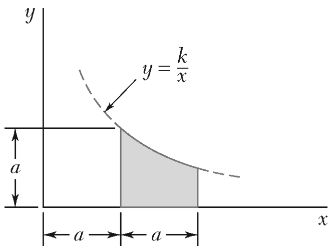


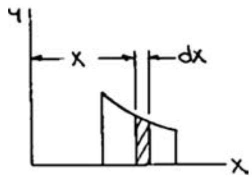
PROBLEM 9.2

Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION

At $x = a, \quad y = a$: $\quad a = \dfrac{k}{a} \quad$ or $\quad k = a^2$

Then $\quad y = \dfrac{a^2}{x}$



Now $\quad dI_y = x^2 \quad dA = x^2(y\,dx)$

$$= x^2\left(\frac{a^2}{x}dx\right) = a^2 x\,dx$$

Then $\quad I_y = \int dI_y = \int_a^{2a} a^2 x\,dx = a^2\left[\frac{1}{2}x^2\right]_a^{2a} = \frac{a^2}{2}[(2a)^2 - (a)^2]$

or $\quad I_y = \dfrac{3}{2}a^4$ ◀

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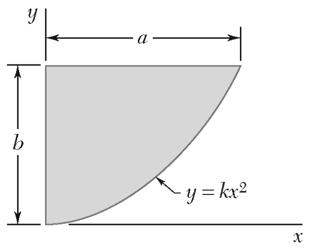
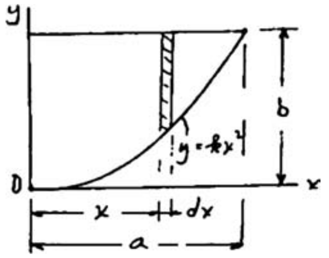


PROBLEM 9.3

Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION



$$y = kx^2$$

For $x = a$:

$$b = ka^2$$

$$k = \frac{b}{a^2}$$

Thus:

$$y = \frac{b}{a^2}x^2$$

$$dA = (b - y)dx$$

$$dI_y = x^2 dA = x^2(b - y)dx = x^2\left(b - \frac{b}{a^2}x^2\right)dx$$

$$I_y = \int dI_y = \int_0^a \left(bx^2 - \frac{b}{a^2}x^4\right)dx = \frac{1}{3}a^3b - \frac{1}{5}a^3b$$

$$I_y = \frac{2}{15}a^3b \quad ◀$$

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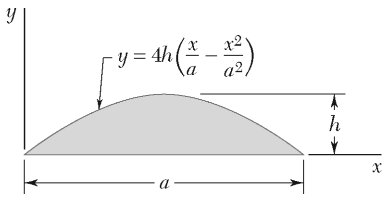
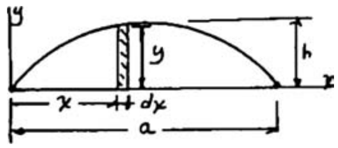


PROBLEM 9.4

Determine by direct integration the moment of inertia of the shaded area with respect to the *y* axis.

SOLUTION



$$y = 4h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)$$

$$dA = y\,dx$$

$$dI_y = x^2 dA = 4hx^2\left(\frac{x}{a} - \frac{x^2}{a^2}\right)dx$$

$$I_y = 4h\int_0^a \left(\frac{x^3}{a} - \frac{x^4}{a^2}\right)dx$$

$$I_y = 4h\left[\frac{x^4}{4a} - \frac{x^5}{5a^2}\right]_0^a = 4h\left(\frac{a^3}{4} - \frac{a^3}{5}\right)$$

$$I_y = \frac{1}{5}ha^3 \quad \blacktriangleleft$$

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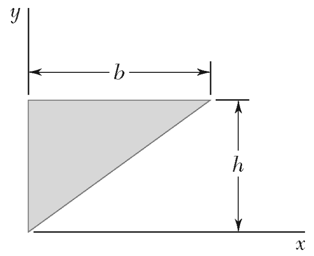
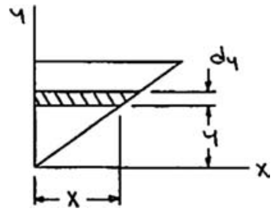


PROBLEM 9.5

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION



By observation $$y = \frac{h}{b}x$$

or $$x = \frac{b}{h}y$$

Now
$$dI_x = y^2 dA = y^2(x\,dy)$$
$$= y^2\left(\frac{b}{h}y\,dy\right)$$
$$= \frac{b}{h}y^3 dy$$

Then
$$I_x = \int dI_x = \int_0^h \frac{b}{h}y^3 dy$$
$$= \frac{b}{h}\left[\frac{1}{4}y^4\right]_0^h$$

or $I_x = \frac{1}{4}bh^3$ ◀

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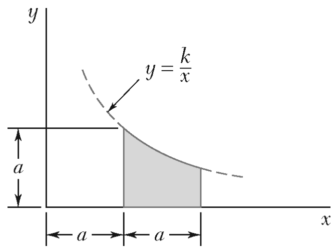
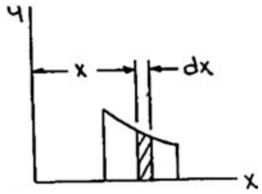


PROBLEM 9.6

Determine by direct integration the moment of inertia of the shaded area with respect to the *x* axis.

SOLUTION

At $x = a, \quad y = a$: $\quad a = \frac{k}{a} \quad$ or $\quad k = a^2$



Then $\quad y = \frac{a^2}{x}$

Now
$$dI_x = \frac{1}{3}y^3 dx = \frac{1}{3}\left(\frac{a^2}{x}\right)^3 dx$$
$$= \frac{1}{3}\frac{a^6}{x^3}dx$$

Then
$$I_x = \int dI_x = \int_a^a \frac{1}{3}\frac{a^6}{x^3}dx = \frac{1}{3}a^6\left[-\frac{1}{2}\frac{1}{x^2}\right]_a^{2a}$$
$$= -\frac{1}{6}a^6\left[\frac{1}{(2a)^2} - \frac{1}{(a)^2}\right]$$

or $\quad I_x = \frac{1}{8}a^4$ ◀

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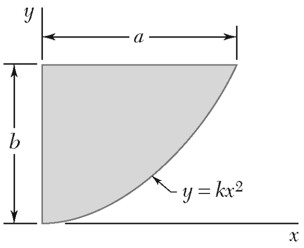
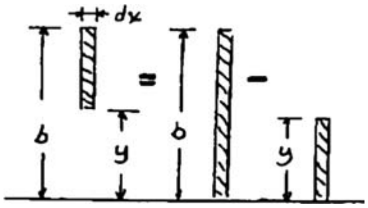


PROBLEM 9.7

Determine by direct integration the moment of inertia of the shaded area with respect to the *x* axis.

SOLUTION

See figure of solution of Problem 9.3.



$$y = \frac{b}{a^2}x$$

$$dI_x = \frac{1}{3}b^3 dx - \frac{1}{3}y^3 dx = \frac{1}{3}b^3 dx - \frac{1}{3}\frac{b^3}{a^6}x^6 dx$$

$$I_x = \int dI_x = \int_0^a \left(\frac{1}{3}b^3 - \frac{1}{3}\frac{b^3}{a^6}x^6\right)dx$$

$$= \frac{1}{3}b^3 a - \frac{1}{3}\frac{b^3}{a^6}\frac{a^7}{7} = \left(\frac{1}{3} - \frac{1}{21}\right)ab^3$$

$$= \left(\frac{7}{21} - \frac{1}{21}\right)ab^3 = \frac{6}{21}ab^3$$

$$I_x = \frac{2}{7}ab^3 \blacktriangleleft$$

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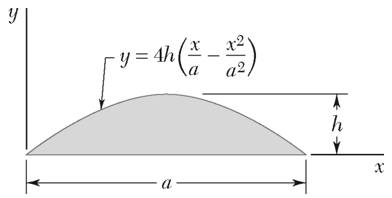

PROBLEM 9.8

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION

See figure of solution of Problem 9.4.

$$y = h_1 + (h_2 - h_1)\frac{x}{a}$$

$$dI_x = \frac{1}{3}y^3 dx$$

$$I_x = \int dI_y = \frac{1}{3}\int_0^a \left[h_1 + (h_2 - h_1)\frac{x}{a}\right]^3 dx$$

$$= \frac{1}{12}\left|\left[h_1 + (h_2 - h_1)\frac{x}{a}\right]^4 \left(\frac{a}{h_2 - h_1}\right)\right|_0^a$$

$$= \frac{a}{12(h_2 - h_1)}\left(h_2^4 - h_1^4\right) = \frac{a}{12} \cdot \frac{\left(h_2^2 + h_1^2\right)(h_2 + h_1)(h_2 - h_1)}{h_2 - h_1}$$

$$I_x = \frac{a}{12}\left(h_1^2 + h_2^2\right)(h_1 + h_2) \blacktriangleleft$$

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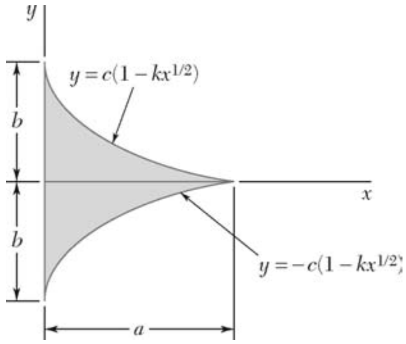
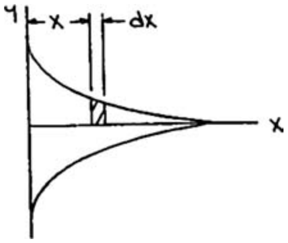


PROBLEM 9.9

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION



At $x = 0, y = b$: $\quad b = c(1-0) \quad$ or $\quad c = b$

At $x = a, y = 0$: $\quad 0 = b(1 - ka^{1/2}) \quad$ or $\quad k = \dfrac{1}{a^{1/2}}$

Then $$y = b\left(1 - \frac{x^{1/2}}{a^{1/2}}\right)$$

Now $$dI_x = \frac{1}{3}y^3 dx = \frac{1}{3}\left[b\left(1 - \frac{x^{1/2}}{a^{1/2}}\right)\right]^3 dx$$

or $$dI_x = \frac{1}{3}b^3\left(1 - 3\frac{x^{1/2}}{a^{1/2}} + 3\frac{x}{a} - \frac{x^{3/2}}{a^{3/2}}\right)dx$$

Then $$I_x = \int dI_x = 2\int_0^a \frac{1}{3}b^3\left(1 - 3\frac{x^{1/2}}{a^{1/2}} + 3\frac{x}{a} - \frac{x^{3/2}}{a^{3/2}}\right)dx$$

$$= \frac{2}{3}b^3\left[x - 2\frac{x^{3/2}}{a^{1/2}} + \frac{3}{2}\frac{x^2}{a} - \frac{2}{5}\frac{x^{5/2}}{a^{3/2}}\right]_0^a$$

or $\quad I_x = \dfrac{1}{15}ab^3$ ◀

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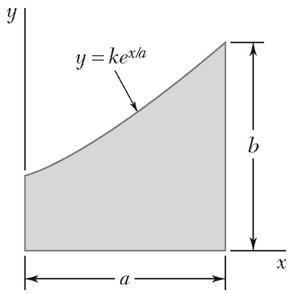
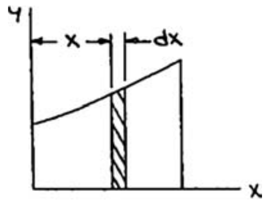


PROBLEM 9.10

Determine by direct integration the moment of inertia of the shaded area with respect to the *x* axis.

SOLUTION



At $x = a,\ y = b$: $\quad b = ke^{a/a} \quad$ or $\quad k = \dfrac{b}{e}$

Then $\quad y = \dfrac{b}{e}e^{x/a} = be^{x/a-1}$

Now
$$dI_x = \frac{1}{3}y^3dx = \frac{1}{3}(be^{x/a-1})^3\,dx$$
$$= \frac{1}{3}b^3e^{3(x/a-1)}dx$$

Then
$$I_x = \int dI_x = \int_0^a \frac{1}{3}b^3e^{3(x/a-1)}dx = \frac{b^3}{3}\left[\frac{a}{3}e^{3(x/a-1)}\right]_0^a$$
$$= \frac{1}{9}ab^3(1-e^{-3}) \qquad \text{or} \quad I_x = 0.1056ab^3 \blacktriangleleft$$

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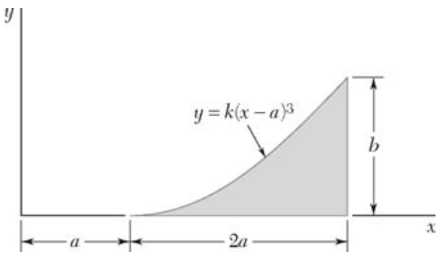


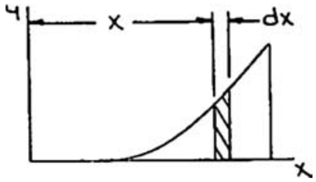
PROBLEM 9.11

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION

At $x = 3a, \quad y = b$: $\quad b = k(3a-a)^3$

or $\quad k = \dfrac{b}{8a^3}$



Then $\quad y = \dfrac{b}{8a^3}(x-a)^3$

Now
$$dI_x = \frac{1}{3}y^3 dx$$
$$= \frac{1}{3}\left[\frac{b}{8a^3}(x-a)^3\right]^3 dx$$
$$= \frac{b^3}{1536a^9}(x-a)^9 dx$$

Then
$$I_x = \int dI_x = \int_a^{3a} \frac{b^3}{1536a^9}(x-a)^9 dx = \frac{b^3}{1536a^9}\left[\frac{1}{10}(x-a)^{10}\right]_a^{3a}$$
$$= \frac{b^3}{15{,}360a^9}[(3a-a)^{10} - 0]$$

or $\quad I_x = \dfrac{1}{15}ab^3$ ◀

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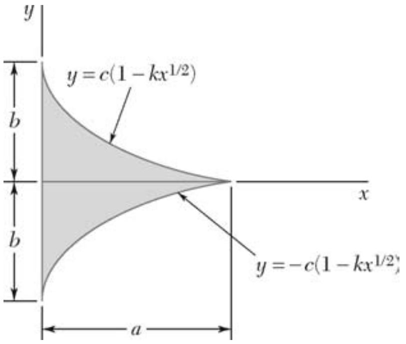


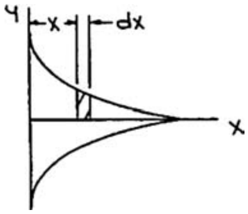
PROBLEM 9.12

Determine by direct integration the moment of inertia of the shaded area with respect to the *y* axis.

SOLUTION

At $x = 0, \quad y = b$: $\quad b = c(1 - 0) \quad$ or $\quad c = b$

$x = a, \quad y = 0$: $\quad 0 = c(1 - ka^{1/2}) \quad$ or $\quad k = \dfrac{1}{a^{1/2}}$



Then

$$y = b\left(1 - \frac{x^{1/2}}{a^{1/2}}\right) \blacktriangleleft$$

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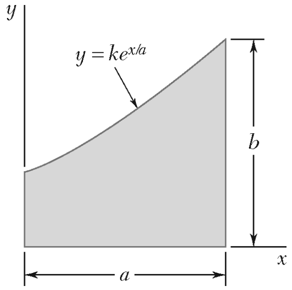
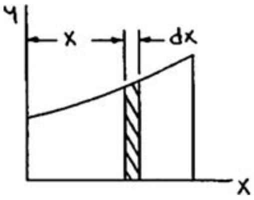


PROBLEM 9.13

Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION



At $x = a, \quad y = b$: $\qquad b = ke^{a/a}$

or $\qquad k = \dfrac{b}{e}$

Then $\qquad y = \dfrac{b}{e}e^{x/a} = be^{x/a-1}$

Now
$$dI_y = x^2 dA = x^2(y\,dx)$$
$$= x^2(be^{x/a-1}dx)$$

Then $\qquad I_y = \int dI_y = \int_0^a bx^2 e^{x/a-1}dx$

Now use integration by parts with

$$u = x^2 \qquad dv = e^{x/a-1}dx$$
$$du = 2x\,dx \qquad v = ae^{x/a-1}$$

Then
$$\int_0^a x^2 e^{x/a-1}dx = \left[x^2 ae^{x/a-1}\right]_0^a - \int_0^a (ae^{x/a-1})2x\,dx$$
$$= a^3 - 2a\int_0^a xe^{x/a-1}dx$$

Using integration by parts with

$$u = x \qquad dv = e^{x/a-1}dx$$
$$du = dx \qquad v = ae^{x/a-1}$$

Then
$$I_y = b\left\{a^3 - 2a\left[(xae^{x/a-1})\big|_0^a - \int_0^a (ae^{x/a-1})dx\right]\right\}$$
$$= b\left\{a^3 - 2a\left[a^2 - (a^2e^{x/a-1})\big|_0^a\right]\right\}$$
$$= b\left\{a^3 - 2a\left[a^2 - (a^2 - a^2e^{-1})\right]\right\}$$

or $\quad I_y = 0.264a^3b$ ◀

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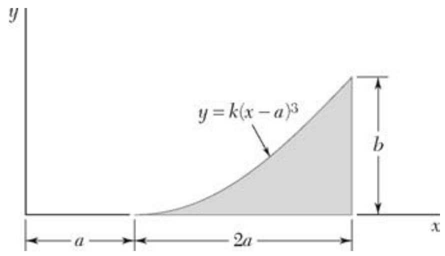
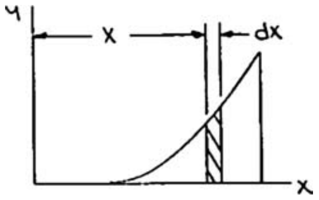


PROBLEM 9.14

Determine by direct integration the moment of inertia of the shaded area with respect to the *y* axis.

SOLUTION



At $x = 3a,\quad y = b$: $\qquad b = k(3a-a)^3$

or $\qquad k = \dfrac{b}{8a^3}$

Then $\qquad y = \dfrac{b}{8a^3}(x-a)^3$

Now $\qquad dI_y = x^2 dA = x^2(y\,dx) = x^2\left[\dfrac{b}{8a^3}(x-a)^3 dx\right]$

$$= \frac{b}{8a^3}x^2(x^3 - 3x^2a + 3xa^2 - a^3)dx$$

Then $\qquad I_y = \int dI_y = \int_a^{3a} \dfrac{b}{8a^3}(x^5 - 3x^4a + 3x^3a^2 - a^3x^2)dx$

$$= \frac{b}{8a^3}\left[\frac{1}{6}x^6 - \frac{3}{5}ax^5 + \frac{3}{4}a^2x^4 - \frac{1}{3}a^3x^3\right]_a^{3a}$$

$$= \frac{b}{8a^3}\left\{\left[\frac{1}{6}(3a)^6 - \frac{3}{5}a(3a)^5 + \frac{3}{4}a^2(3a)^4 - \frac{1}{3}a^3(3a)^3\right]\right.$$

$$\left. - \left[\frac{1}{6}(a)^6 - \frac{3}{5}a(a)^5 + \frac{3}{4}a^2(a)^4 - \frac{1}{3}a^3(a)^3\right]\right\}$$

or $\quad I_y = 3.43a^3b$ ◀

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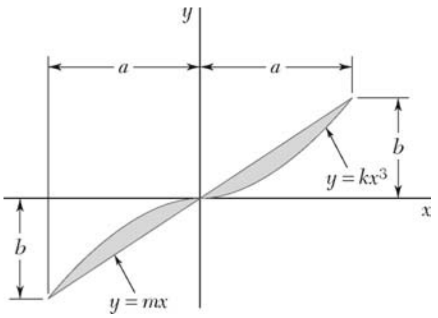
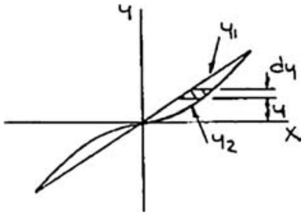


PROBLEM 9.15

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the *x* axis.

SOLUTION



At $x = a, \quad y_1 = y_2 = b$: $\quad y_1: \quad b = ma \quad \text{or} \quad m = \dfrac{b}{a}$

$$y_2: \quad b = ka^3 \quad \text{or} \quad k = \frac{b}{a^3}$$

Then $$y_1 = \frac{b}{a}x$$

or $$x_1 = \frac{a}{b}y$$

and $$y_2 = \frac{b}{a^3}x^3$$

or $$x_2 = \frac{a}{b^{1/3}}y^{1/3}$$

Now $$dA = (x_2 - x_1)dy$$
$$= \left(\frac{a}{b^{1/3}}y^{1/3} - \frac{a}{b}y\right)dy$$

Then $$A = \int dA = 2\int_0^b a\left(\frac{y^{1/3}}{b^{1/3}} - \frac{1}{b}y\right)dy$$
$$= 2a\left[\frac{3}{4}\frac{1}{b^{1/3}}y^{4/3} - \frac{1}{2b}y^2\right]_0^b = \frac{1}{2}ab$$

Now $$dI_x = y^2 dA = y^2\left[\left(\frac{a}{b^{1/3}}y^{1/3} - \frac{a}{b}y\right)dy\right]$$

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PROBLEM 9.15 (Continued)

Then $$I_x = 2\int_0^b a\left(\frac{1}{b^{1/3}}y^{7/3} - \frac{1}{b}y^3\right)dy$$

$$= 2a\left[\frac{3}{10}\frac{1}{b^{1/3}}y^{10/3} - \frac{1}{4b}y^4\right]_0^b$$ or $I_x = \frac{1}{10}ab^3$ ◀

and $$k_x^2 = \frac{I_x}{A} = \frac{\frac{1}{10}ab^3}{\frac{1}{2}ab} = \frac{1}{5}b^2$$ or $k_x = \frac{b}{\sqrt{5}}$ ◀

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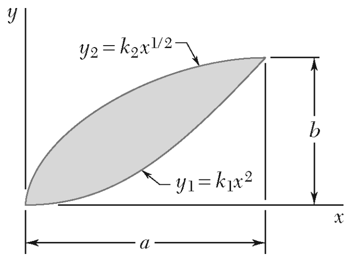
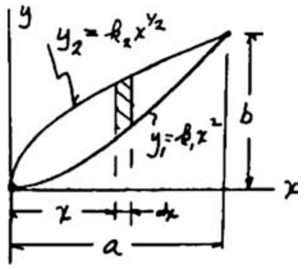


PROBLEM 9.16

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the *x* axis.

SOLUTION



$$y_1 = k_1 x^2 \qquad y_2 = k_2 x^{1/2}$$

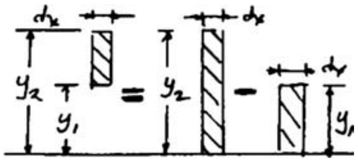
For $\quad x = 0 \quad \text{and} \quad y_1 = y_2 = b$

$$b = k_1 a^2 \qquad b = k_2 a^{1/2}$$

$$k_1 = \frac{b}{a^2} \qquad k_2 = \frac{b}{a^{1/2}}$$

Thus,

$$y_1 = \frac{b}{a^2} x^2 \qquad y_2 = \frac{b}{a^{1/2}} x^{1/2}$$



$$dA = (y_2 - y_1)dx$$

$$A = \int_0^a \left[\frac{b}{a^{1/2}} x^{1/2} - \frac{b}{a^2} x^2 \right] dx$$

$$A = \frac{2}{3} \frac{ba^{3/2}}{a^{1/2}} - \frac{ba^3}{3a^2}$$

$$A = \frac{1}{3} ab$$

$$dI_x = \frac{1}{3} y_2^3 dx - \frac{1}{3} y_1^3 dx$$

$$= \frac{1}{3} \frac{b^3}{a^{3/2}} x^{3/2} dx - \frac{1}{3} \frac{b^3}{a^6} x^6 dx$$

$$I_x = \int dI_x = \frac{b^3}{3a^{3/2}} \int_0^a x^{3/2} dx - \frac{b^3}{3a^6} \int_0^a x^6 dx$$

$$= \frac{b^3}{3a^{3/2}} \frac{a^{5/2}}{\left(\frac{5}{2}\right)} - \frac{b^3}{3a^6} \frac{a^7}{7} = \left(\frac{2}{15} - \frac{1}{21} \right) ab^3 \qquad I_x = \frac{3}{35} ab^3 \blacktriangleleft$$

$$k_x^2 = \frac{I_x}{A} = \frac{\left(\frac{3}{35} ab^3\right)}{\frac{ab}{b}} \qquad k_x = b\sqrt{\frac{9}{35}} \blacktriangleleft$$

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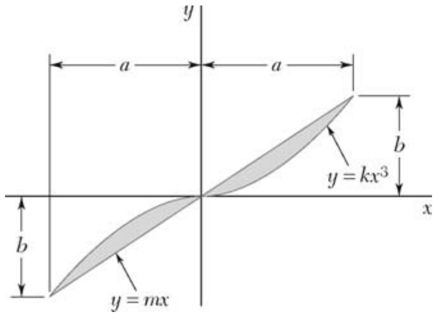
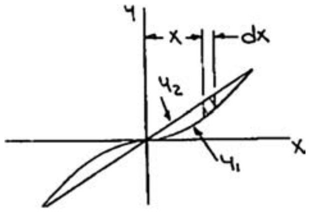


PROBLEM 9.17

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the *y* axis.

SOLUTION



At $x = a, \quad y_1 = y_2 = b$:

$$y_1: \quad b = ka^3 \quad \text{or} \quad k = \frac{b}{a^3}$$

$$y_2: \quad b = ma \quad \text{or} \quad m = \frac{b}{a}$$

Then

$$y_1 = \frac{b}{a^3}x^3$$

$$y_2 = \frac{b}{a}x$$

Now

$$dA = (y_2 - y_1)dx = \left(\frac{b}{a}x - \frac{b}{a^3}x^3\right)dx$$

Then

$$A = \int dA = 2\int_0^a \frac{b}{a}\left(x - \frac{1}{a^2}x^3\right)dx$$

$$= 2\frac{b}{a}\left[\frac{1}{2}x^2 - \frac{1}{4a^2}x^4\right]_0^a = \frac{1}{2}ab$$

Now

$$dI_y = x^2 dA = x^2\left[\left(\frac{b}{a}x - \frac{b}{a^3}x^3\right)dx\right]$$

Then

$$I_y = \int dI_y = 2\int_0^a \frac{b}{a}x^2\left(x - \frac{1}{a^2}x^3\right)dx$$

$$= 2\frac{b}{a}\left[\frac{1}{4}x^4 - \frac{1}{6}\frac{1}{a^2}x^6\right]_0^a \qquad \text{or} \quad I_y = \frac{1}{6}a^3b \blacktriangleleft$$

and

$$k_y^2 = \frac{I_y}{A} = \frac{\frac{1}{6}a^3b}{\frac{1}{2}ab} = \frac{1}{3}a^2 \qquad \text{or} \quad k_y = \frac{a}{\sqrt{3}} \blacktriangleleft$$

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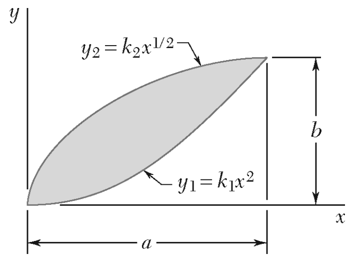

PROBLEM 9.18

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the *y* axis.

SOLUTION

See figure of solution on Problem 9.16.

$$A = \frac{1}{3}ab \qquad dI_y = x^2 dA = x^2(y_2 - y_1)dx$$

$$I_y = \int_0^a x^2\left(\frac{b}{a^{1/2}}x^{1/2} - \frac{b}{a^2}x^2\right)dx = \frac{b}{a^{1/2}}\int_0^a x^{5/2}dx - \frac{b}{a^2}\int_0^a x^4 dx$$

$$I_y = \frac{b}{a^{1/2}} \cdot \frac{b^{7/2}}{\left(\frac{7}{2}\right)} - \frac{b}{a^2} \cdot \frac{a^5}{5} = \left(\frac{2}{7} - \frac{1}{5}\right)a^3 b$$

$$I_y = \frac{3}{35}a^3 b \blacktriangleleft$$

$$k_y^2 = \frac{I_y}{A} = \frac{\left(\frac{3}{35}a^3 b\right)}{\frac{ab}{3}}$$

$$k_y = a\sqrt{\frac{9}{35}} \blacktriangleleft$$

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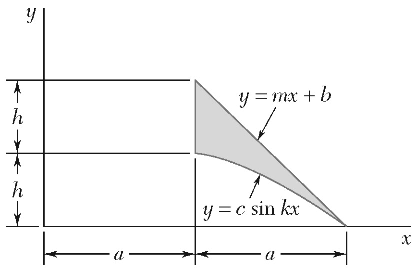
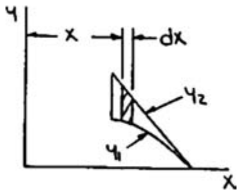


PROBLEM 9.19

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the *x* axis.

SOLUTION



y_1: At $x = 2a, \quad y = 0$: $\qquad 0 = c \sin k(2a)$

$$2ak = \pi \quad \text{or} \quad k = \frac{\pi}{2a}$$

At $x = a, \quad y = h$: $\qquad h = c \sin \frac{\pi}{2a}(a) \quad \text{or} \quad c = h$

y_2: At $x = a, \quad y = 2h$: $\qquad 2h = ma - b$

At $x = 2a, \quad y = 0$: $\qquad 0 = m(2a) + b$

Solving yields $\qquad m = -\frac{2h}{a}, \quad b = 4h$

Then $\qquad y_1 = h \sin \frac{\pi}{2a} x \qquad y_2 = -\frac{2h}{a} x + 4h$

$$= \frac{2h}{a}(-x + 2a)$$

Now $\qquad dA = (y_2 - y_1)dx = \left[\frac{2h}{a}(-x + 2a) - h \sin \frac{\pi}{2a} x\right] dx$

Then $\qquad A = \int dA = \int_a^{2a} h \left[\frac{2}{a}(-x + 2a) - \sin \frac{\pi}{2a} x\right] dx$

$$= h\left[-\frac{1}{a}(-x + 2a)^2 + \frac{2a}{\pi} \cos \frac{\pi}{2a} x\right]_a^{2a}$$

$$= h\left[\left(-\frac{2a}{\pi}\right) + \frac{1}{a}(-a + 2a)^2\right] = ah\left(1 - \frac{2}{\pi}\right)$$

$$= 0.36338ah$$

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PROBLEM 9.19 (Continued)

Find: I_x and k_x

We have
$$dI_x = \left(\frac{1}{3}y_2 - \frac{1}{3}y_1\right)dx = \frac{1}{3}\left\{\left[\frac{2h}{a}(-x+2a)\right]^3 - \left(h\sin\frac{\pi}{2a}x\right)^3\right\}dx$$
$$= \frac{h^3}{3}\left[\frac{8}{a^3}(-x+2a)^3 - \sin^3\frac{\pi}{2a}x\right]$$

Then
$$I_x = \int dI_x = \int_a^{2a}\frac{h^3}{3}\left[\frac{8}{a^3}(-x+2a)^3 - \sin^3\frac{\pi}{2a}x\right]dx$$

Now
$$\sin^3\theta = \sin\theta(1-\cos^2\theta) = \sin\theta - \sin\theta\cos^2\theta$$

Then
$$I_x = \frac{h^3}{3}\int_a^{2a}\left[\frac{8}{a^3}(-x+2a)^3 - \left(\sin\frac{\pi}{2a}x - \sin\frac{\pi}{2a}x\cos^2\frac{\pi}{2a}x\right)\right]dx$$
$$= \frac{h^3}{3}\left[-\frac{2}{a^3}(-x+2a)^4 + \frac{2a}{\pi}\cos\frac{\pi}{2a}x - \frac{2a}{3\pi}\cos^3\frac{\pi}{2a}x\right]_a^{2a}$$
$$= \frac{h^3}{3}\left[\left(-\frac{2a}{\pi} + \frac{2a}{3\pi}\right) + \frac{2}{a^3}(-a+2a)^4\right]$$
$$= \frac{2}{3}ah^3\left(1 - \frac{2}{3\pi}\right)$$

$$I_x = 0.52520ah^3$$ or $I_x = 0.525ah^3$ ◀

and
$$k_x^2 = \frac{I_x}{A} = \frac{0.52520ah^3}{0.36338ah}$$ or $k_x = 1.202h$ ◀

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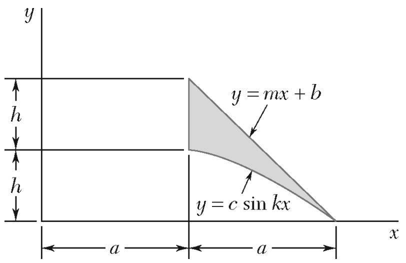
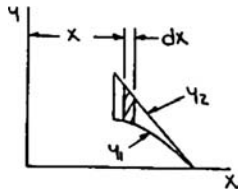


PROBLEM 9.20

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the y axis.

SOLUTION



y_1: At $x = 2a$, $y = 0$: $\quad 0 = c\sin k(2a)$

$$2ak = \pi \quad \text{or} \quad k = \frac{\pi}{2a}$$

At $x = a$, $y = h$: $\quad h = c\sin\frac{\pi}{2a}(a) \quad \text{or} \quad c = h$

y_2: At $x = a$, $y = 2h$: $\quad 2h = ma - b$

At $x = 2a$, $y = 0$: $\quad 0 = m(2a) + b$

Solving yields $\quad m = -\frac{2h}{a}, \quad b = 4h$

Then $\quad y_1 = h\sin\frac{\pi}{2a}x \qquad y_2 = -\frac{2h}{a}x + 4h$

$$= \frac{2h}{a}(-x + 2a)$$

Now $\quad dA = (y_2 - y_1)dx = \left[\frac{2h}{a}(-x + 2a) - h\sin\frac{\pi}{2a}x\right]dx$

Then $\quad A = \int dA = \int_a^{2a} h\left[\frac{2}{a}(-x + 2a) - \sin\frac{\pi}{2a}x\right]dx$

$$= h\left[-\frac{1}{a}(-x + 2a)^2 + \frac{2a}{\pi}\cos\frac{\pi}{2a}x\right]_a^{2a}$$

$$= h\left[\left(-\frac{2a}{\pi}\right) + \frac{1}{a}(-a + 2a)^2\right] = ah\left(1 - \frac{2}{\pi}\right)$$

$$= 0.36338ah$$

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PROBLEM 9.20 (Continued)

Find: I_y and k_y

We have
$$dI_y = x^2 dA = x^2\left\{\left[\frac{2h}{a}(-x+2a) - h\sin\frac{\pi}{2a}x\right]dx\right\}$$
$$= h\left[\frac{2}{a}(-x^3+2ax^2) - x^2\sin\frac{\pi}{2a}x\right]dx$$

Then
$$I_y = \int dI_y = \int_a^{2a} h\left[\frac{2}{a}(-x^3+2ax^2) - x^2\sin\frac{\pi}{2a}x\right]dx$$

Now using integration by parts with

$$u = x^2 \qquad dv = \sin\frac{\pi}{2a}x\,dx$$

$$du = 2x\,dx \qquad v = -\frac{2a}{\pi}\cos\frac{\pi}{2a}x$$

Then
$$\int x^2\sin\frac{\pi}{2a}x\,dx = x^2\left(-\frac{2a}{\pi}\cos\frac{\pi}{2a}x\right) - \int\left(-\frac{2a}{\pi}\cos\frac{\pi}{2a}x\right)(2x\,dx)$$

Now let
$$u = x \qquad dv = \cos\frac{\pi}{2a}x\,dx$$

$$du = dx \qquad v = \frac{2a}{\pi}\sin\frac{\pi}{2a}x$$

Then
$$\int x^2\sin\frac{\pi}{2a}x\,dx = -\frac{2a}{\pi}x^2\cos\frac{\pi}{2a}x + \frac{4a}{\pi}\left[x\left(\frac{2a}{\pi}\sin\frac{\pi}{2a}x\right) - \int\left(\frac{2a}{\pi}\sin\frac{\pi}{2a}x\right)dx\right]$$

$$I_y = h\left[\frac{2}{a}\left(-\frac{1}{4}x^4 + \frac{2}{3}ax^3\right)\right.$$
$$\left.-\left(-\frac{2a}{\pi}x^2\cos\frac{\pi}{2a}x + \frac{8a^2}{\pi^2}x\sin\frac{\pi}{2a}x + \frac{16a^3}{\pi^3}\cos\frac{\pi}{2a}x\right)\right]_a^{2a}$$

$$= h\left\{\frac{2}{a}\left[-\frac{1}{4}(2a)^4 + \frac{2}{3}a(2a)^3\right] - \frac{2a}{\pi}(2a)^2 + \frac{16a^3}{\pi^3}\right\}$$
$$-h\left\{\frac{2}{a}\left[-\frac{1}{4}(a)^4 + \frac{2}{3}a(a)^3\right] - \frac{8a^2}{\pi^2}(a)\right\}$$
$$= 0.61345a^3h$$

or $I_y = 0.613a^3h$ ◀

and
$$k_y^2 = \frac{I_y}{A} = \frac{0.61345a^3h}{0.36338ah}$$

or $k_y = 1.299a$ ◀

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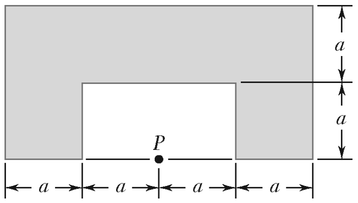
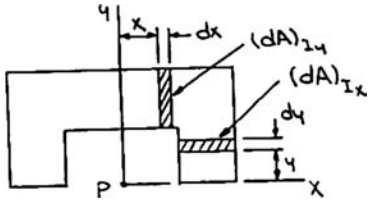


PROBLEM 9.21

Determine the polar moment of inertia and the polar radius of gyration of the shaded area shown with respect to Point *P*.

SOLUTION



We have $$dI_x = y^2 dA = y^2[(x_2 - x_1)dy]$$

Then $$I_x = 2\left[\int_2^a y^2(2a-a)dy + \int_a^{2a} y^2(2a-0)dy\right]$$

$$= 2\left\{a\left[\frac{1}{3}y^3\right]_0^a + 2a\left[\frac{1}{3}y^3\right]_a^{2a}\right\}$$

$$= 2\left\{a\left[\frac{1}{3}(a)^3\right] + \frac{2}{3}a\left[(2a)^3 - (a)^3\right]\right\}$$

$$= 10a^4$$

Also $$dI_y = x^2 dA = x^2[(y_2 - y_1)dx]$$

Then $$I_y = 2\left[\int_0^a x^2(2a-a)dx + \int_a^{2a} x^2(2a-0)dx\right]$$

$$= 10a^4$$

Now $$J_P = I_x + I_y = 10a^4 + 10a^4$$ or $J_P = 20a^4$ ◀

and $$k_P^2 = \frac{J_P}{A} = \frac{20a^4}{(4a)(2a) - (2a)(a)}$$

$$= \frac{10}{3}a^2$$ or $k_P = 1.826a$ ◀

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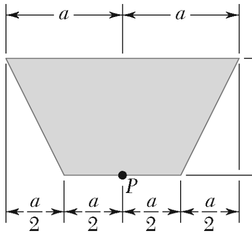
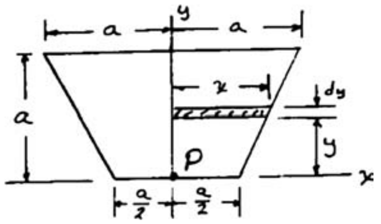


PROBLEM 9.22

Determine the polar moment of inertia and the polar radius of gyration of the shaded area shown with respect to Point *P*.

SOLUTION



$$x = \frac{a}{2} + \frac{a}{2}\frac{y}{a} = \frac{1}{2}(a + y)$$

$$dA = x\,dy = \frac{1}{2}(a + y)dy$$

$$\frac{1}{2}A = \int_0^a dA = \int_0^a \frac{1}{2}(a + y)dy = \frac{1}{2}\left| ay + \frac{y^2}{2} \right|_0^a = \frac{1}{2}\left(a^2 + \frac{a^2}{2}\right) = \frac{3}{4}a^2 \qquad A = \frac{3}{2}a^2 \ \triangleleft$$

$$\frac{1}{2}I_x = \int y^2 dA = \int_0^a y^2 \frac{1}{2}(a + y)dy = \frac{1}{2}\int_0^a (ay^2 + y^3)dy$$

$$= \frac{1}{2}\left| a\frac{y^3}{3} + \frac{y^4}{4} \right|_0^a = \frac{1}{2}\left(\frac{1}{3} + \frac{1}{4}\right)a^4 = \frac{1}{2}\frac{7}{12}a^4 \qquad I_x = \frac{7}{12}a^4 \ \triangleleft$$

$$\frac{1}{2}I_y = \int_0^a \frac{1}{3}x^3 dy = \frac{1}{3}\int_0^a \left(\frac{1}{2}(a + y)\right)^3 dy$$

$$\frac{1}{2}I_y = \frac{1}{24}\int_0^a (a + y)^3 dy = \frac{1}{24}\left| \frac{1}{4}(a + y)^4 \right|_0^a = \frac{1}{96}\left[(2a)^4 - a^4\right] = \frac{15}{96}a^4$$

$$\frac{1}{2}I_y = \frac{5}{32}a^4 \qquad I_y = \frac{5}{16}a^4 \ \triangleleft$$

From Eq. (9.4):
$$J_O = I_x + I_y = \frac{7}{12}a^4 + \frac{5}{16}a^4 = \left(\frac{28 + 15}{48}\right)a^4 \qquad J_O = \frac{43}{48}a^4 \ \blacktriangleleft$$

$$J_O = k_O^2 A \qquad k_O^2 = \frac{J_O}{A} = \frac{\frac{43}{48}a^4}{\frac{3}{2}a^2} = \frac{43}{72}a^2 \qquad k_O = a\sqrt{\frac{43}{72}} \qquad k_O = 0.773a \ \blacktriangleleft$$

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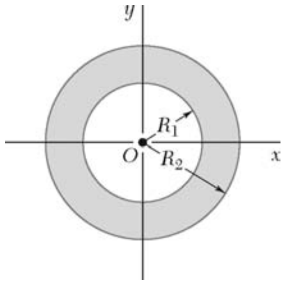
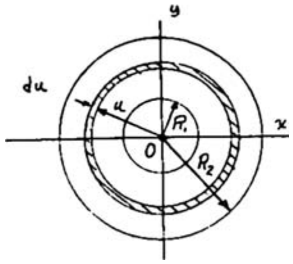


PROBLEM 9.23

(*a*) Determine by direct integration the polar moment of inertia of the annular area shown with respect to Point *O*. (*b*) Using the result of Part *a*, determine the moment of inertia of the given area with respect to the *x* axis.

SOLUTION



(*a*)

$$dA = 2\pi u\,du$$

$$dJ_O = u^2 dA = u^2(2\pi u\,du)$$
$$= 2\pi u^3\,du$$

$$J_O = \int dJ_O = 2\pi\int_{R_1}^{R_2} u^3 du$$
$$= 2\pi\left|\frac{1}{4}u^4\right|_{R_1}^{R_2}$$

$$J_O = \frac{\pi}{2}\left[R_2^4 - R_1^4\right] \blacktriangleleft$$

(*b*) From Eq. (9.4): (*Note* by symmetry.) $I_x = I_y$

$$J_O = I_x + I_y = 2I_x$$

$$I_x = \frac{1}{2}J_O = \frac{\pi}{4}\left[R_2^4 - R_1^4\right]$$

$$I_x = \frac{\pi}{4}\left[R_2^4 - R_1^4\right] \blacktriangleleft$$

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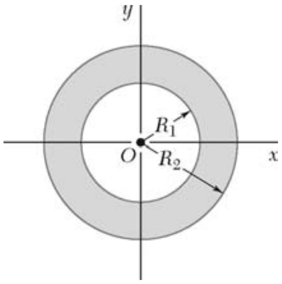
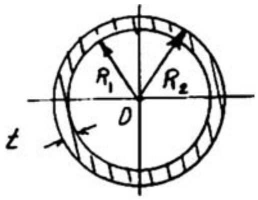


PROBLEM 9.24

(a) Show that the polar radius of gyration k_O of the annular area shown is approximately equal to the mean radius $R_m = (R_1 + R_2)/2$ for small values of the thickness $t = R_2 - R_1$. (b) Determine the percentage error introduced by using R_m in place of k_O for the following values of t/R_m: 1, $\frac{1}{2}$, and $\frac{1}{10}$.

SOLUTION

(a) From Problem 9.23:



$$J_O = \frac{\pi}{2}\left[R_2^4 - R_1^4\right]$$

$$k_O^2 = \frac{J_O}{A} = \frac{\frac{\pi}{2}\left[R_2^4 - R_1^4\right]}{\pi\left(R_2^2 - R_1^2\right)}$$

$$k_O^2 = \frac{1}{2}\left[R_2^2 + R_1^2\right]$$

Thickness: $t = R_2 - R_1$

Mean radius: $R_m = \frac{1}{2}(R_1 + R_2)$

Thus, $R_1 = R_m - \frac{1}{2}t$ and $R_2 = R_m + \frac{1}{2}t$

$$k_O^2 = \frac{1}{2}\left[\left(R_m + \frac{1}{2}t\right)^2 + \left(R_m - \frac{1}{2}t\right)^2\right] = R_m^2 + \frac{1}{4}t^2$$

For t small compared to R_m: $k_O^2 \approx R_m^2$ $\qquad k_O \approx R_m$ ◀

(b) $$\text{Percentage error} = \frac{(\text{exact value}) - (\text{approximation value})}{(\text{exact value})}100$$

$$\text{P.E.} = -\frac{\sqrt{R_m^2 + \frac{1}{4}t^2} - R_m}{\sqrt{R_m^2 + \frac{1}{4}t^2}}(100) = -\frac{\sqrt{1 + \frac{1}{4}\left(\frac{t}{R_m}\right)^2} - 1}{\sqrt{1 + \frac{1}{4}\left(\frac{t}{R_m}\right)^2}}100$$

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PROBLEM 9.24 (Continued)

For $\dfrac{t}{R_m} = 1$: $$\text{P.E.} = \frac{\sqrt{1+\frac{1}{4}} - 1}{\sqrt{1+\frac{1}{4}}}(100) = -10.56\% \quad \blacktriangleleft$$

For $\dfrac{t}{R_m} = \dfrac{1}{2}$: $$\text{P.E.} = -\frac{\sqrt{1+\frac{1}{4}\left(\frac{1}{2}\right)^2} - 1}{\sqrt{1+\frac{1}{4}\left(\frac{1}{2}\right)^2}}(100) = -2.99\% \quad \blacktriangleleft$$

For $\dfrac{t}{R_m} = \dfrac{1}{10}$: $$\text{P.E.} = \frac{\sqrt{1+\frac{1}{4}\left(\frac{1}{10}\right)^2} - 1}{\sqrt{1+\frac{1}{4}\left(\frac{1}{10}\right)^2}}(100) = -0.125\% \quad \blacktriangleleft$$

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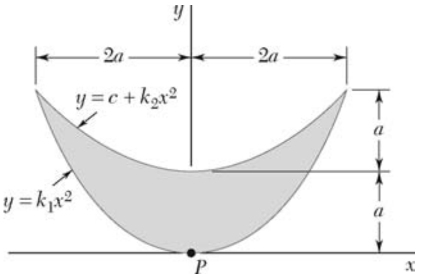
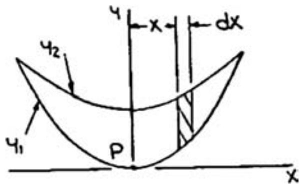


PROBLEM 9.25

Determine the polar moment of inertia and the polar radius of gyration of the shaded area shown with respect to Point *P*.

SOLUTION



y_1: At $x = 2a$, $y = 2a$:

$$2a = k_1(2a)^2 \quad \text{or} \quad k_1 = \frac{1}{2a}$$

y_2: At $x = 0$, $y = a$: $\quad a = c$

At $x = 2a$, $y = 2a$:

$$2a = a + k_2(2a)^2 \quad \text{or} \quad k_2 = \frac{1}{4a}$$

Then
$$y_1 = \frac{1}{2a}x^2 \qquad y_2 = a + \frac{1}{4a}x^2$$
$$= \frac{1}{4a}(4a^2 + x^2)$$

Now
$$dA = (y_2 - y_1)dx = \left[\frac{1}{4a}(4a^2 + x^2) - \frac{1}{2a}x^2\right]dx$$
$$= \frac{1}{4a}(4a^2 - x^2)dx$$

Then
$$A = \int dA = 2\int_0^{2a} \frac{1}{4a}(4a^2 - x^2)dx = \frac{1}{2a}\left[4a^2x - \frac{1}{3}x^3\right]_0^{2a} = \frac{8}{3}a^2$$

Now
$$dI_x = \left(\frac{1}{3}y_2^3 - \frac{1}{3}y_1^3\right)dx = \frac{1}{3}\left\{\left[\frac{1}{4a}(4a^2 + x^2)\right]^3 - \left[\frac{1}{2a}x^2\right]^3\right\}dx$$
$$= \frac{1}{3}\left[\frac{1}{64a^3}(64a^6 + 48a^4x^2 + 12a^2x^4 + x^6) - \frac{1}{8a^3}x^6\right]dx$$
$$= \frac{1}{192a^3}(64a^6 + 48a^4x^2 + 12a^2x^4 - 7x^6)dx$$

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PROBLEM 9.25 (Continued)

Then $$I_x = \int dI_x = 2\int_0^{2a} \frac{1}{192a^3}(64a^6 + 48a^4x^2 + 12a^2x^4 - 7x^6)dx$$

$$= \frac{1}{96a^3}\left[64a^6x + 16a^4x^3 + \frac{12}{5}a^2x^5 - x^7\right]_0^{2a}$$

$$= \frac{1}{96a^3}\left[64a^6(2a) + 16a^4(2a)^3 + \frac{12}{5}a^2(2a)^5 - (2a)^7\right]$$

$$= \frac{1}{96}a^4\left(128 + 128 + \frac{12}{5}\times 32 - 128\right) = \frac{32}{15}a^4$$

Also $$dI_y = x^2 dA = x^2\left[\frac{1}{4a}(4a^2 - x^2)dx\right]$$

Then $$I_y = \int dI_y = 2\int_0^{2a} \frac{1}{4a}x^2(4a^2 - x^2)dx = \frac{1}{2a}\left[\frac{4}{3}a^2x^3 - \frac{1}{5}x^5\right]_0^{2a}$$

$$= \frac{1}{2a}\left[\frac{4}{3}a^2(2a)^3 - \frac{1}{5}(2a)^5\right] = \frac{32}{2}a^4\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{32}{15}a^4$$

Now $$J_P = I_x + I_y = \frac{32}{15}a^4 + \frac{32}{15}a^4$$ or $J_P = \dfrac{64}{15}a^4$ ◀

and $$k_P^2 = \frac{J_P}{A} = \frac{\frac{64}{15}a^4}{\frac{8}{3}a^2} = \frac{8}{5}a^2$$ or $k_P = 1.265a$ ◀

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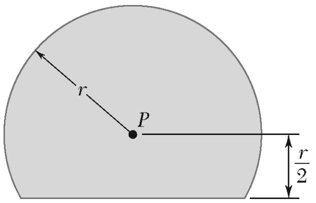
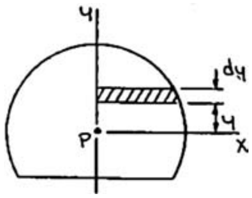


PROBLEM 9.26

Determine the polar moment of inertia and the polar radius of gyration of the shaded area shown with respect to Point *P*.

SOLUTION



The equation of the circle is

$$x^2 + y^2 = r^2$$

So that $\quad x = \sqrt{r^2 - y^2}$

Now $\quad dA = x\,dy = \sqrt{r^2 - y^2}\,dy$

Then $\quad A = \int dA = 2\int_{-r/2}^{r} \sqrt{r^2 - y^2}\,dy$

Let $\quad y = r\sin\theta; \quad dy = r\cos\theta\, d\theta$

Then
$$A = 2\int_{-\pi/6}^{\pi/2} \sqrt{r^2 - (r\sin\theta)^2}\, r\cos\, d\theta$$
$$= 2\int_{-\pi/6}^{\pi/2} r^2\cos^2\theta\, d\theta = 2r^2\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_{-\pi/6}^{\pi/2}$$
$$= 2r^2\left[\frac{\frac{\pi}{2}}{2} - \left(\frac{-\frac{\pi}{6}}{2} + \frac{\sin -\frac{\pi}{3}}{4}\right)\right] = 2r^2\left(\frac{\pi}{3} + \frac{\sqrt{3}}{8}\right)$$
$$= 2.5274r^2$$

Now $\quad dI_x = y^2 dA = y^2\left(\sqrt{r^2 - y^2}\,dy\right)$

Then $\quad I_x = \int dI_x = 2\int_{-r/2}^{r} y^2\sqrt{r^2 - y^2}\,dy$

Let $\quad y = r\sin\theta; \quad dy = r\cos\theta\, d\theta$

Then
$$I_x = 2\int_{-\pi/6}^{\pi/2} (r\sin\theta)^2 \sqrt{r^2 - (r\sin\theta)^2}\, r\cos\theta\, d\theta$$
$$= 2\int_{-\pi/6}^{\pi/2} r^2\sin^2\theta (r\cos\theta) r\cos\theta\, d\theta$$

Now $\quad \sin 2\theta = 2\sin\theta\cos\theta \Rightarrow \sin^2\theta\cos^2\theta = \frac{1}{4}\sin 2\theta$

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PROBLEM 9.26 (Continued)

Then $$I_x = 2\int_{-\pi/6}^{\pi/2} r^4\left(\frac{1}{4}\sin^2 2\theta\right)d\theta = \frac{r^4}{2}\left[\frac{\theta}{2} - \frac{\sin 4\theta}{8}\right]_{-\pi/6}^{\pi/2}$$

$$= \frac{r^4}{2}\left[\frac{\frac{\pi}{2}}{2} - \left(\frac{\frac{\pi}{6}}{2} - \frac{\sin - \frac{2\pi}{3}}{8}\right)\right]$$

$$= \frac{r^4}{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{16}\right)$$

Also $$dI_y = \frac{1}{3}x^3 dy = \frac{1}{3}\left(\sqrt{r^2 - y^2}\right)^3 dy$$

Then $$I_y = \int dI_y = 2\int_{-r/2}^{r} \frac{1}{3}(r^2 - y^2)^{3/2} dy$$

Let $$y = r\sin\theta; \quad dy = r\cos\theta\, d\theta$$

Then $$I_y = \frac{2}{3}\int_{-\pi/6}^{\pi/2} [r^2 - (r\sin\theta)^2]^{3/2} r\cos\theta\, d\theta$$

$$I_y = \frac{2}{3}\int_{-\pi/6}^{\pi/2} (r^3\cos^3\theta) r\cos\theta\, d\theta$$

Now $$\cos^4\theta = \cos^2\theta(1 - \sin^2\theta) = \cos^2\theta - \frac{1}{4}\sin^2 2\theta$$

Then $$I_y = \frac{2}{3}\int_{-\pi/6}^{\pi/2} r^4\left(\cos^2\theta - \frac{1}{4}\sin^2 2\theta\right)d\theta$$

$$= \frac{2}{3}r^4\left[\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right) - \frac{1}{4}\left(\frac{\theta}{2} - \frac{\sin 4\theta}{8}\right)\right]_{-\pi/6}^{\pi/2}$$

$$= \frac{2}{3}r^4\left\{\left[\frac{\frac{\pi}{2}}{2} - \frac{1}{4}\left(\frac{\frac{\pi}{2}}{2}\right)\right] - \left[\frac{-\frac{\pi}{6}}{2} + \frac{\sin - \frac{\pi}{3}}{4} - \frac{1}{4}\left(\frac{-\frac{\pi}{6}}{2} - \frac{\sin - \frac{2\pi}{3}}{8}\right)\right]\right\}$$

$$= \frac{2}{3}r^4\left[\frac{\pi}{4} - \frac{\pi}{16} + \frac{\pi}{12} + \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right) - \frac{\pi}{48} + \frac{1}{32}\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{2}{3}r^4\left(\frac{\pi}{4} + \frac{9\sqrt{3}}{64}\right)$$

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PROBLEM 9.26 (Continued)

Now $$J_P = I_x + I_y = \frac{r^4}{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{16}\right) + \frac{2}{3}r^4\left(\frac{\pi}{4} + \frac{9\sqrt{3}}{64}\right)$$

$$= r^4\left(\frac{\pi}{3} + \frac{\sqrt{3}}{16}\right) = 1.15545r^4$$

or $J_P = 1.155r^4$ ◀

and $$k_P^2 = \frac{J_P}{A} = \frac{1.15545r^4}{2.5274r^2}$$

or $k_P = 0.676r$ ◀

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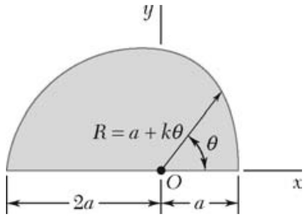
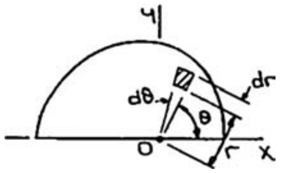


PROBLEM 9.27

Determine the polar moment of inertia and the polar radius of gyration of the shaded area shown with respect to Point *O*.

SOLUTION



At $\theta = \pi, \quad R = 2a$: $\qquad 2a = a + k(\pi)$

or $\qquad k = \dfrac{a}{\pi}$

Then $\qquad R = a + \dfrac{a}{\pi}\theta = a\left(1 + \dfrac{\theta}{\pi}\right)$

Now
$$dA = (dr)(r\,d\theta) = r dr\,d\theta$$

Then
$$A = \int dA = \int_0^{\pi}\int_0^{a(1+\theta/\pi)} r dr\,d\theta = \int_0^{\pi}\left[\frac{1}{2}r^2\right]_0^{a(1+\theta/\pi)} d\theta$$

$$A = \int_0^{\pi}\frac{1}{2}a^2\left(1+\frac{\theta}{\pi}\right)^2 d\theta = \frac{1}{2}a^2\left[\frac{\pi}{3}\left(1+\frac{\theta}{\pi}\right)^3\right]_0^{\pi}$$

$$= \frac{1}{6}\pi a^2\left[\left(1+\frac{\pi}{\pi}\right)^3 - (1)^3\right] = \frac{7}{6}\pi a^2$$

Now $\qquad dJ_O = r^2 dA = r^2(r dr\,d\theta)$

Then
$$J_O = \int dJ_O = \int_0^{\pi}\int_0^{a(1+\theta/\pi)} r^3 dr\,d\theta$$

$$= \int_0^{\pi}\left[\frac{1}{4}r^4\right]_0^{a(1+\theta/\pi)} d\theta = \int_0^{\pi}\frac{1}{4}a^4\left(1+\frac{\theta}{\pi}\right)^4 d\theta$$

$$= \frac{1}{4}a^4\left[\frac{\pi}{5}\left(1+\frac{\theta}{\pi}\right)^5\right]_0^{\pi} = \frac{1}{20}\pi a^4\left[\left(1+\frac{\pi}{\pi}\right)^5 - (1)^5\right]$$

or $\quad J_O = \dfrac{31}{20}\pi a^4$ ◀

and
$$k_O^2 = \frac{J_O}{A} = \frac{\frac{31}{20}\pi a^4}{\frac{7}{6}\pi a^2} = \frac{93}{70}a^2$$

or $\quad k_O = 1.153a$ ◀

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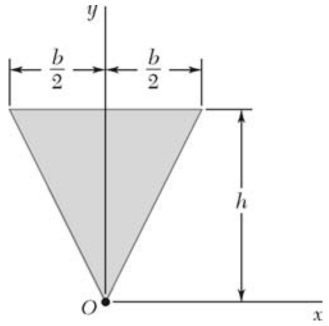
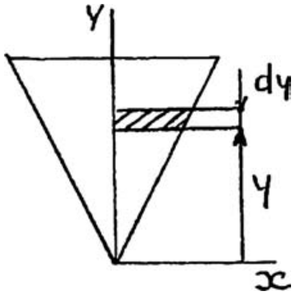


PROBLEM 9.28

Determine the polar moment of inertia and the polar radius of gyration of the isosceles triangle shown with respect to Point O.

SOLUTION



By observation: $y = \dfrac{h}{\frac{b}{2}} x$

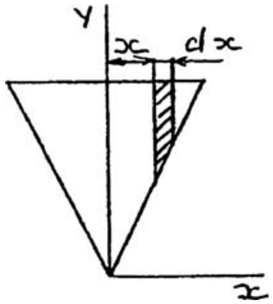
or $x = \dfrac{b}{2h} y$

Now $dA = x dy = \left(\dfrac{b}{2h} y\right) dy$

and $dI_x = y^2 dA = \dfrac{b}{2h} y^3 dy$

Then $I_x = \int dI_x = 2\int_0^h \dfrac{b}{2h} y^3 dy$

$$= \frac{b}{h} \frac{y^4}{4}\bigg|_0^h = \frac{1}{4} bh^3$$



From above: $y = \dfrac{2h}{b} x$

Now $dA = (h - y)dx = \left(h - \dfrac{2h}{b} x\right) dx$

$$= \frac{h}{b}(b - 2x)dx$$

and $dI_y = x^2 dA = x^2 \dfrac{h}{b}(b - 2x)dx$

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PROBLEM 9.28 (Continued)

Then
$$I_y = \int dI_y = 2\int_0^{b/2} \frac{h}{b} x^2 (b - 2x) dx$$

$$= 2\frac{h}{b}\left[\frac{1}{3}bx^3 - \frac{1}{2}x^4\right]_0^{b/2}$$

$$= 2\frac{h}{b}\left[\frac{b}{3}\left(\frac{b}{2}\right)^3 - \frac{1}{2}\left(\frac{b}{2}\right)^4\right] = \frac{1}{48}b^3 h$$

Now
$$J_O = I_x + I_y = \frac{1}{4}bh^3 + \frac{1}{48}b^3 h \qquad \text{or} \qquad J_O = \frac{bh}{48}(12h^2 + b^2) \blacktriangleleft$$

and
$$k_O^2 = \frac{J_O}{A} = \frac{\frac{bh}{48}(12h^2 + b^2)}{\frac{1}{2}bh} = \frac{1}{24}(12h^2 + b^2) \qquad \text{or} \qquad k_O = \frac{\sqrt{12h^2 + b^2}}{24} \blacktriangleleft$$

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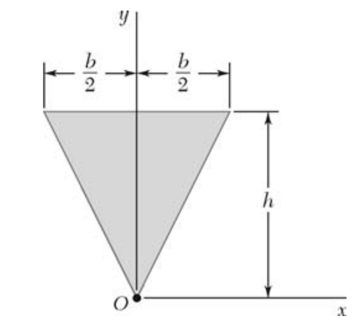


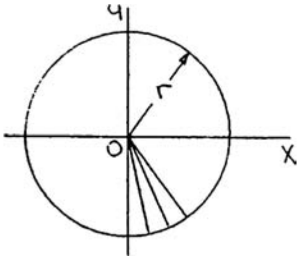
PROBLEM 9.29*

Using the polar moment of inertia of the isosceles triangle of Problem 9.28, show that the centroidal polar moment of inertia of a circular area of radius r is $\pi r^4/2$. (*Hint:* As a circular area is divided into an increasing number of equal circular sectors, what is the approximate shape of each circular sector?)

PROBLEM 9.28 Determine the polar moment of inertia and the polar radius of gyration of the isosceles triangle shown with respect to Point *O*.

SOLUTION

First the circular area is divided into an increasing number of identical circular sectors. The sectors can be approximated by isosceles triangles. For a large number of sectors the approximate dimensions of one of the isosceles triangles are as shown.



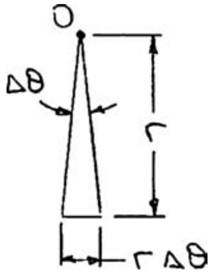
For an isosceles triangle (see Problem 9.28):

$$J_O = \frac{bh}{48}(12h^2 + b^2)$$

Then with $\quad b = r\Delta\theta \quad$ and $\quad h = r$

$$(\Delta J_O)_{\text{sector}} \simeq \frac{1}{48}(r\Delta\theta)(r)\left[12r^2 + (r\Delta\theta)^2\right]$$

$$= \frac{1}{48}r^4\Delta\theta\left[(12 + \Delta\theta^2)\right]$$



Now

$$\frac{dJ_{O\,\text{sector}}}{d\theta} = \lim_{\Delta\theta\to 0}\left(\frac{\Delta J_{O\,\text{sector}}}{\Delta\theta}\right) = \lim_{\Delta\theta\to 0}\left\{\frac{1}{48}r^4\left[12 + (\Delta\theta)^2\right]\right\}$$

$$= \frac{1}{4}r^4$$

Then

$$(J_O)_{\text{circle}} = \int dJ_{O\,\text{sector}} = \int_0^{2\pi}\frac{1}{4}r^4 d\theta = \frac{1}{4}r^4\left[\theta\right]_0^{2\pi}$$

or $\quad (J_O)_{\text{circle}} = \dfrac{\pi}{2}r^4$ ◀

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PROBLEM 9.30*

Prove that the centroidal polar moment of inertia of a given area A cannot be smaller than $A^2/2\pi$. (*Hint:* Compare the moment of inertia of the given area with the moment of inertia of a circle that has the same area and the same centroid.)

SOLUTION

From the solution to sample Problem 9.2, the centroidal polar moment of inertia of a circular area is

$$(J_C)_{\text{cir}} = \frac{\pi}{2} r^4$$

The area of the circle is

$$A_{\text{cir}} = \pi r^2$$

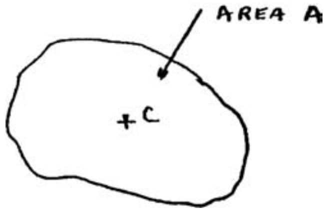
So that

$$[J_C(A)]_{\text{cir}} = \frac{A^2}{2\pi}$$

Two methods of solution will be presented. However, both methods depend upon the observation that as a given element of area dA is moved closer to some Point C, The value of J_C will be decreased $(J_C = \int r^2 dA$; as r decreases, so must J_C).

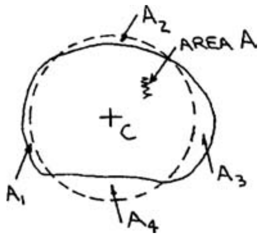
Solution 1

Imagine taking the area A and drawing it into a thin strip of negligible width and of sufficient length so that its area is equal to A. To minimize the value of $(J_C)_A$, the area would have to be distributed as closely as possible about C. This is accomplished by winding the strip into a tightly wound roll with C as its center; any voids in the roll would place the corresponding area farther from C than is necessary, thus increasing the value of $(J_C)_A$. (The process is analogous to rewinding a length of tape back into a roll.) Since the shape of the roll is circular, with the centroid of its area at C, it follows that



$$(J_C)_A \geq \frac{A^2}{2\pi} \quad \text{Q.E.D.} \blacktriangleleft$$

where the equality applies when the original area is circular.



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PROBLEM 9.30* (Continued)

Solution 2

Consider an area A with its centroid at Point C and a circular area of area A with its center (and centroid) at Point C. Without loss of generality, assume that

$$A_1 = A_2 \qquad A_3 = A_4$$

It then follows that

$$(J_C)_A = (J_C)_{\text{cir}} + [J_C(A_1) - J_C(A_2) + J_C(A_3) - J_C(A_4)]$$

Now observe that

$$J_C(A_1) - J_C(A_2) \geq 0$$

$$J_C(A_3) - J_C(A_4) \geq 0$$

since as a given area is moved farther away from C its polar moment of inertia with respect to C must increase.

$$(J_C)_A \geq (J_C)_{\text{cir}} \qquad \text{or} \quad (J_C)_A \geq \frac{A^2}{2\pi} \quad \text{Q.E.D.} \blacktriangleleft$$

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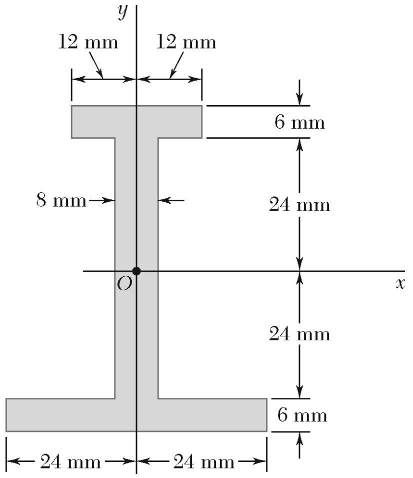


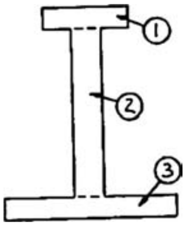
PROBLEM 9.31

Determine the moment of inertia and the radius of gyration of the shaded area with respect to the *x* axis.

SOLUTION

First note that

$$A = A_1 + A_2 + A_3$$
$$= [(24)(6) + (8)(48) + (48)(6)]\ \text{mm}^2$$
$$= (144 + 384 + 288)\ \text{mm}^2$$
$$= 816\ \text{mm}^2$$



Now $I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$

where

$$(I_x)_1 = \frac{1}{12}(24\ \text{mm})(6\ \text{mm})^3 + (144\ \text{mm}^2)(27\ \text{mm})^2$$
$$= (432 + 104,976)\ \text{mm}^4$$
$$= 105,408\ \text{mm}^4$$

$$(I_x)_2 = \frac{1}{12}(8\ \text{mm})(48\ \text{mm})^3 = 73,728\ \text{mm}^4$$

$$(I_x)_3 = \frac{1}{12}(48\ \text{mm})(6\ \text{mm})^3 + (288\ \text{mm}^2)(27\ \text{mm})^2$$
$$= (864 + 209,952)\ \text{mm}^4 = 210,816\ \text{mm}^4$$

Then $I_x = (105,408 + 73,728 + 210,816)\ \text{mm}^4$

$= 389,952\ \text{mm}^4$ or $I_x = 390 \times 10^3\ \text{mm}^4$ ◀

and $k_x^2 = \dfrac{I_x}{A} = \dfrac{389,952\ \text{mm}^4}{816\ \text{mm}^2}$ or $k_x = 21.9\ \text{mm}$ ◀

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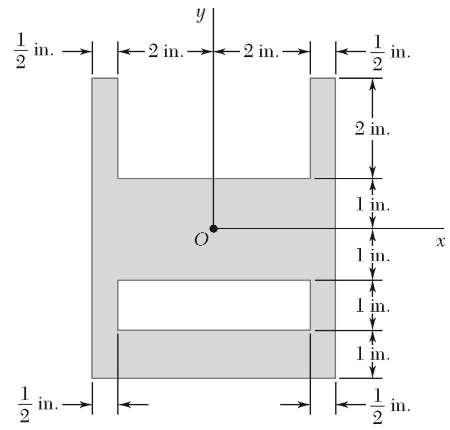


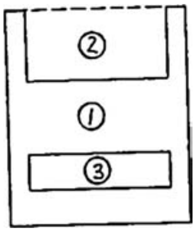
PROBLEM 9.32

Determine the moment of inertia and the radius of gyration of the shaded area with respect to the *x* axis.

SOLUTION

First note that

$$A = A_1 - A_2 - A_3$$
$$= [(5)(6) - (4)(2) - (4)(1)] \text{ in.}^2$$
$$= (30 - 8 - 4) \text{ in.}^2$$
$$= 18 \text{ in.}^2$$



Now $\qquad I_x = (I_x)_1 - (I_x)_2 - (I_x)_3$

where $\qquad (I_x)_1 = \frac{1}{12}(5 \text{ in.})(6 \text{ in.})^3 = 90 \text{ in.}^4$

$$(I_x)_2 = \frac{1}{12}(4 \text{ in.})(2 \text{ in.})^3 + (8 \text{ in.}^2)(2 \text{ in.})^2$$
$$= 34\frac{2}{3} \text{ in.}^4$$

$$(I_x)_3 = \frac{1}{12}(4 \text{ in.})(1 \text{ in.})^3 + (4 \text{ in.}^2)\left(\frac{3}{2} \text{ in.}\right)^2$$
$$= 9\frac{1}{3} \text{ in.}^4$$

Then $\qquad I_x = \left(90 - 34\frac{2}{3} - 9\frac{1}{3}\right) \text{in.}^4 \qquad$ or $\quad I_x = 46.0 \text{ in.}^4$ ◀

and $\qquad k_x^2 = \frac{I_x}{A} = \frac{46.0 \text{ in.}^4}{18 \text{ in.}^4} \qquad$ or $\quad k_x = 1.599 \text{ in.}$ ◀

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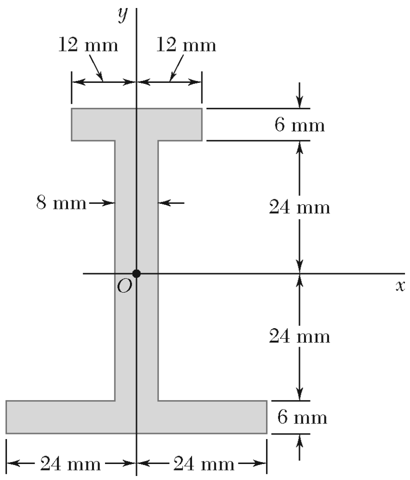


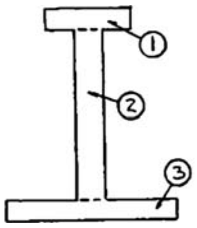
PROBLEM 9.33

Determine the moment of inertia and the radius of gyration of the shaded area with respect to the *y* axis.

SOLUTION

First note that

$$A = A_1 + A_2 + A_3$$
$$= [(24 \times 6) + (8)(48) + (48)(6)] \text{ mm}^2$$
$$= (144 + 384 + 288) \text{ mm}^2$$
$$= 816 \text{ mm}^2$$



Now $\quad I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$

where

$$(I_y)_1 = \frac{1}{12}(6 \text{ mm})(24 \text{ mm})^3 = 6912 \text{ mm}^4$$
$$(I_y)_2 = \frac{1}{12}(48 \text{ mm})(8 \text{ mm})^3 = 2048 \text{ mm}^4$$
$$(I_y)_3 = \frac{1}{12}(6 \text{ mm})(48 \text{ mm})^3 = 55,296 \text{ mm}^4$$

Then $\quad I_y = (6912 + 2048 + 55,296) \text{ mm}^4 = 64,256 \text{ mm}^4$

or $\quad I_y = 64.3 \times 10^3 \text{ mm}^4$ ◀

and $\quad k_y^2 = \dfrac{I_y}{A} = \dfrac{64,256 \text{ mm}^4}{816 \text{ mm}^2}$ $\quad$ or $\quad k_y = 8.87 \text{ mm}$ ◀

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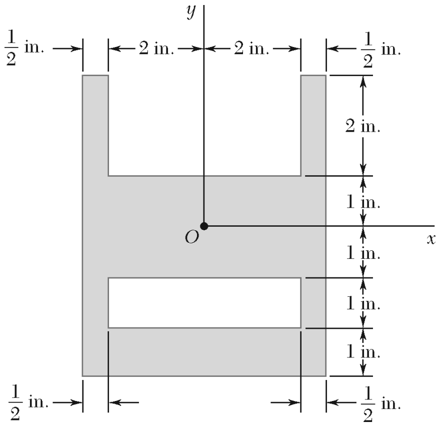


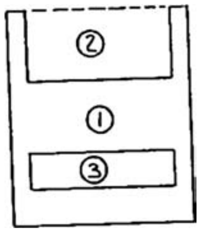
PROBLEM 9.34

Determine the moment of inertia and the radius of gyration of the shaded area with respect to the *y* axis.

SOLUTION

First note that

$$A = A_1 - A_2 - A_3$$
$$= [(5)(6) - (4)(2) - (4)(1)] \text{ in.}^2$$
$$= (30 - 8 - 4) \text{ in.}^2$$
$$= 18 \text{ in.}^2$$



Now $\quad I_y = (I_y)_1 - (I_y)_2 - (I_y)_3$

where

$$(I_y)_1 = \frac{1}{12}(6 \text{ in.})(5 \text{ in.})^3 = 62.5 \text{ in.}^4$$

$$(I_y)_2 = \frac{1}{12}(2 \text{ in.})(4 \text{ in.})^3 = 10\frac{2}{3} \text{ in.}^4$$

$$(I_y)_3 = \frac{1}{12}(1 \text{ in.})(4 \text{ in.})^3 = 5\frac{1}{3} \text{ in.}^4$$

Then $\quad I_y = \left(62.5 - 10\frac{2}{3} - 5\frac{1}{3}\right) \text{in.}^4 \qquad$ or $\quad I_y = 46.5 \text{ in.}^4$ ◀

and $\quad k_y^2 = \frac{I_y}{A} = \frac{46.5 \text{ in.}^4}{18 \text{ in.}^2} \qquad$ or $\quad k_y = 1.607 \text{ in.}$ ◀

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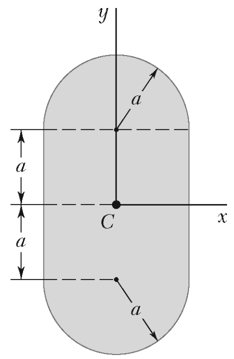


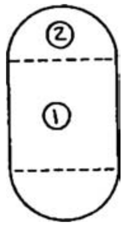
PROBLEM 9.35

Determine the moments of inertia of the shaded area shown with respect to the x and y axes when $a = 20$ mm.

SOLUTION

We have $$I_x = (I_x)_1 + 2(I_x)_2$$

where $$(I_x)_1 = \frac{1}{12}(40 \text{ mm})(40 \text{ mm})^3$$
$$= 213.33\times 10^3 \text{ mm}^4$$



$$(I_x)_2 = \left[\frac{\pi}{8}(20 \text{ mm})^4 - \frac{\pi}{2}(20 \text{ mm})^2\left(\frac{4\times 20}{3\pi} \text{ mm}\right)^2\right]$$
$$+\frac{\pi}{2}(20 \text{ mm})^2\left[\left(\frac{4\times 20}{3\pi}+20\right)\text{mm}\right]^2$$
$$= 527.49\times 10^3 \text{ mm}^4$$

Then $$I_x = [213.33 + 2(527.49)]\times 10^3 \text{ mm}^4$$

or $I_x = 1.268\times 10^6 \text{ mm}^4$ ◀

Also $$I_y = (I_y)_1 + 2(I_y)_2$$

where $$(I_y)_1 = \frac{1}{12}(40 \text{ mm})(40 \text{ mm})^3 = 213.33\times 10^3 \text{ mm}^4$$

$$(I_y)_2 = \frac{\pi}{8}(20 \text{ mm})^4 = 62.83\times 10^3 \text{ mm}^4$$

Then $$I_y = [213.33 + 2(62.83)]\times 10^3 \text{ mm}^4$$

or $I_y = 339\times 10^3 \text{ mm}^4$ ◀

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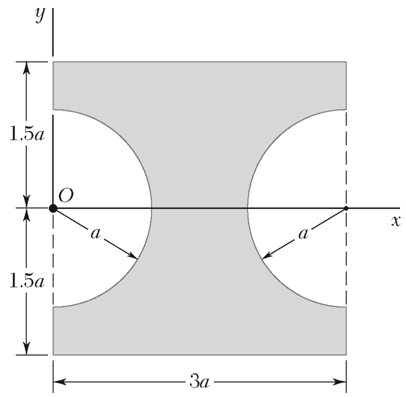


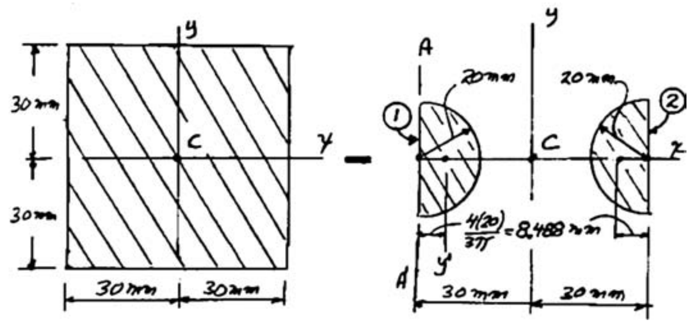
PROBLEM 9.36

Determine the moments of inertia of the shaded area shown with respect to the *x* and *y* axes when $a = 20$ mm.

SOLUTION

Given: Area = Square − 2(Semicircles)

For $a = 20$ mm, we have



Square: $I_x = I_y = \frac{1}{12}(60)^4 = 1080 \times 10^3 \text{ mm}^4$

Semicircle ①:

$$I_x = \frac{\pi}{8}(20)^4 = 62.83 \times 10^3 \text{ mm}^4$$

$$I_{AA'} = \bar{I}_{y'} + Ad^2; \quad \frac{\pi}{8}(20)^4 = \bar{I}_{y'} + \left(\frac{\pi}{2}\right)(20)^2(8.488)^2$$

$$\bar{I}_{y'} = 62.83 \times 10^3 - 45.27 \times 10^3$$

$$\bar{I}_{y'} = 17.56 \times 10^3 \text{ mm}^4$$

$$I_y = \bar{I}_{y'} + A(30 - 8.488)^2 = 17.56 \times 10^3 + \frac{\pi}{2}(20)^2(21.512)^2$$

$$I_y = 308.3 \times 10^3 \text{ mm}^4$$

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PROBLEM 9.36 (Continued)

Semicircle ②: Same as semicircle ①.

Entire Area:

$$I_x = 1080\times10^3 - 2(62.83\times10^3)$$

$$= 954.3\times10^3 \text{ mm}^4 \qquad I_x = 954\times10^3 \text{ mm}^4 \blacktriangleleft$$

$$I_y = 1080\times10^3 - 2(308.3\times10^3)$$

$$= 463.3\times10^3 \text{ mm}^4 \qquad I_y = 463\times10^3 \text{ mm}^4 \blacktriangleleft$$

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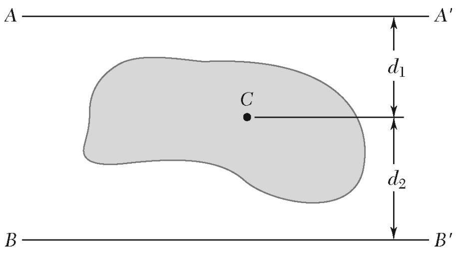

PROBLEM 9.37

For the 4000-mm^2 shaded area shown, determine the distance d_2 and the moment of inertia with respect to the centroidal axis parallel to AA' knowing that the moments of inertia with respect to AA' and BB' are $12\times10^6\text{ mm}^4$ and $23.9\times10^6\text{ mm}^4$, respectively, and that $d_1 = 25$ mm.

SOLUTION

We have $\quad I_{AA'} = \bar{I} + Ad_1^2 \qquad (1)$

and $\quad I_{BB'} = \bar{I} + Ad_2^2$

Subtracting $\quad I_{AA'} - I_{BB'} = A\left(d_1^2 - d_2^2\right)$

or
$$d_2^2 = d_1^2 - \frac{I_{AA'} - I_{BB'}}{A}$$
$$= (25\text{ mm})^2 - \frac{(12 - 23.9)10^6\text{ mm}^4}{4000\text{ mm}^2}$$
$$= 3600\text{ mm}^2 \qquad \text{or } d_2 = 60.0\text{ mm}$$

Using Eq. (1): $\quad \bar{I} = 12\times10^6\text{ mm}^4 - (4000\text{ mm}^2)(25\text{ mm})^2 \qquad$ or $\bar{I} = 9.50\times10^6\text{ mm}^4$ ◀

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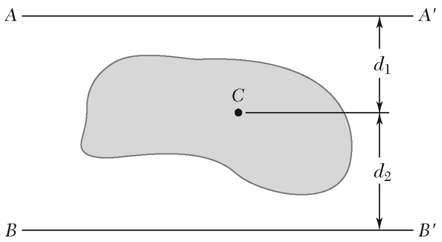

PROBLEM 9.38

Determine for the shaded region the area and the moment of inertia with respect to the centroidal axis parallel to BB', knowing that $d_1 = 25$ mm and $d_2 = 15$ mm and that the moments of inertia with respect to AA' and BB' are $7.84 \times 10^6 \text{ mm}^4$ and $5.20 \times 10^6 \text{ mm}^4$, respectively.

SOLUTION

We have $$I_{AA'} = \bar{I} + Ad_1^2 \qquad (1)$$

and $$I_{BB'} = \bar{I} + Ad_2^2$$

Subtracting $$I_{AA'} - I_{BB'} = A\left(d_1^2 - d_2^2\right)$$

or $$A = \frac{I_{AA'} - I_{BB'}}{d_1^2 - d_2^2} = \frac{(7.84 - 5.20)10^6 \text{ mm}^4}{(25 \text{ mm})^2 - (15 \text{ mm})^2}$$

or $A = 6600 \text{ mm}^2$ ◀

Using Eq. (1): $$\bar{I} = 7.84 \times 10^6 \text{ mm}^4 - (6600 \text{ mm}^2)(25 \text{ mm})^2$$
$$= 3.715 \times 10^6 \text{ mm}^4$$

or $\bar{I} = 3.72 \times 10^6 \text{ mm}^4$ ◀

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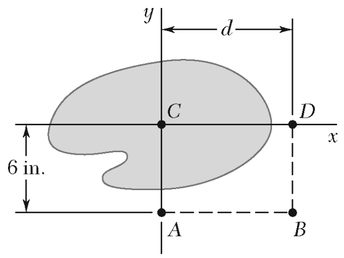
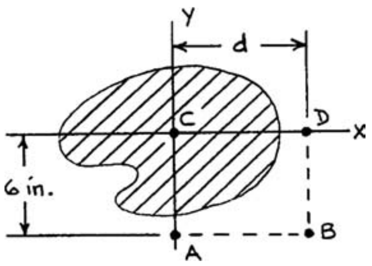


PROBLEM 9.39

The shaded area is equal to 50 in.^2. Determine its centroidal moments of inertia $\bar{I}_x$ and $\bar{I}_y$, knowing that $\bar{I}_y = 2\bar{I}_x$ and that the polar moment of inertia of the area about Point *A* is $J_A = 2250 \text{ in.}^4$.

SOLUTION



Given: $A = 50 \text{ in.}^2 \quad \bar{I}_y = 2\bar{I}_x, \quad J_A = 2250 \text{ in.}^4$

$$J_A = \bar{J}_C + A(6 \text{ in.})^2$$

$$2250 \text{ in.}^4 = \bar{J}_C + (50 \text{ in.}^2)(6 \text{ in.})^2$$

$$\bar{J}_C = 450 \text{ in.}^4$$

$$\bar{J}_C = \bar{I}_x + \bar{I}_y \quad \text{with} \quad \bar{I}_y = 2\bar{I}_x$$

$$450 \text{ in.}^4 = \bar{I}_x + 2\bar{I}_x$$

$\bar{I}_x = 150.0 \text{ in.}^4$ ◀

$\bar{I}_y = 2\bar{I}_x = 300 \text{ in.}^4$ ◀

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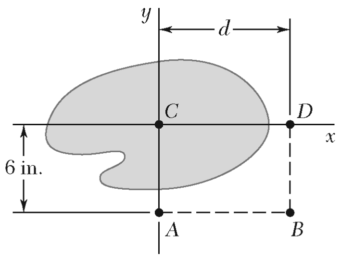

PROBLEM 9.40

The polar moments of inertia of the shaded area with respect to Points A, B, and D are, respectively, $J_A = 2880\text{ in.}^4$, $J_B = 6720\text{ in.}^4$, and $J_D = 4560\text{ in.}^4$. Determine the shaded area, its centroidal moment of inertia $\bar{J}_C$, and the distance d from C to D.

SOLUTION

See figure at solution of Problem 9.39.

Given: $J_A = 2880\text{ in.}^4, \quad J_B = 6720\text{ in.}^4, \quad J_D = 4560\text{ in.}^4$

$$J_B = \bar{J}_C + A(CB)^2; \quad 6720\text{ in.}^4 = \bar{J}_C + A(6^2 + d^2) \qquad (1)$$

$$J_D = \bar{J}_C + A(CD)^2; \quad 4560\text{ in.}^4 = \bar{J}_C + Ad^2 \qquad (2)$$

Eq. (1) subtracted by Eq. (2): $J_B - J_D = 2160\text{ in.}^4 = A(6)^2$ $\qquad A = 60.0\text{ in.}^2$ ◀

$J_A = \bar{J}_C + A(AC)^2; \quad 2880\text{ in.}^4 = \bar{J}_C + (60\text{ in.}^2)(6\text{ in.})^2$ $\qquad \bar{J}_C = 720\text{ in.}^4$ ◀

Eq. (2): $4560\text{ in.}^4 = 720\text{ in.}^4 + (60\text{ in.}^2)d^2$ $\qquad d = 8.00\text{ in.}$ ◀

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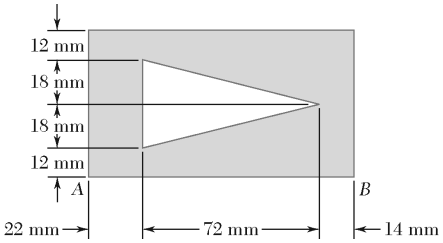
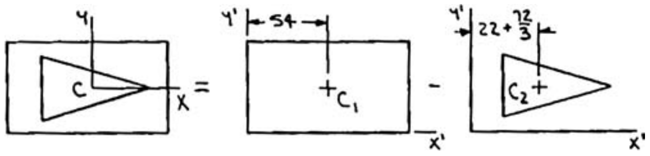


PROBLEM 9.41

Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.

SOLUTION



Dimensions in mm

First locate centroid *C* of the area.

Symmetry implies $\bar{Y} = 30$ mm.

	A, mm^2	$\bar{x}$, mm	$\bar{x}A$, mm^3
1	$108 \times 60 = 6480$	54	349,920
2	$-\frac{1}{2} \times 72 \times 36 = -1296$	46	–59,616
Σ	5184		290,304

Then $\quad \bar{X}\Sigma A = \Sigma \bar{x}A: \quad \bar{X}(5184 \text{ mm}^2) = 290,304 \text{ mm}^3$

or $\quad \bar{X} = 56.0 \text{ mm}$

Now $\quad \bar{I}_x = (I_x)_1 - (I_x)_2$

where $\quad (I_x)_1 = \frac{1}{12}(108 \text{ mm})(60 \text{ mm})^3 = 1.944 \times 10^6 \text{ mm}^4$

$$(I_x)_2 = 2\left[\frac{1}{36}(72 \text{ mm})(18 \text{ mm})^3 + \left(\frac{1}{2} \times 72 \text{ mm} \times 18 \text{ mm}\right)(6 \text{ mm})^2\right]$$

$$= 2(11,664 + 23,328) \text{ mm}^4 = 69.984 \times 10^3 \text{ mm}^4$$

[$(I_x)_2$ is obtained by dividing A_2 into ⊳]

Then $\quad \bar{I}_x = (1.944 - 0.069984) \times 10^6 \text{ mm}^4$

or $\quad \bar{I}_x = 1.874 \times 10^6 \text{ mm}^4$ ◀

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PROBLEM 9.41 (Continued)

Also $\bar{I}_y = (I_y)_1 - (I_y)_2$

where $(I_y)_1 = \frac{1}{12}(60 \text{ mm})(108 \text{ mm})^3 + (6480 \text{ mm}^2)[(56.54) \text{ mm}]^2$

$= (6,298,560 + 25,920) \text{ mm}^4 = 6.324 \times 10^6 \text{ mm}^4$

$(I_y)_2 = \frac{1}{36}(36 \text{ mm})(72 \text{ mm})^3 + (1296 \text{ mm}^2)[(56 - 46) \text{ mm}]^2$

$= (373,248 + 129,600) \text{ mm}^4 = 0.502 \times 10^6 \text{ mm}^4$

Then $\bar{I}_y = (6.324 - 0.502)10^6 \text{ mm}^4$

or $\bar{I}_y = 5.82 \times 10^6 \text{ mm}^4$ ◀

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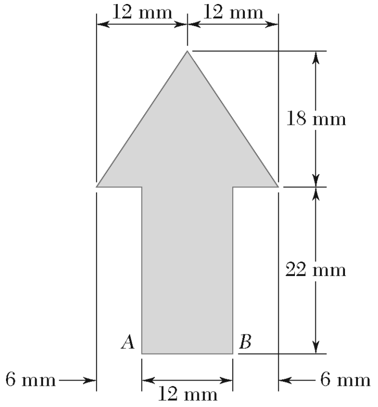


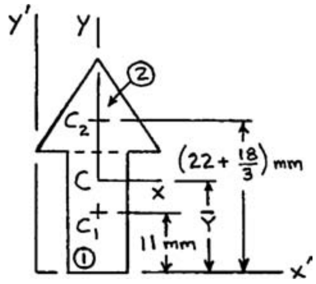
PROBLEM 9.42

Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.

SOLUTION

First locate *C* of the area:

Symmetry implies $\bar{X} = 12$ mm.



	A, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$12 \times 22 = 264$	11	2904
2	$\frac{1}{2}(24)(18) = 216$	28	6048
Σ	480		8952

Then $\quad \bar{Y}\Sigma A = \Sigma \bar{y}A: \quad \bar{Y}(480 \text{ mm}^2) = 8952 \text{ mm}^3$

$$\bar{Y} = 18.65 \text{ mm}$$

Now $\quad \bar{I}_x = (I_x)_1 + (I_x)_2$

where $\quad (I_x)_1 = \frac{1}{12}(12 \text{ mm})(22 \text{ mm})^3 + (264 \text{ mm}^2)[(18.65 - 11) \text{ mm}]^2$

$$= 26{,}098 \text{ mm}^4$$

$$(I_x)_2 = \frac{1}{36}(24 \text{ mm})(18 \text{ mm})^3 + (216 \text{ mm}^2)[(28 - 18.65) \text{ mm}]^2$$

$$= 22{,}771 \text{ mm}^4$$

Then $\quad \bar{I}_x = (26.098 + 22.771) \times 10^3 \text{ mm}^4$

or $\quad \bar{I}_x = 48.9 \times 10^3 \text{ mm}^4$ ◀

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PROBLEM 9.42 (Continued)

Also $$\bar{I}_y = (I_y)_1 + (I_y)_2$$

where $$(I_y)_1 = \frac{1}{12}(22\text{ mm})(12\text{ mm})^3 = 3168\text{ mm}^4$$

$$(I_y)_2 = 2\left[\frac{1}{36}(18\text{ mm})(12\text{ mm})^3 + \left(\frac{1}{2}\times 18\text{ mm}\times 12\text{ mm}\right)(4\text{ mm})^2\right]$$

$$= 5184\text{ mm}^4$$

[$(I_y)_2$ is obtained by dividing A_2 into]

Then $$\bar{I}_y = (3168 + 5184)\text{ mm}^4$$ or $\bar{I}_y = 8.35\times 10^3\text{ mm}^4$ ◀

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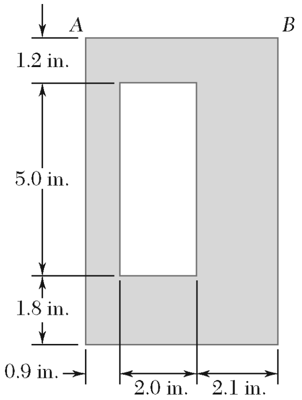
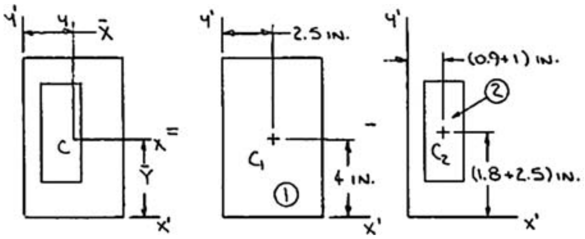


PROBLEM 9.43

Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.

SOLUTION



First locate centroid *C* of the area.

	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in.^{3}	$\bar{y}A$, in.^{3}
1	$5\times 8 = 40$	2.5	4	100	160
2	$-2\times 5 = -10$	1.9	4.3	–19	–43
Σ	30			81	117

Then $\quad \bar{X}\Sigma A = \Sigma \bar{x}A: \quad \bar{X}(30\text{ in.}^2) = 81\text{ in.}^3$

or $\quad \bar{X} = 2.70\text{ in.}$

and $\quad \bar{Y}\Sigma A = \Sigma \bar{y}A: \quad \bar{Y}(30\text{ in.}^2) = 117\text{ in.}^3$

or $\quad \bar{Y} = 3.90\text{ in.}$

Now $\quad \bar{I}_x = (I_x)_1 - (I_x)_2$

where
$$(I_x)_1 = \frac{1}{12}(5\text{ in.})(8\text{ in.})^3 + (40\text{ in.}^2)[(4-3.9)\text{ in.}]^2$$
$$= (213.33 + 0.4)\text{ in.}^4 = 213.73\text{ in.}^4$$
$$(I_x)_2 = \frac{1}{12}(2\text{ in.})(5\text{ in.})^3 + (10\text{ in.}^2)[(4.3-3.9)\text{ in.}]^2$$
$$= (20.83 + 1.60) = 22.43\text{ in.}^4$$

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PROBLEM 9.43 (Continued)

Then $\bar{I}_x = (213.73 - 22.43)\text{ in.}^4$ or $\bar{I}_x = 191.3\text{ in.}^4$ ◀

Also $\bar{I}_y = (I_y)_1 - (I_y)_2$

where
$$(I_y)_1 = \frac{1}{12}(8\text{ in.})(5\text{ in.})^3 + (40\text{ in.}^2)[(2.7 - 2.5)\text{ in.}]^2$$
$$= (83.333 + 1.6)\text{ in.}^4 = 84.933\text{ in.}^4$$
$$(I_y)_2 = \frac{1}{12}(5\text{ in.})(2\text{ in.})^3 + (10\text{ in.}^2)[(2.7 - 1.9)\text{ in.}]^2$$
$$= (3.333 + 6.4)\text{ in.}^4 = 9.733\text{ in.}^4$$

Then $\bar{I}_y = (84.933 - 9.733)\text{ in.}^4$ or $\bar{I}_y = 75.2\text{ in.}^4$ ◀

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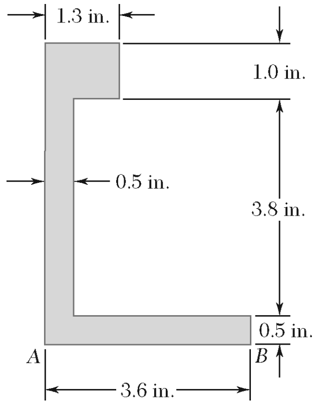
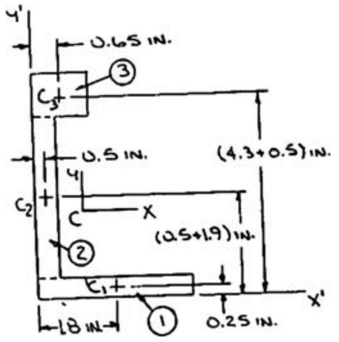


PROBLEM 9.44

Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.

SOLUTION



First locate centroid *C* of the area.

	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in.^{3}	$\bar{y}A$, in.^{3}
1	$3.6\times0.5=1.8$	1.8	0.25	3.24	0.45
2	$0.5\times3.8=1.9$	0.25	2.4	0.475	4.56
3	$1.3\times1=1.3$	0.65	4.8	0.845	6.24
Σ	5.0			4.560	11.25

Then $\quad \bar{X}\Sigma A=\Sigma\bar{x}A: \quad \bar{X}(5\text{ in.}^2)=4.560\text{ in.}^3$

or $\quad \bar{X}=0.912\text{ in.}$

and $\quad \bar{Y}\Sigma A=\Sigma\bar{y}A: \quad \bar{Y}(5\text{ in.}^2)=11.25\text{ in.}^3$

or $\quad \bar{Y}=2.25\text{ in.}$

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PROBLEM 9.44 (Continued)

Now $\bar{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3$

where
$$(I_x)_1 = \frac{1}{12}(3.6\text{ in.})(0.5\text{ in.})^3 + (1.8\text{ in.}^2)[(2.25-0.25)\text{ in.}]^2$$
$$= (0.0375 + 7.20)\text{ in.}^4 = 7.2375\text{ in.}^4$$
$$(I_x)_2 = \frac{1}{12}(0.5\text{ in.})(3.8\text{ in.})^3 + (1.9\text{ in.}^2)[(2.4-2.25)\text{ in.}]^2$$
$$= (2.2863 + 0.0428)\text{ in.}^4 = 2.3291\text{ in.}^4$$
$$(I_x)_3 = \frac{1}{12}(1.3\text{ in.})(1\text{ in.})^3 + (1.3\text{ in.}^2)[(4.8-2.25\text{ in.})]^2$$
$$= (0.1083) + 8.4533)\text{ in.}^4 = 8.5616\text{ in.}^4$$

Then $\bar{I}_x = (7.2375 + 2.3291 + 8.5616)\text{ in.}^4 = 18.1282\text{ in.}^4$

or $\bar{I}_x = 18.13\text{ in.}^4$ ◀

Also $\bar{I}_y = (I_y)_1 + (I_y)_2 + (I_y)_3$

where
$$(I_y)_1 = \frac{1}{12}(0.5\text{ in.})(3.6\text{ in.})^3 + (1.8\text{ in.}^2)[(1.8-0.912)\text{ in.}]^2$$
$$= (1.9440 + 1.4194)\text{ in.}^4 = 3.3634\text{ in.}^4$$
$$(I_y)_2 = \frac{1}{12}(3.8\text{ in.})(0.5\text{ in.})^3 + (1.9\text{ in.}^2)[(0.912-0.25)\text{ in.}]^2$$
$$= (0.0396 + 0.8327)\text{ in.}^4 = 0.8723\text{ in.}^4$$
$$(I_y)_3 = \frac{1}{12}(1\text{ in.})(1.3\text{ in.})^3 + (1.3\text{ in.}^2)[(0.912-0.65)\text{ in.}]^2$$
$$= (0.1831 + 0.0892)\text{ in.}^4 = 0.2723\text{ in.}^4$$

Then $\bar{I}_y = (3.3634 + 0.8723 + 0.2723)\text{ in.}^4 = 4.5080\text{ in.}^4$

or $\bar{I}_y = 4.51\text{ in.}^4$ ◀

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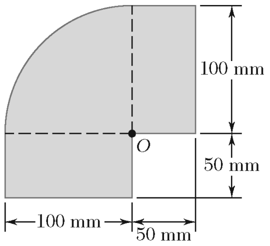
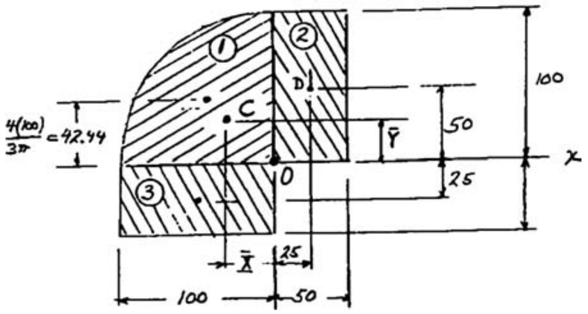


PROBLEM 9.45

Determine the polar moment of inertia of the area shown with respect to (*a*) Point *O*, (*b*) the centroid of the area.

SOLUTION

Dimensions in mm   Symmetry: $\bar{X} = \bar{Y}$



Determination of centroid, *C*, of entire section.

Section	Area, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$\frac{\pi}{4}(100)^2 = 7.854\times10^3$	42.44	333.3×10^3
2	$(50)(100) = 5\times10^3$	50	250×10^3
3	$(100)(50) = 5\times10^3$	–25	-125×10^3
Σ	17.854×10^3		458.3×10^3

$$\bar{Y}\Sigma A = \Sigma\bar{y}A: \quad \bar{Y}(17.854\times10^3 \text{ mm}^2) = 458.3\times10^3 \text{ mm}^3$$

$$\bar{Y} = 25.67 \text{ mm} \qquad \bar{X} = \bar{Y} = 25.67 \text{ mm}$$

Distance *O* to *C*: $\overline{OC} = \sqrt{2}\bar{Y} = \sqrt{2}(25.67) = 36.30 \text{ mm}$

(*a*) Section 1: $J_O = \frac{\pi}{8}(100)^4 = 39.27\times10^6 \text{ mm}^4$

Section 2: $J_O = J + A(\overline{OD})^2 = \frac{1}{12}(50)(100)[50^2 + 100^2] + (50)(100)\left[\left(\frac{50}{2}\right)^2 + \left(\frac{100}{2}\right)^2\right]$

$$J_O = 5.208\times10^6 + 15.625\times10^6 = 20.83\times10^6 \text{ mm}^4$$

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PROBLEM 9.45 (Continued)

Section 3: Same as Section 2; $J_O = 20.83\times10^6\ \text{mm}^4$

Entire section:

$$J_O = 39.27\times10^6 + 2(20.83\times10^6)$$
$$= 80.94\times10^6 \qquad J_O = 80.9\times10^6\ \text{mm}^4 \blacktriangleleft$$

(*b*) Recall that, $\overline{OC} = 36.30\ \text{mm}$ and $A = 17.854\times10^3\ \text{mm}^2$

$$J_O = \bar{J}_C + A(\overline{OC})^2$$
$$80.94\times10^6\ \text{mm}^4 = \bar{J}_C + (17.854\times10^3\ \text{mm}^2)(36.30\ \text{mm})^2$$

$$\bar{J}_C = 57.41\times10^6\ \text{mm}^4 \qquad \bar{J}_C = 57.4\times10^6\ \text{mm}^4 \blacktriangleleft$$

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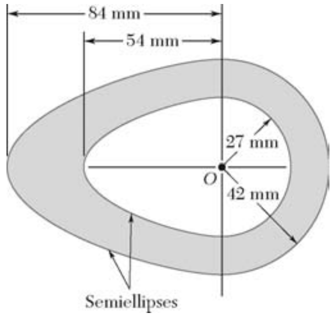


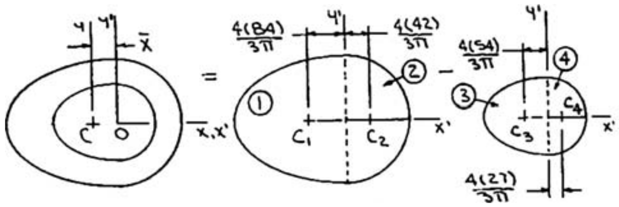
PROBLEM 9.46

Determine the polar moment of inertia of the area shown with respect to (*a*) Point *O*, (*b*) the centroid of the area.

SOLUTION

First locate centroid *C* of the figure.

Note that symmetry implies $\bar{Y} = 0$.



Dimensions in mm

	A, mm^2	$\bar{X}$, mm	$\bar{X}A$, mm^3
1	$\frac{\pi}{2}(84)(42) = 5541.77$	$-\frac{112}{\pi} = -35.6507$	$-197,568$
2	$\frac{\pi}{2}(42)^2 = 2770.88$	$\frac{56}{\pi} = 17.8254$	49,392
3	$-\frac{\pi}{2}(54)(27) = -2290.22$	$-\frac{72}{\pi} = -22.9183$	52,488
4	$-\frac{\pi}{2}(27)^2 = -1145.11$	$\frac{36}{\pi} = 11.4592$	–13,122
Σ	4877.32		–108,810

Then $\bar{X}\Sigma A = \Sigma \bar{x} A$: $\quad \bar{X}(4877.32 \text{ mm}^2) = -108,810 \text{ mm}^3$

or $\bar{X} = -22.3094$

(*a*) $J_O = (J_O)_1 + (J_O)_2 - (J_O)_3 - (J_O)_4$

where
$$(J_O)_1 = \frac{\pi}{8}(84 \text{ mm})(42 \text{ mm})[(84 \text{ mm})^2 + (42 \text{ mm})^2]$$
$$= 12.21960 \times 10^6 \text{ mm}^4$$

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PROBLEM 9.46 (Continued)

$$(J_O)_2 = \frac{\pi}{4}(42 \text{ mm})^4$$
$$= 2.44392 \times 10^6 \text{ mm}^4$$
$$(J_O)_3 = \frac{\pi}{8}(54 \text{ mm})(27 \text{ mm})[(54 \text{ mm})^2 + (27 \text{ mm})^2]$$
$$= 2.08696 \times 10^6 \text{ mm}^4$$
$$(J_O)_4 = \frac{\pi}{4}(27 \text{ mm})^4$$
$$= 0.41739 \times 10^6 \text{ mm}^4$$

Then $$J_O = (12.21960 + 2.44392 - 2.08696 - 0.41739) \times 10^6 \text{ mm}^4$$
$$= 12.15917 \times 10^6 \text{ mm}^4$$

or $J_O = 12.16 \times 10^6 \text{ mm}^4$ ◀

(*b*) $$J_O = \bar{J}_C + A\bar{X}^2$$

or $$\bar{J}_C = 12.15917 \times 10^6 \text{ mm}^4 - (4877.32 \text{ mm}^2)(-22.3094 \text{ mm})^2$$

or $\bar{J}_C = 9.73 \times 10^6 \text{ mm}^4$ ◀

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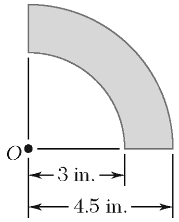
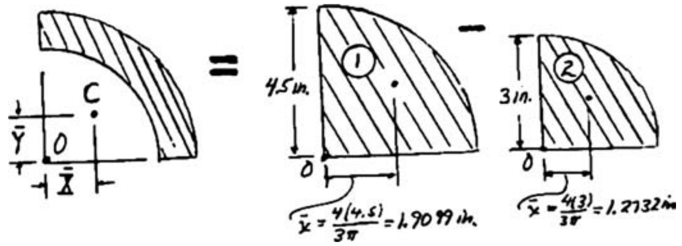


PROBLEM 9.47

Determine the polar moment of inertia of the area shown with respect to (*a*) Point *O*, (*b*) the centroid of the area.

SOLUTION



Section	Area, in.^{2}	$\bar{x}$, in.	$\bar{x}A$, in.^{3}
1	$\dfrac{\pi}{4}(4.5)^2 = 15.904$	1.9099	30.375
2	$-\dfrac{\pi}{4}(3)^2 = -7.069$	1.2732	–9.00
Σ	8.835		21.375

Then $\qquad \bar{X}A = \Sigma\bar{x}A: \quad \bar{X}(8.835\text{ in.}^2) = 21.375\text{ in.}^3$

$$\bar{X} = 2.419\text{ in.}$$

Then $\qquad J_O = \dfrac{\pi}{8}(4.5\text{ in.})^4 - \dfrac{\pi}{8}(3\text{ in.})^4 = 129.22\text{ in.}^4 \qquad\qquad J_O = 129.2\text{ in.}^4$ ◀

$$\overline{OC} = \sqrt{2}\bar{X} = \sqrt{2}(2.419\text{ in.}) = 3.421\text{ in.}$$

$$J_O = \bar{J}_C + A(\overline{OC})^2:$$

$129.22\text{ in.}^4 = \bar{J}_C + (8.835\text{ in.})(3.421\text{ in.})^2 \qquad\qquad \bar{J}_C = 25.8\text{ in.}^4$ ◀

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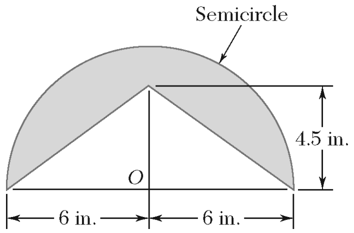
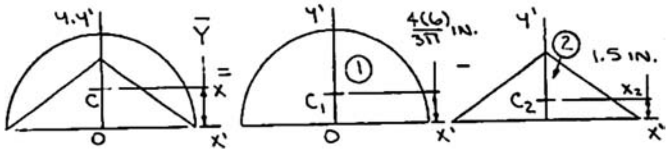


PROBLEM 9.48

Determine the polar moment of inertia of the area shown with respect to (*a*) Point *O*, (*b*) the centroid of the area.

SOLUTION

First locate centroid *C* of the figure.



	A, in.^{2}	$\bar{y}$, in.	$\bar{y}A$, in.^{3}
1	$\frac{\pi}{2}(6)^2 = 56.5487$	$\frac{8}{\pi} = 2.5465$	144
2	$-\frac{1}{2}(12)(4.5) = -27$	1.5	–40.5
Σ	29.5487		103.5

Then $$\bar{Y}\Sigma A = \Sigma\bar{y}A: \quad \bar{Y}(29.5487 \text{ in.}^2) = 103.5 \text{ in.}^3$$

or $$\bar{Y} = 3.5027 \text{ in.}$$

(*a*) $$J_O = (J_O)_1 - (J_O)_2$$

where $$(J_O)_1 = \frac{\pi}{4}(6 \text{ in.})^4 = 107.876 \text{ in.}^4$$

$$(J_O)_2 = (I_{x'})_2 + (I_{y'})_2$$

Now $$(I_{x'})_2 = \frac{1}{12}(12 \text{ in.})(4.5 \text{ in.})^3 = 91.125 \text{ in.}^4$$

and $$(I_{y'})_2 = 2\left[\frac{1}{12}(4.5 \text{ in.})(6 \text{ in.})^3\right] = 162.0 \text{ in.}^4$$

[*Note:* $(I_{y'})_2$ is obtained using]

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PROBLEM 9.48 (Continued)

Then $(J_O)_2 = (91.125 + 162.0)\text{ in.}^4$
$= 253.125\text{ in.}^4$

Finally $J_O = (1017.876 - 253.125)\text{ in.}^4 = 764.751\text{ in.}^4$

or $J_O = 765\text{ in.}^4$ ◀

(*b*) $J_O = \bar{J}_C + A\bar{Y}^2$

or $J_C = 764.751\text{ in.}^4 - (29.5487\text{ in.}^2)(3.5027\text{ in.})^2$

or $\bar{J}_C = 402\text{ in.}^4$ ◀

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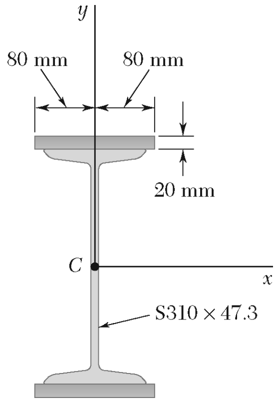
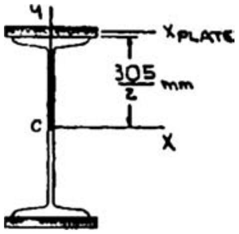


PROBLEM 9.49

Two 20-mm steel plates are welded to a rolled S section as shown. Determine the moments of inertia and the radii of gyration of the combined section with respect to the centroidal *x* and *y* axes.

SOLUTION



S section: $A = 6010 \text{ mm}^2$

$$\bar{I}_x = 90.3 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 3.88 \times 10^6 \text{ mm}^4$$

Note: $A_{\text{total}} = A_S + 2A_{\text{plate}}$

$$= 6010 \text{ mm}^2 + 2(160 \text{ mm})(20 \text{ mm})$$

$$= 12,410 \text{ mm}^2$$

Now $\bar{I}_x = (\bar{I}_x)_S + 2(I_x)_{\text{plate}}$

where $(I_x)_{\text{plate}} = \bar{I}_{x_{\text{plate}}} + Ad^2$

$$= \frac{1}{12}(160 \text{ mm})(20 \text{ mm})^3 + (3200 \text{ mm}^2)[(152.5 + 10) \text{ mm}]^2$$

$$= 84.6067 \times 10^6 \text{ mm}^4$$

Then $\bar{I}_x = (90.3 + 2 \times 84.6067) \times 10^6 \text{ mm}^4$

$$= 259.5134 \times 10^6 \text{ mm}^4$$

or $\bar{I}_x = 260 \times 10^6 \text{ mm}^4$ ◀

and $\bar{k}_x^2 = \dfrac{\bar{I}_x}{A_{\text{total}}} = \dfrac{259.5134 \times 10^6 \text{ mm}^4}{12410 \text{ mm}^2}$

or $\bar{k}_x = 144.6 \text{ mm}$ ◀

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PROBLEM 9.49 (Continued)

Also

$$\bar{I}_y = (\bar{I}_y)_S + 2(\bar{I}_y)_{\text{plate}}$$

$$= 3.88\times10^6\,\text{mm}^4 + 2\left[\frac{1}{12}(20\text{ mm})(160\text{ mm})^3\right]$$

$$= 17.5333\times10^6\,\text{mm}^4$$

or $\bar{I}_y = 17.53\times10^6\,\text{mm}^4$ ◀

and

$$\bar{k}_y^2 = \frac{\bar{I}_y}{A_{\text{total}}} = \frac{17.5333\times10^6\,\text{mm}^4}{12,410\text{ mm}^2}$$

or $\bar{k}_y = 37.6\text{ mm}$ ◀

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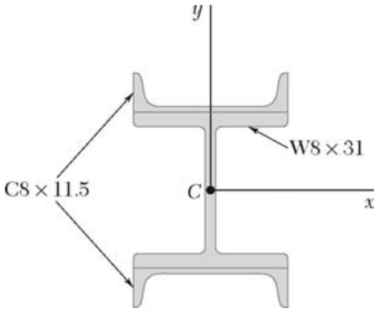


PROBLEM 9.50

Two channels are welded to a rolled W section as shown. Determine the moments of inertia and the radii of gyration of the combined section with respect to the centroidal x and y axes.

SOLUTION

W section: $A = 9.12\text{ in.}^2$

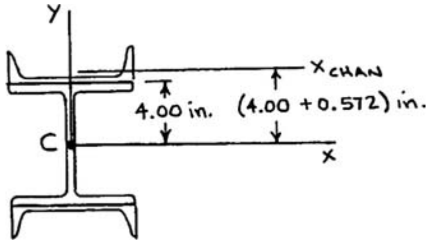
$\bar{I}_x = 110\text{ in.}^4$

$\bar{I}_y = 37.1\text{ in.}^4$

Channel: $A = 3.37\text{ in.}^2$

$\bar{I}_x = 1.31\text{ in.}^4$

$\bar{I}_y = 32.5\text{ in.}^4$



$$A_{\text{total}} = A_{\text{W}} + 2A_{\text{chan}}$$
$$= 9.12 + 2(3.37) = 15.86\text{ in.}^2$$

Now $\bar{I}_x = (\bar{I}_x)_{\text{W}} + 2(I_x)_{\text{chan}}$

where $(I_x)_{\text{chan}} = \bar{I}_{x_{\text{chan}}} + Ad^2$

$$= 1.31\text{ in.}^4 + (3.37\text{ in.}^2)(4.572\text{ in.})^2 = 71.754\text{ in.}^4$$

Then $\bar{I}_x = (110 + 2\times 71.754)\text{ in.}^4 = 253.51\text{ in.}^4$ $\qquad \bar{I}_x = 254\text{ in.}^4$ ◀

and $\bar{k}_x^2 = \dfrac{\bar{I}_x}{A_{\text{total}}} = \dfrac{253.51\text{ in.}^4}{15.86\text{ in.}^2}$ $\qquad \bar{k}_x = 4.00\text{ in.}$ ◀

Also $\bar{I}_y = (\bar{I}_y)_{\text{W}} + 2(\bar{I}_y)_{\text{chan}}$

$= (37.1 + 2\times 32.5)\text{ in.}^4 = 102.1\text{ in.}^4$ $\qquad \bar{I}_y = 102.1\text{ in.}^4$ ◀

and $\bar{k}_y^2 = \dfrac{\bar{I}_y}{A_{\text{total}}} = \dfrac{102.1\text{ in.}^4}{15.86\text{ in.}^2}$ $\qquad \bar{k}_y = 2.54\text{ in.}$ ◀

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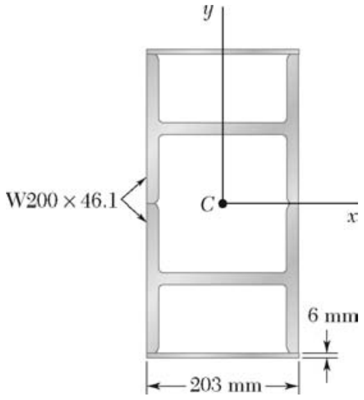
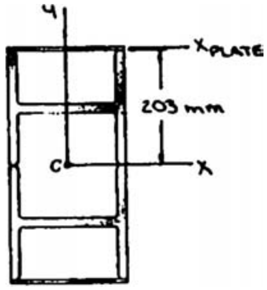


PROBLEM 9.51

To form a reinforced box section, two rolled W sections and two plates are welded together. Determine the moments of inertia and the radii of gyration of the combined section with respect to the centroidal axes shown.

SOLUTION



W section:
$$A = 5880\text{ mm}^2$$
$$\bar{I}_x = 15.4\times10^6\text{ mm}^4$$
$$\bar{I}_y = 45.8\times10^6\text{ mm}^4$$

Note:
$$A_{\text{total}} = 2A_{\text{W}} + 2A_{\text{plate}}$$
$$= 2(5880\text{ mm}^2) + 2(203\text{ mm})(6\text{ mm})$$
$$= 14{,}196\text{ mm}^2$$

Now
$$\bar{I}_x = 2(I_x)_{\text{W}} + 2(I_x)_{\text{plate}}$$

where
$$(I_x)_{\text{W}} = \bar{I}_x + Ad^2 = 15.4\times10^6\text{ mm}^4 + (5880\text{ mm}^2)\left(\frac{203}{2}\text{ mm}\right)^2$$
$$= 75.9772\times10^6\text{ mm}^4$$
$$(I_x)_{\text{plate}} = \bar{I}_{x_{\text{plate}}} + Ad^2$$
$$= \frac{1}{12}(203\text{ mm})(6\text{ mm})^3 + (1218\text{ mm}^2)[(203+3)\text{ mm}]^2$$
$$= 51.6907\times10^6\text{ mm}^4$$

Then
$$\bar{I}_x = [2(75.9772) + 2(51.6907)]\times10^6\text{ mm}^4$$
$$= 255.336\times10^6\text{ mm}^4$$

or $\bar{I}_x = 255\times10^6\text{ mm}^4$ ◀

and
$$\bar{k}_x^2 = \frac{\bar{I}_x}{A_{\text{total}}} = \frac{255.336\times10^6\text{ mm}^4}{14{,}196\text{ mm}^2}$$

or $\bar{k}_x = 134.1\text{ mm}$ ◀

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PROBLEM 9.51 (Continued)

also $$\bar{I}_y = 2(I_y)_W + 2(I_y)_{\text{plate}}$$

where $$(I_y)_W = I_y$$

$$(I_y)_{\text{plate}} = \frac{1}{12}(6\text{ mm})(203\text{ mm})^3 = 4.1827\times10^6\text{ mm}^4$$

Then $$\bar{I}_y = [2(45.8) + 2(4.1827)]\times10^6\text{ mm}^4$$

$$= 99.9654\times10^6\text{ mm}^4$$

or $\bar{I}_y = 100.0\times10^6\text{ mm}^4$ ◀

and $$\bar{k}_y^2 = \frac{\bar{I}_y}{A_{\text{total}}} = \frac{99.9654\times10^6\text{ mm}^4}{14{,}196\text{ mm}^2}$$

or $\bar{k}_y = 83.9\text{ mm}$ ◀

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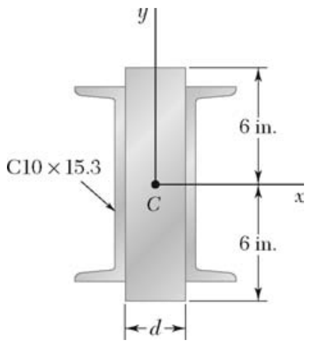
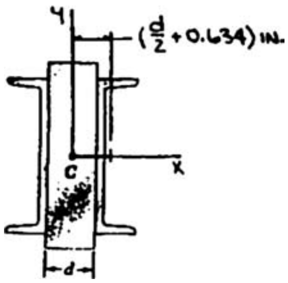


PROBLEM 9.52

Two channels are welded to a $d \times 12$-in. steel plate as shown. Determine the width d for which the ratio $\bar{I}_x/\bar{I}_y$ of the centroidal moments of inertia of the section is 16.

SOLUTION



Channel: $$A = 4.48 \text{ in.}^2$$
$$\bar{I}_x = 67.3 \text{ in.}^4$$
$$\bar{I}_y = 2.27 \text{ in.}^4$$

Now $$\bar{I}_x = 2(\bar{I}_x)_C + (\bar{I}_x)_{\text{plate}}$$
$$= 2(67.3 \text{ in.}^4) + \frac{1}{12}(d \text{ in.})(12 \text{ in.})^3$$
$$= (134.6 + 144d) \text{ in.}^4$$

and $$\bar{I}_y = 2(\bar{I}_y)_C + (\bar{I}_y)_{\text{plate}}$$

where $$(I_y)_C = \bar{I}_y + Ad^2$$

$$(I_y)_C = 2.27 \text{ in.}^4 + (4.48 \text{ in.}^2)\left[\left(\frac{d}{2} + 0.634\right) \text{in.}\right]^2$$
$$= (1.1200d^2 + 2.84032d + 4.07076) \text{ in.}^4$$
$$(\bar{I}_y)_{\text{plate}} = \frac{1}{12}(12 \text{ in.})(d \text{ in.})^3 = d^3 \text{ in.}^4$$

Then $$\bar{I}_y = [2(1.1200d^2 + 2.84032d + 4.07076) + d^3] \text{ in.}^4$$
$$= (d^3 + 2.240d^2 + 5.68064d + 8.14152) \text{ in.}^4$$

Now $$\frac{\bar{I}_x}{\bar{I}_y} = 16: \quad (134.6 + 144d) = 16(d^3 + 2.240d^2 + 5.68064d + 8.14152)$$

or $$d^3 + 2.240d^2 - 3.31936d - 0.27098 = 0$$

Solving yields $d = 1.07669$ in. (the other roots are negative).

$d = 1.077$ in. ◀

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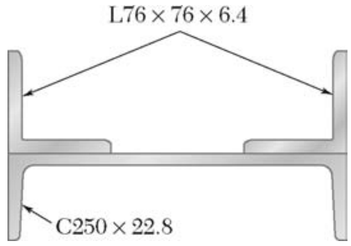
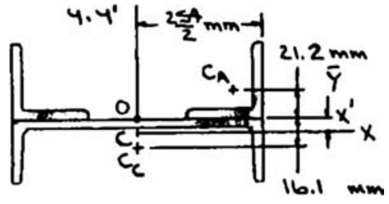


PROBLEM 9.53

Two L76 × 76 × 6.4-mm angles are welded to a C250 × 22.8 channel. Determine the moments of inertia of the combined section with respect to centroidal axes respectively parallel and perpendicular to the web of the channel.

SOLUTION



Angle: $A = 929\text{ mm}^2$

$\bar{I}_x = \bar{I}_y = 0.512 \times 10^6\text{ mm}^4$

Channel: $A = 2890\text{ mm}^2$

$\bar{I}_x = 0.945 \times 10^6\text{ mm}^4$

$\bar{I}_y = 28.0 \times 10^6\text{ mm}^4$

First locate centroid C of the section

	A, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
Angle	$2(929) = 1858$	21.2	39,389.6
Channel	2890	−16.1	−46,529
Σ	4748		−7139.4

Then $\bar{Y}\Sigma A = \Sigma \bar{y} A: \quad \bar{Y}(4748\text{ mm}^2) = -7139.4\text{ mm}^3$

or $\bar{Y} = -1.50366\text{ mm}$

Now $\bar{I}_x = 2(I_x)_L + (I_x)_C$

where $(I_x)_L = \bar{I}_x + Ad^2 = 0.512 \times 10^6\text{ mm}^4 + (929\text{ mm}^2)[(21.2 + 1.50366)\text{ mm}]^2$

$= 0.990859 \times 10^6\text{ mm}^4$

$(I_x)_C = \bar{I}_x + Ad^2 = 0.949 \times 10^6\text{ mm}^4 + (2890\text{ mm}^2)[(16.1 - 1.50366)\text{ mm}]^2$

$= 1.56472 \times 10^6\text{ mm}^4$

Then $\bar{I}_x = [2(0.990859) + 1.56472 \times 10^6\text{ mm}^4$

or $\bar{I}_x = 3.55 \times 10^6\text{ mm}^4$ ◀

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PROBLEM 9.53 (Continued)

Also $\bar{I}_y = 2(I_y)_L + (I_y)_C$

where $(I_y)_L = \bar{I}_y + Ad^2 = 0.512\times10^6\text{ mm}^4 + (929\text{ mm}^2)[(127-21.2)\text{ mm}]^2$

$= 10.9109\times10^6\text{ mm}^4$

$(I_y)_C = \bar{I}_y$

Then $\bar{I}_y = [2(10.9109) + 28.0]\times10^6\text{ mm}^4$

or $\bar{I}_y = 49.8\times10^6\text{ mm}^4$ ◀

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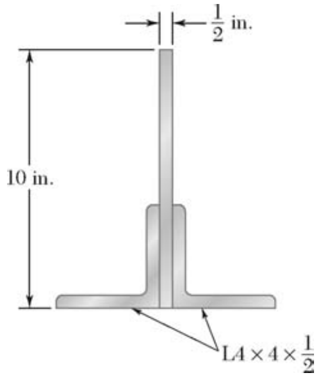
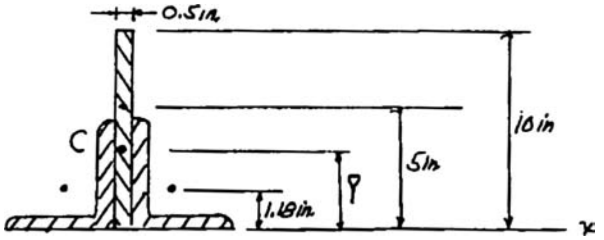


PROBLEM 9.54

Two L4 × 4 × $\frac{1}{2}$-in. angles are welded to a steel plate as shown. Determine the moments of inertia of the combined section with respect to centroidal axes respectively parallel and perpendicular to the plate.

SOLUTION



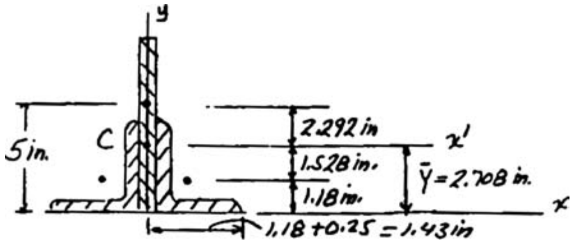
For $4 \times 4 \times \dfrac{1}{2}$-in. angle: $A = 3.75 \text{ in.}^2, \quad \bar{I}_x = \bar{I}_y = 5.52 \text{ in.}^4$

$$\bar{Y}A = \Sigma \bar{y}\,A$$

$$\bar{Y}(12.5 \text{ in.}^2) = 33.85 \text{ in.}^3$$

$$\bar{Y} = 2.708 \text{ in.}$$

Section	Area, in.^{2}	$\bar{y}$ in.	$\bar{y}A$, in.^{3}
Plate	$(0.5)(10) = 5$	5	25
Two angles	$2(3.75) = 7.5$	1.18	8.85
Σ	12.5		33.85



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PROBLEM 9.54 (Continued)

Entire section:

$$\bar{I}_x = \Sigma(\bar{I}_{x'} + Ad^2) = \left[\frac{1}{12}(0.5)(10)^3 + (0.5)(10)(2.292)^2\right] + 2[5.52 + (3.75)(1.528)^2]$$

$$= 41.667 + 26.266 + 1604 + 17.511 = 96.48 \text{ in.}^4 \qquad \bar{I}_x = 96.5 \text{ in.}^4 \blacktriangleleft$$

$$\bar{I}_y = \frac{1}{12}(10)(0.5)^3 + 2[5.52 + (3.75)(1.43)^2]$$

$$= 0.104 + 11.04 + 15.367 = 26.51 \text{ in.}^4 \qquad \bar{I}_y = 26.5 \text{ in.}^4 \blacktriangleleft$$

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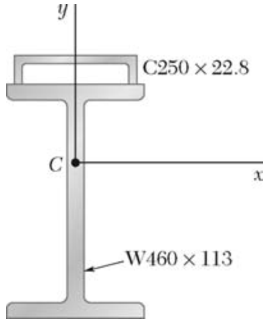
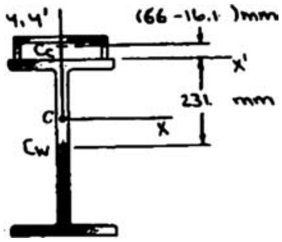


PROBLEM 9.55

The strength of the rolled W section shown is increased by welding a channel to its upper flange. Determine the moments of inertia of the combined section with respect to its centroidal x and y axes.

SOLUTION



W Section: $A = 14,400\text{ mm}^2$

$\bar{I}_x = 554 \times 10^6\text{ mm}^4$

$\bar{I}_y = 63.3 \times 10^6\text{ mm}^4$

Channel: $A = 2890\text{ mm}^2$

$\bar{I}_x = 0.945 \times 10^6\text{ mm}^4$

$\bar{I}_y = 28.0 \times 10^6\text{ mm}^4$

First locate centroid C of the section.

	A, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
W Section	14400	−231	−3326400
Channel	2890	49.9	144211
Σ	17290		−3182189

Then $\bar{Y}\Sigma A = \Sigma \bar{y} A$: $\bar{Y}(17,290\text{ mm}^2) = -3,182,189\text{ mm}^3$

or $\bar{Y} = -184.047\text{ mm}$

Now $\bar{I}_x = (I_x)_{\text{W}} + (I_x)_{\text{C}}$

where $(I_x)_{\text{W}} = \bar{I}_x + Ad^2$

$= 554 \times 10^6\text{ mm}^4 + (14,400\text{ mm}^2)(231 - 184.047)^2\text{ mm}^2$

$= 585.75 \times 10^6\text{ mm}^4$

$(I_x)_{\text{C}} = \bar{I}_x - Ad^2$

$= 0.945 \times 10^6\text{ mm}^4 + (2890\text{ mm}^2)(49.9 + 184.047)^2\text{ mm}^2$

$= 159.12 \times 10^6\text{ mm}^4$

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PROBLEM 9.55 (Continued)

Then $$\bar{I}_x = (585.75 + 159.12) \times 10^6 \text{ mm}^4$$

or $$\bar{I}_x = 745 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

also $$\bar{I}_y = (I_y)_\text{W} + (I_y)_\text{C}$$
$$= (63.3 + 28.0) \times 10^6 \text{ mm}^4$$

or $$\bar{I}_y = 91.3 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

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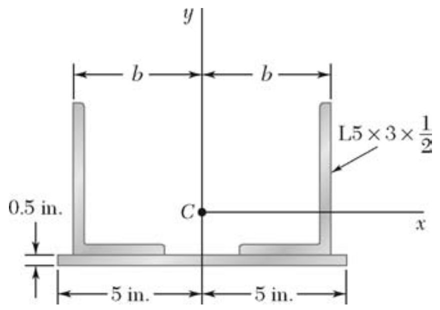
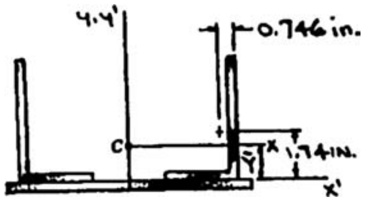


PROBLEM 9.56

Two L5 × 3 × $\frac{1}{2}$-in. angles are welded to a $\frac{1}{2}$-in. steel plate. Determine the distance b and the centroidal moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the combined section, knowing that $\bar{I}_y = 4\bar{I}_x$.

SOLUTION



Angle: $A = 3.75\,\text{in.}^2$

$\bar{I}_x = 9.43\,\text{in.}^4$

$\bar{I}_y = 2.55\,\text{in.}^4$

First locate centroid C of the section.

	Area, in.^{2}	$\bar{y}$, in.	$\bar{y}A$, in.^{3}
Angle	$2(3.75) = 7.50$	1.74	13.05
Plate	$(10)(0.5) = 5$	–0.25	−1.25
Σ	12.50		11.80

Then $\bar{Y}\Sigma A = \Sigma\bar{y}A$: $\quad \bar{Y}(12.50\,\text{in.}^2) = 11.80\,\text{in.}^3$

or $\bar{Y} = 0.944\,\text{in.}$

Now $\bar{I}_x = 2(I_x)_{\text{angle}} + (I_x)_{\text{plate}}$

where $(I_x)_{\text{angle}} = I_x + Ad^2 = 9.43\,\text{in.}^4 + (3.75\,\text{in.}^2)[(1.74 - 0.944)\,\text{in.}]^2$

$= 11.8061\,\text{in.}^4$

$(I_x)_{\text{plate}} = \bar{I}_x + Ad^2 = \frac{1}{12}(10\,\text{in.})(0.5\,\text{in.})^3 + (5\,\text{in.}^2)[(0.25 + 0.944)\,\text{in.}]^2$

$= 7.2323\,\text{in.}^4$

Then $\bar{I}_x = [2(11.8061) + 7.2323]\,\text{in.}^4 = 30.8445\,\text{in.}^4$

or $\bar{I}_x = 30.8\,\text{in.}^4$ ◀

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PROBLEM 9.56 (Continued)

We have $\bar{I}_y = 4\bar{I}_x = 4(30.8445 \text{ in.}^4) = 123.378 \text{ in.}^4$

or $\bar{I}_y = 123.4 \text{ in.}^4$ ◀

Now $\bar{I}_y = 2(I_y)_{\text{angle}} + (\bar{I}_y)_{\text{plate}}$

where $(\bar{I}_y)_{\text{angle}} = \bar{I}_y + Ad^2 = 2.55 \text{ in.}^4 + (3.75 \text{ in.}^2)[(b - 0.746) \text{ in.}]^2$

$$(\bar{I}_y)_{\text{plate}} = \frac{1}{12}(0.5 \text{ in.})(10 \text{ in.})^3 = 41.6667 \text{ in.}^4$$

Then $123.378 \text{ in.}^4 = 2[2.55 + 3.75(b - 0.746)^2] \text{ in.}^4 + 41.6667 \text{ in.}^4$

or $b = 3.94 \text{ in.}$ ◀

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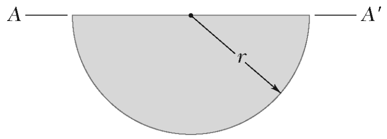


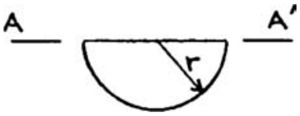
PROBLEM 9.57

The panel shown forms the end of a trough that is filled with water to the line AA'. Referring to Section 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION

From Section 9.2: $R = \gamma \int y\, dA, \quad M_{AA'} = \gamma \int y^2 dA$

Let y_P = distance of center of pressure from AA'. We must have



$$Ry_P = M_{AA'}: \quad y_P = \frac{M_{AA'}}{R} = \frac{\gamma \int y^2 dA}{\gamma \int y\, dA} = \frac{I_{AA'}}{\bar{y}A} \qquad (1)$$

<u>For semicircular panel</u>:

$$I_{AA'} = \frac{\pi}{8} r^4 \qquad \bar{y} = \frac{4r}{3\pi} \qquad A = \frac{\pi}{2} r^2$$

$$y_P = \frac{I_{AA'}}{\bar{y}A} = \frac{\frac{\pi}{8} r^4}{\left(\frac{4r}{3\pi}\right)\frac{\pi}{2} r^2}$$

$$y_P = \frac{3\pi}{16} r \blacktriangleleft$$

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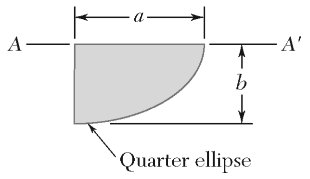
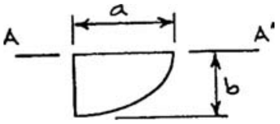


PROBLEM 9.58

The panel shown forms the end of a trough that is filled with water to the line AA'. Referring to Section 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION

See solution of Problem 9.57 for derivation of Eq. (1):



$$y_P = \frac{I_{AA'}}{\bar{y}A} \tag{1}$$

For quarter ellipse panel:

$$I_{AA'} = \frac{\pi}{16}ab^3 \qquad \bar{y} = \frac{4b}{3\pi} \qquad A = \frac{\pi}{4}ab$$

$$y_P = \frac{I_{AA'}}{\bar{y}A} = \frac{\frac{\pi}{16}ab^3}{\left(\frac{4b}{3\pi}\right)\left(\frac{\pi}{4}ab\right)} \qquad y_P = \frac{3\pi}{16}b \blacktriangleleft$$

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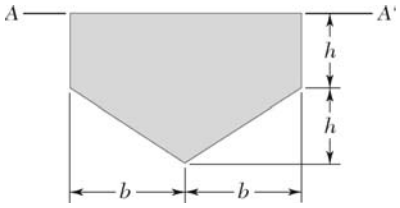


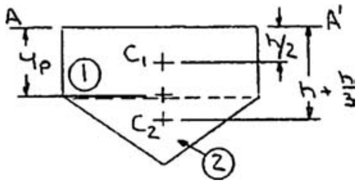
PROBLEM 9.59

The panel shown forms the end of a trough that is filled with water to the line AA'. Referring to Section 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION

Using the equation developed on page 491 of the text:

$$y_P = \frac{I_{AA'}}{\bar{y}A}$$



Now $\qquad \bar{Y}A = \Sigma \bar{y}A$

$$= \frac{h}{2}(2b \times h) + \frac{4}{3}h\left(\frac{1}{2} \times 2b \times h\right)$$

$$= \frac{7}{3}bh^2$$

and $\qquad I_{AA'} = (I_{AA'})_1 + (I_{AA'})_2$

where

$$(I_{AA'})_1 = \frac{1}{3}(2b)(h)^3 = \frac{2}{3}bh^3$$

$$(I_{AA'})_2 = \bar{I}_x + Ad^2 = \frac{1}{36}(2b)(h)^3 + \left(\frac{1}{2} \times 2b \times h\right)\left(\frac{4}{3}h\right)^2$$

$$= \frac{11}{6}bh^3$$

Then $\qquad I_{AA'} = \frac{2}{3}bh^3 + \frac{11}{6}bh^3 = \frac{5}{2}bh^3$

Finally, $\qquad y_P = \frac{\frac{5}{2}bh^3}{\frac{7}{3}bh^2}$

or $\qquad y_P = \frac{15}{14}h$ ◀

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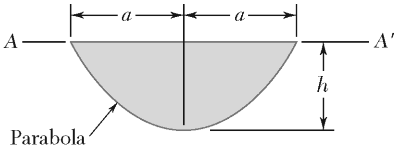


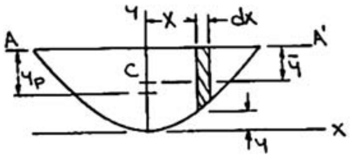
PROBLEM 9.60*

The panel shown forms the end of a trough that is filled with water to the line AA'. Referring to Section 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION

Using the equation developed on page 491 of the text:

$$y_P = \frac{I_{AA'}}{\bar{y}A}$$



For a parabola: $\bar{y} = \frac{2}{5}h \qquad A = \frac{4}{3}ah$

Now $dI_{AA'} = \frac{1}{3}(h-y)^3\,dx$

By observation $y = \frac{h}{a^2}x^2$

So that

$$dI_{AA'} = \frac{1}{3}\left(h - \frac{h}{a^2}x^2\right)^3 dx = \frac{1}{3}\frac{h^3}{a^6}(a^2 - x^2)^3\,dx$$

$$= \frac{1}{3}\frac{h}{a^6}(a^6 - 3a^4x^2 + 3a^2x^4 - x^6)\,dx$$

Then

$$I_{AA'} = 2\int_0^a \frac{1}{3}\frac{h^3}{a^6}(a^6 - 3a^4x^2 + 3a^2x^4 - x^6)\,dx$$

$$= \frac{2}{3}\frac{h^3}{a^6}\left[a^6x - a^4x^3 + \frac{3}{5}a^2x^5 - \frac{1}{7}x^7\right]_0^a$$

$$= \frac{32}{105}ah^3$$

Finally,

$$y_p = \frac{\frac{32}{105}ah^3}{\frac{2}{5}h \times \frac{4}{3}ah}$$

or $\qquad y_P = \frac{4}{7}h$ ◀

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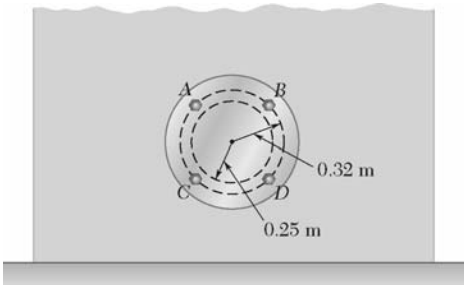
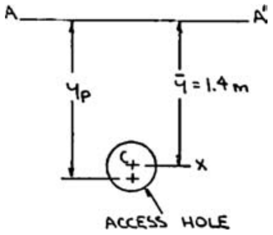


PROBLEM 9.61

The cover for a 0.5-m-diameter access hole in a water storage tank is attached to the tank with four equally spaced bolts as shown. Determine the additional force on each bolt due to the water pressure when the center of the cover is located 1.4 m below the water surface.

SOLUTION

From Section 9.2:



$$R = \gamma \bar{y} A \qquad y_P = \frac{I_{AA'}}{\bar{y}A}$$

where R is the resultant of the hydrostatic forces acting on the cover and y_P is the depth to the point of application of R.

Recalling that $\gamma = p \cdot y,$ we have

$$R = (10^3 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2)(1.4 \text{ m})[\pi(0.25 \text{ m})^2]$$
$$= 2696.67 \text{ N}$$

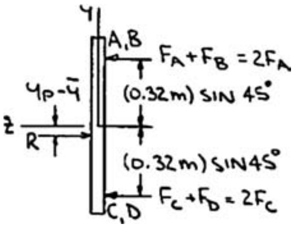
Also
$$I_{AA'} = \bar{I}_x + A\bar{y}^2 = \frac{\pi}{4}(0.25 \text{ m})^4 + [\pi(0.25 \text{ m})^2](1.4 \text{ m})^2$$
$$= 0.387913 \text{ m}^4$$

Then
$$y_P = \frac{0.387913 \text{ m}^4}{(1.4 \text{ m})[\pi(0.25 \text{ m})^2]} = 1.41116 \text{ m}$$

Now note that symmetry implies

$$F_A = F_B \qquad F_C = F_D$$

Next consider the free-body of the cover.



We have $+\circlearrowleft \Sigma M_{CD} = 0$: $[2(0.32 \text{ m}) \sin 45°](2F_A)$
$-[0.32 \sin 45° - (1.41116 - 1.4)] \text{ m}$
$\times (2696.67 \text{ N}) = 0$

or
$$F_A = 640.92 \text{ N}$$

Then $\overset{+}{\longleftarrow} \Sigma F_z = 0$: $2(640.92 \text{ N}) + 2F_C - 2696.67 \text{ N} = 0$

or
$$F_C = 707.42 \text{ N}$$

$$F_A = F_B = 641 \text{ N} \blacktriangleleft$$

and
$$F_C = F_D = 707 \text{ N} \blacktriangleleft$$

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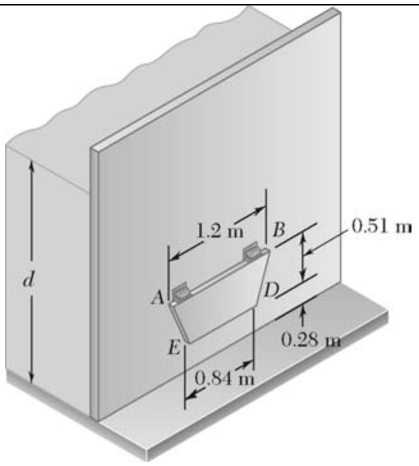
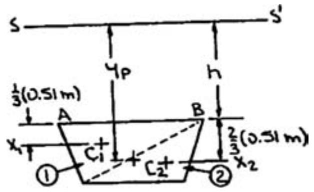


PROBLEM 9.62

A vertical trapezoidal gate that is used as an automatic valve is held shut by two springs attached to hinges located along edge *AB*. Knowing that each spring exerts a couple of magnitude 1470 N · m, determine the depth *d* of water for which the gate will open.

SOLUTION



From Section 9.2:

$$R = \gamma \bar{y} A \qquad y_P = \frac{I_{SS'}}{\bar{y}A}$$

where *R* is the resultant of the hydrostatic forces acting on the gate and y_P is the depth to the point of application of *R*. Now

$$\bar{y}A = \Sigma \bar{y}A = [(h+0.17)\text{ m}]\left(\frac{1}{2}\times 1.2\text{ m}\times 0.51\text{ m}\right) + [(h+0.34)\text{ m}]\left(\frac{1}{2}\times 0.84\times 0.51\text{ m}\right)$$

$$= (0.5202h + 0.124848)\text{ m}^3$$

Recalling that $\gamma = py$, we have

$$R = (10^3\text{ kg/m}^3 \times 9.81\text{ m/s}^2)(0.5202h + 0.124848)\text{ m}^3$$

$$= 5103.162(h+0.24)\text{ N}$$

Also $$I_{SS'} = (I_{SS'})_1 + (I_{SS'})_2$$

where $$(I_{SS'})_1 = I_x + Ad^2$$

$$= \frac{1}{36}(1.2\text{ m})(0.51\text{ m})^3 + \left(\frac{1}{2}\times 1.2\text{ m}\times 0.51\text{ m}\right)[(h+0.17)\text{ m}]^2$$

$$= [0.0044217 + 0.306(h+0.17)^2]\text{ m}^4$$

$$= (0.306h^2 + 0.10404h + 0.0132651)\text{ m}^4$$

$$(I_{SS'})_2 = \bar{I}_{X_2} + Ad^2$$

$$= \frac{1}{36}(0.84\text{ m})(0.51\text{ m})^3 + \left(\frac{1}{2}\times 0.84\text{ m}\times 0.51\text{ m}\right)[(h+0.34)\text{ m}]^2$$

$$= [0.0030952 + 0.2142(h+0.34)^2]\text{ m}^4$$

$$= (0.2142h^2 + 0.145656h + 0.0278567)\text{ m}^4$$

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PROBLEM 9.62 (Continued)

Then $$I_{SS'} = (I_{SS'})_1 + (I_{SS'})_2$$
$$= (0.5202h^2 + 0.249696h + 0.0411218)\ \text{m}^4$$

and $$y_P = \frac{(0.5202h^2 + 0.244696h + 0.0411218)\ \text{m}^4}{(0.5202h + 0.124848)\ \text{m}^3}$$
$$= \frac{h^2 + 0.48h + 0.07905}{h + 0.24}\ \text{m}$$

For the gate to open, require that

$$\Sigma M_{AB}: \quad M_{\text{open}} = (y_P - h)R$$

Substituting $$2940\ \text{N} \cdot \text{m} = \left(\frac{h^2 + 0.48h + 0.07905}{h + 0.24} - h \right) \text{m} \times 5103.162(h + 0.24)\ \text{N}$$

or $$5103.162(0.24h + 0.07905) = 2940$$

or $$h = 2.0711\ \text{m}$$

Then $$d = (2.0711 + 0.79)\ \text{m}$$

or $d = 2.86\ \text{m}$ ◀

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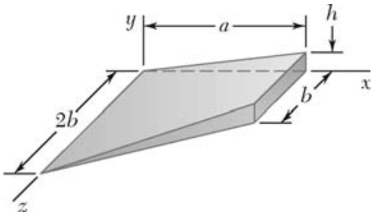
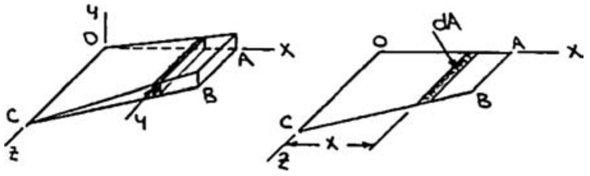


PROBLEM 9.63*

Determine the x coordinate of the centroid of the volume shown. (*Hint:* The height y of the volume is proportional to the x coordinate; consider an analogy between this height and the water pressure on a submerged surface.)

SOLUTION



First note that $$y = \frac{h}{a}x$$

Now $$\bar{x}\int dV = \int \bar{x}_{EL}\, dV$$

where $$\bar{x}_{EL} = x \quad dV = y\,dA = \left(\frac{h}{a}x\right)dA$$

Then $$\bar{x} = \frac{\int x\left(\frac{h}{a}x\,dA\right)}{\int \frac{h}{a}x\,dA} = \frac{\int x^2 dA}{\int x\,dA} = \frac{(I_z)_A}{(\bar{x}A)_A}$$

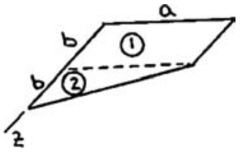
where $(I_z)_A$ and $(\bar{x}_A)_A$ pertain to area.

OABC: $(I_z)_A$ is the moment of inertia of the area with respect to the z axis, $\bar{x}_A$ is the x coordinate of the centroid of the area, and A is the area of *OABC*. Then

$$(I_z)_A = (I_z)_{A_1} + (I_z)_{A_2}$$

$$= \frac{1}{3}(b)(a)^3 + \frac{1}{12}(b)(a)^3$$

$$= \frac{5}{12}a^3 b$$



and $$(\bar{x}A)_A = \Sigma\bar{x}A$$

$$= \left[\left(\frac{a}{2}\right)(a\times b)\right] + \left[\left(\frac{a}{3}\right)\left(\frac{1}{2}\times a\times b\right)\right]$$

$$= \frac{2}{3}a^2 b$$

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PROBLEM 9.63* (Continued)

Finally, $\bar{x} = \dfrac{\frac{5}{12}a^3b}{\frac{2}{3}a^2b}$

or $\bar{x} = \dfrac{5}{8}a$ ◀

Analogy with hydrostatic pressure on a submerged plate:

Recalling that $P = \gamma y,$ it follows that the following analogies can be established.

Height $y \sim P$

$$dV = y\,dA \sim p\,dA = dF$$
$$xdV = x(y\,dA) \sim y\,dF = dM$$

Recalling that $y_P = \dfrac{M_x}{R}\left(= \dfrac{\int dM}{\int dF}\right)$

It can then be concluded that $x \sim y_P$

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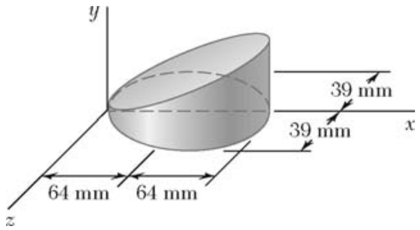

PROBLEM 9.64*

Determine the *x* coordinate of the centroid of the volume shown; this volume was obtained by intersecting an elliptic cylinder with an oblique plane. (*Hint:* The height *y* of the volume is proportional to the *x* coordinate; consider an analogy between this height and the water pressure on a submerged surface.)

SOLUTION

Following the "Hint," it can be shown that (see solution to Problem 9.63)

$$\bar{x} = \frac{(I_z)_A}{(\bar{x}A)_A} x$$

where $(I_z)_A$ and $(\bar{x}A)_A$ are the moment of inertia and the first moment of the area, respectively, of the elliptical area of the base of the volume with respect to the *z* axis. Then

$$(I_z)_A = \bar{I}_z + Ad^2$$
$$= \frac{\pi}{4}(39\text{ mm})(64\text{ mm})^3 + [\pi(64\text{ mm})(39\text{ mm})](64\text{ mm})^2$$
$$= 12.779520\pi \times 10^6\text{ mm}^4$$

$$(\bar{x}A)_A = (64\text{ mm})[\pi(64\text{ mm})(39\text{ mm})]$$
$$= 0.159744\pi \times 10^6\text{ mm}^3$$

Finally
$$\bar{x} = \frac{12.779520\pi \times 10^6\text{ mm}^4}{0.159744\pi \times 10^6\text{ mm}^4}$$

or $\bar{x} = 80.0\text{ mm}$ ◀

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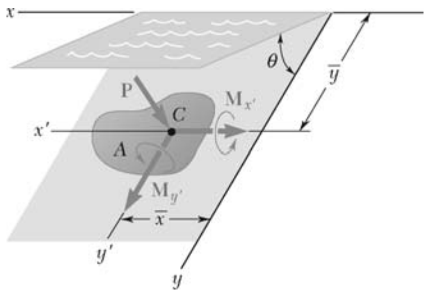
PROBLEM 9.65*

Show that the system of hydrostatic forces acting on a submerged plane area A can be reduced to a force $\mathbf{P}$ at the centroid C of the area and two couples. The force $\mathbf{P}$ is perpendicular to the area and is of magnitude $P = \gamma A\bar{y}\sin\theta$, where γ is the specific weight of the liquid, and the couples are $\mathbf{M}_{x'} = (\gamma\bar{I}_{x'}\sin\theta)\mathbf{i}$ and $\mathbf{M}_{y'} = (\gamma\bar{I}_{x'y'}\sin\theta)\mathbf{j}$, where $\bar{I}_{x'y'} = \int x'y'\,dA$ (see Section 9.8). Note that the couples are independent of the depth at which the area is submerged.



SOLUTION

The pressure p at an arbitrary depth $(y\sin\theta)$ is

$$p = \gamma(y\sin\theta)$$

so that the hydrostatic force dF exerted on an infinitesimal area dA is

$$dF = (\gamma y\sin\theta)dA$$

Equivalence of the force $\mathbf{P}$ and the system of infinitesimal forces dF requires

$$\Sigma F: \quad P = \int dF = \int \gamma y\sin\theta\,dA = \gamma\sin\theta\int y\,dA$$

or $\quad P = \gamma A\bar{y}\sin\theta$ ◀

Equivalence of the force and couple $(\mathbf{P},\ \mathbf{M}_{x'} + \mathbf{M}_{y'})$ and the system of infinitesimal hydrostatic forces requires

$$\Sigma M_x: \quad -\bar{y}P - M_{x'} = \int(-y\,dF)$$

Now

$$-\int y\,dF = -\int y(\gamma y\sin\theta)dA = -\gamma\sin\theta\int y^2 dA$$
$$= -(\gamma\sin\theta)I_x$$

Then

$$-\bar{y}P - M_{x'} = -(\gamma\sin\theta)I_x$$

or

$$M_{x'} = (\gamma\sin\theta)I_x - \bar{y}(\gamma A\bar{y}\sin\theta)$$
$$= \gamma\sin\theta(I_x - A\bar{y}^2)$$

or $\quad M_{x'} = \gamma\bar{I}_{x'}\sin\theta$ ◀

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PROBLEM 9.65* (Continued)

$$\Sigma M_y: \quad \bar{x}P + M_{y'} = \int x\,dF$$

Now $$\int x\,dF = \int x(\gamma y \sin\theta)dA = \gamma \sin\theta \int xy\,dA$$

$$= (\gamma \sin\theta) I_{xy} \qquad \text{(Equation 9.12)}$$

Then $$\bar{x}P + M_{y'} = (\gamma \sin\theta) I_{xy}$$

or $$M_{y'} = (\gamma \sin\theta) I_{xy} - \bar{x}(\gamma A \bar{y} \sin\theta)$$

$$= \gamma \sin\theta (I_{xy} - A\bar{x}\,\bar{y})$$

or $$M_{y'} = \gamma \bar{I}_{x'y'} \sin\theta \quad ◀$$

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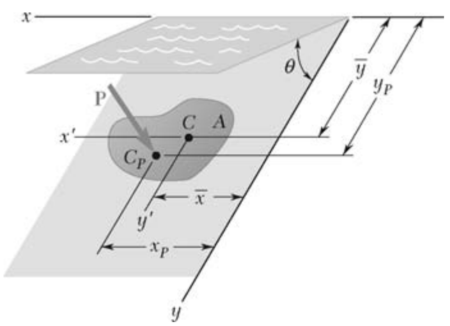

PROBLEM 9.66*

Show that the resultant of the hydrostatic forces acting on a submerged plane area A is a force **P** perpendicular to the area and of magnitude $P = \gamma A\overline{y}\sin\theta = \overline{p}A$, where γ is the specific weight of the liquid and $\overline{p}$ is the pressure at the centroid C of the area. Show that **P** is applied at a Point C_P, called the center of pressure, whose coordinates are $x_P = I_{xy}/A\overline{y}$ and $y_P = I_x/A\overline{y}$, where $\overline{I}_{xy} = \int xy\,dA$ (see Section 9.8). Show also that the difference of ordinates $y_P - \overline{y}$ is equal to $\overline{k}_{x'}^2/\overline{y}$ and thus depends upon the depth at which the area is submerged.

SOLUTION

The pressure P at an arbitrary depth $(y\sin\theta)$ is

$$P = \gamma(y\sin\theta)$$

so that the hydrostatic force dP exerted on an infinitesimal area dA is

$$dP = (\gamma y\sin\theta)dA$$

The magnitude **P** of the resultant force acting on the plane area is then

$$P = \int dP = \int \gamma y\sin\theta\,dA = \gamma\sin\theta\int y\,dA$$

$$= \gamma\sin\theta(\overline{y}A)$$

Now $\quad \overline{p} = \gamma\overline{y}\sin\theta \qquad P = \overline{p}A$ ◀

Next observe that the resultant **P** is equivalent to the system of infinitesimal forces d**P**. Equivalence then requires

$$\Sigma M_x:\quad -y_P P = -\int y\,dP$$

Now $\quad \int y\,dP = \int y(\gamma y\sin\theta)dA = \gamma\sin\theta\int y^2\,dA$

$$= (\gamma\sin\theta)I_x$$

Then $\quad y_P P = (\gamma\sin\theta)I_x$

or $\quad y_P = \dfrac{(\gamma\sin\theta)I_x}{\gamma\sin\theta(\overline{y}A)}$

or $\quad y_P = \dfrac{I_x}{A\overline{y}}$ ◀

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PROBLEM 9.66* (Continued)

$$\Sigma M_y\text{:}\quad x_P P = \int x\,dP$$

Now $$\int x\,dP = \int x(\gamma y \sin\theta)dA = \gamma \sin\theta \int xy\,dA$$

$$= (\gamma \sin\theta) I_{xy} \qquad \text{(Equation 9.12)}$$

Then $$x_P P = (\gamma \sin\theta) I_{xy}$$

or $$x_P = \frac{(\gamma \sin\theta) I_{xy}}{\gamma \sin\theta(\bar{y}A)}$$

or $$x_P = \frac{I_{xy}}{A\bar{y}} \blacktriangleleft$$

Now $$I_x = \bar{I}_{x'} + A\bar{y}^2$$

From above $$I_x = (A\bar{y}) y_P$$

By definition $$\bar{I}_{x'} = \bar{k}_{x'}^2 A$$

Substituting $$(A\bar{y}) y_P = \bar{k}_{x'}^2 A + A\bar{y}^2$$

Rearranging yields $$y_P - \bar{y} = \frac{\bar{k}_{x'}^2}{\bar{y}} \blacktriangleleft$$

Although $\bar{k}_{x'}$ is not a function of the depth of the area (it depends only on the shape of A), $\bar{y}$ is dependent on the depth.

$$(y_P - \bar{y}) = f\,(\text{depth})$$

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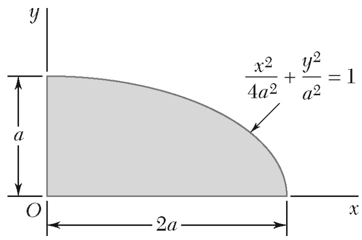
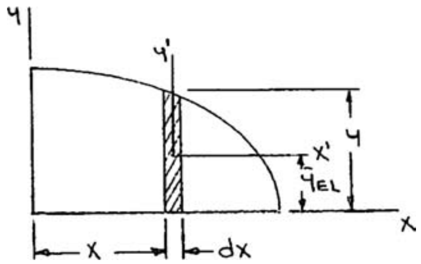


PROBLEM 9.67

Determine by direct integration the product of inertia of the given area with respect to the *x* and *y* axes.

SOLUTION



First note
$$y = a\sqrt{1-\frac{x^2}{4a^2}}$$
$$=\frac{1}{2}\sqrt{4a^2-x^2}$$

We have
$$dI_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}dA$$

where
$$d\bar{I}_{x'y'} = 0 \quad \text{(symmetry)} \quad \bar{x}_{EL} = x$$
$$\bar{y}_{EL} = \frac{1}{2}y = \frac{1}{4}\sqrt{4a^2-x^2}$$
$$dA = ydx = \frac{1}{2}\sqrt{4a^2-x^2dx}$$

Then
$$I_{xy} = \int dI_{xy} = \int_0^{2a} x\left(\frac{1}{4}\sqrt{4a^2-x^2}\right)\left(\frac{1}{2}\sqrt{4a^2-x^2}\right)dx$$
$$=\frac{1}{8}\int_0^{2a}(4a^2x - x^3)dx = \frac{1}{8}\left[2a^2x^2 - \frac{1}{4}x^4\right]_0^{2a}$$
$$=\frac{a^4}{8}\left[2(2)^2 - \frac{1}{4}(2)^4\right]$$

or $\quad I_{xy} = \frac{1}{2}a^4$ ◀

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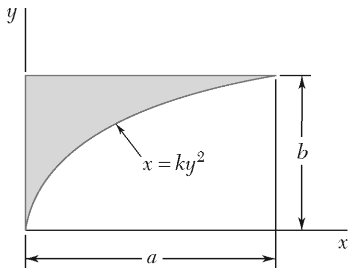
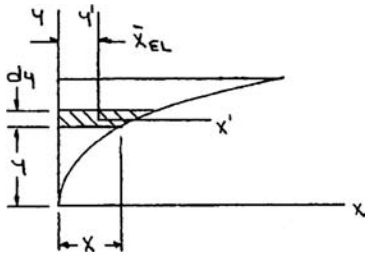


PROBLEM 9.68

Determine by direct integration the product of inertia of the given area with respect to the x and y axes.

SOLUTION



At $\quad x = a, \; y = b: \quad a = kb^2$

or $\quad k = \dfrac{a}{b^2}$

Then $\quad x = \dfrac{a}{b^2} y^2$

We have $\quad dI_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}dA$

where $\quad d\bar{I}_{x'y'} = 0 \quad \text{(symmetry)} \quad \bar{x}_{EL} = \dfrac{1}{2}x = \dfrac{a}{2b^2}y^2$

$$\bar{y}_{EL} = y \quad dA = x\,dy = \frac{a}{b^2}y^2 dy$$

Then
$$I_{xy} = \int dI_{xy} = \int_0^b \left(\frac{a}{2b^2}y^2\right)(y)\left(\frac{a}{b^2}y^2 dy\right)$$

$$= \frac{a^2}{2b^4}\int_0^b y^5 dy = \frac{a^2}{2b^4}\left[\frac{1}{6}y^6\right]_0^b$$

or $\quad I_{xy} = \dfrac{1}{12}a^2b^2$ ◀

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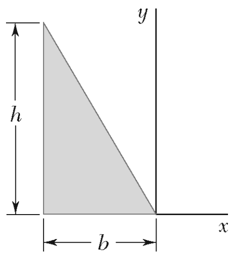
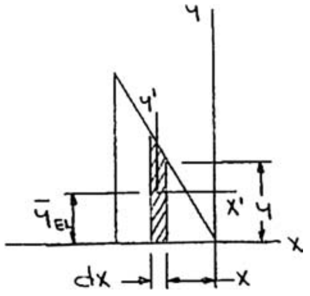


PROBLEM 9.69

Determine by direct integration the product of inertia of the given area with respect to the *x* and *y* axes.

SOLUTION



First note that

$$y = -\frac{h}{b}x$$

Now $$dI_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}\,dA$$

where $$d\bar{I}_{x'y'} = 0 \quad \text{(symmetry)}$$

$$\bar{x}_{EL} = x \qquad \bar{y}_{EL} = \frac{1}{2}y = -\frac{1}{2}\frac{h}{b}x$$

$$dA = y\,dx = -\frac{h}{b}x\,dx$$

Then $$I_{xy} = \int dI_{xy} = \int_{-b}^{0} x\left(-\frac{1}{2}\frac{h}{b}x\right)\left(-\frac{h}{b}x\,dx\right)$$

$$= \frac{1}{2}\frac{h^2}{b^2}\int_{-b}^{0} x^3 dx = \frac{1}{2}\frac{h^2}{b^2}\left[\frac{1}{4}x^4\right]_{-b}^{0}$$

or $$I_{xy} = -\frac{1}{8}b^2h^2 \blacktriangleleft$$

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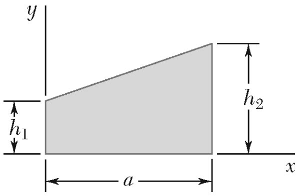
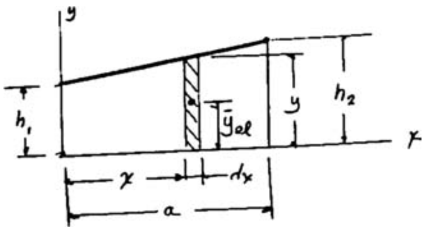


PROBLEM 9.70

Determine by direct integration the product of inertia of the given area with respect to the *x* and *y* axes.

SOLUTION



$$y = h_1 + (h_2 - h_1)\frac{x}{a}$$

$$dI_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}\,dA$$

$$d\bar{I}_{x'y'} = 0 \quad \text{(by symmetry)}$$

$$\bar{x}_{EL} = x \qquad \bar{y}_{EL} = \frac{1}{2}y \qquad dA = y\,dx$$

$$I_{xy} = \int dI_{xy} - \int_0^a x\left(\frac{1}{2}y\right)y\,dx = \frac{1}{2}\int_0^a xy^2 dx$$

$$= \frac{1}{2}\int_0^a x\left[h_1 + (h_2 - h_1)\frac{x}{a}\right]^2 dx$$

$$= \frac{1}{2}\int_0^a \left[h_1^2 x + 2h_1(h_2 - h_1)\frac{x^2}{a} + (h_2 - h_1)^2\frac{x^3}{a^2}\right]dx$$

$$= \frac{1}{2}\left[h_1^2\frac{a^2}{2} + 2h_1(h_2 - h_1)\frac{a^3}{3a} + (h_2 - h_1)^2\frac{a^4}{4a^2}\right]$$

$$= \frac{1}{2}\left[h_1^2\frac{a^2}{2} + \frac{2}{3}h_1h_2a^2 - \frac{2}{3}h_1^2a^2 + \frac{1}{4}h_2^2a^2 - \frac{1}{2}h_2h_1a + \frac{1}{4}h_1^2a^2\right]$$

$$= \frac{a^2}{2}\left[h_1^2\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right) + h_1h_2\left(\frac{2}{3} - \frac{1}{2}\right) + h_2^2\left(\frac{1}{4}\right)\right]$$

$$= \frac{a^2}{2}\left[h_1^2\left(\frac{1}{12}\right) + h_1h_2\left(\frac{1}{6}\right) + h_2^2\left(\frac{1}{4}\right)\right]$$

$$I_{xy} = \frac{a^2}{24}\left(h_1^2 + 2h_1h_2 + 3h_2^2\right) \blacktriangleleft$$

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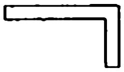
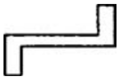
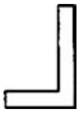
PROBLEM 9.70 (Continued)

The following table is provided for the convenience of the instructor, as many problems in this and the next lesson are related.

Type of Problem						
Compute I_x and I_y	Figure 9.12			Figure 9.13B		Figure 9.13A
Compute I_{xy}	9.67	9.72	9.73	9.74	9.75	9.78
$I_{x'}, I_{y'}, I_{x'y'}$ by equations	9.79	9.80	9.81	9.83	9.82	9.84
Principal axes by equations	9.85	9.86	*9.87*	9.89	9.88	9.90
$I_{x'}, I_{y'}, I_{x'y'}$ by Mohr's circle	9.91	9.92	*9.93*	9.95	9.94	9.96
Principal axes by Mohr's circle	9.97	9.98	9.100	9.101	9.103	9.106

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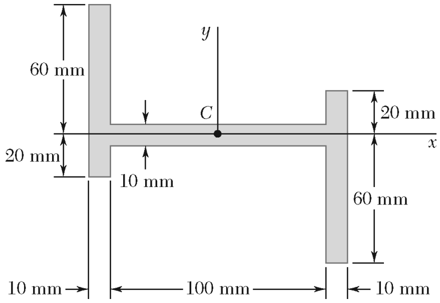
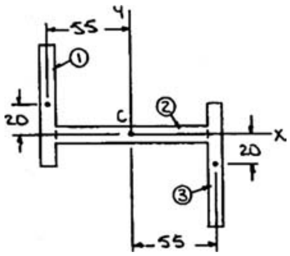


PROBLEM 9.71

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

SOLUTION

Dimensions in mm



We have $\bar{I}_{xy} = (I_{xy})_1 + (\bar{I}_{xy})_2 + (I_{xy})_3$

Now symmetry implies

$$(\bar{I}_{xy})_2 = 0$$

and for the other rectangles

$$I_{xy} = \bar{I}_{x'y'} + \bar{x}\bar{y}A$$

where $I_{x'y'} = 0$ (symmetry)

Thus $\bar{I}_{xy} = (\bar{x}\bar{y}A)_1 + (\bar{x}\bar{y}A)_3$

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}\bar{y}A$, mm^4
1	$10 \times 80 = 800$	–55	20	–880,000
3	$10 \times 80 = 800$	55	–20	–880,000
Σ				–1,760,000

$\bar{I}_{xy} = -1.760 \times 10^6 \text{ mm}^4$ ◀

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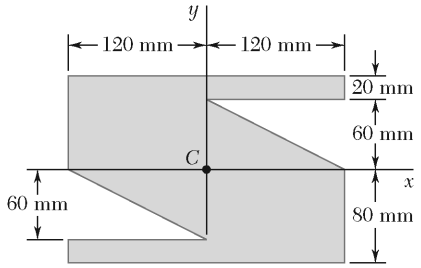
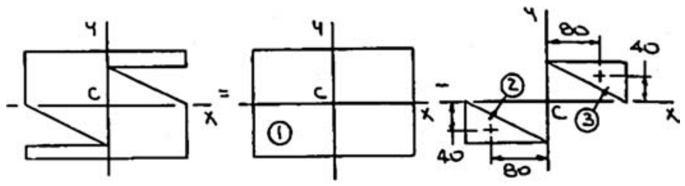


PROBLEM 9.72

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal *x* and *y* axes.

SOLUTION



Dimensions in mm

We have $\quad \bar{I}_{xy} = (\bar{I}_{xy})_1 - (I_{xy})_2 - (I_{xy})_3$

Now symmetry implies $\quad (I_{xy})_1 = 0$

and for each triangle $\quad I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$

where, using the results of Sample Problem 9.6, $\bar{I}_{x'y'} = -\frac{1}{72}b^2h^2$ for both triangles. Note that the sign of $\bar{I}_{x'y'}$ is unchanged because the angles of rotation are $0°$ and $180°$ for triangles 2 and 3, respectively. Now

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}\,\bar{y}A$, mm^4
2	$\frac{1}{2}(120)(60) = 3600$	-80	-40	11.520×10^6
3	$\frac{1}{2}(120)(60) = 3600$	80	40	11.520×10^6
Σ				23.040×10^6

Then $\quad \bar{I}_{xy} = -\left\{2\left[-\frac{1}{72}(120\text{ mm})^2(60\text{ mm})^2\right] + 23.040 \times 10^6\text{ mm}^4\right\}$

or $\quad \bar{I}_{xy} = -21.6 \times 10^6\text{ mm}^4$ ◀

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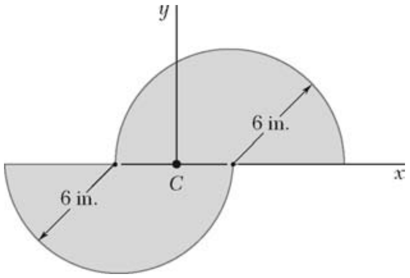
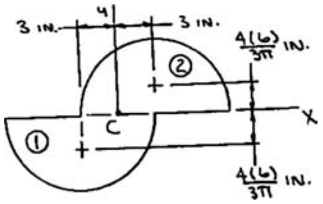


PROBLEM 9.73

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal *x* and *y* axes.

SOLUTION



We have $\bar{I}_{xy} = (I_{xy})_1 + (I_{xy})_2$

For each semicircle $I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$

and $I_{x'y'} = 0 \quad \text{(symmetry)}$

Thus $\bar{I}_{xy} = \Sigma \bar{x}\,\bar{y}A$

	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}\bar{y}A$
1	$\frac{\pi}{2}(6)^2 = 18\pi$	–3	$-\frac{8}{\pi}$	432
2	$\frac{\pi}{2}(6)^2 = 18\pi$	3	$\frac{8}{\pi}$	432
Σ				864

$\bar{I}_{xy} = 864 \text{ in.}^4$ ◀

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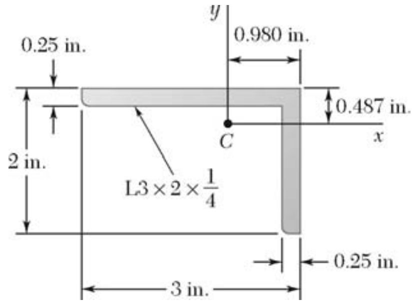
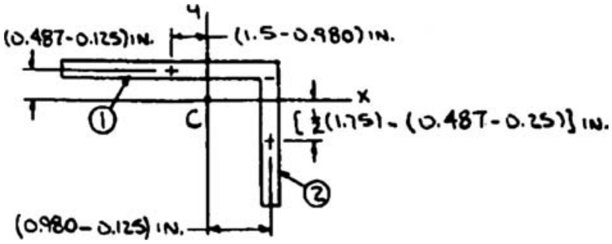


PROBLEM 9.74

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal *x* and *y* axes.

SOLUTION



We have $\bar{I}_{xy} = (I_{xy})_1 + (I_{xy})_2$

For each rectangle

$$I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$$

and $\bar{I}_{x'y'} = 0$ (symmetry)

Thus $\bar{I}_{xy} = \Sigma\bar{x}\,\bar{y}A$

	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}\bar{y}A$, in.^{4}
1	$3\times0.25 = 0.75$	–0.520	0.362	–0.141180
2	$0.25\times1.75 = 0.4375$	0.855	–0.638	–0.238652
Σ				–0.379832

$\bar{I}_{xy} = -0.380$ in.^{4} ◀

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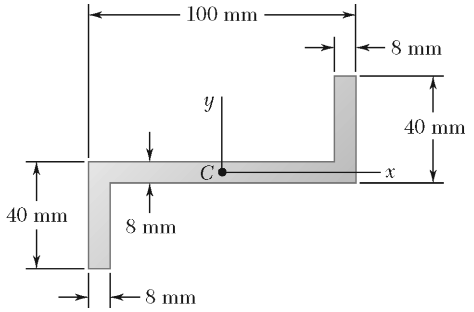
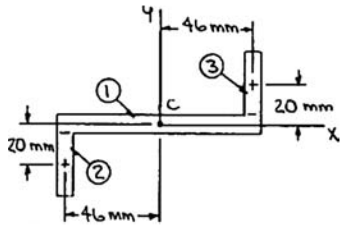


PROBLEM 9.75

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal *x* and *y* axes.

SOLUTION



We have $I_{xy} = (\bar{I}_{xy})_1 + (I_{xy})_2 + (I_{xy})_3$

Now symmetry implies $(\bar{I}_{xy})_1 = 0$

and for the other rectangles

$$I_{xy} = \bar{I}_{x'y'} + \bar{x}\bar{y}A$$

where $\bar{I}_{x'y'} = 0$ (symmetry)

Thus $\bar{I}_{xy} = (\bar{x}\bar{y}A)_2 + (\bar{x}\bar{y}A)_3$

	A, mm^2	$\bar{X}$, mm	$\bar{y}$, mm	$\bar{x}\bar{y}A$, mm^4
2	$8\times32=256$	–46	–20	235,520
3	$8\times32=256$	46	20	235,520
Σ				471,040

$\bar{I}_{xy} = 471\times10^3$ mm^4 ◀

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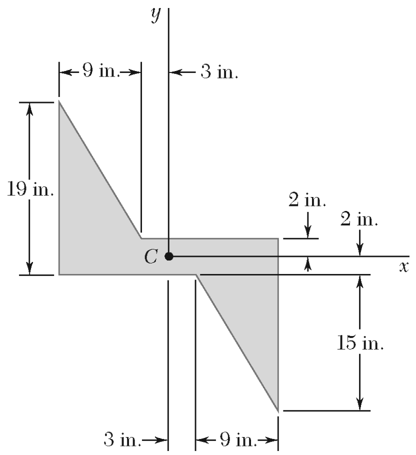
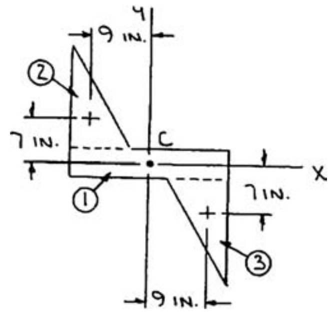


PROBLEM 9.76

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal *x* and *y* axes.

SOLUTION



We have $\bar{I}_{xy} = (\bar{I}_{xy})_1 + (I_{xy})_2 + (I_{xy})_3$

Now, symmetry implies

$$(\bar{I}_{xy})_1 = 0$$

and for each triangle $I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$

where, using the results of Sample Problem 9.6, $\bar{I}_{x'y'} = -\frac{1}{72}b^2h^2$ for both triangles. Note that the sign of $\bar{I}_{x'y'}$ is unchanged because the angles of rotation are $0°$ and $180°$ for triangles 2 and 3, respectively.

Now

	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}\,\bar{y}A$, in.^{4}
2	$\frac{1}{2}(9)(15) = 67.5$	–9	7	–4252.5
3	$\frac{1}{2}(9)(15) = 67.5$	9	–7	–4252.5
Σ				–8505

Then
$$\bar{I}_{xy} = 2\left[-\frac{1}{72}(9\text{ in.})^2\,(15\text{ in.})^2\right] - 8505\text{ in.}^4$$
$$= -9011.25\text{ in.}^4$$

or $\bar{I}_{xy} = -9010\text{ in.}^4$ ◀

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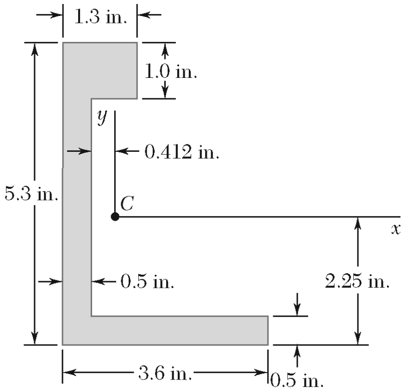
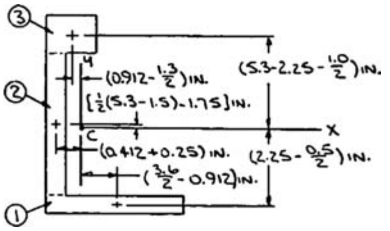


PROBLEM 9.77

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

SOLUTION



We have $\quad \bar{I}_{xy} = (\bar{I}_{xy})_1 + (I_{xy})_2 + (I_{xy})_3$

For each rectangle $\quad I_{xy} = \bar{I}_{x'y'} + \bar{x}\bar{y}A$

and $\quad \bar{I}_{x'y'} = 0 \;$ (symmetry)

Thus $\quad I_{xy} = \Sigma \bar{x}\bar{y}A$

	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}\bar{y}A$, in.^{4}
1	$3.6 \times 0.5 = 1.8$	0.888	–2.00	–3.196
2	$0.5 \times 3.8 = 1.9$	–0.662	0.15	–0.18867
3	$1.3 \times 1.0 = 1.3$	–0.262	2.55	–0.86853
Σ				–4.25320

$\bar{I}_{xy} = -4.25 \text{ in.}^4$ ◀

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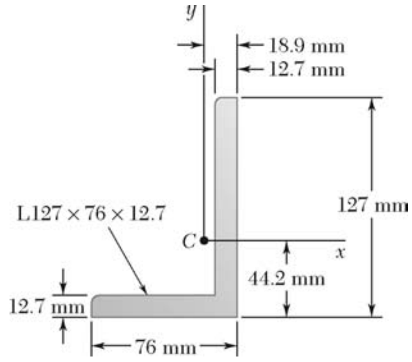
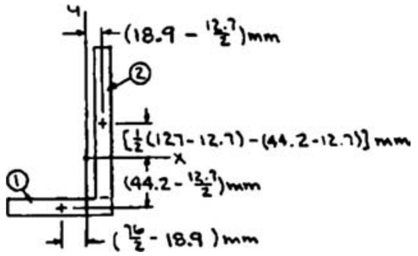


PROBLEM 9.78

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

SOLUTION



We have $\bar{I}_{xy} = (I_{xy})_1 + (I_{xy})_2$

For each rectangle $I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$

and $\bar{I}_{x'y'} = 0$ (symmetry)

Thus $I_{xy} = \Sigma\bar{x}\,\bar{y}A$

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}\,\bar{y}A$, mm^4
1	$76\times 12.7 = 965.2$	–19.1	–37.85	697,777
2	$12.7\times(127-12.7) = 1451.61$	12.55	25.65	467,284
Σ				1,165,061

$$\bar{I}_{xy} = 1.165\times 10^6\ \text{mm}^4 \ \blacktriangleleft$$

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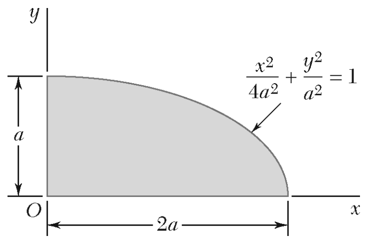
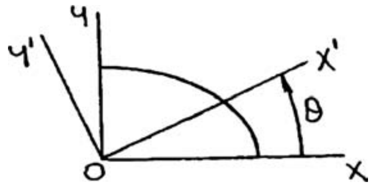


PROBLEM 9.79

Determine for the quarter ellipse of Problem 9.67 the moments of inertia and the product of inertia with respect to new axes obtained by rotating the *x* and *y* axes about *O* (*a*) through 45° counterclockwise, (*b*) through 30° clockwise.

SOLUTION



From Figure 9.12:

$$I_x = \frac{\pi}{16}(2a)(a)^3 = \frac{\pi}{8}a^4$$

$$I_y = \frac{\pi}{16}(2a)^3(a) = \frac{\pi}{2}a^4$$

From Problem 9.67:

$$I_{xy} = \frac{1}{2}a^4$$

First note

$$\frac{1}{2}(I_x + I_y) = \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right) = \frac{5}{16}\pi a^4$$

$$\frac{1}{2}(I_x - I_y) = \frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right) = -\frac{3}{16}\pi a^4$$

Now use Equations (9.18), (9.19), and (9.20).

Equation (9.18):

$$I_{x'} = \frac{1}{2}(I_x + I_y) + \frac{1}{2}(I_x - I_y)\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{5}{16}\pi a^4 - \frac{3}{16}\pi a^4 \cos 2\theta - \frac{1}{2}a^4 \sin 2\theta$$

Equation (9.19):

$$I_{y'} = \frac{1}{2}(I_x + I_y) - \frac{1}{2}(I_x - I_y)\cos 2\theta + I_{xy}\sin 2\theta$$

$$= \frac{5}{16}\pi a^4 + \frac{3}{16}\pi a^4 \cos 2\theta + \frac{1}{2}a^4 \sin 2\theta$$

Equation (9.20):

$$I_{x'y'} = \frac{1}{2}(I_x - I_y)\sin 2\theta + I_{xy}\cos 2\theta$$

$$= -\frac{3}{16}\pi a^4 \sin 2\theta + \frac{1}{2}a^4 \cos 2\theta$$

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PROBLEM 9.79 (Continued)

(*a*) $\theta = +45°$:

$$I_{x'} = \frac{5}{16}\pi a^4 - \frac{3}{16}\pi a^4 \cos 90° - \frac{1}{2}a^4 \sin 90°$$

or $I_{x'} = 0.482a^4$ ◀

$$I_{y'} = \frac{5}{16}\pi + \frac{3}{16}\pi a^4 \cos 90° + \frac{1}{2}a^4$$

or $I_{y'} = 1.482a^4$ ◀

$$I_{x'y'} = -\frac{3}{16}\pi a^4 \sin 90° + \frac{1}{2}a^4 \cos 90°$$

or $I_{x'y'} = -0.589a^4$ ◀

(*b*) $\theta = -30°$:

$$I_{x'} = \frac{5}{16}\pi a^4 - \frac{3}{16}\pi a^4 \cos(-60°) - \frac{1}{2}a^4 \sin(-60°)$$

or $I_{x'} = 1.120a^4$ ◀

$$I_{y'} = \frac{5}{16}\pi a^4 + \frac{3}{16}\pi a^4 \cos(-60°) + \frac{1}{2}a^4 \sin(-60°)$$

or $I_{y'} = 0.843a^4$ ◀

$$I_{x'y'} = -\frac{3}{16}\pi a^4 \sin(-60°) + \frac{1}{2}a^4 \cos(-60°)$$

or $I_{x'y'} = 0.760a^4$ ◀

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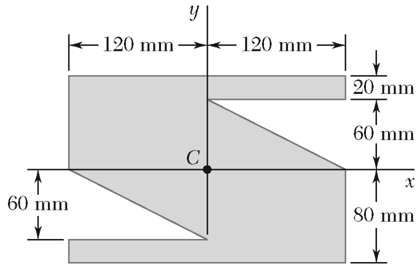
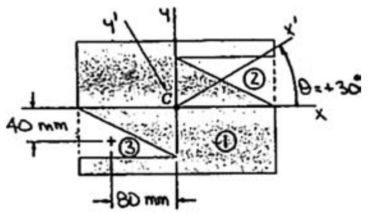


PROBLEM 9.80

Determine the moments of inertia and the product of inertia of the area of Problem 9.72 with respect to new centroidal axes obtained by rotating the *x* and *y* axes 30° counterclockwise.

SOLUTION



From Problem 9.72: $\bar{I}_{xy} = -21.6\times10^6\,\text{mm}^4$

Now $\bar{I}_x = (\bar{I}_x)_1 - (I_x)_2 - (I_x)_3$

where

$$(\bar{I}_x)_1 = \frac{1}{12}(240\text{ mm})(160\text{ mm})^3$$
$$= 81.920\times10^6\,\text{mm}^4$$

$$(I_x)_2 = (I_x)_3 = \frac{1}{36}(120\text{ mm})(60\text{ mm})^3 + \left[\frac{1}{2}(120\text{ mm})(60\text{ mm})\right](40\text{ mm})^2$$
$$= 6.480\times10^6\,\text{mm}^4$$

Then $\bar{I}_x = [81.920 - 2(6.480)]\times10^6\,\text{mm}^4 = 68.96\times10^6\,\text{mm}^4$

Also $\bar{I}_y = (\bar{I}_y)_1 - (I_y)_2 - (I_y)_3$

where $(\bar{I}_y)_1 = \dfrac{1}{12}(160\text{ mm})(240\text{ mm})^3 = 184.320\times10^6\,\text{mm}^4$

$$(I_y)_2 = (I_y)_3 = \frac{1}{36}(60\text{ mm})(120\text{ mm})^3 + \left[\frac{1}{2}(120\text{ mm})(60\text{ mm})\right](80\text{ mm})^2$$
$$= 25.920\times10^6\,\text{mm}^4$$

Then $\bar{I}_y = [184.320 - 2(25.920)]\times10^6\,\text{mm}^4 = 132.48\times10^6\,\text{mm}^4$

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PROBLEM 9.80 (Continued)

Now $$\frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(68.96 + 132.48)\times 10^6 = 100.72\times 10^6 \text{ mm}^4$$

$$\frac{1}{2}(\bar{I}_x - \bar{I}_y) = \frac{1}{2}(68.96 - 132.48)\times 10^6 = -31.76\times 10^6 \text{ mm}^4$$

Using Eqs. (9.18), (9.19), and (9.20):

Eq. (9.18): $$I_x = \frac{1}{2}(\bar{I}_x + \bar{I}_y) + \frac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta - \bar{I}_{xy}\sin 2\theta$$

$$= [100.72 + (-31.76)\cos 60° - (-21.6)\sin 60°]\times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'} = 103.5\times 10^6 \text{ mm}^4$ ◀

Eq. (9.19): $$\bar{I}_{y'} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) - \frac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$

$$= [100.72 - (-31.76)\cos 60° + (-21.6)\sin 60°]\times 10^6 \text{ mm}^4$$

or $\bar{I}_{y'} = 97.9\times 10^6 \text{ mm}^4$ ◀

Eq. (9.20): $$\bar{I}_{x'y'} = \frac{1}{2}(\bar{I}_x - \bar{I}_y)\sin 2\theta + \bar{I}_{xy}\sin 2\theta$$

$$= [-31.76\sin 60° + (-21.6)\cos 60°]\times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'y'} = -38.3\times 10^6 \text{ mm}^4$ ◀

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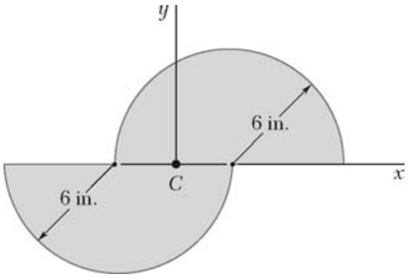


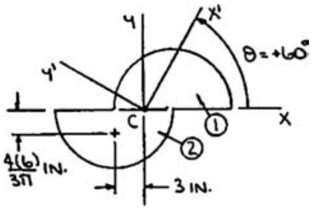
PROBLEM 9.81

Determine the moments of inertia and the product of inertia of the area of Problem 9.73 with respect to new centroidal axes obtained by rotating the *x* and *y* axes 60° counterclockwise.

SOLUTION

From Problem 9.73:

$$\bar{I}_{xy} = 864 \text{ in.}^4$$



Now $\quad \bar{I}_x = (I_x)_1 + (I_x)_2$

where $\quad (I_x)_1 = (I_x)_2 = \dfrac{\pi}{8}(6 \text{ in.})^4$

$$= 162\pi \text{ in.}^4$$

Then $\quad \bar{I}_x = 2(162\pi \text{ in.}^4) = 324\pi \text{ in.}^4$

Also $\quad \bar{I}_y = (I_y)_1 + (I_y)_2$

where $\quad (I_y)_1 = (I_y)_2 = \dfrac{\pi}{8}(6 \text{ in.})^4 + \left[\dfrac{\pi}{2}(6 \text{ in.})^2\right](3 \text{ in.})^2 = 324\pi \text{ in.}^4$

Then $\quad \bar{I}_y = 2(324\pi \text{ in.}^4) = 648\pi \text{ in.}^4$

Now $\quad \dfrac{1}{2}(\bar{I}_x + \bar{I}_y) = \dfrac{1}{2}(324\pi + 648\pi) = 486\pi \text{ in.}^4$

$$\frac{1}{2}(\bar{I}_x - \bar{I}_y) = \frac{1}{2}(324\pi - 648\pi) = -162\pi \text{ in.}^4$$

Using Eqs. (9.18), (9.19), and (9.20):

Eq. (9.18): $\quad \bar{I}_{x'} = \dfrac{1}{2}(\bar{I}_x + \bar{I}_y) + \dfrac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta - \bar{I}_{xy}\sin 2\theta$

$$= [486\pi + (-162\pi)\cos 120° - 864 \sin 120°] \text{ in.}^4$$

or $\quad \bar{I}_{x'} = 1033 \text{ in.}^4$ ◀

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PROBLEM 9.81 (Continued)

Eq. (9.19):

$$\bar{I}_{y'} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) - \frac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$

$$= [486\pi - (-162\pi)\cos 120° + 864\sin 120°]\text{ in.}^4$$

or $\bar{I}_{y'} = 2020\text{ in.}^4$ ◀

Eq. (9.20):

$$\bar{I}_{x'y'} = \frac{1}{2}(\bar{I}_x - \bar{I}_y)\sin 2\theta + \bar{I}_{xy}\cos 2\theta$$

$$= [(-162\pi)\sin 120° + 864\cos 120°]\text{ in.}^4$$

or $\bar{I}_{x'y'} = -873\text{ in.}^4$ ◀

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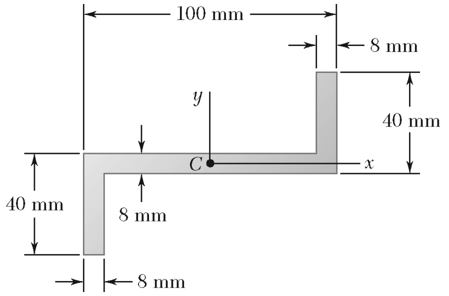
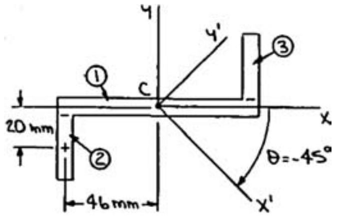


PROBLEM 9.82

Determine the moments of inertia and the product of inertia of the area of Problem 9.75 with respect to new centroidal axes obtained by rotating the x and y axes 45° clockwise.

SOLUTION

From Problem 9.75:



$$\bar{I}_{xy} = 471,040 \text{ mm}^4$$

Now $$\bar{I}_x = (\bar{I}_x)_1 + (I_x)_2 + (I_x)_3$$

where $$(\bar{I}_x)_1 = \frac{1}{12}(100 \text{ mm})(8 \text{ mm})^3 = 4266.67 \text{ mm}^4$$

$$(I_x)_2 = (I_x)_3 = \frac{1}{12}(8 \text{ mm})(32 \text{ mm})^3 + [(8 \text{ mm})(32 \text{ mm})](20 \text{ mm})^2 = 124,245.33$$

Then $$\bar{I}_x = [4266.67 + 2(124,245.33)] \text{ mm}^4 = 252,757 \text{ mm}^4$$

Also $$\bar{I}_y = (\bar{I}_y)_1 + (I_y)_2 + (I_y)_3$$

where $$(\bar{I}_y)_1 = \frac{1}{12}(8 \text{ mm})(100 \text{ mm})^3 = 666,666.7 \text{ mm}^4$$

$$(\bar{I}_y)_2 = (I_y)_3 = \frac{1}{12}(32 \text{ mm})(8 \text{ mm})^3 + [(8 \text{ mm})(32 \text{ mm})](46 \text{ mm})^2 = 543,061.3 \text{ mm}^4$$

Then $$\bar{I}_y = [666,666.7 + 2(543,061.3)] \text{ mm}^4 = 1,752,789 \text{ mm}^4$$

Now $$\frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(252,757 + 1,752,789) \text{ mm}^4 = 1,002,773 \text{ mm}^4$$

$$\frac{1}{2}(\bar{I}_x - \bar{I}_y) = \frac{1}{2}(252,757 - 1,752,789) \text{ mm}^4 = -750,016 \text{ mm}^4$$

Using Eqs. (9.18), (9.19), and (9.20):

Eq. (9.18): $$\bar{I}_{x'} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) + \frac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta - \bar{I}_{xy}\sin 2\theta$$

$$= [1,002,773 + (-750,016)\cos(-90°) - 471,040\sin(-90°)]$$

or $$\bar{I}_{x'} = 1.474 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

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PROBLEM 9.82 (Continued)

Eq. (9.19):
$$\bar{I}_{y'} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) - \frac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$
$$= [1,002,773 - (-750,016)\cos(-90°) + 471,040\sin(-90°)]$$

or $\bar{I}_{y'} = 0.532 \times 10^6 \text{ mm}^4$ ◀

Eq. (9.20):
$$\bar{I}_{x'y'} = \frac{1}{2}(\bar{I}_x - \bar{I}_y)\sin 2\theta + \bar{I}_{xy}\cos 2\theta$$
$$= [(-750,016)\sin(-90°) + 471,040\cos(-90°)]$$

or $\bar{I}_{x'y'} = 0.750 \times 10^6 \text{ mm}^4$ ◀

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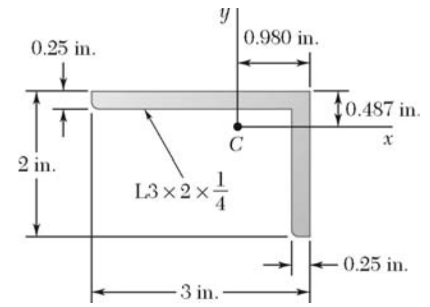


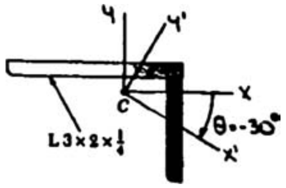
PROBLEM 9.83

Determine the moments of inertia and the product of inertia of the L3×2×$\frac{1}{4}$-in. angle cross section of Problem 9.74 with respect to new centroidal axes obtained by rotating the *x* and *y* axes 30° clockwise.

SOLUTION

From Figure 9.13:

$$\bar{I}_x = 0.390 \text{ in.}^4$$
$$\bar{I}_y = 1.09 \text{ in.}^4$$



From Problem 9.74:

$$\bar{I}_{xy} = -0.37983 \text{ in.}^4$$

Now
$$\frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(0.390 + 1.09) \text{ in.}^4 = 0.740 \text{ in.}^4$$
$$\frac{1}{2}(\bar{I}_x - \bar{I}_y) = \frac{1}{2}(0.390 - 1.09) \text{ in.}^4 = -0.350 \text{ in.}^4$$

Using Eqs. (9.18), (9.19), and (9.20):

Eq. (9.18):
$$\bar{I}_{x'} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) + \frac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta - \bar{I}_{xy}\sin 2\theta$$
$$= [0.740 + (-0.350)\cos(-60°) - (-0.37983)\sin(-60°)]$$

or $\bar{I}_{x'} = 0.236 \text{ in.}^4$ ◀

Eq. (9.19):
$$\bar{I}_{y'} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) - \frac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$
$$= [0.740 - (-0.350)\cos(-60°) + (-0.37983)\sin(-60°)]$$

or $\bar{I}_{y'} = 1.244 \text{ in.}^4$ ◀

Eq. (9.20):
$$\bar{I}_{x'y'} = \frac{1}{2}(\bar{I}_x - \bar{I}_y)\sin 2\theta + \bar{I}_{xy}\cos 2\theta$$
$$= [(-0.350)\sin(-60°) + (-0.37983)\cos(-60°)]$$

or $I_{x'y'} = 0.1132 \text{ in.}^4$ ◀

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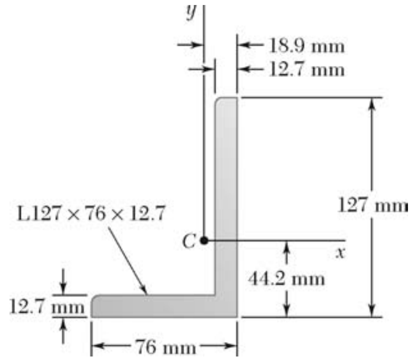


PROBLEM 9.84

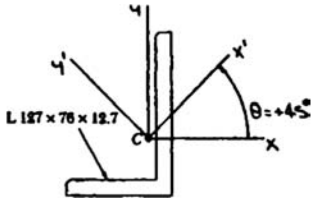
Determine the moments of inertia and the product of inertia of the L127×76×12.7-mm angle cross section of Problem 9.78 with respect to new centroidal axes obtained by rotating the *x* and *y* axes 45° counterclockwise.

SOLUTION

From Figure 9.13:

$$\bar{I}_x = 3.93\times10^6\,\text{mm}^4$$

$$\bar{I}_y = 1.06\times10^6\,\text{mm}^4$$



From Problem 9.78:

$$\bar{I}_{xy} = 1.165061\times10^6\,\text{mm}^4$$

Now $\qquad \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(3.93+1.06)\times10^6\ \text{mm}^4$

$$= 2.495\times10^6\,\text{mm}^4$$

$$\frac{1}{2}(\bar{I}_x - \bar{I}_y) = \frac{1}{2}(3.93-1.06)\times10^6\,\text{mm}^4 = 1.435\times10^6\,\text{mm}^4$$

Using Eqs. (9.18), (9.19), and (9.20):

Eq. (9.18): $\qquad \bar{I}_{x'} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) + \frac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta - \bar{I}_{xy}\sin 2\theta$

$$= [2.495 + 1.435\cos 90° - 1.165061\sin 90°]\times10^6\,\text{mm}^4$$

or $\qquad \bar{I}_{x'} = 1.330\times10^6\,\text{mm}^4$ ◀

Eq. (9.19): $\qquad \bar{I}_{y'} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) - \frac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta + \bar{I}_{xy}\sin 2\theta$

$$= [2.495 - 1.435\cos 90° + 1.165061\sin 90°]\times10^6\,\text{mm}^4$$

or $\qquad \bar{I}_{y'} = 3.66\times10^6\,\text{mm}^4$ ◀

Eq. (9.20): $\qquad \bar{I}_{x'y'} = \frac{1}{2}(\bar{I}_x - \bar{I}_y)\sin 2\theta + \bar{I}_{xy}\cos 2\theta$

$$= [(1.435\sin 90° + 1.165061\cos 90°]\times10^6\,\text{mm}^4$$

or $\qquad \bar{I}_{x'y'} = 1.435\times10^6\,\text{mm}^4$ ◀

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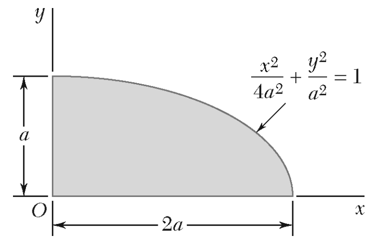


PROBLEM 9.85

For the quarter ellipse of Problem 9.67, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

SOLUTION

From Problem 9.79: $I_x = \frac{\pi}{8}a^4 \qquad I_y = \frac{\pi}{2}a^4$

Problem 9.67: $I_{xy} = \frac{1}{2}a^4$

Now Eq. (9.25):

$$\tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y} = -\frac{2\left(\frac{1}{2}a^4\right)}{\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4}$$

$$= \frac{8}{3\pi} = 0.84883$$

Then $2\theta_m = 40.326° \quad \text{and} \quad 220.326°$

or $\theta_m = 20.2° \text{ and } 110.2°$ ◀

Also Eq. (9.27):

$$I_{\max,\min} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$= \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right)$$

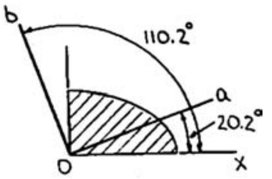
$$\pm\sqrt{\left[\frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right)\right]^2 + \left(\frac{1}{2}a^4\right)^2}$$

$$= (0.981,748 \pm 0.772,644)a^4$$

or $I_{\max} = 1.754a^4$ ◀

and $I_{\min} = 0.209a^4$ ◀

By inspection, the *a* axis corresponds to $I_{\min}$ and the *b* axis corresponds to $I_{\max}$.



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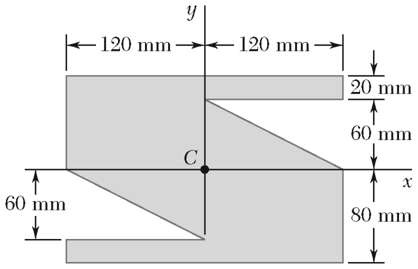


PROBLEM 9.86

For the area indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

Area of Problem 9.72.

SOLUTION

From Problem 9.80:

$$\bar{I}_x = 68.96\times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 132.48\times 10^6 \text{ mm}^4$$

Problem 9.72:

$$\bar{I}_{xy} = -21.6\times 10^6 \text{ mm}^4$$

Now Eq. (9.25):

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(-21.6\times 10^6)}{(68.96-132.48)\times 10^6}$$

$$= -0.68010$$

Then

$$2\theta_m = -34.220° \quad \text{and} \quad 145.780°$$

or $\theta_m = -17.11°$ and $72.9°$ ◀

Also Eq. (9.27):

$$\bar{I}_{\max,\min} = \frac{\bar{I}_x + \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + I_{xy}^2}$$

Then

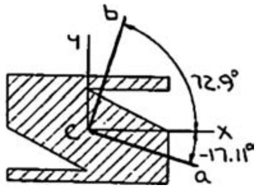
$$\bar{I}_{\max,\min} = \left[\frac{68.96+132.48}{2} \pm \sqrt{\left(\frac{68.96-132.48}{2}\right)^2 + (-21.6)^2}\right]\times 10^6$$

$$= (100.72 \pm 38.409)\times 10^6 \text{ mm}^4$$

or $\bar{I}_{\max} = 139.1\times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\min} = 62.3\times 10^6 \text{ mm}^4$ ◀

By inspection, the *a* axis corresponds to $\bar{I}_{\min}$ and the *b* axis corresponds to $\bar{I}_{\max}$.



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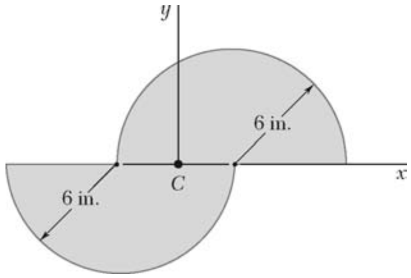


PROBLEM 9.87

For the area indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

Area of Problem 9.73.

SOLUTION

From Problem 9.81: $\bar{I}_x = 324\pi \text{ in.}^4 \qquad \bar{I}_y = 648\pi \text{ in.}^4$

Problem 9.73: $\bar{I}_{xy} = 864 \text{ in.}^4$

Now Eq. (9.25):

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(864)}{324\pi - 648\pi}$$

$$= 1.69765$$

Then $2\theta_m = 59.500° \quad \text{and} \quad 239.500°$

or $\theta_m = 29.7° \quad \text{and} \quad 119.7°$ ◀

Also Eq. (9.27):

$$\bar{I}_{\text{max, min}} = \frac{\bar{I}_x + \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

Then

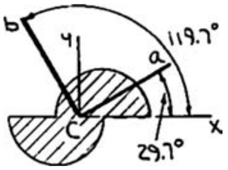
$$\bar{I}_{\text{max,min}} = \frac{324\pi + 648\pi}{2} \pm \sqrt{\left(\frac{324\pi - 648\pi}{2}\right)^2 + 864^2}$$

$$= (1526.81 \pm 1002.75) \text{ in.}^4$$

or $\bar{I}_{\text{max}} = 2530 \text{ in.}^4$ ◀

and $\bar{I}_{\text{min}} = 524 \text{ in.}^4$ ◀

By inspection, the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.



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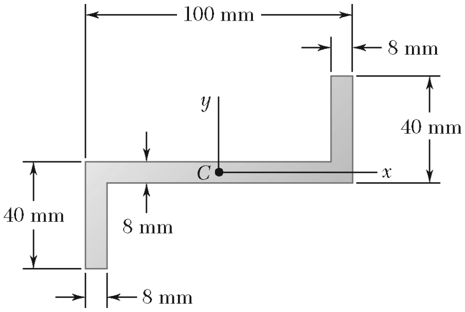


PROBLEM 9.88

For the area indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

Area of Problem 9.75.

SOLUTION

From Problem 9.82: $\bar{I}_x = 252{,}757 \text{ mm}^4$

$\bar{I}_y = 1{,}752{,}789 \text{ mm}^4$

Problem 9.75: $\bar{I}_{xy} = 471{,}040 \text{ mm}^4$

Now Eq. (9.25):

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(471{,}040)}{252{,}757 - 1{,}752{,}789}$$

$$= 0.62804$$

Then $2\theta_m = 32.130° \quad \text{and} \quad 212.130°$

or $\theta_m = 16.07° \text{ and } 106.1°$ ◀

Also Eq. (9.27):

$$\bar{I}_{\text{max, min}} = \frac{\bar{I}_x - \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

Then

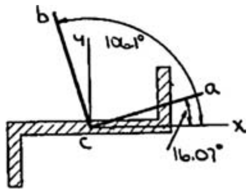
$$\bar{I}_{\text{max, min}} = \frac{252{,}757 + 1{,}752{,}789}{2} \pm \sqrt{\left(\frac{252{,}757 - 1{,}752{,}789}{2}\right)^2 + 471040^2}$$

$$= (1{,}002{,}773 \pm 885{,}665) \text{ mm}^4$$

or $\bar{I}_{\text{max}} = 1.888 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\text{min}} = 0.1171 \times 10^6 \text{ mm}^4$ ◀

By inspection, the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.



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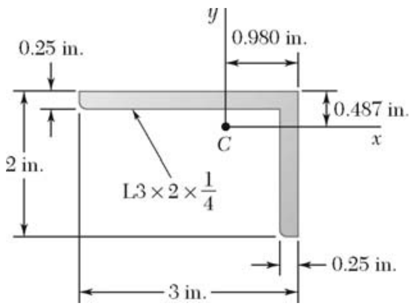


PROBLEM 9.89

For the angle cross section indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

The $L3\times2\times\frac{1}{4}$-in. angle cross section of Problem 9.74.

SOLUTION

From Problem 9.83: $\bar{I}_x = 0.390 \text{ in.}^4 \qquad \bar{I}_y = 1.09 \text{ in.}^4$

Problem 9.74: $\bar{I}_{xy} = -0.37983 \text{ in.}^4$

Now Eq. (9.25):

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(-0.37983)}{0.390 - 1.09}$$

$$= -1.08523$$

Then $2\theta_m = -47.341° \text{ and } 132.659°$

or $\theta_m = -23.7° \text{ and } 66.3°$ ◀

Also Eq. (9.27):

$$\bar{I}_{\text{max, min}} = \frac{\bar{I}_x + \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

Then

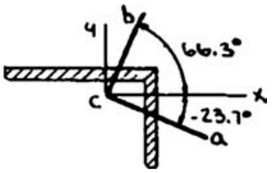
$$\bar{I}_{\text{max, min}} = \frac{0.390 + 1.09}{2} \pm \sqrt{\left(\frac{0.390 - 1.09}{2}\right)^2 + (-0.37983)^2}$$

$$= (0.740 \pm 0.51650)^2 \text{ in.}^4$$

or $\bar{I}_{\text{max}} = 1.257 \text{ in.}^4$ ◀

and $\bar{I}_{\text{min}} = 0.224 \text{ in.}^4$ ◀

By inspection, the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.



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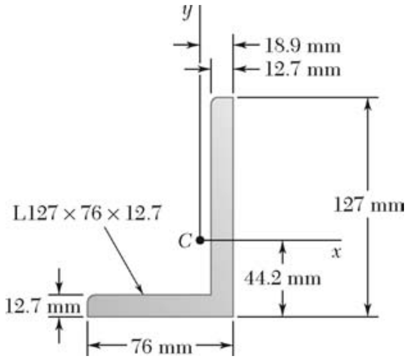


PROBLEM 9.90

For the angle cross section indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

The $L127 \times 76 \times 12.7\text{-mm}$ angle cross section of Problem 9.78.

SOLUTION

From Problem 9.84: $\bar{I}_x = 3.93 \times 10^6 \text{ mm}^4$

$\bar{I}_y = 1.06 \times 10^6 \text{ mm}^4$

Problem 9.78: $\bar{I}_{xy} = 1.165061 \times 10^6 \text{ mm}^4$

Now Eq. (9.25):
$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(1.165061 \times 10^6)}{(3.93 - 1.06) \times 10^6}$$
$$= -0.81189$$

Then $2\theta_m = -39.073°$ and $140.927°$

or $\theta_m = -19.54°$ and $70.5°$ ◀

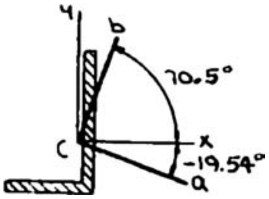
Also Eq. (9.27):
$$\bar{I}_{\text{max, min}} = \frac{\bar{I}_x + \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

Then
$$\bar{I}_{\text{max, min}} = \left[\frac{3.93 + 1.06}{2} \pm \sqrt{\left(\frac{3.93 - 1.06}{2}\right)^2 + 1.165061^2}\right] \times 10^6 \text{ mm}^4$$
$$= (2.495 \pm 1.84840) \times 10^6 \text{ mm}^4$$

or $\bar{I}_{\max} = 4.34 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\min} = 0.647 \times 10^6 \text{ mm}^4$ ◀

By inspection, the a axis corresponds to $\bar{I}_{\max}$ and the b axis corresponds to $\bar{I}_{\min}$.



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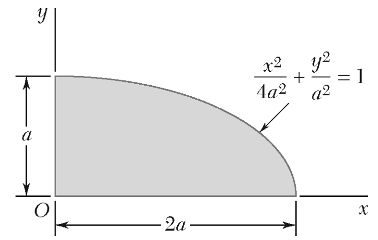
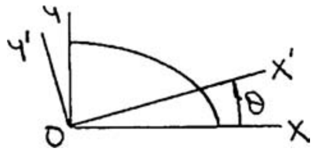


PROBLEM 9.91

Using Mohr's circle, determine for the quarter ellipse of Problem 9.67 the moments of inertia and the product of inertia with respect to new axes obtained by rotating the *x* and *y* axes about *O* (*a*) through 45° counterclockwise, (*b*) through 30° clockwise.

SOLUTION



From Problem 9.79: $$I_x = \frac{\pi}{8}a^4$$

$$I_y = \frac{\pi}{2}a^4$$

Problem 9.67: $$I_{xy} = \frac{1}{2}a^4$$

The Mohr's circle is defined by the diameter *XY*, where

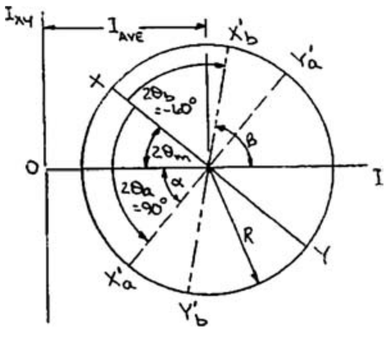
$$X\left(\frac{\pi}{8}a^4, \frac{1}{2}a^4\right) \quad \text{and} \quad Y\left(\frac{\pi}{2}a^4, -\frac{1}{2}a^4\right)$$

Now $$I_{\text{ave}} = \frac{1}{2}(I_x + I_y) = \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right) = \frac{5}{16}\pi a^4 = 0.98175a^4$$

and $$R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} = \sqrt{\left[\frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right)\right]^2 + \left(\frac{1}{2}a^4\right)^2}$$

$$= 0.77264a^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y}$$

$$= -\frac{2\left(\frac{1}{2}a^4\right)}{\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4}$$

$$= 0.84883$$

or $$2\theta_m = 40.326°$$

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PROBLEM 9.91 (Continued)

Then

$$\alpha = 90° - 40.326°$$
$$= 49.674°$$

$$\beta = 180° - (40.326° + 60°)$$
$$= 79.674°$$

(*a*) $\theta = +45°$:

$$I_{x'} = I_{\text{ave}} - R\cos\alpha = 0.98175a^4 - 0.77264a^4 \cos 49.674°$$

or $I_{x'} = 0.482a^4$ ◀

$$I_{y'} = I_{\text{ave}} + R\cos\alpha = 0.98175a^4 + 0.77264a^4 \cos 49.674°$$

or $I_{y'} = 1.482a^4$ ◀

$$I_{x'y'} = -R\sin\alpha = -0.77264a^4 \sin 49.674°$$

or $I_{x'y'} = -0.589a^4$ ◀

(*b*) $\theta = -30°$:

$$I_{x'} = I_{\text{ave}} + R\cos\beta = 0.98175a^4 + 0.77264a^4 \cos 79.674°$$

or $I_{x'} = 1.120a^4$ ◀

$$I_{y'} = I_{\text{ave}} - R\cos\beta = 0.98175a^4 - 0.77264a^4 \cos 79.674°$$

or $I_{y'} = 0.843a^4$ ◀

$$I_{x'y'} = R\sin\beta = 0.77264a^4 \sin 79.674°$$

or $I_{x'y'} = 0.760a^4$ ◀

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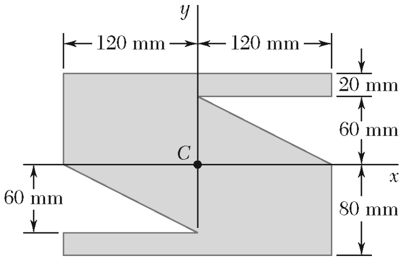
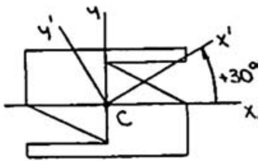


PROBLEM 9.92

Using Mohr's circle, determine the moments of inertia and the product of inertia of the area of Problem 9.72 with respect to new centroidal axes obtained by rotating the *x* and *y* axes 30° counterclockwise.

SOLUTION



From Problem 9.80: $\bar{I}_x = 68.96\times 10^6\,\text{mm}^4$

$\bar{I}_y = 132.48\times 10^6\,\text{mm}^4$

Problem 9.72: $\bar{I}_{xy} = -21.6\times 10^6\,\text{mm}^4$

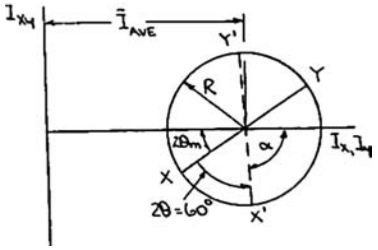
The Mohr's circle is defined by the diameter *XY*, where $X(68.96\times 10^6, -21.6\times 10^6)$ and $Y(132.48\times 10^6,\ 21.6\times 10^6)$.

Now $$I_{\text{ave}} = \frac{1}{2}(\bar{I}_x - \bar{I}_y) = \frac{1}{2}(68.96 + 132.48)\times 10^6 = 100.72\times 10^6\ \text{mm}^4$$

and $$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + I_{xy}^2} = \left\{\sqrt{\left[\frac{1}{2}(68.96 - 132.48)\right]^2 + (-21.6)^2}\right\}\times 10^6\,\text{mm}^4$$

$$= 38.409\times 10^6\,\text{mm}^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(-21.6\times 10^6)}{(68.96 - 132.48)\times 10^6}$$

$$= -0.68010$$

or $$2\theta_m = -34.220°$$

Then $$\alpha = 180° - (34.220° + 60°)$$
$$= 85.780°$$

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PROBLEM 9.92 (Continued)

Then $\bar{I}_{x'} = \bar{I}_{\text{ave}} + R\cos\alpha = (100.72 + 38.409\cos 85.780°)\times 10^6$

or $\bar{I}_{x'} = 103.5\times 10^6\,\text{mm}^4$ ◀

$\bar{I}_{y'} = \bar{I}_{\text{ave}} - R\cos\alpha = (100.72 - 38.409\cos 85.780°)\times 10^6$

or $\bar{I}_{y'} = 97.9\times 10^6\,\text{mm}^4$ ◀

$\bar{I}_{x'y'} = -R\sin\alpha = -(38.409\times 10^6)\sin 85.780°$

or $\bar{I}_{x'y'} = -38.3\times 10^6\,\text{mm}^4$ ◀

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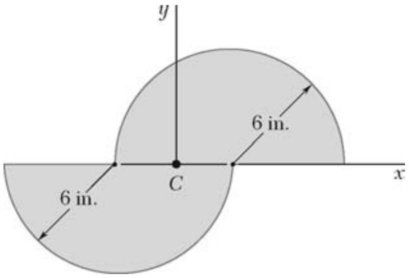


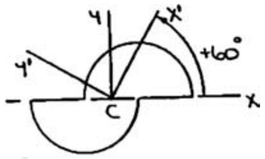
PROBLEM 9.93

Using Mohr's circle, determine the moments of inertia and the product of inertia of the area of Problem 9.73 with respect to new centroidal axes obtained by rotating the *x* and *y* axes 60° counterclockwise.

SOLUTION

From Problem 9.81: $\bar{I}_x = 324\pi \text{ in.}^4$

$\bar{I}_y = 648\pi \text{ in.}^4$



Problem 9.73: $\bar{I}_{xy} = 864 \text{ in.}^4$

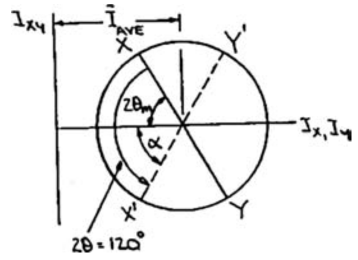
The Mohr's circle is defined by the diameter *XY*, where $X(324\pi,\ 864)$ and $Y(648\pi,\ -864)$.

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(324\pi + 648\pi) = 1526.81 \text{ in.}^4$$

and $$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2} = \sqrt{\left[\frac{1}{2}(324\pi + 648\pi)\right]^2 + 864^2}$$

$$= 1002.75 \text{ in.}^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(864)}{324\pi - 648\pi}$$

$$= 1.69765$$

or $$2\theta_m = 59.500°$$

Then $$\alpha = 120° - 59.500°$$

$$= 60.500°$$

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PROBLEM 9.93 (Continued)

Then $$\bar{I}_{x'} = \bar{I}_{\text{ave}} - R\cos\alpha = 1526.81 - 1002.75\cos 60.500°$$

or $\bar{I}_{x'} = 1033 \text{ in.}^4$ ◀

$$\bar{I}_{y} = \bar{I}_{\text{ave}} + R\cos\alpha = 1526.81 + 1002.75\cos 60.500°$$

or $\bar{I}_{y'} = 2020 \text{ in.}^4$ ◀

$$\bar{I}_{x'y'} = -R\sin\alpha = -1002.75\sin 60.500°$$

or $\bar{I}_{x'y'} = -873 \text{ in.}^4$ ◀

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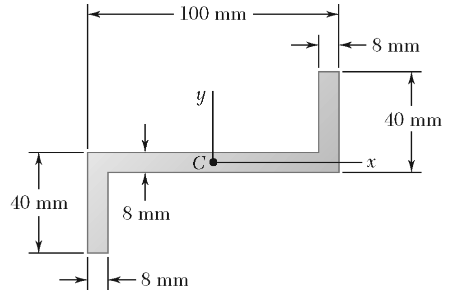


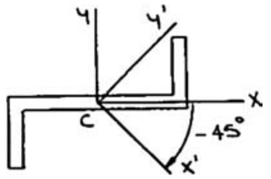
PROBLEM 9.94

Using Mohr's circle, determine the moments of inertia and the product of inertia of the area of Problem 9.75 with respect to new centroidal axes obtained by rotating the *x* and *y* axes 45° clockwise.

SOLUTION

From Problem 9.82: $\bar{I}_x = 252,757 \text{ mm}^4$

$\bar{I}_y = 1,752,789 \text{ mm}^4$



Problem 9.75: $\bar{I}_{xy} = 471,040 \text{ mm}^4$

The Mohr's circle is defined by the diameter *XY*, where *X* (252,757; 471,040) and *Y* (1,752,789; $-471,040$).

Now

$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(252,757 + 1,752,789)$$

$$= 1,002,773 \text{ mm}^4$$

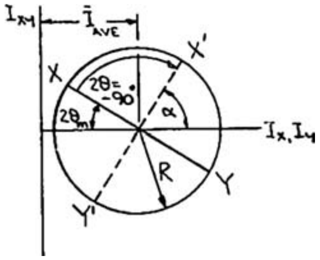
and

$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$$

$$= \sqrt{\left[\frac{1}{2}(252,757 - 1,752,789)^2\right] + 471,040^2}$$

$$= 885,665 \text{ mm}^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(471,040)}{252,757 - 1,752,789}$$

$$= 0.62804$$

or $2\theta_m = 32.130°$

Then $\alpha = 180° - (32.130 + 90°)$

$$= 57.870°$$

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PROBLEM 9.94 (Continued)

Then $\bar{I}_{x'} = \bar{I}_{\text{ave}} + R\cos\alpha = 1,002,773 + 885,665\cos 57.870°$

or $\bar{I}_{x'} = 1.474 \times 10^6\ \text{mm}^4$ ◀

$\bar{I}_{y'} = \bar{I}_{\text{ave}} - R\cos\alpha = 1,002,773 - 885,665\cos 57.870°$

or $\bar{I}_{y'} = 0.532 \times 10^6\ \text{mm}^4$ ◀

$\bar{I}_{x'y'} = R\sin\alpha = 885,665\sin 57.870°$

or $\bar{I}_{x'y'} = 0.750 \times 10^6\ \text{mm}^4$ ◀

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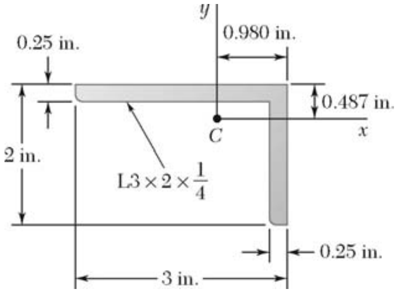
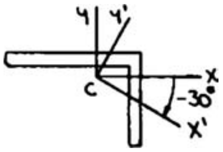


PROBLEM 9.95

Using Mohr's circle, determine the moments of inertia and the product of inertia of the $L3\times 2\times \frac{1}{4}$-in. angle cross section of Problem 9.74 with respect to new centroidal axes obtained by rotating the *x* and *y* axes 30° clockwise.

SOLUTION



From Problem 9.83: $\bar{I}_x = 0.390 \text{ in.}^4$

$\bar{I}_y = 1.09 \text{ in.}^4$

Problem 9.74:

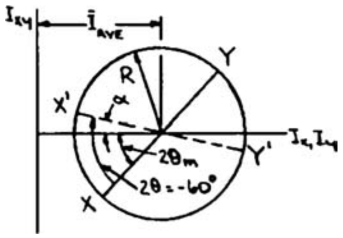
$\bar{I}_{xy} = -0.37983 \text{ in.}^4$

The Mohr's circle is defined by the diameter *XY*, where $X(0.390, -0.37983)$ and $Y(1.09,\ 0.37983)$.

Now
$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y)$$
$$= \frac{1}{2}(0.390 + 1.09)$$
$$= 0.740 \text{ in.}^4$$

and
$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$$
$$= \sqrt{\left[\frac{1}{2}(0.390 - 1.09)\right]^2 + (-0.37983)^2}$$
$$= 0.51650 \text{ in.}^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y}$$
$$= -\frac{2(-0.37983)}{0.390 - 1.09}$$
$$= -1.08523$$

or
$$2\theta_m = -47.341°$$

Then
$$\alpha = 60° - 47.341° = 12.659°$$

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PROBLEM 9.95 (Continued)

Then $$\overline{I}_{x'} = \overline{I}_{\text{ave}} - R\cos\alpha = 0.740 - 0.51650\cos 12.659°$$

or $\overline{I}_{x'} = 0.236\text{ in.}^4$ ◀

$$\overline{I}_{y'} = \overline{I}_{\text{ave}} + R\cos\alpha = 0.740 + 0.51650\cos 12.659°$$

or $\overline{I}_{y} = 1.244\text{ in.}^4$ ◀

$$\overline{I}_{x'y'} = R\sin\alpha = 0.51650\sin 12.659°$$

or $\overline{I}_{x'y'} = 0.1132\text{ in.}^4$ ◀

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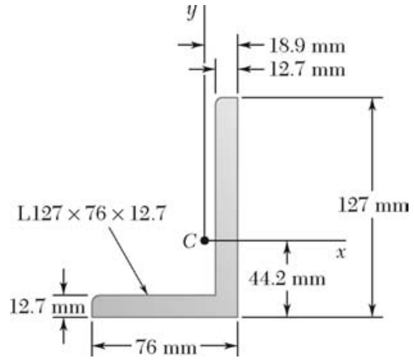


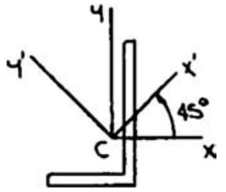
PROBLEM 9.96

Using Mohr's circle, determine the moments of inertia and the product of inertia of the $L127 \times 76 \times 12.7$-mm angle cross section of Problem 9.78 with respect to new centroidal axes obtained by rotating the *x* and *y* axes 45° counterclockwise.

SOLUTION

From Problem 9.84: $\bar{I}_x = 3.93 \times 10^6 \text{ mm}^4$

$\bar{I}_y = 1.06 \times 10^6 \text{ mm}^4$



Problem 9.78: $\bar{I}_{xy} = 1.165061 \times 10^6 \text{ mm}^4$

The Mohr's circle is defined by the diameter *XY*, where $X(3.93 \times 10^6,\ 1.165061 \times 10^6)$, $Y(1.06 \times 10^6,\ -1.165061 \times 10^6)$.

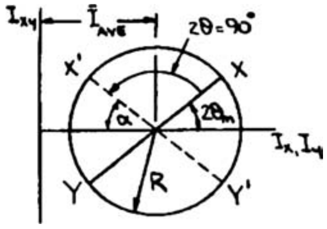
Now
$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(3.93 + 1.06) \times 10^6 = 2.495 \times 10^6 \text{ mm}^4$$

and
$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$$

$$= \left\{\sqrt{\left[\frac{1}{2}(3.93 - 1.06)\right]^2 + 1.165061^2}\right\} \times 10^6 \text{ mm}^4$$

$$= 1.84840 \times 10^6 \text{ mm}^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(1.165061 \times 10^6)}{(3.93 - 1.06) \times 10^6}$$

$$= -0.81189$$

or
$$2\theta_m = -39.073°$$

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PROBLEM 9.96 (Continued)

Then $$\alpha = 180° - (39.073° + 90°)$$
$$= 50.927°$$

Then $$\bar{I}_{x'} = \bar{I}_{\text{ave}} - R\cos\alpha = (2.495 - 1.84840\cos 50.927°)\times 10^6$$

or $\bar{I}_{x'} = 1.330\times 10^6\ \text{mm}^4$ ◀

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} + R\cos\alpha = (2.495 + 1.84840\cos 50.927°)\times 10^6$$

or $\bar{I}_{y'} = 3.66\times 10^6\ \text{mm}^4$ ◀

$$\bar{I}_{x'y'} = R\sin\alpha = (1.84840\times 10^6)\sin 50.927°$$

or $\bar{I}_{x'y'} = 1.435\times 10^6\ \text{mm}^4$ ◀

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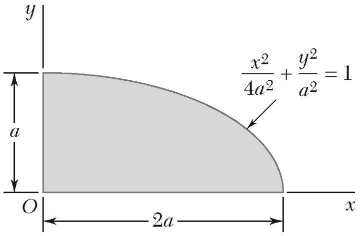


PROBLEM 9.97

For the quarter ellipse of Problem 9.67, use Mohr's circle to determine the orientation the principal axes at the origin and the corresponding values of the moments of inertia.

SOLUTION

From Problem 9.79: $$I_x = \frac{\pi}{8}a^4 \qquad I_y = \frac{\pi}{2}a^4$$

Problem 9.67: $$I_{xy} = \frac{1}{2}a^4$$

The Mohr's circle is defined by the diameter *XY*, where

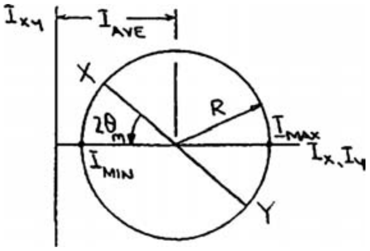
$$X\left(\frac{\pi}{8}a^4,\ \frac{1}{2}a^4\right) \quad \text{and} \quad Y\left(\frac{\pi}{2}a^4,\ -\frac{1}{2}a^4\right)$$

Now $$I_{\text{ave}} = \frac{1}{2}(I_x + I_y) = \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right) = 0.98175a^4$$

and

$$R = \sqrt{\left[\frac{1}{2}(I_x - I_y)\right]^2 + I_{xy}^2}$$
$$= \sqrt{\left[\frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right)\right]^2 + \left(\frac{1}{2}a^4\right)^2}$$
$$= 0.77264a^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y}$$
$$= -\frac{2\left(\frac{1}{2}a^4\right)}{\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4}$$
$$= 0.84883$$

or $$2\theta_m = 40.326°$$

and $$\theta_m = 20.2°$$

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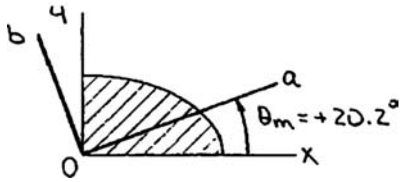
PROBLEM 9.97 (Continued)

The Principal axes are obtained by rotating the xy axes through

20.2° counterclockwise ◀

about O.



Now $$I_{max,\,min} = I_{ave} \pm R = 0.98175a^4 \pm 0.77264a^4$$

or $I_{max} = 1.754\,a^4$ ◀

and $I_{min} = 0.209\,a^4$ ◀

From the Mohr's circle it is seen that the a axis corresponds to I_{min} and the b axis corresponds to I_{max}.

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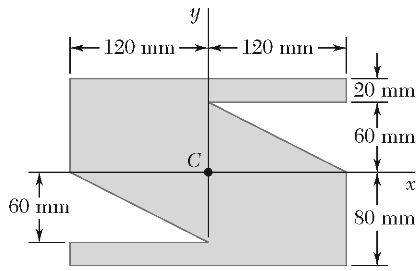


PROBLEM 9.98

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.72.

SOLUTION

From Problem 9.80: $\bar{I}_x = 68.96 \times 10^6 \text{ mm}^4$

$\bar{I}_y = 132.48 \times 10^6 \text{ mm}^4$

Problem 9.72: $\bar{I}_{xy} = -21.6 \times 10^6 \text{ mm}^4$

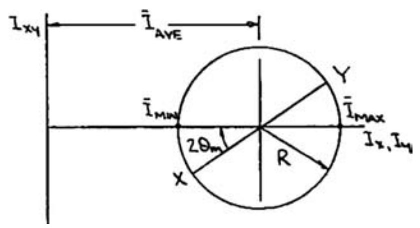
The Mohr's circle is defined by the diameter XY, where $X(68.96 \times 10^6, -21.6 \times 10^6)$ and $Y(132.48 \times 10^6, 21.6 \times 10^6)$

Now
$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y)$$
$$= \frac{1}{2}(68.96 + 132.48) \times 10^6$$
$$= 100.72 \times 10^6 \text{ mm}^4$$

and
$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$$
$$= \left\{\sqrt{\left[\frac{1}{2}(68.96 - 132.48)\right]^2 + (-21.6)^2}\right\} \times 10^6$$
$$= 38.409 \times 10^6 \text{ mm}^4$$

The Mohr's circle is then drawn as shown.

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$
$$= -\frac{2(-21.6 \times 10^6)}{(68.96 - 132.48) \times 10^6}$$
$$= -0.68010$$



or $2\theta_m = -34.220°$

and $\theta_m = -17.11°$

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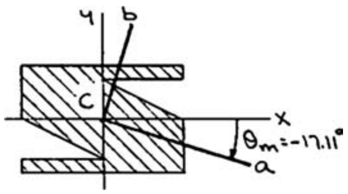
PROBLEM 9.98 (Continued)

The principal axes are obtained by rotating the xy axes through

17.11° clockwise ◀

about C.



Now $$\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = (100.72 + 38.409) \times 10^6$$

or $\bar{I}_{\text{max}} = 139.1 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\text{min}} = 62.3 \times 10^6 \text{ mm}^4$ ◀

From the Mohr's circle it is seen that the a axis corresponds to $\bar{I}_{\text{min}}$ and the b axis corresponds to $\bar{I}_{\text{max}}$.

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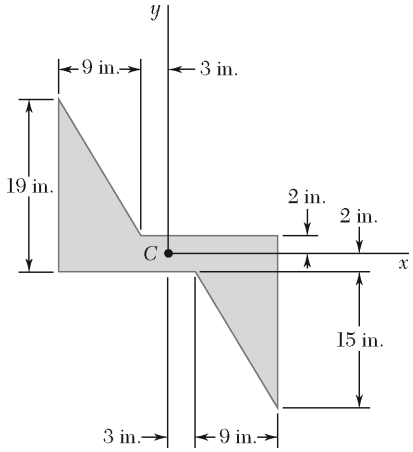
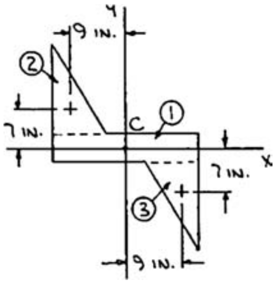


PROBLEM 9.99

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.76.

SOLUTION



From Problem 9.76:

$$\bar{I}_{xy} = -9011.25 \text{ in.}^4$$

Now $\quad \bar{I}_x = (\bar{I}_x)_1 + (I_x)_2 + (I_x)_3$

where $\quad (\bar{I}_x)_1 = \frac{1}{12}(24 \text{ in.})(4 \text{ in.})^3 = 128 \text{ in.}^4$

$$(I_x)_2 = (I_x)_3 = \frac{1}{36}(9 \text{ in.})(15 \text{ in.})^3 + \left[\frac{1}{2}(9 \text{ in.})(15 \text{ in.})\right](7 \text{ in.})^2$$

$$= 4151.25 \text{ in.}^4$$

Then $\quad \bar{I}_x = [128 + 2(4151.25)] \text{ in.}^4$

$$= 8430.5 \text{ in.}^4$$

Also $\quad \bar{I}_y = (\bar{I}_y)_1 + (I_y)_2 + (I_y)_3$

where $\quad (\bar{I}_y)_1 = \frac{1}{12}(4 \text{ in.})(24 \text{ in.}) = 4608 \text{ in.}^4$

$$(I_y)_2 = (I_y)_3 = \frac{1}{36}(15 \text{ in.})(9 \text{ in.})^3 + \left[\frac{1}{2}(9 \text{ in.})(15 \text{ in.})\right](9 \text{ in.})^2$$

$$= 5771.25 \text{ in.}^4$$

Then $\quad \bar{I}_y = [4608 + 2(5771.25)] \text{ in.}^4 = 16150.5 \text{ in.}^4$

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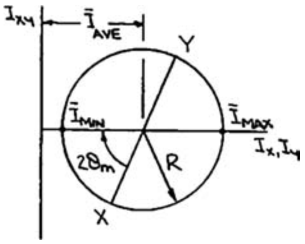
PROBLEM 9.99 (Continued)

The Mohr's circle is defined by the diameter *XY*, where $X(8430.5, \ -9011.25)$ and $Y(16150.5, \ 9011.25)$.

Now
$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(8430.5 + 16150.5) = 12290.5 \text{ in.}^4$$

and
$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$$
$$= \sqrt{\left[\frac{1}{2}(8430.5 - 16150.5)\right]^2 + (-9011.25)^2}$$
$$= 9803.17 \text{ in.}^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$
$$= -\frac{2(-9011.25)}{8430.5 - 16150.5}$$
$$= -2.33452$$

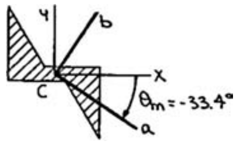
or
$$2\theta_m = -66.812°$$

and
$$\theta_m = -33.4°$$

The principal axes are obtained by rotating the *xy* axes through

33.4° clockwise ◀

about *C*.



Now
$$\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = 12290.5 \pm 9803.17$$

or $\bar{I}_{\text{max}} = 22.1 \times 10^3 \text{in.}^4$ ◀

and $\bar{I}_{\text{min}} = 2490 \text{ in.}^4$ ◀

From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.

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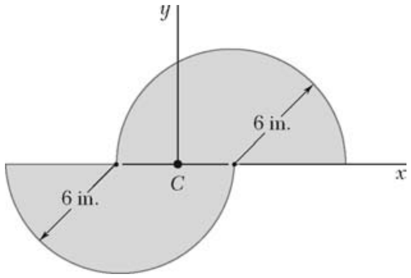


PROBLEM 9.100

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.73

SOLUTION

From Problem 9.81: $\bar{I}_x = 324\pi \text{ in.}^4 \qquad \bar{I}_y = 648\pi \text{ in.}^4$

Problem 9.73: $\bar{I}_{xy} = 864 \text{ in.}^4$

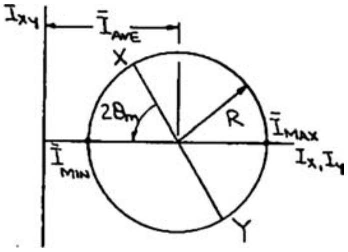
The Mohr's circle is defined by the diameter XY, where $X(324\pi,\ 864)$ and $Y(648\pi, -864)$.

Now
$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(324\pi + 648\pi) = 1526.81 \text{ in.}^4$$

and
$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)^2\right] + I_{xy}^2}$$
$$= \sqrt{\left[\frac{1}{2}(324\pi - 648\pi)\right]^2 + 864^2}$$
$$= 1002.75 \text{ in.}^4$$

The Mohr's circle is then drawn as shown.

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$
$$= -\frac{2(864)}{324\pi - 648\pi}$$
$$= 1.69765$$



or $\qquad 2\theta_m = 59.4998°$

and $\qquad \theta_m = 29.7°$

The principal axes are obtained by rotating the xy axes through

29.7° counterclockwise ◀

about C.

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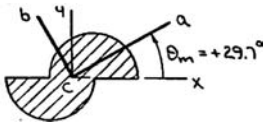
PROBLEM 9.100 (Continued)

Now $$\bar{I}_{\max,\min} = \bar{I}_{\text{ave}} \pm R = 1526.81 \pm 1002.75$$

or $I_{\max} = 2530 \text{ in.}^4$ ◀

and $\bar{I}_{\min} = 524 \text{ in.}^4$ ◀



From the Mohr's circle it is seen that the *a* axis corresponds to $I_{\min}$ and the *b* axis corresponds to $\bar{I}_{\max}$.

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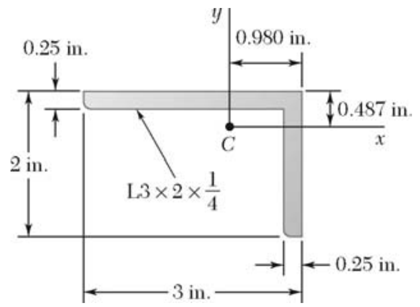


PROBLEM 9.101

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.74.

SOLUTION

From Problem 9.83: $\bar{I}_x = 0.390 \text{ in.}^4$

$\bar{I}_y = 1.09 \text{ in.}^4$

Problem 9.74: $\bar{I}_{xy} = -0.37983 \text{ in.}^4$

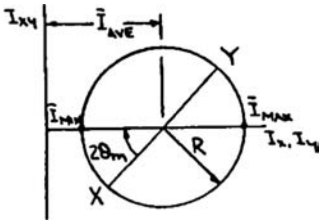
The Mohr's circle is defined by the diameter XY, where $X(0.390, -0.37983)$ and $Y(1.09, 0.37983)$.

Now $\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(0.390 + 1.09) = 0.740 \text{ in.}^4$

and
$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$$
$$= \sqrt{\left[\frac{1}{2}(0.390 - 1.09)\right]^2 + (-0.37983)^2}$$
$$= 0.51650 \text{ in.}^4$$

The Mohr's circle is then drawn as shown.

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$
$$= -\frac{2(-0.37983)}{0.390 - 1.09}$$
$$= -1.08523$$



Then $2\theta_m = -47.341°$

and $\theta_m = -23.7°$

The principal axes are obtained by rotating the *xy* axes through 23.7° clockwise ◀

about *C*.

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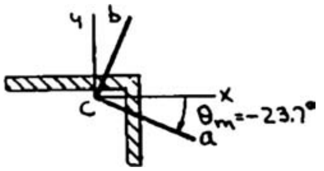
PROBLEM 9.101 (Continued)

Now $$\bar{I}_{\max,\min} = \bar{I}_{\text{ave}} \pm R = 0.740 \pm 0.51650$$

or $\bar{I}_{\max} = 1.257 \text{ in.}^4$ ◀

and $\bar{I}_{\min} = 0.224 \text{ in.}^4$ ◀



From the Mohr's circle it is seen that the *a* axis corresponds to $I_{\min}$ and the *b* axis corresponds to $I_{\max}$.

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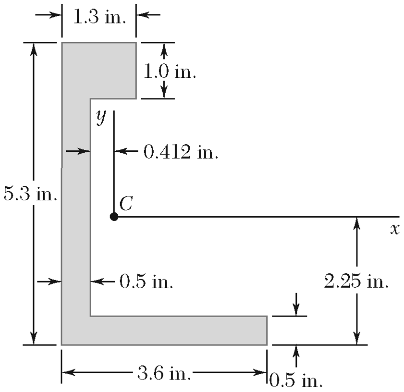


PROBLEM 9.102

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.77

(The moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area of Problem 9.102 were determined in Problem 9.44).

SOLUTION

From Problem 9.44: $\bar{I}_x = 18.1282 \text{ in.}^4$

$\bar{I}_y = 4.5080 \text{ in.}^4$

Problem 9.77: $\bar{I}_{xy} = -4.25320 \text{ in.}^4$

The Mohr's circle is defined by the diameter XY, where $X(18.1282, -4.25320)$ and $Y(4.5080, 4.25320)$.

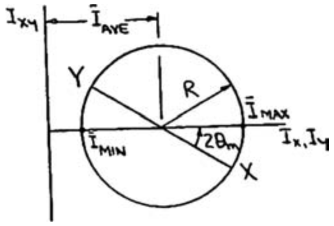
Now $$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(18.1282 + 4.5080) = 11.3181 \text{ in.}^4$$

and $$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$$

$$= \sqrt{\left[\frac{1}{2}(18.1282 - 4.5080)\right]^2 + (-4.25320)^2}$$

$$= 8.02915 \text{ in.}^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(-4.25320)}{18.1282 - 4.5080}$$

$$= 0.62454$$

or $$2\theta_m = 31.986°$$

and $$\theta_m = 15.99°$$

The principal axes are obtained by rotating the xy axes through

15.99° counterclockwise ◀

about C.

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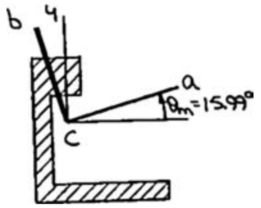
PROBLEM 9.102 (Continued)

Now $$\bar{I}_{\max,\min} = \bar{I}_{\text{ave}} \pm R = 11.3181 \pm 8.02915$$

or $$\bar{I}_{\max} = 19.35 \text{ in.}^4 \blacktriangleleft$$

and $$\bar{I}_{\min} = 3.29 \text{ in.}^4 \blacktriangleleft$$



From the Mohr's circle it is seen that the *a* axis corresponds to $I_{\max}$ and the *b* axis corresponds to $I_{\min}$.

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PROBLEM 9.103

The moments and product of inertia of an L4 × 3 × $\frac{1}{4}$-in. angle cross section with respect to two rectangular axes *x* and *y* through *C* are, respectively, $\bar{I}_x = 1.33$ in.^{4}, $\bar{I}_y = 2.75$ in.^{4}, and $\bar{I}_{xy} < 0$, with the minimum value of the moment of inertia of the area with respect to any axis through *C* being $\bar{I}_{\min} = 0.692$ in.^{4}. Using Mohr's circle, determine (*a*) the product of inertia $\bar{I}_{xy}$ of the area, (*b*) the orientation of the principal axes, (*c*) the value of $\bar{I}_{\max}$.

SOLUTION

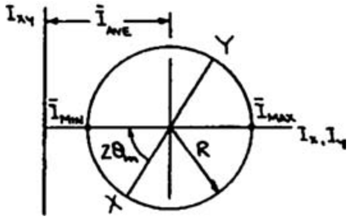
(*Note:* A review of a table of rolled-steel shapes reveals that the given values of $\bar{I}_x$ and $\bar{I}_y$ are obtained when the 4-in. leg of the angle is parallel to the *x* axis. Further, for $\bar{I}_{xy} < 0$, the angle must be oriented as shown.)

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(1.33 + 2.75) = 2.040 \text{ in.}^4$$

and $$\bar{I}_{\min} = \bar{I}_{\text{ave}} - R \quad \text{or} \quad R = 2.040 - 0.692$$

$$= 1.348 \text{ in.}^4$$

Using $\bar{I}_{\text{ave}}$ and *R*, the Mohr's circle is then drawn as shown; note that for the diameter *XY*, $X(1.33,\ \bar{I}_{xy})$ and $Y(2.75,\ |\bar{I}_{xy}|)$.



(*a*) We have $$R^2 = \left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2$$

or $$\bar{I}_{xy}^2 = 1.348^2 - \left[\frac{1}{2}(1.33 - 2.75)\right]^2$$

Solving for $\bar{I}_{xy}$ and taking the negative root (since $\bar{I}_{xy} < 0$) yields $\bar{I}_{xy} = -1.14586$ in.^{4}.

$$\bar{I}_{xy} = -1.146 \text{ in.}^4 \blacktriangleleft$$

(*b*) We have $$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(-1.14586)}{1.33 - 2.75}$$
$$= -1.61389$$

or $$2\theta_m = -58.217° \qquad \theta_m = -29.1°$$

The principal axes are obtained by rotating the *xy* axes through

29.1° clockwise ◀

about *C*.

(*c*) We have $$\bar{I}_{\max} = \bar{I}_{\text{ave}} + R = 2.040 + 1.348$$

or $$\bar{I}_{\max} = 3.39 \text{ in.}^4 \blacktriangleleft$$

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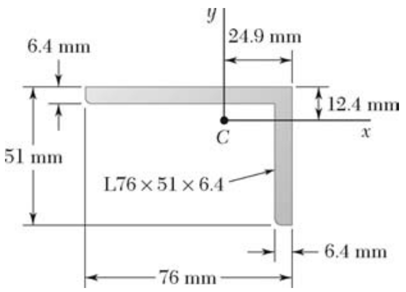


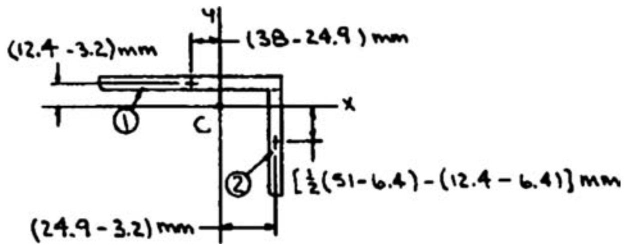
PROBLEM 9.104

Using Mohr's circle, determine for the cross section of the rolled-steel angle shown the orientation of the principal centroidal axes and the corresponding values of the moments of inertia. (Properties of the cross sections are given in Figure 9.13.)

SOLUTION

From Figure 9.13B: $\bar{I}_x = 0.162 \times 10^6 \text{ mm}^4$

$$\bar{I}_y = 0.454 \times 10^6 \text{ mm}^4$$



We have $\bar{I}_{xy} = (I_{xy})_1 + (I_{xy})_2$

For each rectangle $I_{xy} = \bar{I}_{x'y'} + \bar{x}\bar{y}A$

and $\bar{I}_{x'y'} = 0$ (symmetry) $I_{xy} = \Sigma \bar{x}\bar{y}A$

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}\bar{y}A$, mm^4
1	$76 \times 6.4 = 486.4$	–13.1	9.2	–58620.93
2	$6.4 \times (51 - 6.4) = 285.44$	21.7	–16.3	–100962.98
Σ				–159583.91

$$\bar{I}_{xy} = -159584 \text{ mm}^4$$

The Mohr's circle is defined by the diameter XY where $X(0.162 \times 10^6, -0.159584 \times 10^6)$ and $Y(0.454 \times 10^6, 0.159584 \times 10^6)$

Now $\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(0.162 + 0.454) \times 10^6$

$$= 0.3080 \times 10^6 \text{ mm}^4$$

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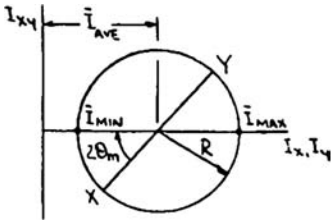
PROBLEM 9.104 (Continued)

and $$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + I_{xy}^2} = \left\{\sqrt{\left[\frac{1}{2}(0.162 - 0.454)\right]^2 + (-0.159584)^2}\right\} \times 10^6$$

$$= 0.21629 \times 10^6 \text{ mm}^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(-0.159584 \times 10^6)}{(0.162 - 0.454) \times 10^6}$$

$$= -1.09304$$

or $$2\theta_m = -47.545$$

and $$\theta_m = -23.8°$$

The principal axes are obtained by rotating the *xy* axes through

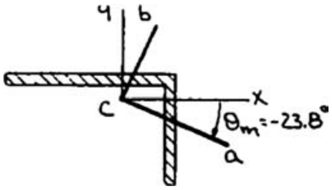
23.8° clockwise ◀

About *C*.

Now $$\bar{I}_{\max,\min} = \bar{I}_{\text{ave}} \pm R = (0.3080 \pm 0.21629) \times 10^6$$

or $$\bar{I}_{\max} = 0.524 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

and $$\bar{I}_{\min} = 0.0917 \times 10^6 \text{ mm}^4 \blacktriangleleft$$



From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\min}$ and the *b* axis corresponds to $\bar{I}_{\max}$.

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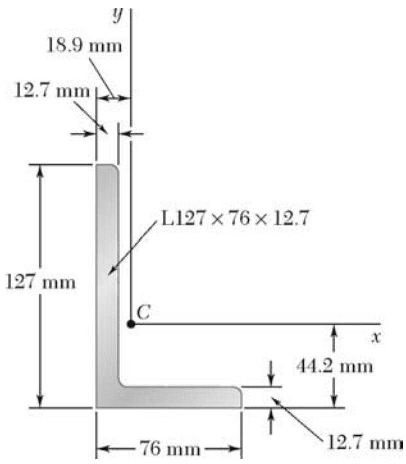

PROBLEM 9.105

Using Mohr's circle, determine for the cross section of the rolled-steel angle shown the orientation of the principal centroidal axes and the corresponding values of the moments of inertia. (Properties of the cross sections are given in Figure 9.13.)

SOLUTION

From Figure 9.13B:

$$\bar{I}_x = 3.93 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 1.06 \times 10^6 \text{ mm}^4$$

Problem 9.7B:

$$\bar{I}_{xy} = -1.165061 \times 10^6 \text{ mm}^4$$

(Note that the figure of Problem 9.105 is obtained by replacing x with $-x$ in the figure of Problem 9.78; thus the change in sign of $\bar{I}_{xy}$.)

The Mohr's circle is defined by the diameter XY, where $X(3.93 \times 10^6, -1.165061 \times 10^6)$ and $Y(1.06 \times 10^6, 1.165061 \times 10^6)$.

Now

$$I_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y)$$

$$= \frac{1}{2}(3.93 + 1.06) \times 10^6$$

$$= 2.495 \times 10^6 \text{ mm}^4$$

and

$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$$

$$= \left\{\sqrt{\left[\frac{1}{2}(3.93 - 1.06)\right]^2 + (-1.165061)^2}\right\} \times 10^6$$

$$= 1.84840 \times 10^6 \text{ mm}^4$$

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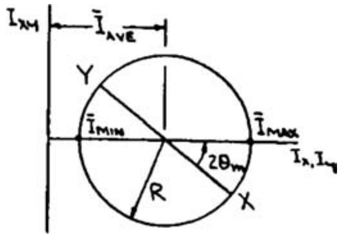
PROBLEM 9.105 (Continued)

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(-1.165061\times 10^6)}{(3.93-1.06)\times 10^6}$$

$$= 0.81189$$

or $$2\theta_m = 39.073°$$

and $$\theta_m = 19.54°$$

The principal axes are obtained by rotating the *xy* axes through

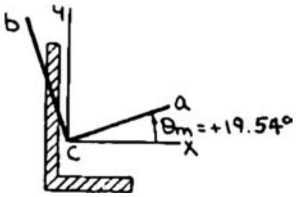
19.54° counterclockwise ◀

about *C*.

Now $$\bar{I}_{\max,\min} = \bar{I}_{\text{ave}} \pm R = (2.495 \pm 1.84840)\times 10^6$$

or $\bar{I}_{\max} = 4.34\times 10^6\ \text{mm}^4$ ◀

and $\bar{I}_{\min} = 0.647\times 10^6\ \text{mm}^4$ ◀



From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\max}$ and the *b* axis corresponds to $\bar{I}_{\min}$.

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PROBLEM 9.106*

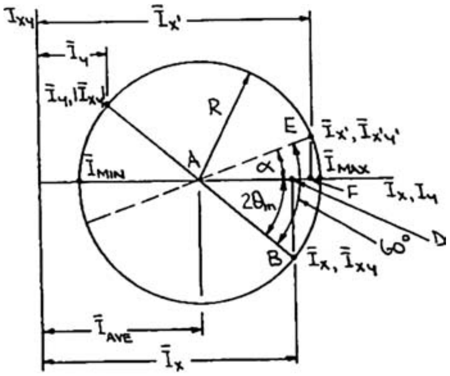
For a given area the moments of inertia with respect to two rectangular centroidal x and y axes are $\bar{I}_x = 1200$ in.^{4} and $\bar{I}_y = 300$ in.^{4}, respectively. Knowing that after rotating the x and y axes about the centroid 30° counterclockwise, the moment of inertia relative to the rotated x axis is 1450 in.^{4}, use Mohr's circle to determine (a) the orientation of the principal axes, (b) the principal centroidal moments of inertia.

SOLUTION

We have $$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(1200 + 300) = 750 \text{ in.}^4$$

Now observe that $\bar{I}_x > \bar{I}_{\text{ave}}$, $\bar{I}_{x'} > \bar{I}_x$, and $2\theta = +60°$. This is possible only if $\bar{I}_{xy} < 0$. Therefore, assume $\bar{I}_{xy} < 0$ and (for convenience) $\bar{I}_{x'y'} > 0$. Mohr's circle is then drawn as shown.

We have $$2\theta_m + \alpha = 60°$$



Now using ΔABD: $$R = \frac{\bar{I}_x - \bar{I}_{\text{ave}}}{\cos 2\theta_m} = \frac{1200 - 750}{\cos 2\theta_m} = \frac{450}{\cos 2\theta_m} \quad (\text{in.}^4)$$

Using ΔAEF: $$R = \frac{\bar{I}_{x'} - \bar{I}_{\text{ave}}}{\cos \alpha} = \frac{1450 - 750}{\cos \alpha} = \frac{700}{\cos \alpha} \quad (\text{in.}^4)$$

Then $$\frac{450}{\cos 2\theta_m} = \frac{700}{\cos \alpha} \qquad \alpha = 60° - 2\theta_m$$

or $$9\cos(60° - 2\theta_m) = 14\cos 2\theta_m$$

Expanding: $$9(\cos 60° \cos 2\theta_m + \sin 60° \sin 2\theta_m) = 14\cos 2\theta_m$$

or $$\tan 2\theta_m = \frac{14 - 9\cos 60°}{9\sin 60°} = 1.21885$$

or $$2\theta_m = 50.633° \quad \text{and} \quad \theta_m = 25.3°$$

(*Note:* $2\theta_m < 60°$ implies assumption $\bar{I}_{x'y'} > 0$ is correct.)

Finally, $$R = \frac{450}{\cos 50.633°} = 709.46 \text{ in.}^4$$

(a) From the Mohr's circle it is seen that the principal axes are obtained by rotating the given centroidal x and y axes through θ_m about the centroid C or

25.3° counterclockwise ◀

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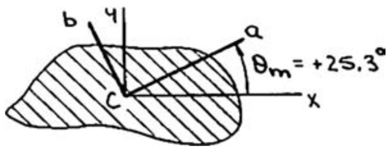
PROBLEM 9.106* (Continued)



(*b*) We have $\bar{I}_{max,\,min} = \bar{I}_{ave} \pm R = 750 \pm 709.46$

or $\bar{I}_{max} = 1459 \text{ in.}^4$ ◀

and $\bar{I}_{min} = 40.5 \text{ in.}^4$ ◀

From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{max}$ and the *b* axis corresponds to $\bar{I}_{min}$.

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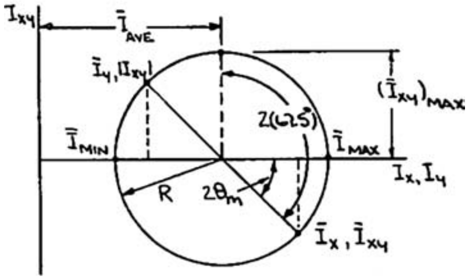
PROBLEM 9.107

It is known that for a given area $\bar{I}_y = 48 \times 10^6$ mm^4 and $\bar{I}_{xy} = -20 \times 10^6$ mm^4, where the x and y axes are rectangular centroidal axes. If the axis corresponding to the maximum product of inertia is obtained by rotating the x axis 67.5° counterclockwise about C, use Mohr's circle to determine (*a*) the moment of inertia $\bar{I}_x$ of the area, (*b*) the principal centroidal moments of inertia.

SOLUTION

First assume $\bar{I}_x > \bar{I}_y$ and then draw the Mohr's circle as shown. (*Note:* Assuming $\bar{I}_x < \bar{I}_y$ is not consistent with the requirement that the axis corresponding to $(\bar{I}_{xy})_{\max}$ is obtained after rotating the x axis through 67.5° CCW.)



From the Mohr's circle we have

$$2\theta_m = 2(67.5°) - 90° = 45°$$

(*a*) From the Mohr's circle we have

$$\bar{I}_x = \bar{I}_y + 2\frac{|I_{xy}|}{\tan 2\theta_m} = 48\times10^6 + 2\frac{20\times10^6}{\tan 45°}$$

or $\bar{I}_x = 88.0\times10^6\,\text{mm}^4$ ◀

(*b*) We have

$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(88.0 + 48)\times10^6$$
$$= 68.0\times10^6\,\text{mm}^4$$

and

$$R = \frac{|I_{xy}|}{\sin 2\theta_m} = \frac{20\times10^6}{\sin 45°} = 28.284\times10^6\,\text{mm}^4$$

Now

$$\bar{I}_{\max,\min} = \bar{I}_{\text{ave}} \pm R = (68.0 \pm 28.284)\times10^6$$

or $\bar{I}_{\max} = 96.3\times10^6\,\text{mm}^4$ ◀

and $\bar{I}_{\min} = 39.7\times10^6\,\text{mm}^4$ ◀

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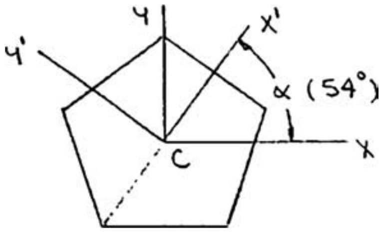
PROBLEM 9.108

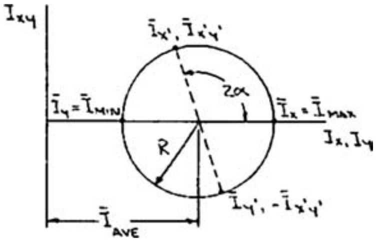
Using Mohr's circle, show that for any regular polygon (such as a pentagon) (*a*) the moment of inertia with respect to every axis through the centroid is the same, (*b*) the product of inertia with respect to every pair of rectangular axes through the centroid is zero.

SOLUTION

Consider the regular pentagon shown, with centroidal axes *x* and *y*.



Because the *y* axis is an axis of symmetry, it follows that $\bar{I}_{xy} = 0.$ Since $\bar{I}_{xy} = 0,$ the *x* and *y* axes must be principal axes. Assuming $\bar{I}_x = \bar{I}_{\max}$ and $\bar{I}_y = \bar{I}_{\min},$ the Mohr's circle is then drawn as shown.



Now rotate the coordinate axes through an angle α as shown; the resulting moments of inertia, $\bar{I}_{x'}$ and $\bar{I}_{y'}$, and product of inertia, $\bar{I}_{x'y'}$, are indicated on the Mohr's circle. However, the x' axis is an axis of symmetry, which implies $\bar{I}_{x'y'} = 0.$ For this to be possible on the Mohr's circle, the radius *R* must be equal to zero (thus, the circle degenerates into a point). With $R = 0,$ it immediately follows that

(*a*) $\bar{I}_x = \bar{I}_y = \bar{I}_{x'} = \bar{I}_{y'} = \bar{I}_{\text{ave}}$ (for all moments of inertia with respect to an axis through *C*) ◀

(*b*) $\bar{I}_{xy} = \bar{I}_{x'y'} = 0$ (for all products of inertia with respect to all pairs of rectangular axes with origin at *C*) ◀

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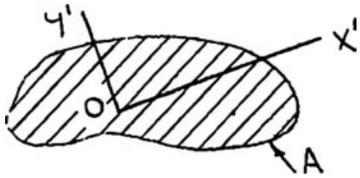
PROBLEM 9.109

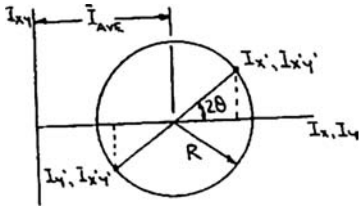
Using Mohr's circle, prove that the expression $I_{x'}I_{y'} - I_{x'y'}^2$ is independent of the orientation of the x' and y' axes, where $I_{x'}$, $I_{y'}$, and $I_{x'y'}$ represent the moments and product of inertia, respectively, of a given area with respect to a pair of rectangular axes x' and y' through a given Point O. Also show that the given expression is equal to the square of the length of the tangent drawn from the origin of the coordinate system to Mohr's circle.

SOLUTION

First observe that for a given area A and origin O of a rectangular coordinate system, the values of I_{ave} and R are the same for all orientations of the coordinate axes. Shown below is a Mohr's circle, with the moments of inertia, $I_{x'}$ and $I_{y'}$, and the product of inertia, $I_{x'y'}$, having been computed for an arbitrary orientation of the $x'y'$ axes.



From the Mohr's circle



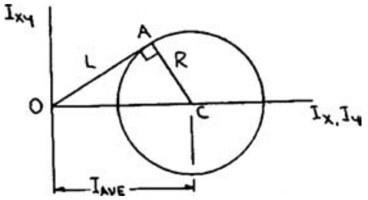
$$I_{x'} = I_{\text{ave}} + R\cos 2\theta$$
$$I_{y'} = I_{\text{ave}} - R\cos 2\theta$$
$$I_{x'y'} = R\sin 2\theta$$

Then, forming the expression $I_{x'}I_{y'} - I_{x'y'}^2$

$$\begin{aligned} I_{x'}I_{y'} - I_{x'y'}^2 &= (I_{\text{ave}} + R\cos 2\theta)(I_{\text{ave}} - R\cos 2\theta) - (R\sin 2\theta)^2 \\ &= \left(I_{\text{ave}}^2 - R^2\cos^2 2\theta\right) - (R^2\sin^2 2\theta) \\ &= I_{\text{ave}}^2 - R^2 \quad \text{which is a constant} \end{aligned}$$

$I_{x'}I_{y'} - I_{x'y'}^2$ is independent of the orientation of the coordinate axes Q.E.D. ◀

Shown is a Mohr's circle, with line $\overline{OA}$, of length L, the required tangent.



Noting that ∠ OAC is a right angle, it follows that

$$L^2 = I_{\text{ave}}^2 - R^2$$

or $\quad L^2 = I_{x'}I_{y'} - I_{x'y'}^2$ Q.E.D. ◀

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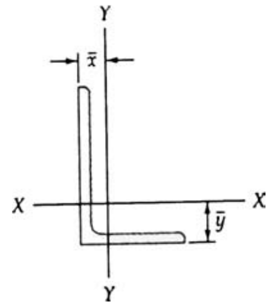


PROBLEM 9.110

Using the invariance property established in the preceding problem, express the product of inertia I_{xy} of an area A with respect to a pair of rectangular axes through O in terms of the moments of inertia I_x and I_y of A and the principal moments of inertia $I_{\min}$ and $I_{\max}$ of A about O. Use the formula obtained to calculate the product of inertia I_{xy} of the L3 × 2 × $\frac{1}{4}$-in. angle cross section shown in Figure 9.13A, knowing that its maximum moment of inertia is 1.257 in^4.

SOLUTION

Consider the following two sets of moments and products of inertia, which correspond to two different orientations of the coordinate axes whole origin is at Point O.

Case 1: $I_{x'} = I_x, \quad I_{y'} = I_y, \quad I_{x'y'} = I_{xy}$

Case 2: $I_{x'} = I_{\max}, \quad I_{y'} = I_{\min}, \quad I_{x'y'} = 0$

The invariance property then requires

$$I_x I_y - I_{xy}^2 = I_{\max} I_{\min} \qquad \text{or} \quad I_{xy} = \pm\sqrt{I_x I_y - I_{\max} I_{\min}} \ ◀$$

From Figure 9.13A: $\bar{I}_x = 1.09 \text{ in.}^4$

$\bar{I}_y = 0.390 \text{ in.}^4$

Using Eq. (9.21): $\bar{I}_x + \bar{I}_y = \bar{I}_{\max} + \bar{I}_{\min}$

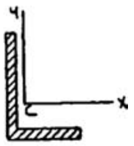
Substituting $1.09 + 0.390 = 1.257 + \bar{I}_{\min}$

or $\bar{I}_{\min} = 0.223 \text{ in.}^4$

Then
$$\bar{I}_{xy} = \sqrt{(1.09)(0.390) - (1.257)(0.223)}$$
$$= \pm 0.381 \text{ in.}^4$$

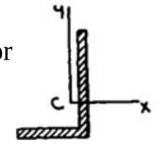
The two roots correspond to the following two orientations of the cross section.

For



$\bar{I}_{xy} = -0.381 \text{ in.}^4$ ◀

and for



$\bar{I}_{xy} = 0.381 \text{ in.}^4$ ◀

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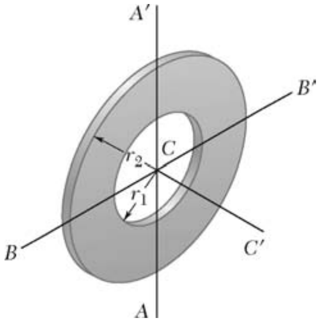
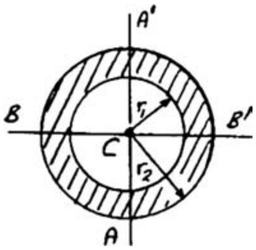


PROBLEM 9.111

Determine the mass moment of inertia of a ring of mass *m*, cut from a thin uniform plate, with respect to (*a*) the axis *AA*′, (*b*) the centroidal axis *CC*′ that is perpendicular to the plane of the ring.

SOLUTION



$$\text{Mass} = m = \rho V = \rho t A$$

$$I_{\text{mass}} = \rho t I_{\text{area}} = \frac{m}{A} I_{\text{area}}$$

We first determine

$$A = \pi r_2^2 - \pi r_1^2 = \pi\left(r_2^2 - r_1^2\right)$$

$$I_{AA',\text{area}} = \frac{\pi}{4} r_2^4 - \frac{\pi}{4} r_1^4 = \frac{\pi}{4}\left(r_2^4 - r_1^4\right)$$

(*a*)

$$I_{AA',\text{ mass}} = \frac{m}{A} I_{AA',\text{ area}} = \frac{m}{\pi\left(r_2^2 - r_1^2\right)} \frac{\pi}{4}\left(r_2^4 - r_1^4\right) = \frac{1}{4} m\left(r_2^2 + r_1^2\right)$$

$$I_{AA'} = \frac{1}{4} m\left(r_1^2 + r_2^2\right) \blacktriangleleft$$

(*b*) By symmetry: $I_{BB'} = I_{AA'}$

Eq. (9.38): $I_{CC'} = I_{AA'} + I_{BB'} = 2I_{AA'}$

$$I_{CC'} = \frac{1}{2} m\left(r_1^2 + r_2^2\right) \blacktriangleleft$$

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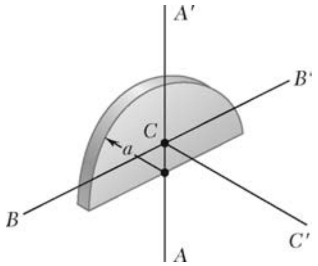
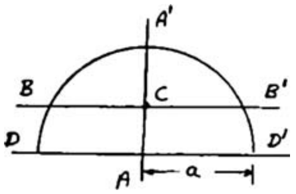


PROBLEM 9.112

A thin semicircular plate has a radius a and a mass m. Determine the mass moment of inertia of the plate with respect to (a) the centroidal axis BB', (b) the centroidal axis CC' that is perpendicular to the plate.

SOLUTION



$$\text{mass} = m = \rho t A$$

$$I_{\text{mass}} = \rho t I_{\text{area}} = \frac{m}{A} I_{\text{area}}$$

Area: $$A = \frac{1}{2}\pi a^2$$

$$I_{AA',\text{area}} = I_{DD',\text{area}} = \frac{1}{2}\left(\frac{\pi}{4}a^4\right) = \frac{1}{8}\pi a^4$$

$$I_{AA',\text{mass}} = I_{DD',\text{mass}} = \frac{m}{A} I_{AA',\text{area}} = \frac{m}{\frac{1}{2}\pi a^2}\left(\frac{1}{8}\pi a^4\right) = \frac{1}{4}ma^2$$

(a) $$I_{BB'} = I_{DD'} - m(AC)^2 = \frac{1}{4}ma^2 - m\left(\frac{4a}{3\pi}\right)^2$$

$$= (0.25 - 0.1801)ma^2$$

$$I_{BB'} = 0.0699ma^2 \blacktriangleleft$$

(b) Eq. (9.38): $$I_{CC'} = I_{AA'} + I_{BB'} = \frac{1}{4}ma^2 + 0.0699ma^2$$

$$I_{CC'} = 0.320ma^2 \blacktriangleleft$$

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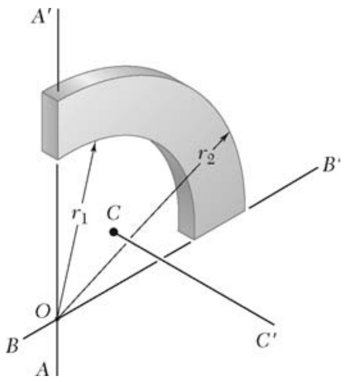


PROBLEM 9.113

The quarter ring shown has a mass m and was cut from a thin, uniform plate. Knowing that $r_1 = \frac{3}{4} r_2$, determine the mass moment of inertia of the quarter ring with respect to (*a*) the axis AA', (*b*) the centroidal axis CC' that is perpendicular to the plane of the quarter ring.

SOLUTION

First note
$$\text{mass} = m = \rho V = \rho t A$$
$$= \rho t \frac{\pi}{4}\left(r_2^2 - r_1^2\right)$$

Also
$$I_{\text{mass}} = \rho t I_{\text{area}}$$
$$= \frac{m}{\frac{\pi}{4}\left(r_2^2 - r_1^2\right)} I_{\text{area}}$$

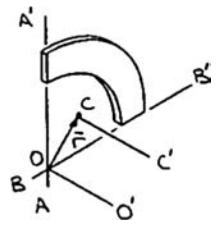
(*a*) Using Figure 9.12,
$$I_{AA',\text{area}} = \frac{\pi}{16}\left(r_2^4 - r_1^4\right)$$

Then
$$I_{AA',\text{mass}} = \frac{m}{\frac{\pi}{4}\left(r_2^2 - r_1^2\right)} \times \frac{\pi}{16}\left(r_2^4 - r_1^4\right)$$
$$= \frac{m}{4}\left(r_2^2 + r_1^2\right)$$
$$= \frac{m}{4}\left[r_2^2 + \left(\frac{3}{4} r_2\right)^2\right]$$

or $I_{AA'} = \frac{25}{64} m r_2^2$ ◀

(*b*) Symmetry implies
$$I_{BB',\text{mass}} = I_{AA',\text{mass}}$$

Then
$$I_{DD'} = I_{AA'} + I_{BB'}$$
$$= 2\left(\frac{25}{64} m r_2^2\right)$$
$$= \frac{25}{32} m r_2^2$$



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PROBLEM 9.113 (Continued)

Now locate centroid *C*.

$$\bar{X}\,\Sigma A = \Sigma \bar{x} A$$

or
$$\bar{X}\left(\frac{\pi}{4}r_2^2 - \frac{\pi}{4}r_1^2\right) = \frac{4r_2}{3\pi}\left(\frac{\pi}{4}r_2^2\right) - \frac{4r_1}{3\pi}\left(\frac{\pi}{4}r_1^2\right)$$

or
$$\bar{X} = \frac{4}{3\pi}\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$$

Now
$$\bar{r} = \bar{X}\sqrt{2}$$

$$= \frac{4\sqrt{2}}{3\pi}\frac{r_2^3 - \left(\frac{3}{4}r_2\right)^3}{r_2^2 - \left(\frac{3}{4}r_2\right)^2}$$

$$= \frac{37\sqrt{2}}{21\pi}r_2$$

Finally,
$$I_{DD'} = I_{CC'} + m\bar{r}^2$$

or
$$\frac{25}{32}mr_2^2 = I_{CC'} + m\left(\frac{37\sqrt{2}}{21\pi}r_2\right)^2$$

or $I_{CC'} = 0.1522mr_2^2$ ◀

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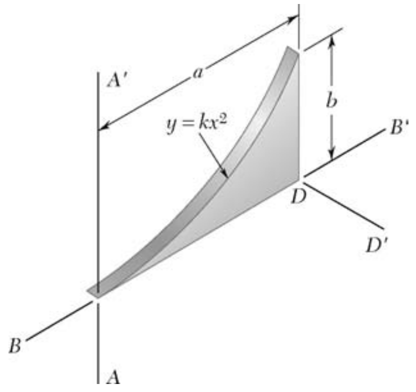


PROBLEM 9.114

The parabolic spandrel shown was cut from a thin, uniform plate. Denoting the mass of the spandrel by m, determine its mass moment of inertia with respect to (a) the axis BB', (b) the axis DD' that is perpendicular to the spandrel. (*Hint:* See Sample Problem 9.3.)

SOLUTION

First note

$$\text{mass} = m = \rho V = \rho t A$$

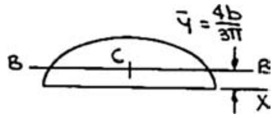
$$= \rho t\left(\frac{\pi}{2}ab\right)$$

Also

$$I_{\text{mass}} = \rho t I_{\text{area}}$$

$$= \frac{2m}{\pi ab} I_{\text{area}}$$

(a) We have



$$I_{x,\text{area}} = I_{BB',\text{area}} + A\bar{y}^2$$

or

$$I_{BB',\text{area}} = \frac{\pi}{8}ab^3 - \left(\frac{\pi}{2}ab\right)\left(\frac{4b}{3\pi}\right)^2$$

(a) From Sample Problem 9.3:

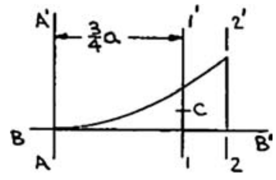
$$I_{BB',\text{area}} = \frac{1}{21}ab^3$$

Then

$$I_{BB',\text{mass}} = \frac{3m}{ab} \times \frac{1}{21}ab^3$$

or $I_{BB'} = \frac{1}{7}mb^2$ ◀

(b) From Sample Problem 9.3:



$$I_{AA',\text{area}} = \frac{1}{5}a^3 b$$

Now

$$I_{AA',\text{area}} = I_{11',\text{area}} + A\left(\frac{3}{4}a\right)^2$$

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PROBLEM 9.114 (Continued)

and $$I_{22',\text{area}} = I_{11',\text{area}} + A\left(\frac{1}{4}a\right)^2$$

Then $$I_{22',\text{area}} = I_{AA',\text{area}} + A\left[\left(\frac{1}{4}a\right)^2 - \left(\frac{3}{4}a\right)^2\right]$$

$$= \frac{1}{5}a^3b + \frac{1}{3}ab\left(\frac{1}{16}a^2 - \frac{9}{16}a^2\right)$$

$$= \frac{1}{30}a^3b$$

and $$I_{22',\text{mass}} = \frac{3m}{ab} \times \frac{1}{30}a^3b$$

$$= \frac{1}{10}ma^2$$

Finally, $$I_{DD',\text{mass}} = I_{BB',\text{mass}} + I_{22',\text{mass}}$$

$$= \frac{1}{7}mb^2 + \frac{1}{10}ma^2$$

or $$I_{DD'} = \frac{1}{70}m(7a^2 + 10b^2)$$ ◀

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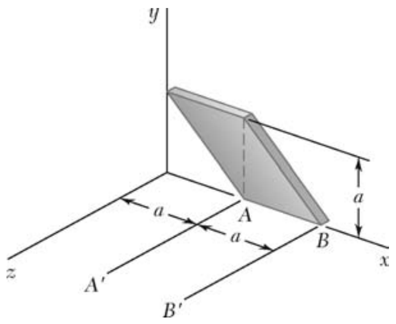
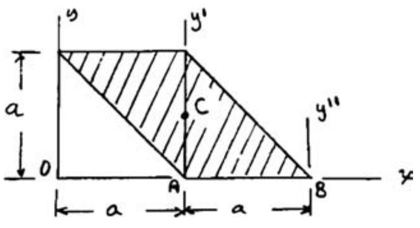


PROBLEM 9.115

A thin plate of mass m was cut in the shape of a parallelogram as shown. Determine the mass moment of inertia of the plate with respect to (a) the x axis, (b) the axis BB', which is perpendicular to the plate.

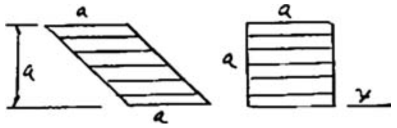
SOLUTION



$$\text{mass} = m = \rho t A$$

$$I_{\text{mass}} = \rho t I_{\text{area}} = \frac{m}{A} I_{\text{area}}$$

(a) Consider parallelogram as made of horizontal strips and slide strips to form a square since distance from each strip to x axis is unchanged.



$$I_{x,\text{area}} = \frac{1}{3}a^4$$

$$I_{x,\text{mass}} = \frac{m}{A} I_{x,\text{area}} = \frac{m}{a^2}\left(\frac{1}{3}a^4\right) \qquad I_x = \frac{1}{3}ma^2 \blacktriangleleft$$

(b) For centroidal axis y':

$$\bar{I}_{y',\text{area}} = 2\left[\frac{1}{12}a^4\right] = \frac{1}{6}a^4$$

$$\bar{I}_{y',\text{mass}} = \frac{m}{A} I_{y',\text{area}} = \frac{m}{a^2}\left(\frac{1}{6}a^4\right) = \frac{1}{6}ma^2$$

$$I_{y''} = \bar{I}_{y'} + ma^2 = \frac{1}{6}ma^2 + ma^2 = \frac{7}{6}ma^2$$

For axis $BB' \perp$ to plate, Eq. (9.38):

$$I_{BB'} = I_x + I_{y''} = \frac{1}{3}ma^2 + \frac{7}{6}ma^2 \qquad I_{BB'} = \frac{3}{2}ma^2 \blacktriangleleft$$

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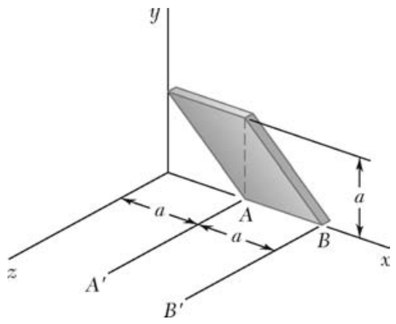

PROBLEM 9.116

A thin plate of mass m was cut in the shape of a parallelogram as shown. Determine the mass moment of inertia of the plate with respect to (a) the y axis, (b) the axis AA', which is perpendicular to the plate.

SOLUTION

See sketch of solution of Problem 9.115.

(a) From Part b of solution of Problem 9.115:

$$\bar{I}_{y'} = \frac{1}{6}ma^2$$

$$I_y = \bar{I}_{y'} + ma^2 = \frac{1}{6}ma^2 + ma^2 \qquad I_y = \frac{7}{6}ma^2 \blacktriangleleft$$

(b) From solution of Problem 9.115:

$$\bar{I}_{y'} = \frac{1}{6}ma^2 \quad \text{and} \quad I_x = \frac{1}{3}ma^2$$

Eq. (9.38): $I_{AA'} = \bar{I}_{y'} + I_x$

$$= \frac{1}{6}ma^2 + \frac{1}{3}ma^2 \qquad I_{AA'} = \frac{1}{2}ma^2 \blacktriangleleft$$

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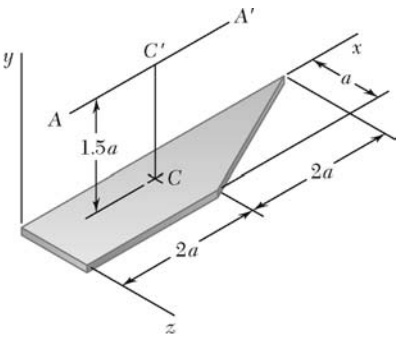


PROBLEM 9.117

A thin plate of mass m has the trapezoidal shape shown. Determine the mass moment of inertia of the plate with respect to (a) the x axis, (b) the y axis.

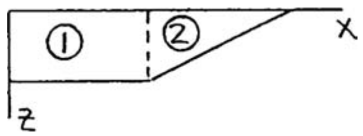
SOLUTION

First note $\quad \text{mass} = m = \rho V = \rho t A$

$$= \rho t\left[(2a)(a) + \frac{1}{2}(2a)(a)\right] = 3\rho t a^2$$

Also $\quad I_{\text{mass}} = \rho t I_{\text{area}} = \dfrac{m}{3a^2} I_{\text{area}}$

(a) Now



$$I_{x,\text{area}} = (I_x)_{1,\text{area}} + (I_x)_{2,\text{area}}$$

$$= \frac{1}{3}(2a)(a)^3 + \frac{1}{12}(2a)(a)^3$$

$$= \frac{5}{6}a^4$$

Then $\quad I_{x,\text{mass}} = \dfrac{m}{3a^2} \times \dfrac{5}{6} a^4$ $\quad$ or $\quad I_{x,\text{ mass}} = \dfrac{5}{18} ma^2$ ◀

(b) We have

$$I_{z,\text{area}} = (I_z)_{1,\text{area}} + (I_z)_{2,\text{ area}}$$

$$= \left[\frac{1}{3}(a)(2a)^3\right] + \left[\frac{1}{36}(a)(2a)^3 + \frac{1}{2}(2a)(a)\left(2a + \frac{1}{3}\times 2a\right)^2\right] = 10a^4$$

Then $\quad I_{x,\text{mass}} = \dfrac{m}{3a^2} \times 10a^4 = \dfrac{10}{3} ma^2$

Finally, $\quad I_{y,\text{mass}} = I_{x,\text{mass}} + I_{z,\text{mass}}$

$$= \frac{5}{18}ma^2 + \frac{10}{3}ma^2$$

$$= \frac{65}{18}ma^2$$

or $\quad I_{y,\text{mass}} = 3.61 ma^2$ ◀

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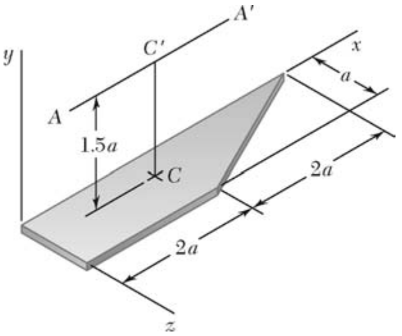

PROBLEM 9.118

A thin plate of mass m has the trapezoidal shape shown. Determine the mass moment of inertia of the plate with respect to (a) the centroidal axis CC' that is perpendicular to the plate, (b) the axis AA' that is parallel to the x axis and is located at a distance $1.5a$ from the plate.

SOLUTION

First locate the centroid C.

$$\bar{X}\Sigma A = \Sigma \bar{x} A: \quad \bar{X}(2a^2 + a^2) = a(2a^2) + \left(2a + \frac{1}{3}\times 2a\right)(a^2)$$

or

$$\bar{X} = \frac{14}{9}a$$

$$\bar{Z}\Sigma A = \Sigma \bar{z} A: \quad \bar{Z}(2a^2 + a^2) = \left(\frac{1}{2}a\right)(2a^2) + \left(\frac{1}{3}a\right)(a^2)$$

or

$$\bar{Z} = \frac{4}{9}a$$

(a) We have $\quad I_{y,\,\text{mass}} = \bar{I}_{CC',\,\text{mass}} + m(\bar{X}^2 + \bar{Z}^2)$

From the solution to Problem 9.117:

$$I_{y,\text{mass}} = \frac{65}{18}ma^2$$

Then $\quad \bar{I}_{cc',\text{mass}} = \frac{65}{18}ma^2 - m\left[\left(\frac{14}{9}a\right)^2 + \left(\frac{4}{9}a\right)^2\right]$

or $\quad \bar{I}_{cc'} = 0.994ma^2$ ◀

(b) We have $\quad I_{x,\text{mass}} = \bar{I}_{BB',\text{mass}} + m(\bar{Z})^2$

and $\quad I_{AA',\,\text{mass}} = \bar{I}_{BB',\,\text{mass}} + m(1.5a)^2$

Then $\quad I_{AA',\,\text{mass}} = I_{x,\,\text{mass}} + m\left[(1.5a)^2 - \left(\frac{4}{9}a\right)^2\right]$

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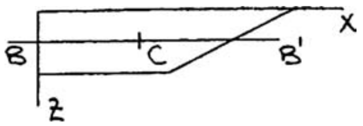
PROBLEM 9.118 (Continued)

From the solution to Problem 9.117:



$$I_{x,\text{mass}} = \frac{5}{18}ma^2$$

Then
$$I_{AA',\text{mass}} = \frac{5}{18}ma^2 + m\left[(1.5a)^2 - \left(\frac{4}{9}a\right)^2\right]$$

or $I_{AA'} = 2.33ma^2$ ◀

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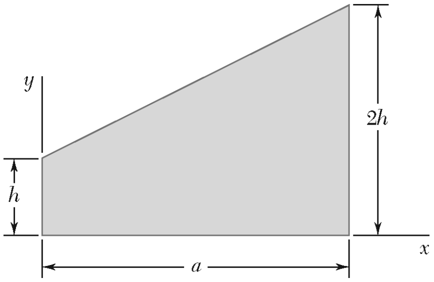
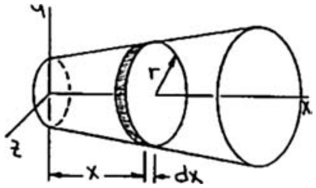


PROBLEM 9.119

The area shown is revolved about the *x* axis to form a homogeneous solid of revolution of mass *m*. Using direct integration, express the mass moment of inertia of the solid with respect to the *x* axis in terms of *m* and *h*.

SOLUTION



We have $y = \frac{2h-h}{a}x + h$

so that $r = \frac{h}{a}(x+a)$

For the element shown:

$$dm = \rho\pi r^2 dx \quad dI_x = \frac{1}{2}r^2 dm$$

$$= \rho\pi\left[\frac{h}{a}(x+a)\right]^2 dx$$

Then $m = \int dm = \int_0^a \rho\pi\frac{h^2}{a^2}(x+a)^2 dx = \frac{1}{3}\rho\pi\frac{h^2}{a^2}[(x+a)^3]_0^a$

$$= \frac{1}{3}\rho\pi\frac{h^2}{a^2}(8a^3 - a^3) = \frac{7}{3}\rho\pi a h^2$$

Now $I_x = \int dI_x = \int \frac{1}{2}r^2(\rho\pi r^2 dx) = \frac{1}{2}\rho\pi\int_0^a \left[\frac{h}{a}(x+a)\right]^4 dx$

$$= \frac{1}{2}\rho\pi \times \frac{1}{5}\frac{h^4}{a^4}[(x+a)^5]_0^a = \frac{1}{10}\rho\pi\frac{h^4}{a^4}(32a^5 - a^5)$$

$$= \frac{31}{10}\rho\pi a h^4$$

From above: $\rho\pi a h^2 = \frac{3}{7}m$

Then $I_x = \frac{31}{10}\left(\frac{3}{7}m\right)h^2 = \frac{93}{70}mh^2$

or $I_x = 1.329\, mh^2$ ◀

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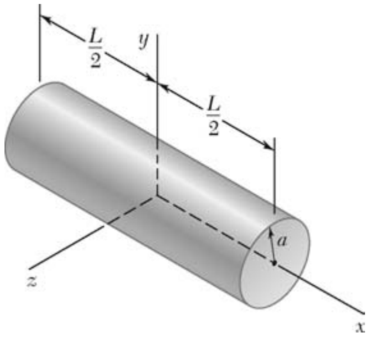
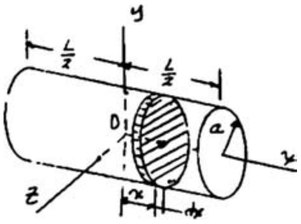


PROBLEM 9.120

Determine by direct integration the mass moment of inertia with respect to the *y* axis of the right circular cylinder shown, assuming that it has a uniform density and a mass *m*.

SOLUTION

For element shown: $dm = \rho dV = \rho \pi a^2 dx$



$$dI_y = d\bar{I}_y + x^2 dm$$

$$= \frac{1}{4}a^2 dm + x^2 dm$$

$$= \left(\frac{1}{4}a^2 + x^2\right)\rho\pi a^2 dx$$

$$I_y = \int dI_y = \int_{-L/2}^{+L/2} \rho\pi a^2 \left(\frac{1}{4}a^2 + x^2\right) dx$$

$$= \rho\pi a^2 \left|\frac{1}{4}a^2 x + \frac{x^3}{3}\right|_{-L/2}^{+L/2} = 2\rho\pi a^2 \left[\frac{1}{4}a^2 \frac{L}{2} + \frac{1}{3}\left(\frac{L}{2}\right)^3\right]$$

$$I_y = \frac{1}{12}\rho\pi a^2 L(3a^2 + L^2) \quad \triangleleft$$

But for whole cylinder: $m = \rho V = \rho \pi a^2 L$

Thus, $$I_y = \frac{1}{12}m(3a^2 + L^2) \quad \blacktriangleleft$$

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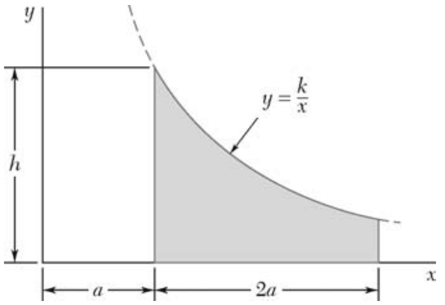
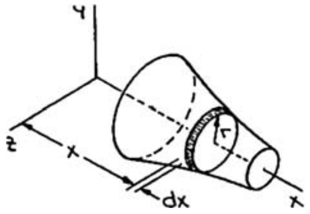


PROBLEM 9.121

The area shown is revolved about the x axis to form a homogeneous solid of revolution of mass m. Determine by direct integration the mass moment of inertia of the solid with respect to (a) the x axis, (b) the y axis. Express your answers in terms of m and the dimensions of the solid.

SOLUTION



We have at (a, h): $h = \dfrac{k}{a}$

or $k = ah$

For the element shown:

$$r = 4$$

$$dm = \rho\pi r^2 dx$$

$$= \rho\pi \left(\frac{ah}{x}\right)^2 dx$$

Then

$$m = \int dm = \int_a^{3a} \rho\pi \left(\frac{ah}{x}\right)^2 dx$$

$$= \rho\pi a^2 h^2 \left[-\frac{1}{x}\right]_a^{3a}$$

$$= \rho\pi a^2 h^2 \left[-\frac{1}{3a} - \left(-\frac{1}{a}\right)\right] = \frac{2}{3}\rho\pi a h^2$$

(a) For the element: $dI_x = \dfrac{1}{2} r^2 dm = \dfrac{1}{2}\rho\pi r^4 dx$

Then

$$I_x = \int dI_x = \int_a^{3a} \frac{1}{2}\rho\pi\left(\frac{ah}{x}\right)^4 dx = \frac{1}{2}\rho\pi a^4 h^4 \left[-\frac{1}{3}\frac{1}{x^3}\right]_a^{3a}$$

$$= -\frac{1}{6}\rho\pi a^4 h^4 \left[\left(\frac{1}{3a}\right)^3 - \left(\frac{1}{a}\right)^3\right] = \frac{1}{6} \times \frac{26}{27}\rho\pi a h^4$$

$$= \frac{1}{6} \times \frac{2}{3}\rho\pi a h^2 \times \frac{13}{9} h^2 = \frac{13}{54} m h^2$$

or $I_x = 0.241\, mh^2$ ◀

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PROBLEM 9.121 (Continued)

(*b*) For the element: $dI_y = dI_y + x^2 dm$

$$= \frac{1}{4} r^2 dm + x^2 dm$$

Then $$I_y = \int dI_y = \int_a^{3a} \left[\frac{1}{4}\left(\frac{ah}{x}\right)^2 + x^2 \right] \rho\pi \left(\frac{ah}{x}\right)^2 dx$$

$$= \rho\pi a^2 h^2 \int_a^{3a} \left(\frac{1}{4} \frac{a^2 h^2}{x^4} + 1 \right) dx = \rho\pi a^2 h^2 \left[-\frac{1}{12} \frac{a^2 h^2}{x^3} + x \right]_a^{3a}$$

$$= \left(\frac{3}{2} m \right) a \left\{ \left[-\frac{1}{12} \frac{a^2 h^2}{(3a)^3} + 3a \right] - \left[-\frac{1}{12} \frac{a^2 h^2}{(a)^3} + a \right] \right\}$$

$$= \frac{3}{2} ma \left(\frac{1}{12} \times \frac{26}{27} \frac{h^2}{a} + 2a \right) = m \left(\frac{13}{108} h^2 + 3a^2 \right)$$

or $I_y = m(3a^2 + 0.1204h^2)$ ◀

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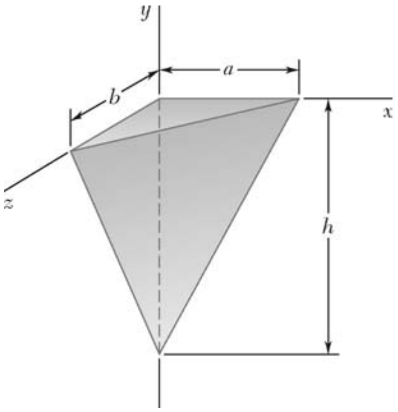


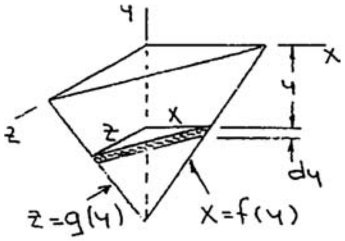
PROBLEM 9.122

Determine by direct integration the mass moment of inertia with respect to the x axis of the tetrahedron shown, assuming that it has a uniform density and a mass m.

SOLUTION

We have $$x = \frac{a}{h}y + a = a\left(1 + \frac{y}{h}\right)$$

and $$z = \frac{b}{h}y + b = b\left(1 + \frac{y}{h}\right)$$



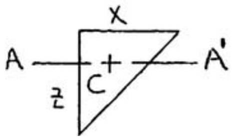
For the element shown: $$dm = \rho\left(\frac{1}{2}xz\,dy\right) = \frac{1}{2}\rho ab\left(1 + \frac{y}{h}\right)^2 dy$$

Then $$m = \int dm = \int_{-h}^{0} \frac{1}{2}\rho ab\left(1 + \frac{y}{h}\right)^2 dy$$

$$= \frac{1}{2}\rho ab \times \frac{h}{3}\left[\left(1 + \frac{y}{h}\right)^3\right]_{-h}^{0}$$

$$= \frac{1}{6}\rho abh\left[(1)^3 - (1-1)^3\right]$$

$$= \frac{1}{6}\rho abh$$



Now, for the element: $$I_{AA',\text{area}} = \frac{1}{36}xz^3 = \frac{1}{36}ab^3\left(1 + \frac{y}{h}\right)^4$$

Then $$dI_{AA',\text{mass}} = \rho t I_{AA',\text{area}} = \rho(dy)\left[\frac{1}{3}ab^3\left(1 + \frac{y}{h}\right)^4\right]$$

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PROBLEM 9.122 (Continued)

Now

$$dI_x = dI_{AA',\text{mass}} + \left[y^2 + \left(\frac{1}{3}z\right)^2\right]dm$$

$$= \frac{1}{36}\rho ab^3\left(1+\frac{y}{h}\right)^4 dy$$

$$+\left\{y^2 + \left[\frac{1}{3}b\left(1+\frac{y}{h}\right)\right]^2\right\}\left[\frac{1}{2}\rho ab\left(1+\frac{y}{h}\right)^2 dy\right]$$

$$= \frac{1}{12}\rho ab^3\left(1+\frac{y}{h}\right)^4 dy + \frac{1}{2}\rho ab\left(y^2 + 2\frac{y^3}{h} + \frac{y^4}{h^2}\right)dy$$

Now $$m = \frac{1}{6}\rho abh$$

Then $$dI_x = \left[\frac{1}{2}m\frac{b^2}{h}\left(1+\frac{y}{h}\right)^4 + \frac{3m}{h}\left(y^2 + 2\frac{y^3}{h} + \frac{y^4}{h^2}\right)\right]dy$$

and $$I_x = \int dI_x = \int_{-h}^{0}\frac{m}{2h}\left[b^2\left(1+\frac{y}{h}\right)^4 + 6\left(y^2 + 2\frac{y^3}{h} + \frac{y^4}{h^2}\right)\right]dy$$

$$= \frac{m}{2h}\left[b^2 \times \frac{h}{5}\left(1+\frac{y}{h}\right)^5 + 6\left(\frac{1}{3}y^3 + \frac{1}{2}\frac{y^4}{h} + \frac{y^5}{5h^2}\right)\right]_{-h}^{0}$$

$$= \frac{m}{2h}\left\{\frac{1}{5}b^2h(1)^5 - 6\left[\frac{1}{3}(-h)^3 + \frac{1}{2h}(-h)^4 + \frac{1}{5h^2}(-h)^5\right]\right\}$$

or $$I_x = \frac{1}{10}m(b^2 + h^2)$$ ◀

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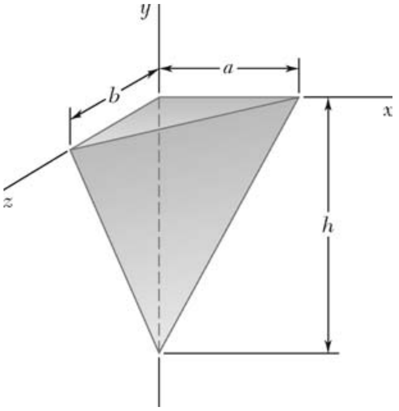
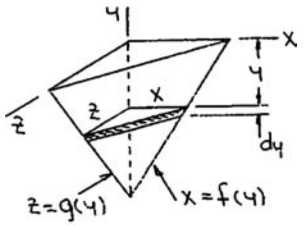


PROBLEM 9.123

Determine by direct integration the mass moment of inertia with respect to the y axis of the tetrahedron shown, assuming that it has a uniform density and a mass m.

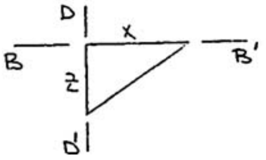
SOLUTION



We have $$x = \frac{a}{h}y + a = a\left(1 + \frac{y}{h}\right)$$

and $$z = \frac{b}{h}y + b = b\left(1 + \frac{y}{h}\right)$$

For the element shown: $$dm = \rho\left(\frac{1}{2}xz\,dy\right) = \frac{1}{2}\rho ab\left(1 + \frac{y}{h}\right)^2 dy$$



Then $$m = \int dm = \int_{-h}^{0} \frac{1}{2}\rho ab\left(1 + \frac{y}{h}\right)^2 dy$$

$$= \frac{1}{2}\rho ab \times \frac{h}{3}\left[\left(1 + \frac{y}{h}\right)^3\right]_{-h}^{0}$$

$$= \frac{1}{6}\rho abh\left[(1)^3 - (1-1)^3\right]$$

$$= \frac{1}{6}\rho abh$$

Also $$I_{BB',\text{area}} = \frac{1}{12}xz^3 \qquad I_{DD',\text{area}} = \frac{1}{12}zx^3$$

Then, using $$I_{\text{mass}} = \rho t I_{\text{area}}$$

we have $$dI_{BB',\text{mass}} = \rho(dy)\left(\frac{1}{12}xz^3\right) \qquad dI_{DD',\text{mass}} = \rho(dy)\left(\frac{1}{12}zx^3\right)$$

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PROBLEM 9.123 (Continued)

Now

$$dI_y = dI_{BB',\,\text{mass}} + dI_{DD',\,\text{mass}}$$
$$= \frac{1}{12}\rho xz(x^2 + z^2)dy$$
$$= \frac{1}{12}\rho ab\left(1+\frac{y}{h}\right)^2\left[(a^2+b^2)\left(1+\frac{y}{h}\right)^2\right]dy$$

We have $$m = \frac{1}{6}\rho abh \Rightarrow dI_y = \frac{m}{2h}(a^2+b^2)\left(1+\frac{y}{h}\right)^4 dy$$

Then

$$I_y = \int dI_y = \int_{-h}^{0}\frac{m}{2h}(a^2+b^2)\left(1+\frac{y}{h}\right)^4 dy$$
$$= \frac{m}{2h}\left(a^2+b^2\right)\times\frac{h}{5}\left[\left(1+\frac{y}{h}\right)^5\right]_{-h}^{0}$$
$$= \frac{m}{10}\left(a^2+b^2\right)\left[(1)^5 - (1-1)^5\right]$$

or $$I_y = \frac{1}{10}m(a^2+b^2)$$ ◀

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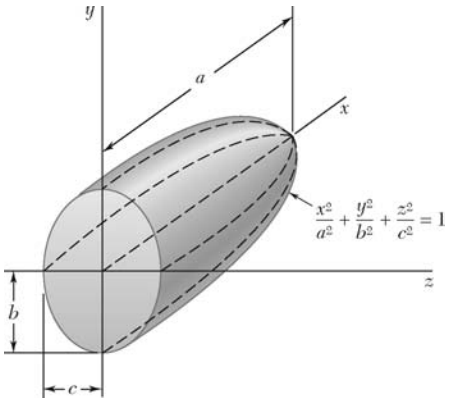


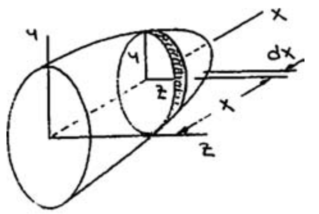
PROBLEM 9.124*

Determine by direct integration the mass moment of inertia with respect to the *z* axis of the semiellipsoid shown, assuming that it has a uniform density and a mass *m*.

SOLUTION

First note that when

$$z = 0: \quad y = b\left(1 - \frac{x^2}{a^2}\right)^{1/2}$$



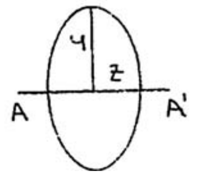
$$y = 0: \quad z = c\left(1 - \frac{x^2}{a^2}\right)^{1/2}$$

For the element shown:

$$dm = \rho(\pi yzdx) = \pi\rho bc\left(1 - \frac{x^2}{a^2}\right)dx$$

Then

$$m = \int dm = \int_0^a \pi\rho bc\left(1 - \frac{x^2}{a^2}\right)dx$$



$$= \pi\rho bc\left[x - \frac{1}{3a^2}x^3\right]_0^a = \frac{2}{3}\pi\rho abc$$

For the element:

$$I_{AA',\text{area}} = \frac{\pi}{4}zy^3$$

Then

$$dI_{AA',\text{mass}} = \rho t I_{AA',\text{area}} = \rho(dx)\left(\frac{\pi}{4}zy^3\right)$$

Now

$$dI_z = dI_{AA',\text{mass}} + x^2 dm$$

$$= \frac{\pi}{4}\rho b^3 c\left(1 - \frac{x^2}{a^2}\right)^2 dx + x^2\left[\pi\rho bc\left(1 - \frac{x^2}{a^2}\right)dx\right]$$

$$= \frac{3m}{2a}\left[\frac{b^2}{4}\left(1 - 2\frac{x^2}{a^2} + \frac{x^4}{a^4}\right) + \left(x^2 - \frac{x^4}{a^2}\right)\right]dx$$

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PROBLEM 9.124* (Continued)

Finally,

$$I_z = \int dI_z$$

$$= \frac{3m}{2a}\int_0^a \left[\frac{b^2}{4}\left(1 - 2\frac{x^2}{a^2} + \frac{x^4}{a^4}\right) + \left(x^2 - \frac{x^4}{a^2}\right)\right]dx$$

$$= \frac{3m}{2a}\left[\frac{b^2}{4}\left(x - \frac{2}{3}\frac{x^3}{a^2} + \frac{1}{5}\frac{x^5}{a^4}\right) + \left(\frac{1}{3}x^3 - \frac{1}{5}\frac{x^5}{a^2}\right)\right]_0^a$$

$$= \frac{3}{2}m\left[\frac{b^2}{4}\left(1 - \frac{2}{3} + \frac{1}{5}\right) + a^2\left(\frac{1}{3} - \frac{1}{5}\right)\right]$$

or $I_z = \frac{1}{5}m\left(a^2 + b^2\right)$ ◀

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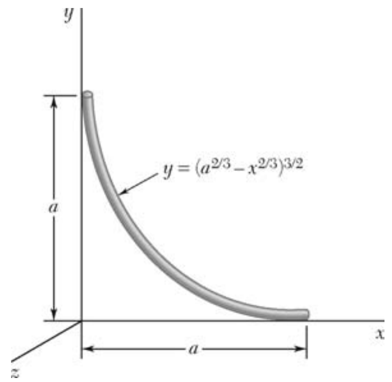


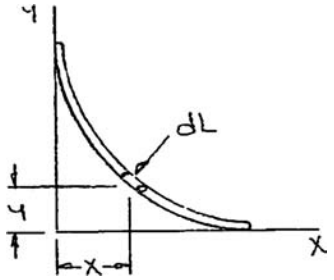
PROBLEM 9.125*

A thin steel wire is bent into the shape shown. Denoting the mass per unit length of the wire by m', determine by direct integration the mass moment of inertia of the wire with respect to each of the coordinate axes.

SOLUTION

First note $$\frac{dy}{dx} = -x^{-1/3}(a^{2/3} - x^{2/3})^{1/2}$$

Then $$1 + \left(\frac{dy}{dx}\right)^2 = 1 + x^{-2/3}(a^{2/3} - x^{2/3})$$
$$= \left(\frac{a}{x}\right)^{2/3}$$



For the element shown: $$dm = m'dL = m'\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$
$$= m'\left(\frac{a}{x}\right)^{1/3} dx$$

Then $$m = \int dm = \int_0^a m' \frac{a^{1/3}}{x^{1/3}} dx = \frac{3}{2} m' a^{1/3} \left[x^{2/3}\right]_0^a = \frac{3}{2} m' a$$

Now $$I_x = \int y^2 dm = \int_0^a (a^{2/3} - x^{2/3})^3 \left(m' \frac{a^{1/3}}{x^{1/3}} dx\right)$$
$$= m' a^{1/3} \int_0^a \left(\frac{a^2}{x^{1/3}} - 3a^{4/3}x^{1/3} + 3a^{2/3}x - x^{5/3}\right) dx$$
$$= m' a^{1/3} \left[\frac{3}{2} a^2 x^{2/3} - \frac{9}{4} a^{4/3} x^{4/3} + \frac{3}{2} a^{2/3} x^2 - \frac{3}{8} x^{8/3}\right]_0^a$$
$$= m' a^3 \left(\frac{3}{2} - \frac{9}{4} + \frac{3}{2} - \frac{3}{8}\right) = \frac{3}{8} m' a^3$$

or $$I_x = \frac{1}{4} ma^2 \blacktriangleleft$$

Symmetry implies $$I_y = \frac{1}{4} ma^2 \blacktriangleleft$$

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PROBLEM 9.125* (Continued)

Alternative solution:

$$I_y = \int x^2 dm = \int_0^a x^2 \left(m' \frac{a^{1/3}}{x^{1/3}} dx \right) = m' a^{1/3} \int_0^a x^{5/3} dx$$

$$= m' a^{1/3} \times \frac{3}{8} \left[x^{8/3} \right]_0^a = \frac{3}{8} m' a^3$$

$$= \frac{1}{4} m a^2$$

Also $$I_z = \int \left(x^2 + y^2 \right) dm = I_y + I_x$$

or $$I_z = \frac{1}{2} m a^2 \blacktriangleleft$$

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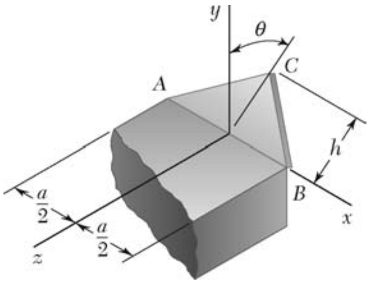
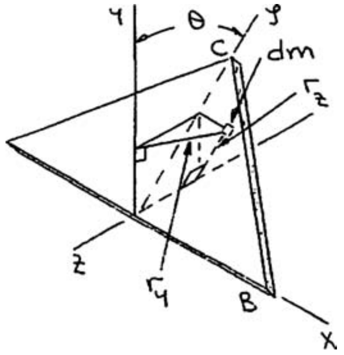


PROBLEM 9.126

A thin triangular plate of mass m is welded along its base AB to a block as shown. Knowing that the plate forms an angle θ with the y axis, determine by direct integration the mass moment of inertia of the plate with respect to (a) the x axis, (b) the y axis, (c) the z axis.

SOLUTION



For line BC:

$$\zeta = -\frac{h}{\frac{a}{2}}x + h$$

$$= \frac{h}{a}(a - 2x)$$

Also

$$m = \rho V = \rho t\left(\frac{1}{2}ah\right)$$

$$= \frac{1}{2}\rho tah$$

(a) We have

$$dI_x = \frac{1}{12}\zeta^2 dm' + \left(\frac{\zeta}{2}\right)^2 dm'$$

$$= \frac{1}{3}\zeta^2 dm'$$

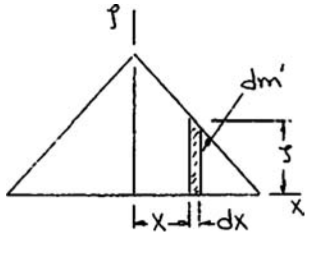
where

$$dm' = \rho t\zeta\, dx$$

Then

$$I_x = \int dI_x = 2\int_0^{a/2}\frac{1}{3}\zeta^2(\rho t\zeta\, dx)$$

$$= \frac{2}{3}\rho t\int_0^{a/2}\left[\frac{h}{a}(a-2x)\right]^3 dx$$



$$= \frac{2}{3}\rho t\frac{h^3}{a^3}\times\frac{1}{4}\left(-\frac{1}{2}\right)\left[(a-2x)^4\right]_0^{a/2}$$

$$= -\frac{1}{12}\rho t\frac{h^3}{a^3}\left[(a-a)^4 - (a)^4\right]$$

$$= \frac{1}{12}\rho tah^3$$

or $\quad I_x = \frac{1}{6}mh^2$ ◀

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PROBLEM 9.126 (Continued)

Now $$I_\zeta = \int x^2 dm$$

and $$I_\zeta = \int x^2 dm' = 2\int_0^{a/2} x^2 (\rho t \zeta dx)$$

$$= 2\rho t \int_0^{a/2} x^2 \left[\frac{h}{a}(a-2x)\right] dx$$

$$= 2\rho t \frac{h}{a}\left[\frac{a}{3}x^3 - \frac{1}{4}x^4\right]_0^{a/2}$$

$$= 2\rho t \frac{h}{a}\left[\frac{a}{3}\left(\frac{a}{2}\right)^3 - \frac{1}{4}\left(\frac{a}{2}\right)^4\right]$$

$$= \frac{1}{48}\rho t a^3 h = \frac{1}{24} m a^2$$

(*b*) We have $$I_y = \int r_y^2 dm = \int \left[x^2 + (\zeta \sin\theta)^2\right] dm$$

$$= \int x^2 dm + \sin^2\theta \int \zeta^2 dm$$

Now $$I_x = \int \zeta^2 dm \Rightarrow I_y = I_\zeta + I_x \sin^2\theta$$

$$= \frac{1}{24} m a^2 + \frac{1}{6} m h^2 \sin^2\theta$$

or $$I_y = \frac{m}{24}(a^2 + 4h^2 \sin^2\theta) \blacktriangleleft$$

(*c*) We have $$I_z = \int r_z^2 dm = \int (x^2 + y^2) dm$$

$$= \int \left[x^2 + (\zeta \cos\theta)^2\right] dm$$

$$= \int x^2 dm + \cos^2\theta \int \zeta^2 dm$$

$$= I_\zeta + I_x \cos^2\theta$$

$$= \frac{1}{24} m a^2 + \frac{1}{6} m h^2 \cos^2\theta$$

or $$I_z = \frac{m}{24}(a^2 + 4h^2 \cos^2\theta) \blacktriangleleft$$

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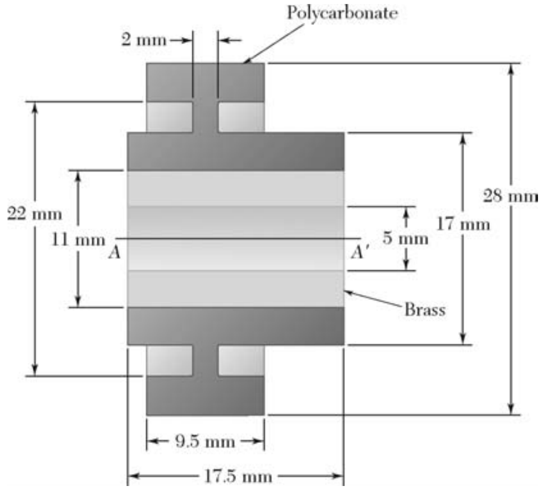


PROBLEM 9.127

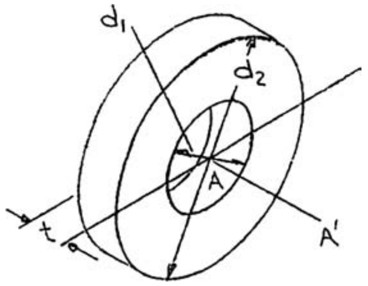
Shown is the cross section of a molded flat-belt pulley. Determine its mass moment of inertia and its radius of gyration with respect to the axis *AA′*. (The density of brass is 8650 kg/m^3 and the density of the fiber-reinforced polycarbonate used is 1250 kg/m^3.)

SOLUTION

First note for the cylindrical ring shown that

$$m = \rho V = \rho t \times \frac{\pi}{4}\left(d_2^2 - d_1^2\right)$$

and, using Figure 9.28, that



$$\begin{aligned}
I_{AA'} &= \frac{1}{2}m_2\left(\frac{d_2}{2}\right)^2 - \frac{1}{2}m_1\left(\frac{d_1}{2}\right)^2 \\
&= \frac{1}{8}\left[\left(\rho t \times \frac{\pi}{4}d_2^2\right)d_2^2 - \left(\rho t \times \frac{\pi}{4}d_1^2\right)d_1^2\right] \\
&= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^4 - d_1^4\right) \\
&= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^2 - d_1^2\right)\left(d_2^2 + d_1^2\right) \\
&= \frac{1}{8}m\left(d_1^2 + d_2^2\right)
\end{aligned}$$

Now treat the pulley as four concentric rings and, working from the brass outward, we have

$$\begin{aligned}
m = \frac{\pi}{4}\Big\{&8650 \text{ kg/m}^3 \times (0.0175 \text{ m}) \times (0.011^2 - 0.005^2) \text{ m}^2 \\
&+ 1250 \text{ kg/m}^3\Big[(0.0175 \text{ m}) \times (0.017^2 - 0.011^2) \text{ m}^2 \\
&+ (0.002 \text{ m}) \times (0.022^2 - 0.017^2) \text{ m}^2 \\
&+ (0.0095 \text{ m}) \times (0.028^2 - 0.022^2) \text{ m}^2\Big]\Big\} \\
&= (11.4134 + 2.8863 + 0.38288 + 2.7980) \times 10^{-3} \text{ kg} \\
&= 17.4806 \times 10^{-3} \text{ kg}
\end{aligned}$$

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PROBLEM 9.127 (Continued)

and

$$I_{AA'} = \frac{1}{8}\Big[(11.4134)(0.005^2 + 0.011^2) + (2.8863)(0.011^2 + 0.017^2)$$
$$+ (0.38288)(0.017^2 + 0.022^2)$$
$$+(2.7980)(0.022^2 + 0.028^2)\Big]\times 10^{-3}\ \text{kg}\times\text{m}^2$$

$$= (208.29 + 147.92 + 37.00 + 443.48)\times 10^{-9}\ \text{kg}\cdot\text{m}^2$$
$$= 836.69\times 10^{-9}\ \text{kg}\cdot\text{m}^2$$

or $I_{AA'} = 837\times 10^{-9}\ \text{kg}\cdot\text{m}^2$ ◀

Now

$$k_{AA'}^2 = \frac{I_{AA'}}{m} = \frac{836.69\times 10^{-9}\ \text{kg}\cdot\text{m}^2}{17.4806\times 10^{-3}\ \text{kg}} = 47.864\times 10^{-6}\ \text{m}^2$$

or $k_{AA'} = 6.92\ \text{mm}$ ◀

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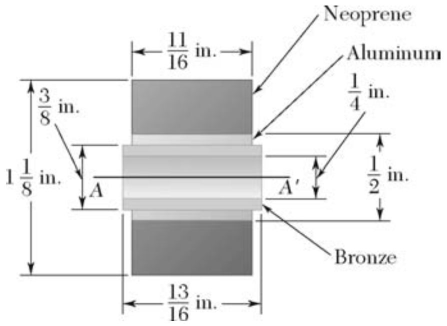


PROBLEM 9.128

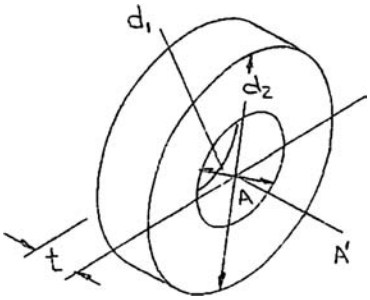
Shown is the cross section of an idler roller. Determine its mass moment of inertia and its radius of gyration with respect to the axis AA'. (The specific weight of bronze is 0.310 lb/in^3; of aluminum, 0.100 lb/in^3; and of neoprene, 0.0452 lb/in^3.)

SOLUTION

First note for the cylindrical ring shown that

$$m = \rho V = \rho t \times \frac{\pi}{4}\left(d_2^2 - d_1^2\right) = \frac{\pi}{4}\rho t\left(d_2^2 - d_1^2\right)$$

and, using Figure 9.28, that



$$\begin{aligned}
I_{AA'} &= \frac{1}{2}m_2\left(\frac{d_2}{2}\right)^2 - \frac{1}{2}m_1\left(\frac{d_1}{2}\right)^2 \\
&= \frac{1}{8}\left[\left(\rho t \times \frac{\pi}{4}d_2^2\right)d_2^2 - \left(\rho t \times \frac{\pi}{4}d_1^2\right)d_1^2\right] \\
&= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^4 - d_1^4\right) \\
&= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^2 - d_1^2\right)\left(d_2^2 + d_1^2\right) \\
&= \frac{1}{8}m\left(d_1^2 + d_2^2\right)
\end{aligned}$$

Now treat the roller as three concentric rings and, working from the bronze outward, we have

$$\begin{aligned}
m = \frac{\pi}{4} \times \frac{1}{32.2}\,\text{ft/s}^2 \Bigg\{&(0.310\ \text{lb/in.}^3)\left(\frac{13}{16}\text{in.}\right)\left[\left(\frac{3}{8}\right)^2 - \left(\frac{1}{4}\right)^2\right]\text{in.}^2 \\
&+ \left(0.100\ \text{lb/in.}^3\right)\left(\frac{11}{16}\text{in.}\right)\left[\left(\frac{1}{2}\right)^2 - \left(\frac{3}{8}\right)^2\right]\text{in.}^2 \\
&+ \left(0.0452\ \text{lb/in.}^3\right)\left(\frac{11}{16}\text{in.}\right)\left[\left(1\frac{1}{8}\right)^2 - \left(\frac{1}{2}\right)^2\right]\text{in.}^2\Bigg\}
\end{aligned}$$

$$= \left(479.96 + 183.41 + 769.80\right) \times 10^{-6}\ \text{lb} \cdot \text{s}^2/\text{ft}$$

$$= 1.4332 \times 10^{-3}\ \text{lb} \cdot \text{s}^2/\text{ft}$$

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PROBLEM 9.128 (Continued)

and
$$I_{AA'} = \frac{1}{8}\left\{(479.96)\left[\left(\frac{1}{4}\right)^2 + \left(\frac{3}{8}\right)^2\right] + (183.41)\left[\left(\frac{3}{8}\right)^2 + \left(\frac{1}{2}\right)^2\right]\right.$$
$$\left. + (769.80)\left[\left(1\frac{1}{8}\right)^2 + \left(\frac{1}{2}\right)^2\right]\right\} \times 10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft} \times \text{in.}^2 \times \frac{1\ \text{ft}^2}{144\ \text{in.}^2}$$
$$= (84.628 + 62.191 + 1012.78) \times 10^{-9}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$
$$= 1.15960 \times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{AA'} = 1.160 \times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

Now
$$k_{AA'}^2 = \frac{I_{AA'}}{m} = \frac{1.15960 \times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2}{1.4332 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}} = 809.09 \times 10^{-6}\ \text{ft}^2$$

Then
$$k_{AA'} = 28.445 \times 10^{-3}\ \text{ft}$$

or $k_{AA'} = 0.341\ \text{in.}$ ◀

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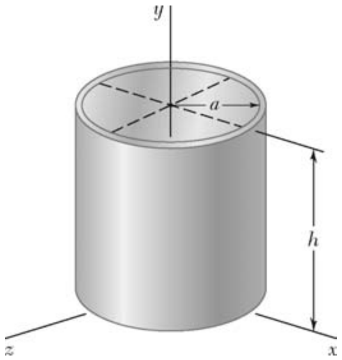
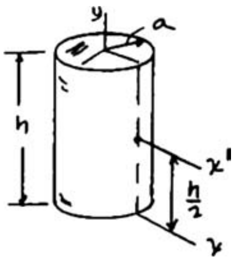


PROBLEM 9.129

Knowing that the thin cylindrical shell shown is of mass m, thickness t, and height h, determine the mass moment of inertia of the shell with respect to the x axis. (*Hint:* Consider the shell as formed by removing a cylinder of radius a and height h from a cylinder of radius $a + t$ and height h; then neglect terms containing t^2 and t^3 and keep those terms containing t.)

SOLUTION

From Figure 9.28 for a solid cylinder (change orientation of axes):

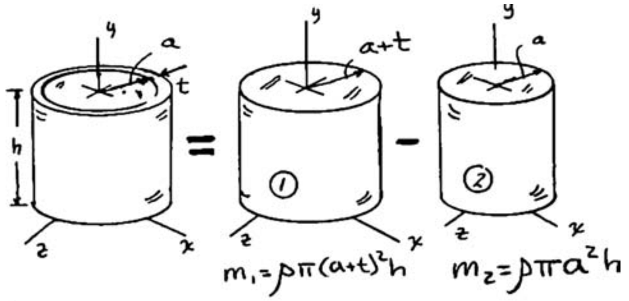


$$\bar{I}_{x'} = \frac{1}{12}m(3a^2 + h^2)$$

$$I_x = \bar{I}_{x'} + m\left(\frac{h}{2}\right)^2$$

$$= \frac{1}{12}m(3a^2 + h^2) + m\frac{h^2}{4}$$

$$= m\left(\frac{3}{12}a^2 + \frac{h^2}{3}\right) \qquad I_x = \frac{1}{12}m(3a^2 + 4h^2) \ \triangleleft$$



Shell = Cylinder ① – Cylinder ②

$$I_x = \frac{1}{12}m_1[3(a+t)^2 + 4h^2] - \frac{1}{12}m_2(3a^2 + 4h^2)$$

$$= \frac{1}{12}[\rho\pi(a+t)^2 h][3(a+t)^2 + 4h^2] - \frac{1}{12}(\rho\pi a^2 h)(3a^2 + 4h^2)$$

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PROBLEM 9.129 (Continued)

Expand $(a+t)$ factors and neglecting terms in $t^2, t^3,$ and t^4:

$$I_x = \frac{\rho\pi h}{12}\left[\left(a^2 + 2at + t^2\right)\left(3a^2 + 6at + 3t^2 + 4h^2\right)\right] - 3a^4 - 4a^2h^2$$

$$= \frac{\rho\pi h}{12}\left(3a^4 + 6a^3t + 3a^2t^2 + 4a^2h^2 + 6a^3t + 8ath^2 - 3a^4 - 4a^2h^2\right)$$

$$= \frac{\rho\pi h}{12}\left(12a^3t + 8ath^2\right)$$

$$= \frac{\rho\pi hat}{12}\left(12a^2 + 8h^2\right)$$

Mass of shell: $m = \rho(2\pi ath) = 2\rho\pi hat$

$$I_x = \frac{2\rho\pi hat}{24}\left(12a^2 + 8h^2\right)$$

$$= \frac{4m}{24}\left(3a^2 + 2h^2\right)$$

or $I_x = \dfrac{m}{6}\left(3a^2 + 2h^2\right)$ ◀

Checks:

$$\left.\begin{aligned} a = 0;\quad I_x &= \frac{1}{3}mh^2 - \text{slender rod} \\ h = 0;\quad I_x &= \frac{1}{2}ma^2 - \text{thin ring} \end{aligned}\right\}\ \text{OK}$$

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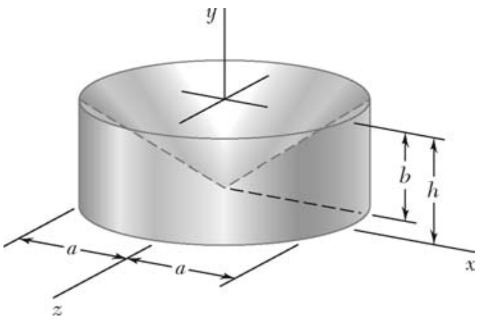
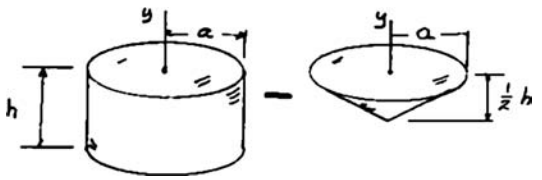


PROBLEM 9.130

The machine part shown is formed by machining a conical surface into a circular cylinder. For $b = \frac{1}{2}h,$ determine the mass moment of inertia and the radius of gyration of the machine part with respect to the y axis.

SOLUTION



Mass: $m_{\text{cyl}} = \rho\pi a^2 h \qquad m_{\text{cone}} = \frac{1}{3}\rho\pi a^2 \frac{h}{2} = \frac{1}{6}\rho\pi a^2 h$

$$I_y:\quad I_{\text{cyl}} = \frac{1}{2}m_{\text{cyl}}a^2 = \frac{1}{2}\rho\pi a^4 h \qquad I_{\text{cone}} = \frac{3}{10}m_{\text{cone}}a^2 = \frac{1}{20}\rho\pi a^4 h$$

For entire machine part:

$$m = m_{\text{cyl}} - m_{\text{cone}} = \rho\pi a^2 h - \frac{1}{6}\rho\pi a^2 h = \frac{5}{6}\rho\pi a^2 h$$

$$I_y = I_{\text{cyl}} - I_{\text{cone}} = \frac{1}{2}\rho\pi a^4 h - \frac{1}{20}\rho\pi a^4 h = \frac{9}{20}\rho\pi a^4 h$$

or $I_y = \left(\frac{5}{6}\rho\pi a^2 h\right)\left(\frac{6}{5}\right)\left(\frac{9}{20}\right)a^2$ $\qquad I_y = \frac{27}{50}ma^2$ ◀

Then $k_y^2 = \frac{I}{m} = \frac{27}{50}a^2$ $\qquad k_y = 0.735a$ ◀

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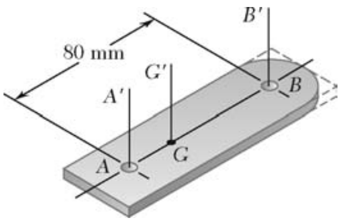
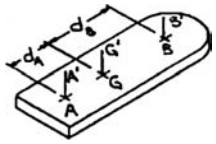


PROBLEM 9.131

After a period of use, one of the blades of a shredder has been worn to the shape shown and is of mass 0.18 kg. Knowing that the mass moments of inertia of the blade with respect to the *AA′* and *BB′* axes are $0.320\ \text{g}\cdot\text{m}^2$ and $0.680\ \text{g}\cdot\text{m}^2$, respectively, determine (*a*) the location of the centroidal axis *GG′*, (*b*) the radius of gyration with respect to axis *GG′*.

SOLUTION



(*a*) We have $d_B = (0.08 - d_A)\ \text{m}$

and, using the parallel axis

$$I_{AA'} = \bar{I}_{GG'} + md_A^2$$
$$I_{BB'} = \bar{I}_{GG'} + md_B^2$$

Then
$$I_{BB'} - I_{AA'} = m\left(d_B^2 - d_A^2\right)$$
$$= m\left[(0.08 - d_A)^2 - d_A^2\right]$$
$$= m(0.0064 - 0.16d_A)$$

Substituting:
$$(0.68 - 0.32)\times 10^{-3}\ \text{kg}\cdot\text{m}^2 = 0.18\ \text{kg}(0.0064 - 0.16d_A)\ \text{m}^2$$

or $d_A = 27.5\ \text{mm}$ ◀

(*b*) We have $I_{AA'} = I_{GG'} + md_A^2$

or
$$I_{GG'} = 0.32\times 10^{-3}\ \text{kg}\cdot\text{m}^2 - 0.18\ \text{kg}\cdot(0.0275\ \text{m})^2$$
$$= 0.183875\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

Then
$$k_{GG'}^2 = \frac{I_{GG'}}{m} = \frac{0.183875\times 10^{-3}\ \text{kg}\cdot\text{m}^2}{0.18\ \text{kg}} = 1.02153\times 10^{-3}\ \text{m}^2$$

or $k_{GG'} = 32.0\ \text{mm}$ ◀

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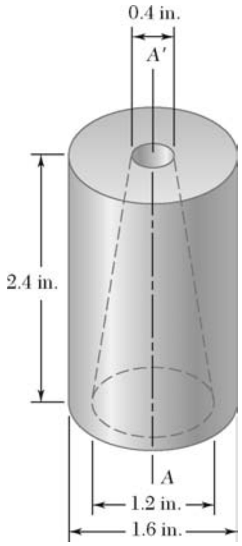
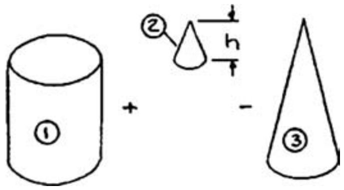


PROBLEM 9.132

Determine the mass moment of inertia of the 0.9-lb machine component shown with respect to the axis AA'.

SOLUTION

First note that the given shape can be formed adding a small cone to a cylinder and then removing a larger cone as indicated.



Now $$\frac{h}{0.4} = \frac{h+2.4}{1.2} \quad \text{or} \quad h = 1.2 \text{ in.}$$

The weight of the body is given by

$$W = mg = g(m_1 + m_2 - m_3) = \rho g(V_1 + V_2 - V_3)$$

or $$0.9 \text{ lb} = \rho \times 32.2 \text{ ft/s}^2$$

$$\left[\pi(0.8)^2(2.4) + \frac{\pi}{3}(0.2)^2(1.2) - \frac{\pi}{3}(0.6)^2(3.6)\right]\text{in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= \rho \times 32.2 \text{ ft/s}^2 (2.79253 + 0.02909 - 0.78540) \times 10^{-3} \text{ ft}^3$$

or $$\rho = 13.7266 \text{ lb} \cdot \text{s}^2/\text{ft}^4$$

Then $$m_1 = (13.7266)(2.79253 \times 10^{-3}) = 0.038332 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_2 = (13.7266)(0.02909 \times 10^{-3}) = 0.000399 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_3 = (13.7266)(0.78540 \times 10^{-3}) = 0.010781 \text{ lb} \cdot \text{s}^2/\text{ft}$$

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PROBLEM 9.132 (Continued)

Finally, using Figure 9.28, we have

$$I_{AA'} = (I_{AA'})_1 + (I_{AA'})_2 - (I_{AA'})_3$$

$$= \frac{1}{2}m_1a_1^2 + \frac{3}{10}m_2a_2^2 - \frac{3}{10}m_3a_3^2$$

$$= \left[\frac{1}{2}(0.038332)(0.8)^2 + \frac{3}{10}(0.000399)(0.2)^2 - \frac{3}{10}(0.010781)(0.6)^2\right](\text{lb}\cdot\text{s}^2/\text{ft})\times\text{in}^2\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= (85.1822 + 0.0333 - 8.0858)\times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{AA'} = 77.1\times 10^{-6}\,\text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

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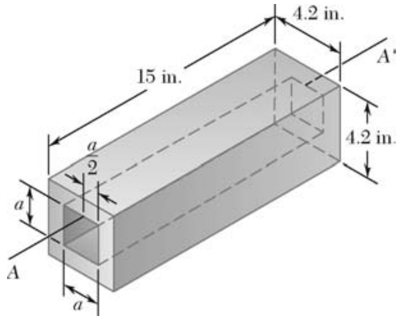
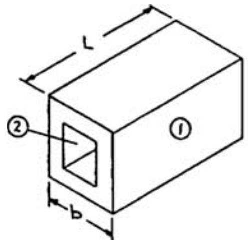


PROBLEM 9.133

A square hole is centered in and extends through the aluminum machine component shown. Determine (*a*) the value of *a* for which the mass moment of inertia of the component with respect to the axis AA', which bisects the top surface of the hole, is maximum, (*b*) the corresponding values of the mass moment of inertia and the radius of gyration with respect to the axis AA'. (The specific weight of aluminum is 0.100 lb/in.^{3}.)

SOLUTION



First note $m_1 = \rho V_1 = \rho b^2 L$

and $m_2 = \rho V_2 = \rho a^2 L$

(*a*) Using Figure 9.28 and the parallel-axis theorem, we have

$$I_{AA'} = (I_{AA'})_1 - (I_{AA'})_2$$

$$= \left[\frac{1}{12}m_1(b^2 + b^2) + m_1\left(\frac{a}{2}\right)^2\right]$$

$$- \left[\frac{1}{12}m_2(a^2 + a^2) + m_2\left(\frac{a}{2}\right)^2\right]$$

$$= (\rho b^2 L)\left(\frac{1}{6}b^2 + \frac{1}{4}a^2\right) - (\rho a^2 L)\left(\frac{5}{12}a^2\right)$$

$$= \frac{\rho L}{12}(2b^4 + 3b^2a^2 - 5a^4)$$

Then $$\frac{dI_{AA'}}{da} = \frac{\rho L}{12}(6b^2 a - 20a^3) = 0$$

or $a = 0$ and $a = b\sqrt{\dfrac{3}{10}}$

Also $$\frac{d^2 I_{AA'}}{da^2} = \frac{\rho L}{12}(6b^2 - 60a^2) = \frac{1}{2}\rho L(b^2 - 10a^2)$$

Now for $a = 0$, $\dfrac{d^2 I_{AA'}}{da^2} > 0$

and for $a = b\sqrt{\dfrac{3}{10}}$, $\dfrac{d^2 I_{AA'}}{da^2} < 0$

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PROBLEM 9.133 (Continued)

$(I_{AA'})_{\max}$ occurs when $$a = b\sqrt{\frac{3}{10}}$$

Then $$a = (4.2\text{ in.})\sqrt{\frac{3}{10}}$$

or $a = 2.30$ in. ◀

(*b*) From Part *a*:

$$(I_{AA'})_{\max} = \frac{\rho L}{12}\left[2b^4 + 3b^2\left(b\sqrt{\frac{3}{10}}\right)^2 - 5\left(b\sqrt{\frac{3}{10}}\right)^4\right] = \frac{49}{240}\rho L b^4$$

$$= \frac{49}{240}\frac{\gamma_{AL}}{g}Lb^4 = \frac{49}{240}\times\frac{0.100\text{ lb/in.}^3}{32.2\text{ ft/s}^2}\times(15\text{ in.})(4.2\text{ in.})^4\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

or $(I_{AA'})_{\max} = 20.6\times10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

Now $$k_{AA'}^2 = \frac{(I_{AA'})_{\max}}{m}$$

where $$m = m_1 - m_2 = \rho L(b^2 - a^2)$$

$$= \rho L\left[b^2 - \left(b\sqrt{\frac{3}{10}}\right)^2\right]$$

$$= \frac{7}{10}\rho L b^2$$

Then $$k_{AA'}^2 = \frac{\frac{49}{240}\rho L b^4}{\frac{7}{10}\rho L b^2} = \frac{7}{24}b^2 = \frac{7}{24}(4.2\text{ in.})^2$$

or $k_{AA'} = 2.27$ in. ◀

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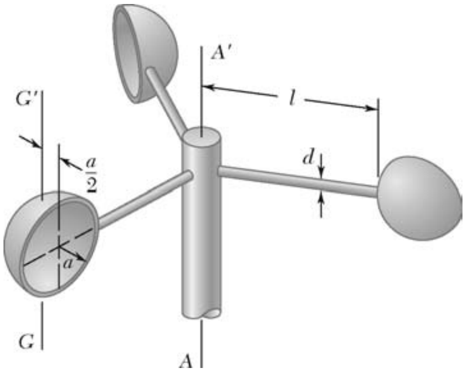
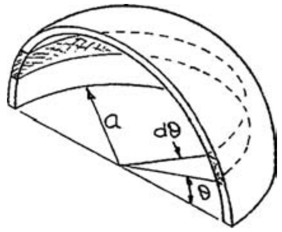


PROBLEM 9.134

The cups and the arms of an anemometer are fabricated from a material of density ρ. Knowing that the mass moment of inertia of a thin, hemispherical shell of mass m and thickness t with respect to its centroidal axis GG' is $5ma^2/12$, determine (*a*) the mass moment of inertia of the anemometer with respect to the axis AA', (*b*) the ratio of a to l for which the centroidal moment of inertia of the cups is equal to 1 percent of the moment of inertia of the cups with respect to the axis AA'.

SOLUTION

(*a*) First note
$$m_{\text{arm}} = \rho V_{\text{arm}} = \rho \times \frac{\pi}{4} d^2 l$$



and
$$dm_{\text{cup}} = \rho dV_{\text{cup}} = \rho[(2\pi a\cos\theta)(t)(a d\theta)]$$

Then
$$m_{\text{cup}} = \int dm_{\text{cup}} = \int_0^{\pi/2} 2\pi\rho a^2 t\cos\theta d\theta = 2\pi\rho a^2 t[\sin\theta]_0^{\pi/2} = 2\pi\rho a^2 t$$

Now
$$(I_{AA'})_{\text{anem}} = (I_{AA'})_{\text{cups}} + (I_{AA'})_{\text{arms}}$$

Using the parallel-axis theorem and assuming the arms are slender rods, we have

$$(I_{AA'})_{\text{anem}} = 3\left[(I_{GG'})_{\text{cup}} + m_{\text{cup}} d_{AG}^2\right] + 3\left[\overline{I}_{\text{arm}} + m_{\text{arm}} d_{AG_{\text{arm}}}\right]$$
$$= 3\left\{\frac{5}{12} m_{\text{cup}} a^2 + m_{\text{cup}}\left[(l+a)^2 + \left(\frac{a}{2}\right)^2\right]\right\} + 3\left[\frac{1}{2} m_{\text{arm}} l^2 + m_{\text{arm}}\left(\frac{l}{2}\right)^2\right]$$
$$= 3m_{\text{cup}}\left(\frac{5}{3}a^2 + 2la + l^2\right) + m_{\text{arm}} l^2$$
$$= 3(2\pi\rho a^2 t)\left(\frac{5}{3}a^2 + 2la + l^2\right) + \left(\frac{\pi}{4}\rho d^2 l\right)(l^2)$$

or
$$(I_{AA'})_{\text{anem}} = \pi\rho l^2\left[6a^2 t\left(\frac{5}{3}\frac{a^2}{l^2} + 2\frac{a}{l} + 1\right) + \frac{d^2 l}{4}\right] \blacktriangleleft$$

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PROBLEM 9.134 (Continued)

(*b*) We have $\dfrac{(I_{GG'})_{\text{cup}}}{(I_{AA'})_{\text{cup}}} = 0.01$

or $\dfrac{5}{12}m_{\text{cup}}a^2 = 0.01m_{\text{cup}}\left(\dfrac{5}{3}a^2 + 2la + l^2\right)$ (from Part *a*)

Now let $\zeta = \dfrac{a}{l}$.

Then $5\zeta^2 = 0.12\left(\dfrac{5}{3}\zeta^2 + 2\zeta + 1\right)$

or $40\zeta^2 - 2\zeta - 1 = 0$

Then $\zeta = \dfrac{2 \pm \sqrt{(-2)^2 - 4(40)(-1)}}{2(40)}$

or $\zeta = 0.1851$ and $\zeta = -0.1351$

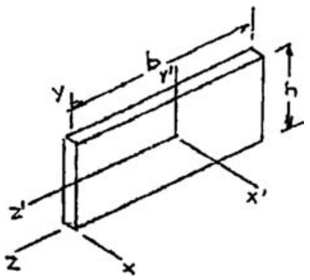
$$\frac{a}{l} = 0.1851 \blacktriangleleft$$

To the instructor:

The following formulas for the mass moment of inertia of thin plates and a half cylindrical shell are derived at this time for use in the solutions of Problems 9.135 through 9.140.

Thin rectangular plate

$$(I_x)_m = (\bar{I}_{x'})_m + md^2$$
$$= \frac{1}{12}m(b^2 + h^2) + m\left[\left(\frac{b}{2}\right)^2 + \left(\frac{h}{2}\right)^2\right]$$
$$= \frac{1}{3}m(b^2 + h^2)$$



$$(I_y)_m = (\bar{I}_{y'})_m + md^2$$
$$= \frac{1}{12}mb^2 + m\left(\frac{b}{2}\right)^2 = \frac{1}{3}mb^2$$

$$I_z = (\bar{I}_{z'})_m + md^2$$
$$= \frac{1}{12}mh^2 + m\left(\frac{h}{2}\right)^2 = \frac{1}{3}mh^2$$

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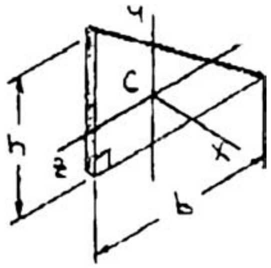
PROBLEM 9.134 (Continued)

Thin triangular plate



We have $$m = \rho V = \rho\left(\frac{1}{2}bht\right)$$

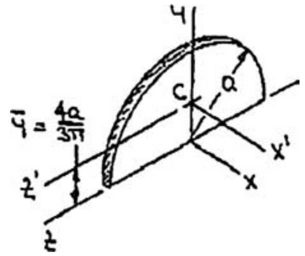
and $$\bar{I}_{z,\text{area}} = \frac{1}{36}bh^3$$

Then $$\bar{I}_{z,\text{mass}} = \rho t I_{z,\text{area}}$$
$$= \rho t \times \frac{1}{36}bh^3$$
$$= \frac{1}{18}mh^2$$

Similarly, $$\bar{I}_{y,\text{mass}} = \frac{1}{18}mb^2$$

Now $$\bar{I}_{x,\text{mass}} = \bar{I}_{y,\text{mass}} + \bar{I}_{z,\text{mass}} = \frac{1}{18}m(b^2 + h^2)$$

Thin semicircular plate



We have $$m = \rho V = \rho\left(\frac{\pi}{2}a^2 t\right)$$

and $$\bar{I}_{y,\text{area}} = I_{z,\text{area}} = \frac{\pi}{8}a^4$$

Then $$\bar{I}_{y,\text{mass}} = I_{z,\text{mass}} = \rho t \bar{I}_{y,\text{area}}$$
$$= \rho t \times \frac{\pi}{8}a^4$$
$$= \frac{1}{4}ma^2$$

Now $$I_{x,\text{mass}} = \bar{I}_{y,\text{mass}} + I_{z,\text{mass}} = \frac{1}{2}ma^2$$

Also $$I_{x,\text{mass}} = \bar{I}_{x',\text{mass}} + m\bar{y}^2 \quad \text{or} \quad \bar{I}_{x',\text{mass}} = m\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)a^2$$

and $$I_{z,\text{mass}} = \bar{I}_{z',\text{mass}} + m\bar{y}^2 \quad \text{or} \quad \bar{I}_{z',\text{mass}} = m\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)a^2$$

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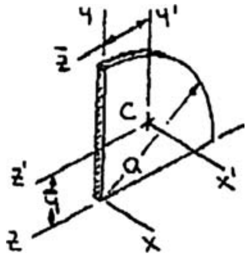
PROBLEM 9.134 (Continued)

$$\bar{y} = \bar{z} = \frac{4a}{3\pi}$$

Thin Quarter-Circular Plate

We have $$m = \rho V = \rho\left(\frac{\pi}{4}a^2 t\right)$$

and $$I_{y,\text{area}} = I_{z,\text{area}} = \frac{\pi}{16}a^4$$



Then $$I_{y,\text{mass}} = I_{z,\text{mass}} = \rho t I_{y,\text{area}}$$

$$= \rho t \times \frac{\pi}{16}a^4$$

$$= \frac{1}{4}ma^2$$

Now $$I_{x,\text{mass}} = I_{y,\text{mass}} + I_{z,\text{mass}} = \frac{1}{2}ma^2$$

Also $$I_{x,\text{mass}} = I_{x',\text{mass}} + m(\bar{y}^2 + \bar{z}^2)$$

or $$\bar{I}_{x',\text{mass}} = m\left(\frac{1}{2} - \frac{32}{9\pi^2}\right)a^2$$

and $$I_{y,\text{mass}} = I_{y',\text{mass}} + m\bar{z}^2$$

or $$\bar{I}_{y',\text{mass}} = m\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)a^2$$

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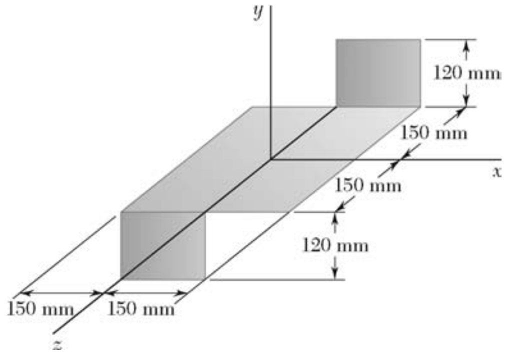
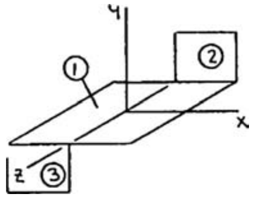


PROBLEM 9.135

A 2-mm thick piece of sheet steel is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3, determine the mass moment of inertia of the component with respect to each of the coordinate axes.

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}}V = \rho_{\text{ST}}tA$$

Then

$$m_1 = (7850 \text{ kg/m}^3)(0.002 \text{ m})(0.3 \text{ m})^2$$
$$= 1.413 \text{ kg}$$

$$m_2 = m_3 = (7850 \text{ kg/m}^3)(0.002 \text{ m}) \times (0.15 \times 0.12)\,\text{m}^2$$
$$= 0.2826 \text{ kg}$$

Using Figure 9.28 and the parallel-axis theorem, we have

$$I_x = (I_x)_1 + 2(I_x)_2$$
$$= \left[\frac{1}{12}(1.413 \text{ kg})(0.3 \text{ m})^2\right]$$
$$+ 2\left[\frac{1}{12}(0.2826 \text{ kg})(0.12 \text{ m})^2 + (0.2828 \text{ kg})(0.15^2 + 0.06^2)\text{m}^2\right]$$
$$= [(0.0105975) + 2(0.0003391 + 0.0073759)] \text{ kg} \cdot \text{m}^2$$
$$= [(0.0105975) + 2(0.0077150)] \text{ kg} \cdot \text{m}^2$$

or $\quad I_x = 26.0 \times 10^{-3} \text{ kg} \cdot \text{m}^2$ ◀

$$I_y = (I_y)_1 + 2(I_y)_2$$
$$= \left[\frac{1}{12}(1.413 \text{ kg})(0.3^2 + 0.3^2)\text{ m}^2\right]$$
$$+ 2\left[\frac{1}{12}(0.2826 \text{ kg})(0.15 \text{ m})^2 + (0.2826 \text{ kg})(0.075^2 + 0.15^2)\text{ m}^2\right]$$
$$= [(0.0211950) + 2(0.0005299 + 0.0079481)] \text{ kg} \cdot \text{m}^2$$
$$= [(0.0211950) + 2(0.0084780)] \text{ kg} \cdot \text{m}^2$$

or $\quad I_y = 38.2 \times 10^{-3} \text{ kg} \cdot \text{m}^2$ ◀

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PROBLEM 9.135 (Continued)

$$I_z = (I_z)_1 + 2(I_z)_2$$

$$= \left[\frac{1}{12}(1.413\text{ kg})(0.3\text{ m})^2\right]$$

$$+ 2\left[\frac{1}{12}(0.2826\text{ kg})(0.15^2 + 0.12^2)\text{ m}^2 + (0.2826\text{ kg})(0.075^2 + 0.06^2)\text{ m}^2\right]$$

$$= [(0.0105975) + 2(0.0008690 + 0.0026070)]\text{ kg}\cdot\text{m}^2$$

$$= [(0.0105975) + 2(0.0034760)]\text{ kg}\cdot\text{m}^2$$

or $I_z = 17.55 \times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

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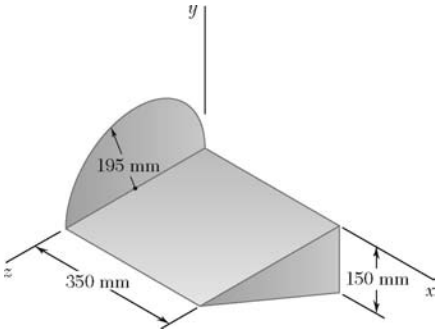


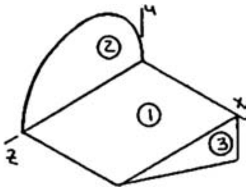
PROBLEM 9.136

A 2-mm thick piece of sheet steel is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3, determine the mass moment of inertia of the component with respect to each of the coordinate axes.

SOLUTION

First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \rho_{\text{ST}} tA$$



Then

$$m_1 = (7850 \text{ kg/m}^3)(0.002 \text{ m})(0.35 \times 0.39) \text{ m}^2$$
$$= 2.14305 \text{ kg}$$

$$m_2 = (7850 \text{ kg/m}^3)(0.002 \text{ m})\left(\frac{\pi}{2} \times 0.195^2\right)\text{m}^2 = 0.93775 \text{ kg}$$

$$m_3 = (7850 \text{ kg/m}^3)(0.002 \text{ m})\left(\frac{1}{2} \times 0.39 \times 0.15\right)\text{m}^2 = 0.45923 \text{ kg}$$

Using Figure 9.28 for component 1 and the equations derived above for components 2 and 3, we have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

$$= \left[\frac{1}{12}(2.14305 \text{ kg})(0.39 \text{ m})^2 + (2.14305 \text{ kg})\left(\frac{0.39}{2} \text{ m}\right)^2\right]$$

$$+ \left\{\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)(0.93775 \text{ kg})(0.195 \text{ m})^2 + (0.93775 \text{ kg})\left[\left(\frac{4 \times 0.195}{3\pi}\right)^2 + (0.195)^2\right]\text{m}^2\right\}$$

$$+ \left\{\frac{1}{18}(0.45923 \text{ kg})[(0.39)^2 + (0.15)^2]\text{ m}^2 + (0.45923 \text{ kg})\left[\left(\frac{0.39}{3}\right)^2 + \left(\frac{0.15}{3}\right)^2\right]\text{m}^2\right\}$$

$$= [(0.027163 + 0.081489) + (0.011406 + 0.042081) + (0.004455 + 0.008909)] \text{ kg} \cdot \text{m}^2$$

$$= (0.108652 + 0.053487 + 0.013364) \text{ kg} \cdot \text{m}^2$$

$$= 0.175503 \text{ kg} \cdot \text{m}^2$$

or $\quad I_x = 175.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2$ ◀

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PROBLEM 9.136 (Continued)

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

$$= \left\{\frac{1}{12}(2.14305 \text{ kg})[(0.35)^2 + (0.39)^2] \text{ m}^2 + (2.14305 \text{ kg})\left[\left(\frac{0.35}{2}\right)^2 + \left(\frac{0.39}{2}\right)^2\right]\text{m}^2\right\}$$

$$+\left[\frac{1}{4}(0.93775 \text{ kg})(0.195 \text{ m})^2 + (0.93775 \text{ kg})(0.195 \text{ m})^2\right]$$

$$+\left\{\frac{1}{18}(0.45923 \text{ kg})(0.39 \text{ m})^2 + (0.45923 \text{ kg})\left[(0.35)^2 + \left(\frac{0.39}{3}\right)^2\right]\text{m}^2\right\}$$

$$= [(0.049040 + 0.147120) + (0.008914 + 0.035658)$$

$$+ (0.003880 + 0.064017)] \text{ kg}\cdot\text{m}^2$$

$$= (0.196160 + 0.044572 + 0.067897) \text{ kg}\cdot\text{m}^2$$

$$= 0.308629 \text{ kg}\cdot\text{m}^2$$

or $I_y = 309\times10^{-3} \text{ kg}\cdot\text{m}^2$ ◀

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3$$

$$= \left[\frac{1}{12}(2.14305 \text{ kg})(0.35 \text{ m})^2 + (2.14305 \text{ kg})\left(\frac{0.35}{2} \text{ m}\right)^2\right]$$

$$+\left[\frac{1}{4}(0.93775 \text{ kg})(0.195 \text{ m})^2\right]$$

$$+\left\{\frac{1}{18}(0.45923 \text{ kg})(0.15 \text{ m})^2 + (0.45923 \text{ kg})\left[(0.35)^2 + \left(\frac{0.15}{3}\right)^2\right]\text{m}^2\right\}$$

$$= [(0.021877 + 0.065631) + 0.008914) + (0.000574 + 0.057404)] \text{ kg}\cdot\text{m}^2$$

$$= (0.087508 + 0.008914 + 0.057978) \text{ kg}\cdot\text{m}^2$$

$$= 0.154400 \text{ kg}\cdot\text{m}^2$$

or $I_z = 154.4\times10^{-3} \text{ kg}\cdot\text{m}^2$ ◀

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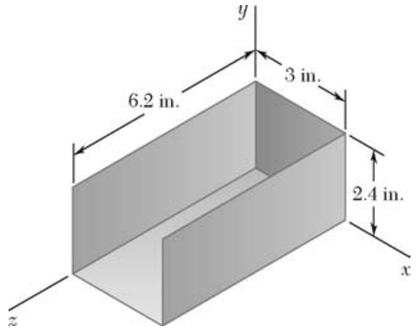
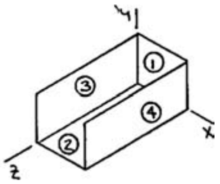


PROBLEM 9.137

The cover for an electronic device is formed from sheet aluminum that is 0.05 in. thick. Determine the mass moment of inertia of the cover with respect to each of the coordinate axes. (The specific weight of aluminum is 0.100 lb/in.^3.)

SOLUTION



First compute the mass of each component. We have

$$m = \rho V = \frac{\gamma}{g} tA$$

Then
$$m_1 = \frac{0.100\text{ lb/in.}^3}{32.2\text{ ft/s}^2} \times 0.05\text{ in.} \times (3 \times 2.4)\text{ in.}^2 = 1.11801 \times 10^{-3}\text{ lb} \cdot \text{s}^2\text{/ft}$$

$$m_2 = \frac{0.100\text{ lb/in.}^3}{32.2\text{ ft/s}^2} \times 0.05\text{ in.} \times (3 \times 6.2)\text{ in.}^2 = 2.88820 \times 10^{-3}\text{ lb} \cdot \text{s}^2\text{/ft}$$

$$m_3 = m_4 = \frac{0.100\text{ lb/in.}^3}{32.2\text{ ft/s}^2} \times 0.05\text{ in.} \times (2.4 \times 6.2)\text{ in.}^2 = 2.31056 \times 10^{-3}\text{ lb} \cdot \text{s}^2\text{/ft}$$

Using Figure 9.28 and the parallel-axis theorem, we have

$$(I_x)_3 = (I_x)_4$$

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4$$

$$= \left[\frac{1}{12}(1.11801 \times 10^{-3}\text{ lb} \cdot \text{s}^2\text{/ft})(2.4\text{ in.})^2 + (1.11801 \times 10^{-3}\text{ lb} \cdot \text{s}^2\text{/ft})\left(\frac{2.4}{2}\text{ in.}\right)^2\right]\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+ \left[\frac{1}{12}(2.88820 \times 10^{-3}\text{ lb} \cdot \text{s}^2\text{/ft})(6.2\text{ in.})^2 + (2.88820 \times 10^{-3}\text{ lb} \cdot \text{s}^2\text{/ft})\left(\frac{6.2}{2}\text{ in.}\right)^2\right]\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+ 2\left\{\frac{1}{12}(2.31056 \times 10^{-3}\text{ lb} \cdot \text{s}^2\text{/ft})\left[(2.4)^2 + (6.2)^2\right]\text{in.}^2 + (2.31056 \times 10^{-3}\text{ lb} \cdot \text{s}^2\text{/ft})\left[\left(\frac{2.4}{2}\right)^2 + \left(\frac{6.2}{2}\right)^2\right]\text{in.}^2\right\}\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

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PROBLEM 9.137 (Continued)

$$= [(3.7267 + 11.1801) + (64.2491 + 192.7472) + 2(59.1011 + 177.3034)] \times 10^{-6} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= [14.9068 + 256.9963 + 2(236.4045)] \times 10^{-6} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $\quad I_x = 745 \times 10^{-6} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4$$

$$= \left[\frac{1}{12}(1.11801\times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft})(3 \text{ in.})^2 + (1.11801\times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft})\left(\frac{3}{2} \text{ in.}\right)^2\right]\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+ \left\{\frac{1}{12}(2.88820\times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft})[(3)^2 + (6.2)^2] \text{ in.}^2 + (2.88820\times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{3}{2}\right)^2 + \left(\frac{6.2}{2}\right)^2\right] \text{ in.}^2\right\}\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+ \left[\frac{1}{12}(2.31056\times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft})(6.2 \text{ in.})^2 + (2.31056\times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft})\left(\frac{6.2}{2} \text{ in.}\right)^2\right]\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+ \left\{\frac{1}{12}(2.31056\times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft})(6.2 \text{ in.})^2 + (2.31056\times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft})\left[(3)^2 + \left(\frac{6.2}{2}\right)^2\right] \text{ in.}^2\right\}\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= [(5.8230 + 17.4689) + (79.2918 + 237.8754) + (51.3993 + 154.1978) + (51.3993 + 298.6078)] \times 10^{-6} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= (23.2919 + 317.1672 + 205.5971 + 350.0071) \times 10^{-6} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $\quad I_y = 896 \times 10^{-6} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

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PROBLEM 9.137 (Continued)

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4$$

$$= \left\{\frac{1}{12}(1.11801\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})[(3)^2 + (2.4)^2]\ \text{in.}^2\right.$$

$$\left. + (1.11801\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{3}{2}\right)^2 + \left(\frac{2.4}{2}\right)^2\right]\text{in.}^2\right\}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+\left[\frac{1}{12}(2.88820\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(3\ \text{in.})^2\right.$$

$$\left. + (2.88820\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left(\frac{3}{2}\ \text{in.}\right)^2\right]\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+\left[\frac{1}{12}(2.31056\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(2.4\ \text{in.})^2\right.$$

$$\left. + (2.31056\times10^{-3}\ \text{lb}\cdot\text{s}^2/\ \text{ft})\left(\frac{2.4}{2}\text{in.}\right)^2\right]\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+\left\{\frac{1}{12}(2.31056\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(2.4\ \text{in.})^2\right.$$

$$\left. + (2.31056\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[(3)^2 + \left(\frac{2.4}{2}\right)^2\right]\text{in.}^2\right\}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= [(9.5497 + 28.6490) + (15.0427 + 45.1281)$$

$$+ (7.7019 + 23.1056) + (7.7019 + 167.5156)]\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= (38.1987 + 60.1708 + 30.8075 + 175.2175)\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_z = 304\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

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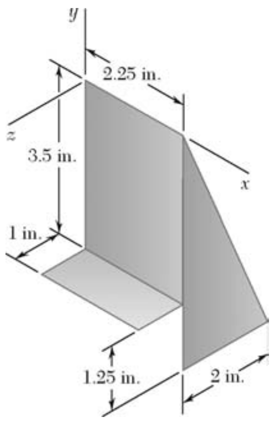
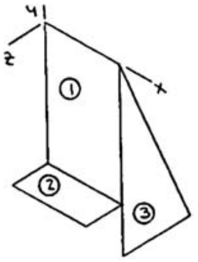


PROBLEM 9.138

A framing anchor is formed of 0.05-in.-thick galvanized steel. Determine the mass moment of inertia of the anchor with respect to each of the coordinate axes. (The specific weight of galvanized steel is 470 lb/ft^3.)

SOLUTION



First compute the mass of each component. We have

$$m = \rho V = \frac{\gamma_{\text{G.S.}}}{g} tA$$

Then

$$m_1 = \frac{470 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times 0.05 \text{ in.} \times (2.25 \times 3.5) \text{ in.}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$
$$= 3325.97 \times 10^{-6} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_2 = \frac{470 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times 0.05 \text{ in.} \times (2.25 \times 1) \text{ in.}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$
$$= 950.28 \times 10^{-6} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_3 = \frac{470 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times 0.05 \text{ in.} \times \left(\frac{1}{2} \times 2 \times 4.75\right) \text{ in.}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$
$$= 2006.14 \times 10^{-6} \text{ lb} \cdot \text{s}^2/\text{ft}$$

Using Figure 9.28 for components 1 and 2 and the equations derived above for component 3, we have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$
$$= \left[\frac{1}{12}(3325.97 \times 10^{-6} \text{ lb} \cdot \text{s}^2/\text{ft})(3.5 \text{ in.})^2\right.$$
$$\left. + (3325.97 \times 10^{-6} \text{ lb} \cdot \text{s}^2/\text{ft})\left(\frac{3.5}{2} \text{ in.}^2\right)\right]\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$
$$+ \left\{\frac{1}{12}(950.28 \times 10^{-6} \text{ lb} \cdot \text{s}^2/\text{ft})(1 \text{ in.})^2\right.$$
$$\left. + (950.28 \times 10^{-6} \text{ lb} \cdot \text{s}^2/\text{ft})\left[(3.5)^2 + \left(\frac{1}{2}\right)^2\right] \text{ in.}^2\right\}\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

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PROBLEM 9.138 (Continued)

$$+\left\{\frac{1}{18}(2006.14\times10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})[(4.75)^2+(2)^2]\text{ in.}^2\right.$$

$$\left.+(2006.14\times10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{2}{3}\times4.75\right)^2+\left(\frac{1}{3}\times2\right)^2\right]\text{ in.}^2\right\}\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$=[(23.578+70.735)+(0.550+82.490)$$

$$+(20.559+145.894)]\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$=(94.313+83.040+166.453)\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_x = 344\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$$I_y=(I_y)_1+(I_y)_2+(I_y)_3$$

$$=\left[\frac{1}{12}(3325.97\times10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})(2.25\text{ in.})^2\right.$$

$$\left.+(3325.97\times10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})\left(\frac{2.25}{2}\text{ in.}\right)^2\right]\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+\left\{\frac{1}{12}(950.28\times10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})[(2.25)^2+(1)^2]\text{ in.}^2\right.$$

$$\left.+(950.28\times10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{2.25}{2}\right)^2+\left(\frac{1}{2}\right)^2\right]\text{ in.}^2\right\}\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+\left\{\frac{1}{18}(2006.14\times10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})(2\text{ in.})^2\right.$$

$$\left.+(2006.14\times10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})\left[(2.25)^2+\left(\frac{1}{3}\times2\right)^2\right]\text{ in.}^2\right\}\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$=[(9.744+29.232)+(3.334+10.002)$$

$$+(3.096+76.720)]\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$=(38.976+13.336+79.816)\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_y = 132.1\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

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PROBLEM 9.138 (Continued)

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3$$

$$= \left\{\frac{1}{12}(3325.97\times10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})[(2.25)^2 + (3.5)^2]\text{ in.}^2\right.$$

$$\left.+ (3325.97\times10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{2.25}{2}\right)^2 + \left(\frac{3.5}{2}\right)^2\right]\text{ in.}^2\right\}\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+ \left\{\frac{1}{12}(950.28\times10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})(2.25\text{ in.})^2\right.$$

$$\left.+ (950.28\times10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{2.25}{2}\right)^2 + (3.5)^2\right]\text{ in.}^2\right\}\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+ \left\{\frac{1}{18}(2006.14\times10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})(4.75\text{ in.})^2\right.$$

$$\left.+ (2006.14\times10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})\left[(2.25)^2 + \left(\frac{2}{3}\times4.75\right)^2\right]\text{ in.}^2\right\}\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= [(33.322 + 99.967) + (2.784 + 89.192)$$

$$+ (17.463 + 210.231)]\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= (133.289 + 91.976 + 227.694)\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_z = 453\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

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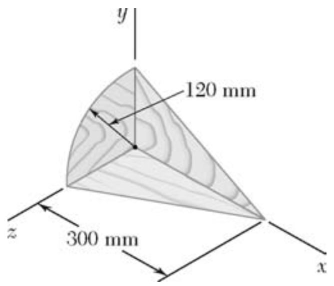
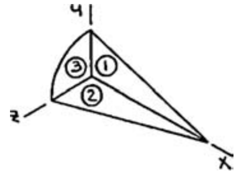


PROBLEM 9.139

A subassembly for a model airplane is fabricated from three pieces of 1.5-mm plywood. Neglecting the mass of the adhesive used to assemble the three pieces, determine the mass moment of inertia of the subassembly with respect to each of the coordinate axes. (The density of the plywood is 780 kg/m^3.)

SOLUTION



First compute the mass of each component. We have

$$m = \rho V = \rho t A$$

Then

$$m_1 = m_2 = (780 \text{ kg/m}^3)(0.0015 \text{ m})\left(\frac{1}{2}\times 0.3\times 0.12\right)\text{m}^2$$

$$= 21.0600\times 10^{-3} \text{ kg}$$

$$m_3 = (780 \text{ kg/m}^3)(0.0015 \text{ m})\left(\frac{\pi}{4}\times 0.12^2\right)\text{m}^2 = 13.2324\times 10^{-3} \text{ kg}$$

Using the equations derived above and the parallel-axis theorem, we have

$$(I_x)_1 = (I_x)_2$$

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

$$= 2\left[\frac{1}{18}(21.0600\times 10^{-3} \text{ kg})(0.12 \text{ m})^2 + (21.0600\times 10^{-3} \text{ kg})\left(\frac{0.12}{3} \text{ m}\right)^2\right]$$

$$+\left[\frac{1}{2}(13.2324\times 10^{-3} \text{ kg})(0.12 \text{ m})^2\right]$$

$$= [2(16.8480 + 33.6960) + (95.2733)]\times 10^{-6} \text{ kg}\cdot\text{m}^2$$

$$= [2(50.5440) + (95.2733)]\times 10^{-6} \text{ kg}\cdot\text{m}^2$$

or $I_x = 196.4\times 10^{-6} \text{ kg}\cdot\text{m}^2$ ◀

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PROBLEM 9.139 (Continued)

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

$$= \left[\frac{1}{18}(21.0600\times10^{-3}\text{ kg})(0.3\text{ m})^2 + (21.0600\text{ kg})\left(\frac{0.3}{3}\text{m}\right)^2\right]$$

$$+ \left\{\frac{1}{18}(21.0600\times10^{-3}\text{ kg})[(0.3)^2 + (0.12)^2]\text{ m}^2\right.$$

$$\left. + (21.0600\times10^{-3}\text{ kg})\left[\left(\frac{0.3}{3}\right)^2 + \left(\frac{0.12}{3}\right)^2\right]\text{m}^2\right\}$$

$$+ \left[\frac{1}{4}(13.2324\times10^{-3}\text{ kg})(0.12\text{ m})^2\right]$$

$$= [(105.300 + 210.600) + (122.148 + 244.296)$$

$$+ (47.637)]\times10^{-6}\text{ kg}\cdot\text{m}^2$$

$$= (315.900 + 366.444 + 47.637)\times10^{-6}\text{ kg}\cdot\text{m}^2$$

or $I_y = 730\times10^{-6}\text{ kg}\cdot\text{m}^2$ ◀

Symmetry implies $I_y = I_z$

$I_z = 730\times10^{-6}\text{ kg}\cdot\text{m}^2$ ◀

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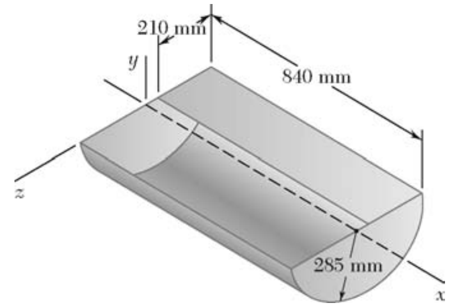
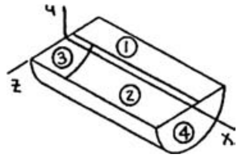


PROBLEM 9.140*

A farmer constructs a trough by welding a rectangular piece of 2-mm-thick sheet steel to half of a steel drum. Knowing that the density of steel is $7850\ \text{kg/m}^3$ and that the thickness of the walls of the drum is 1.8 mm, determine the mass moment of inertia of the trough with respect to each of the coordinate axes. Neglect the mass of the welds.

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \rho_{\text{ST}} t A$$

Then

$$m_1 = (7850\ \text{kg/m}^3)(0.002\ \text{m})(0.84 \times 0.21)\ \text{m}^2$$
$$= 2.76948\ \text{kg}$$

$$m_2 = (7850\ \text{kg/m}^3)(0.0018\ \text{m})(\pi \times 0.285 \times 0.84)\ \text{m}^2$$
$$= 10.62713\ \text{kg}$$

$$m_3 = m_4 = (7850\ \text{kg/m}^3)(0.0018\ \text{m})\left(\frac{\pi}{2} \times 0.285^2\right)\text{m}^2$$
$$= 1.80282\ \text{kg}$$

Using Figure 9.28 for component 1 and the equations derived above for components 2 through 4, we have

$$(I_x)_3 = (I_x)_4$$
$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4$$
$$= \left[\frac{1}{12}(2.76948\ \text{kg})(0.21\ \text{m})^2 + (2.76948\ \text{kg})\left(0.285 - \frac{0.21}{2}\right)^2 \text{m}^2\right]$$
$$+ [(10.62713\ \text{kg})(0.285\ \text{m})^2] + 2\left[\frac{1}{2}(1.80282\ \text{kg})(0.285\ \text{m})^2\right]$$
$$= [(0.01018 + 0.08973) + (0.86319) + 2(0.07322)]\ \text{kg}\cdot\text{m}^2$$
$$= [(0.09991 + 0.86319 + 2(0.07322)]\ \text{kg}\cdot\text{m}^2$$

or $\quad I_x = 1.110\ \text{kg}\cdot\text{m}^2$ ◀

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PROBLEM 9.140* (Continued)

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4$$

$$= \left\{ \frac{1}{12}(2.76948 \text{ kg})[(0.84)^2 + (0.21)^2]\text{ m}^2 \right.$$

$$\left. + (2.76948 \text{ kg})\left[\left(\frac{0.84}{2}\right)^2 + \left(0.285 - \frac{0.21}{2}\right)^2\right]\text{m}^2 \right\}$$

$$+ \left\{ \frac{1}{12}(10.62713 \text{ kg})[(0.84)^2 + 6(0.285)^2]\text{ m}^2 + (10.62713 \text{ kg})\left(\frac{0.84}{2}\text{ m}\right)^2 \right\}$$

$$+ \left[\frac{1}{4}(1.80282 \text{ kg})(0.285 \text{ m})^2\right]$$

$$+ \left[\frac{1}{4}(1.80282 \text{ kg})(0.285 \text{ m})^2 + (1.80282 \text{ kg})(0.84 \text{ m})^2\right]$$

$$= [(0.17302 + 0.57827) + (1.05647 + 1.87463)$$

$$+ (0.03661) + (0.03661 + 1.27207)]\text{ kg} \cdot \text{m}^2$$

$$= (0.75129 + 2.93110 + 0.03661 + 1.30868)\text{ kg} \cdot \text{m}^2$$

or $I_y = 5.03 \text{ kg} \cdot \text{m}^2$ ◀

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4$$

$$= \left[\frac{1}{12}(2.76948 \text{ kg})(0.84 \text{ m})^2 + (2.76948 \text{ kg})\left(\frac{0.84}{2}\text{ m}\right)^2\right]$$

$$+ \left\{ \frac{1}{12}(10.62713 \text{ kg})[(0.84)^2 + 6(0.285)^2]\text{ m}^2 + (10.62713 \text{ kg})\left(\frac{0.84}{2}\text{ m}\right)^2 \right\}$$

$$+ \left[\frac{1}{4}(1.80282 \text{ kg})(0.285 \text{ m})^2\right]$$

$$+ \left[\frac{1}{4}(1.80282 \text{ kg})(0.285 \text{ m})^2 + (1.80282 \text{ kg})(0.84 \text{ m})^2\right]$$

$$= [(0.16285 + 0.48854) + (1.05647 + 1.87463)$$

$$+ (0.03661) + (0.03661 + 1.27207)]\text{ kg} \cdot \text{m}^2$$

$$= (0.65139 + 2.93110 + 0.03661 + 1.30868)\text{ kg} \cdot \text{m}^2$$

or $I_z = 4.93 \text{ kg} \cdot \text{m}^2$ ◀

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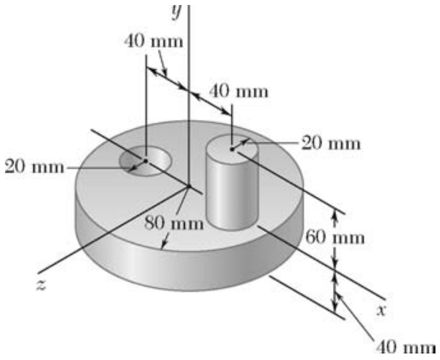
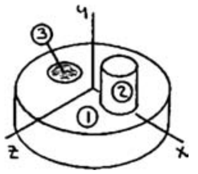


PROBLEM 9.141

The machine element shown is fabricated from steel. Determine the mass moment of inertia of the assembly with respect to (*a*) the *x* axis, (*b*) the *y* axis, (*c*) the *z* axis. (The density of steel is 7850 kg/m^3.)

SOLUTION

First compute the mass of each component. We have



$$m = \rho_{\text{ST}} V$$

Then

$$m_1 = (7850\ \text{kg/m}^3)(\pi(0.08\ \text{m})^2(0.04\ \text{m})]$$
$$= 6.31334\ \text{kg}$$

$$m_2 = (7850\ \text{kg/m}^3)[\pi(0.02\ \text{m})^2(0.06\ \text{m})] = 0.59188\ \text{kg}$$

$$m_3 = (7850\ \text{kg/m}^3)[\pi(0.02\ \text{m})^2(0.04\ \text{m})] = 0.39458\ \text{kg}$$

Using Figure 9.28 and the parallel-axis theorem, we have

(*a*)

$$I_x = (I_x)_1 + (I_x)_2 - (I_x)_3$$
$$= \left\{\frac{1}{12}(6.31334\ \text{kg})[3(0.08)^2 + (0.04)^2]\ \text{m}^2 + (6.31334\ \text{kg})(0.02\ \text{m})^2\right\}$$
$$+ \left\{\frac{1}{12}(0.59188\ \text{kg})[3(0.02)^2 + (0.06)^2]\ \text{m}^2 + (0.59188\ \text{kg})(0.03\ \text{m})^2\right\}$$
$$- \left\{\frac{1}{12}(0.39458\ \text{kg})[3(0.02)^2 + (0.04)^2]\ \text{m}^2 + (0.39458\ \text{kg})(0.02\ \text{m})^2\right\}$$
$$= [(10.94312 + 2.52534) + (0.23675 + 0.53269)$$
$$- (0.09207 + 0.15783)] \times 10^{-3}\ \text{kg} \cdot \text{m}^2$$
$$= (13.46846 + 0.76944 - 0.24990) \times 10^{-3}\ \text{kg} \cdot \text{m}^2$$
$$= 13.98800 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$$

or $I_x = 13.99 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$ ◀

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PROBLEM 9.141 (Continued)

(*b*) $I_y = (I_y)_1 + (I_y)_2 - (I_y)_3$

$$= \left[\frac{1}{2}(6.31334\text{ kg})(0.08\text{ m})^2\right]$$

$$+\left[\frac{1}{2}(0.59188\text{ kg})(0.02\text{ m})^2 + (0.59188\text{ kg})(0.04\text{ m})^2\right]$$

$$-\left[\frac{1}{2}(0.39458\text{ kg})(0.02\text{ m}^2) + (0.39458\text{ kg})(0.04\text{ m})^2\right]$$

$$= [(20.20269) + (0.11838 + 0.94701)$$

$$- (0.07892 + 0.63133)]\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

$$= (20.20269 + 1.06539 - 0.71025)\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

$$= 20.55783\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

or $I_y = 20.6\times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

(*c*) $I_z = (I_z)_1 + (I_z)_2 - (I_z)_3$

$$= \left\{\frac{1}{12}(6.31334\text{ kg})[3(0.08)^2 + (0.04)^2]\text{ m}^2 + (6.31334\text{ kg})(0.02\text{ m})^2\right\}$$

$$+\left\{\frac{1}{12}(0.59188\text{ kg})[3(0.02)^2 + (0.06)^2]\text{ m}^2 + (0.59188\text{ kg})[(0.04)^2 + (0.03)^2]\text{ m}^2\right\}$$

$$-\left\{\frac{1}{12}(0.39458\text{ kg})[3(0.02)^2 + (0.04)^2]\text{ m}^2 + (0.03958\text{ kg})[(0.04)^2 + (0.02)^2]\text{ m}^2\right\}$$

$$= [(10.94312 + 2.52534) + (0.23675 + 1.47970)$$

$$- (0.09207 + 0.78916)]\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

$$= (13.46846 + 1.71645 - 0.88123)\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

$$= 14.30368\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

or $I_z = 14.30\times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

To the Instructor:

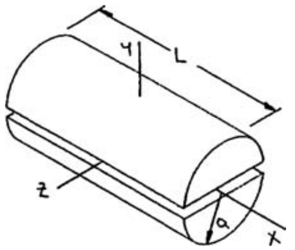
The following formulas for the mass of inertia of a semicylinder are derived at this time for use in the solutions of Problems 9.142 through 9.145.

From Figure 9.28:

Cylinder

$$(I_x)_{\text{cyl}} = \frac{1}{2}m_{\text{cyl}}a^2$$

$$(I_y)_{\text{cyl}} = (I_z)_{\text{cyl}} = \frac{1}{12}m_{\text{cyl}}(3a^2 + L^2)$$



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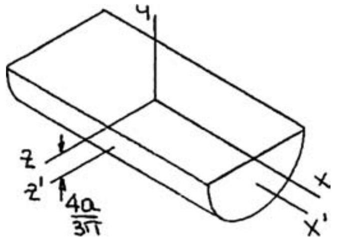
PROBLEM 9.141 (Continued)

Symmetry and the definition of the mass moment of inertia $(I = \int r^2 dm)$ imply

$$(I)_{\text{semicylinder}} = \frac{1}{2}(I)_{\text{cylinder}}$$

$$(I_x)_{\text{sc}} = \frac{1}{2}\left(\frac{1}{2}m_{\text{cyl}}a^2\right)$$

and $$(I_y)_{\text{sc}} = (I_z)_{\text{sc}} = \frac{1}{2}\left[\frac{1}{12}m_{\text{cyl}}(3a^2 + L^2)\right]$$



However, $$m_{\text{sc}} = \frac{1}{2}m_{\text{cyl}}$$

Thus, $$(I_x)_{\text{sc}} = \frac{1}{2}m_{\text{sc}}a^2$$

and $$(I_y)_{\text{sc}} = (I_z)_{\text{sc}} = \frac{1}{12}m_{\text{sc}}(3a^2 + L^2)$$

Also, using the parallel axis theorem find

$$\bar{I}_{x'} = m_{\text{sc}}\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)a^2$$

$$\bar{I}_{z'} = m_{\text{sc}}\left[\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)a^2 + \frac{1}{12}L^2\right]$$

where x' and z' are centroidal axes.

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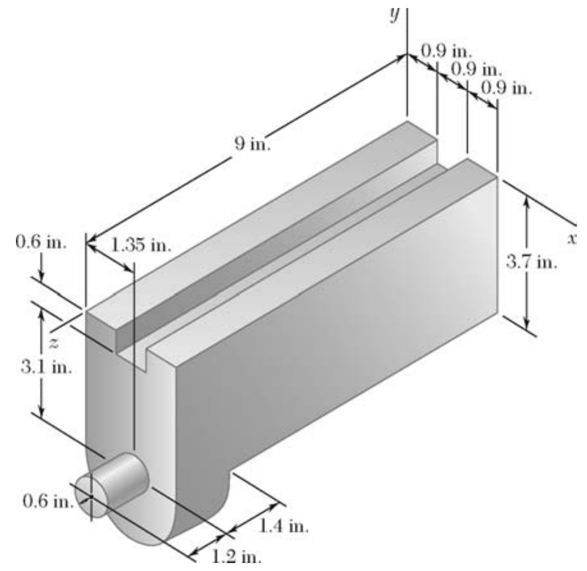
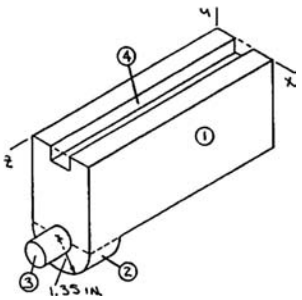


PROBLEM 9.142

Determine the mass moment of inertia of the steel machine element shown with respect to the y axis. (The specific weight of steel is 490 lb/ft^3.)

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \frac{\gamma_{\text{ST}}}{g} V$$

Then

$$m_1 = \frac{490\text{ lb/ft}^3}{32.2\text{ ft/s}^2} \times (2.7 \times 3.7 \times 9)\text{ in.}^3 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3$$
$$= 791.780 \times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = \frac{490\text{ lb/ft}^3}{32.2\text{ ft/s}^2} \times \left(\frac{\pi}{2} \times 1.35^2 \times 1.4\right)\text{ in.}^3 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3$$
$$= 35.295 \times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = \frac{490\text{ lb/ft}^3}{32.2\text{ ft/s}^2} \times (\pi \times 0.6^2 \times 1.2)\text{ in.}^3 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3$$
$$= 11.952 \times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_4 = \frac{490\text{ lb/ft}^3}{32.2\text{ ft/s}^2} \times (0.9 \times 0.6 \times 9)\text{ in.}^3 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3 = 42.799 \times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}$$

The mass moments of inertia are now computed using Figure 9.28 (components 1, 3, and 4) and the equations derived above (component 2).

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PROBLEM 9.142 (Continued)

Find: I_y

We have

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 - (I_y)_4$$

$$= \left\{\frac{1}{12}(791.780\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})[(2.7)^2 + (9)^2]\ \text{in.}^2\right.$$

$$\left. + (791.780\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{2.7}{2}\right)^2 + \left(\frac{9}{2}\right)^2\right]\ \text{in.}^2\right\}\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+ \left\{\frac{1}{12}(35.295\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})[3(1.35)^2 + (1.4)^2]\ \text{in.}^2\right.$$

$$\left. + (35.295\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[(1.35)^2 + \left(9 - \frac{1.4}{2}\right)^2\right]\ \text{in.}^2\right\}\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+ \left\{\frac{1}{12}(11.952\times 10^{-3}\ \text{lb}\cdot\text{s}^2\ /\text{ft})[3(0.6)^2 + (1.2)^2]\ \text{in.}^2\right.$$

$$\left. + (11.952\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})[(1.35)^2 + \left(9 + \frac{1.2}{2}\right)^2\ \text{in.}^2\right\}\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$- \left\{\frac{1}{12}(42.799\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})[(0.9)^2 + (9)^2]\ \text{in.}^2\right.$$

$$\left. + (42.799\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[(1.35)^2 + \left(\frac{9}{2}\right)^2\right]\ \text{in.}^2\right\}\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= [(40.4550 + 121.3650) + (0.1517 + 17.3319)$$

$$+ (0.0174 + 7.8005) - (2.0263 + 6.5603)]\times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= (161.8200 + 17.4836 + 7.8179 - 8.5866)\times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_y = 0.1785\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

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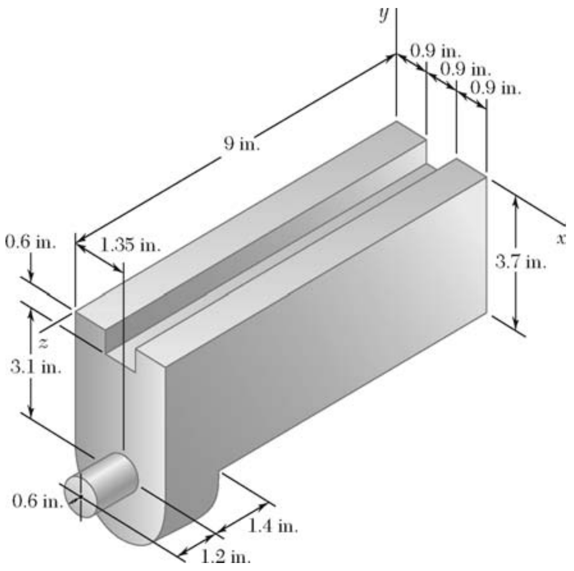


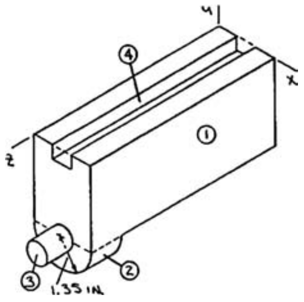
PROBLEM 9.143

Determine the mass moment of inertia of the steel machine element shown with respect to the z axis. (The specific weight of steel is 490 lb/ft^3.)

SOLUTION

First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \frac{\gamma_{\text{ST}}}{g} V$$



Then

$$m_1 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times (2.7 \times 3.7 \times 9) \text{ in.}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 791.780 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_2 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times \left(\frac{\pi}{2} \times 1.35^2 \times 1.4\right) \text{ in.}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 35.295 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_3 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times (\pi \times 0.6^2 \times 1.2) \text{ in.}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 11.952 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_4 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times (0.9 \times 0.6 \times 9) \text{ in.}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3 = 42.799 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

The mass moments of inertia are now computed using Figure 9.28 (components 1, 3, and 4) and the equations derived above (component 2).

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PROBLEM 9.143 (Continued)

Find: I_z

We have $$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 - (I_z)_4$$

$$= \left\{ \frac{1}{12}(791.780\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})[(2.7)^2 + (3.7)^2]\ \text{in.}^2 \right.$$

$$\left. + (791.780\ \text{lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{2.7}{2}\right)^2 + \left(\frac{3.7}{2}\right)^2\right]\text{in.}^2 \right\} \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+ \left\{ (35.295\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)(1.35\ \text{in.})^2 \right.$$

$$\left. + (35.295\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[(1.35)^2 + \left(3.7 + \frac{4\times1.35}{3\pi}\right)^2\right]\text{in.}^2 \right\} \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+ \left\{ \frac{1}{12}(11.952\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(0.6\ \text{in.})^2 \right.$$

$$\left. + (11.952\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})[(1.35)^2 + (3.7)^2]\ \text{in.}^2 \right\} \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$- \left\{ \frac{1}{12}(42.799\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})[(0.9)^2 + (0.6)^2]\ \text{in.}^2 \right.$$

$$\left. + (42.799\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[(1.35)^2 + \left(\frac{0.6}{2}\right)^2\right]\text{in.}^2 \right\} \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$I_z = [(9.6132 + 28.8395) + (0.1429 + 4.9219)$$

$$+ (0.0149 + 1.2875) - (0.0290 + 0.5684)]\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= (38.4527 + 5.0648 + 1.3024 - 0.5974)\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_z = 0.0442\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

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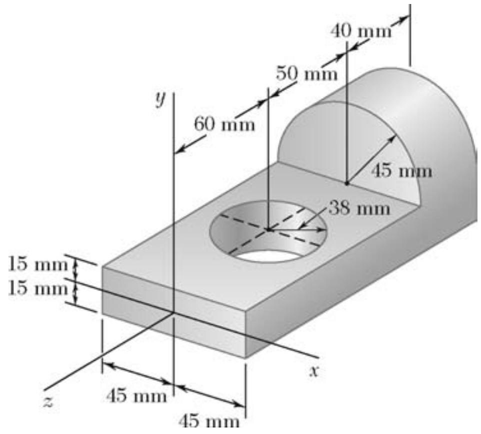
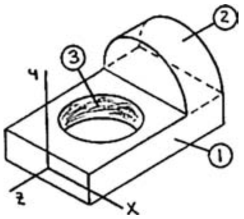


PROBLEM 9.144

Determine the mass moment of inertia and the radius of gyration of the steel machine element shown with respect to the x axis. (The density of steel is 7850 kg/m^3.)

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}}V$$

Then
$$m_1 = (7850\text{ kg/m}^3)(0.09 \times 0.03 \times 0.15)\text{ m}^3$$
$$= 3.17925\text{ kg}$$

$$m_2 = (7850\text{ kg/m}^3)\left[\frac{\pi}{2}(0.045)^2 \times 0.04\right]\text{m}^3$$
$$= 0.998791\text{ kg}$$

$$m_3 = (7850\text{ kg/m}^3)[\pi(0.038)^2 \times 0.03]\text{ m}^3 = 1.06834\text{ kg}$$

Using Figure 9.28 for components 1 and 3 and the equation derived above (before the solution to Problem 9.142) for a semicylinder, we have

$$I_x = (I_x)_1 + (I_x)_2 - (I_x)_3$$
$$= \left[\frac{1}{12}(3.17925\text{ kg})(0.03^2 + 0.15^2)\text{ m}^2 + (3.17925\text{ kg})(0.075\text{ m})^2\right]$$
$$+ \left\{(0.998791\text{ kg})\left[\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)(0.045\text{ m})^2 + \frac{1}{12}(0.04\text{ m})^2\right]\right.$$
$$\left. + (0.998791\text{ kg})\left[(0.13)^2 + \left(\frac{4 \times 0.045}{3\pi} + 0.015\right)^2\right]\text{ m}^2\right\}$$
$$- \left\{\frac{1}{12}(1.06834\text{ kg})[3(0.038\text{ m})^2 + (0.03\text{ m})^2] + (1.06834\text{ kg})(0.06\text{ m})^2\right\}$$

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PROBLEM 9.144 (Continued)

$$= [(0.0061995 + 0.0178833) + (0.0002745 + 0.0180409) - (0.0004658 + 0.0038460)] \text{ kg} \cdot \text{m}^2$$

$$= (0.0240828 + 0.0183154 - 0.0043118) \text{ kg} \cdot \text{m}^2$$

$$= 0.0380864 \text{ kg} \cdot \text{m}^2$$

or $I_x = 38.1 \times 10^{-3} \text{ kg} \cdot \text{m}^2$ ◀

Now

$$m = m_1 + m_2 - m_3 = (3.17925 + 0.998791 - 1.06834) \text{ kg}$$
$$= 3.10970 \text{ kg}$$

and

$$k_x^2 = \frac{I_x}{m} = \frac{0.0380864 \text{ kg} \cdot \text{m}^2}{3.10970 \text{ kg}}$$

or $k x = 110.7 \text{ mm}$ ◀

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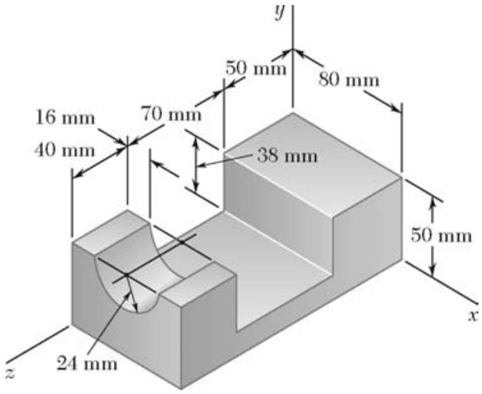
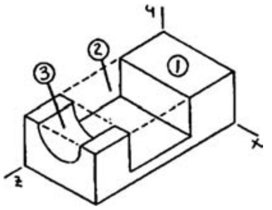


PROBLEM 9.145

Determine the mass moment of inertia of the steel fixture shown with respect to (*a*) the *x* axis, (*b*) the *y* axis, (*c*) the *z* axis. (The density of steel is 7850 kg/m^3.)

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V$$

Then

$$m_1 = 7850\ \text{kg/m}^3 \times (0.08 \times 0.05 \times 0.160)\ \text{m}^3$$
$$= 5.02400\ \text{kg}$$

$$m_2 = 7850\ \text{kg/m}^3 \times (0.08 \times 0.038 \times 0.07)\ \text{m}^3 = 1.67048\ \text{kg}$$

$$m_3 = 7850\ \text{kg/m}^3 \times \left(\frac{\pi}{2} \times 0.024^2 \times 0.04\right) \text{m}^3 = 0.28410\ \text{kg}$$

Using Figure 9.28 for components 1 and 2 and the equations derived above for component 3, we have

(*a*) $\quad I_x = (I_x)_1 - (I_x)_2 - (I_x)_3$

$$= \left\{ \frac{1}{12}(5.02400\ \text{kg})[(0.05)^2 + (0.16)^2]\ \text{m}^2 + (5.02400\ \text{kg})\left[\left(\frac{0.05}{2}\right)^2 + \left(\frac{0.16}{2}\right)^2\right] \text{m}^2 \right\}$$

$$- \left\{ \frac{1}{12}(1.67048\ \text{kg})[(0.038)^2 + (0.07)^2]\ \text{m}^2 + (1.67048\ \text{kg})\left[\left(0.05 - \frac{0.038}{2}\right)^2 + \left(0.05 + \frac{0.07}{2}\right)^2\right] \text{m}^2 \right\}$$

$$- \left\{ (0.28410\ \text{kg})\left[\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)(0.024)^2 + \frac{1}{12}(0.04)^2\right] \text{m}^2 \right.$$

$$\left. + (0.28410\ \text{kg})\left[\left(0.05 - \frac{4 \times 0.024}{3\pi}\right)^2 + \left(0.16 - \frac{0.04}{2}\right)^2\right] \text{m}^2 \right\}$$

$$= [(11.7645 + 35.2936) - (0.8831 + 13.6745) - (0.0493 + 6.0187)] \times 10^{-3}\ \text{kg} \cdot \text{m}^2$$

$$= (47.0581 - 14.5576 - 6.0680) \times 10^{-3}\ \text{kg} \cdot \text{m}^2$$

$$= 26.4325 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$$

or $\quad I_x = 26.4 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$ ◀

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PROBLEM 9.145 (Continued)

(*b*) $I_y = (I_y)_1 - (I_y)_2 - (I_y)_3$

$$= \left\{\frac{1}{12}(5.02400\text{ kg})[(0.08)^2 + (0.16)^2]\text{ m}^2 + (5.02400\text{ kg})\left[\left(\frac{0.08}{2}\right)^2 + \left(\frac{0.16}{2}\right)^2\right]\text{m}^2\right\}$$

$$-\left\{\frac{1}{12}(1.67048\text{ kg})[(0.08)^2 + (0.07)^2]\text{ m}^2 + (1.67048\text{ kg})\left[\left(\frac{0.08}{2}\right)^2 + \left(0.05 + \frac{0.07}{2}\right)^2\right]\text{m}^2\right\}$$

$$-\left\{\frac{1}{12}(0.28410\text{ kg})[3(0.024)^2 + (0.04)^2]\text{ m}^2 + (0.28410\text{ kg})\left[\left(\frac{0.08}{2}\right)^2 + \left(0.16 - \frac{0.04}{2}\right)^2\right]\text{m}^2\right\}$$

$$= [(13.3973 + 40.1920) - (1.5730 + 14.7420) - (0.0788 + 6.0229)]\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

$$= (53.5893 - 16.3150 - 6.1017)\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

$$= 31.1726\times 10^{-3}\text{ kg}\cdot\text{m}^2 \qquad \text{or} \quad I_y = 31.2\times 10^{-3}\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

(*c*) $I_z = (I_z)_1 - (I_z)_2 - (I_z)_3$

$$= \left\{\frac{1}{12}(5.02400\text{ kg})[(0.08)^2 + (0.05)^2]\text{ m}^2 + (5.02400\text{ kg})\left[\left(\frac{0.08}{2}\right)^2 + \left(\frac{0.05}{2}\right)^2\right]\text{m}^2\right\}$$

$$-\left\{\frac{1}{12}(1.67048\text{ kg})[(0.08)^2 + (0.038)^2]\text{ m}^2 + (1.67048\text{ kg})\left[\left(\frac{0.08}{2}\right)^2 + \left(0.05 - \frac{0.038}{2}\right)^2\right]\text{m}^2\right\}$$

$$-\left\{(0.28410\text{ kg})\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)(0.024\text{ m})^2 + (0.28410\text{ kg})\left[\left(\frac{0.08}{2}\right)^2 + \left(0.05 - \frac{4\times 0.024}{3\pi}\right)^2\right]\text{m}^2\right\}$$

$$= [(3.7261 + 11.1784) - (1.0919 + 4.2781) - (0.0523 + 0.9049)]\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

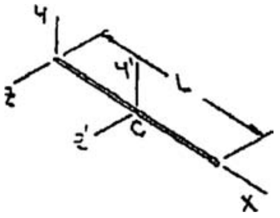
$$= (14.9045 - 5.3700 - 0.9572)\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

$$= 8.5773\times 10^{-3}\text{ kg}\cdot\text{m}^2 \qquad \text{or} \quad I_z = 8.58\times 10^{-3}\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

To the instructor:

The following formulas for the mass moment of inertia of wires are derived or summarized at this time for use in the solutions of Problems 9.146 through 9.148.

Slender Rod



$$I_x = 0 \qquad \bar{I}_{y'} = \bar{I}_{z'} = \frac{1}{12}mL^2 \text{ (Figure 9.28)}$$

$$I_y = I_z = \frac{1}{3}mL^2 \text{ (Sample Problem 9.9)}$$

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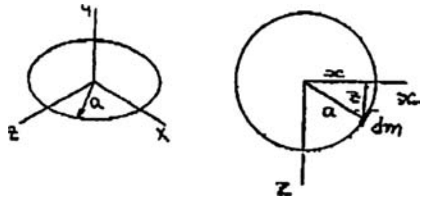
PROBLEM 9.145 (Continued)

Circle

We have $\bar{I}_y = \int r^2 dm = ma^2$



Now $\bar{I}_y = \bar{I}_x + \bar{I}_z$

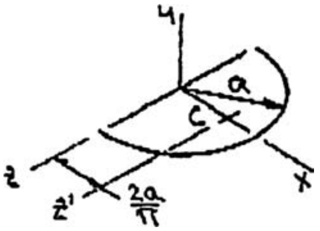
And symmetry implies $\bar{I}_x = \bar{I}_z$

$$\bar{I}_x = \bar{I}_z = \frac{1}{2}ma^2$$

Semicircle

Following the above arguments for a circle, We have

$$\bar{I}_x = I_z = \frac{1}{2}ma^2 \qquad I_y = ma^2$$



Using the parallel-axis theorem

$$I_z = \bar{I}_{z'} + m\bar{x}^2 \qquad x = \frac{2a}{\pi}$$

or $$I_{z'} = m\left(\frac{1}{2} - \frac{4}{\pi^2}\right)a^2$$

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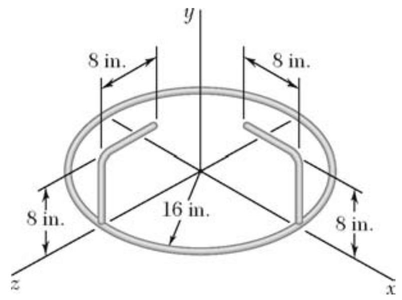
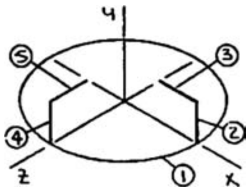


PROBLEM 9.146

Aluminum wire with a weight per unit length of 0.033 lb/ft is used to form the circle and the straight members of the figure shown. Determine the mass moment of inertia of the assembly with respect to each of the coordinate axes.

SOLUTION



First compute the mass of each component. We have

$$m = \frac{W}{g} = \frac{1}{g}\left(\frac{W}{L}\right)_{\text{AL}} L$$

Then
$$m_1 = \frac{1}{32.2\ \text{ft/s}^2} \times 0.033\ \text{lb/ft} \times (2\pi \times 16\ \text{in.}) \times \frac{1\ \text{ft}}{12\ \text{in.}}$$
$$= 8.5857 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$
$$m_2 = m_3 = m_4 = m_5 = \frac{1}{32.2\ \text{ft/s}^2} \times 0.033\ \text{lb/ft} \times 8\ \text{in.} \times \frac{1\ \text{ft}}{12\ \text{in.}}$$
$$= 0.6832\ \text{lb}\cdot\text{s}^2/\text{ft}$$

Using the equations given above and the parallel-axis theorem, we have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 + (I_x)_5$$
$$= \left[\frac{1}{2}(8.5857 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(16\ \text{in.})^2\right] \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$
$$+ \left[\frac{1}{3}(0.6832\ \text{lb}\cdot\text{s}^2/\text{ft})(8\ \text{in.})^2\right] \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$
$$+ [0 + (0.6832\ \text{lb}\cdot\text{s}^2/\text{ft})(8\ \text{in.})^2] \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$
$$+ \left[\frac{1}{12}(0.6832\ \text{lb}\cdot\text{s}^2/\text{ft})(8\ \text{in.})^2 + (0.6832\ \text{lb}\cdot\text{s}^2/\text{ft})(4^2 + 16^2)\ \text{in.}^2\right] \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$
$$+ \left[\frac{1}{12}(0.6832\ \text{lb}\cdot\text{s}^2/\text{ft})(8\ \text{in.})^2 + (0.6832\ \text{lb}\cdot\text{s}^2/\text{ft})(8^2 + 12^2)\ \text{in.}^2\right] \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$
$$= [(7.6315) + (0.1012) + (0.3036) + (0.0253 + 1.2905) + (0.0253 + 0.9868)] \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$
$$= (7.6315 + 0.1012 + 0.3036) + 1.3158 + 1.0121) \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$
$$= 10.3642 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_x = 10.36 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

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PROBLEM 9.146 (Continued)

$$(I_y)_2 = (I_y)_4, \qquad (I_y)_3 = (I_y)_5$$

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4 + (I_y)_5$$

$$= \left[(8.5857\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(16\ \text{in.})^2\right]\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+2[0+(0.6832\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(16\ \text{in.})^2]\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+2\left[\frac{1}{12}(0.6832\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(8\ \text{in.})^2+(0.6832\ \text{lb}\cdot\text{s}^2/\text{ft})(12\ \text{in.})^2\right]\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= [(15.2635)+2(1.2146)+2(0.0253+0.6832)]\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= [15.2635+2(1.2146)+2(0.7085)]\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 19.1097\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_y = 19.11\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

Symmetry implies $I_x = I_z$

$I_z = 10.36\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

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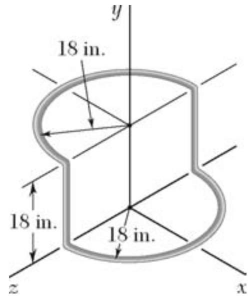
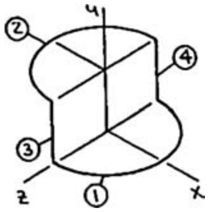


PROBLEM 9.147

The figure shown is formed of $\frac{1}{8}$-in.-diameter steel wire. Knowing that the specific weight of the steel is 490 lb/ft^3, determine the mass moment of inertia of the wire with respect to each of the coordinate axes.

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \frac{\gamma_{\text{ST}}}{g} AL$$

Then

$$m_1 = m_2 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times \left[\frac{\pi}{4}\left(\frac{1}{8}\text{ in.}\right)^2\right] \times (\pi \times 18 \text{ in.}) \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_3 = m_4 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} = \left[\frac{\pi}{4}\left(\frac{1}{8}\text{in.}\right)^2\right] \times 18 \text{ in.} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 1.9453 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

Using the equations given above and the parallel-axis theorem, we have

$$(I_x)_3 = (I_x)_4$$

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4$$

$$= \left[\frac{1}{2}(6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft})(18 \text{ in.})^2\right] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+ \left[\frac{1}{2}(6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft})(18 \text{ in.})^2 + (6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft})(18 \text{ in.})^2\right] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+ 2\left[\frac{1}{12}(1.9453 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft})(18 \text{ in.})^2 + (6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft})(9^2 + 18^2) \text{ in.}^2\right] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= [(6.8751) + (6.8751 + 13.1502) + 2(0.3647 + 5.4712)] \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$= [6.8751 + 20.6252 + 2(5.8359)] \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$= 39.1721 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

or $I_x = 0.0392 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$ ◀

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PROBLEM 9.147 (Continued)

$$(I_y)_1 = (I_y)_2, \quad (I_y)_3 = (I_y)_4$$

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4$$

$$= 2\left[(6.1112\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(18\ \text{in.})^2\right]\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+ 2[(0 + 1.9453\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(18\ \text{in.})^2]\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= [2(13.7502) + 2(4.3769)]\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 36.2542\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \qquad \text{or} \quad I_y = 0.0363\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \ \blacktriangleleft$$

$$(I_z)_3 = (I_z)_4$$

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4$$

$$= \left[\frac{1}{2}(6.1112\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(18\ \text{in.})^2\right]\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+ \left\{(6.1112\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left(\frac{1}{2}-\frac{4}{\pi^2}\right)(18\ \text{in.})^2\right.$$

$$\left. + (6.1112\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{2\times18}{\pi}\right)^2 + (18)^2\right]\text{in.}^2\right\}\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+ 2\left[\frac{1}{3}(1.9453\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(18\ \text{in.})^2\right]\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= [(6.8751) + (1.3024 + 19.3229) + 2(1.4590)]\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= [6.8751 + 20.6253 + 2(1.4590)]\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 30.4184\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \qquad \text{or} \quad I_z = 0.0304\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \ \blacktriangleleft$$

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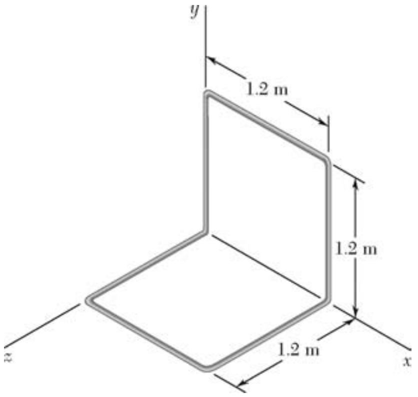
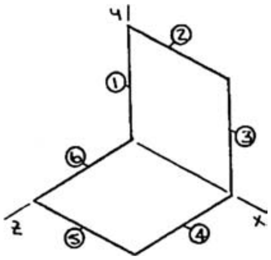


PROBLEM 9.148

A homogeneous wire with a mass per unit length of 0.056 kg/m is used to form the figure shown. Determine the mass moment of inertia of the wire with respect to each of the coordinate axes.

SOLUTION



First compute the mass *m* of each component. We have

$$\begin{aligned} m &= (m/L)L \\ &= 0.056 \text{ kg/m} \times 1.2 \text{ m} \\ &= 0.0672 \text{ kg} \end{aligned}$$

Using the equations given above and the parallel-axis theorem, we have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 + (I_x)_5 + (I_x)_6$$

Now $\qquad (I_x)_1 = (I_x)_3 = (I_x)_4 = (I_x)_6 \quad \text{and} \quad (I_x)_2 = (I_x)_5$

Then
$$\begin{aligned} I_x &= 4\left[\frac{1}{3}(0.0672 \text{ kg})(1.2 \text{ m})^2\right] + 2[0 + (0.0672 \text{ kg})(1.2 \text{ m})^2] \\ &= [4(0.03226) + 2(0.09677)] \text{ kg}\cdot\text{m}^2 \\ &= 0.32258 \text{ kg}\cdot\text{m}^2 \end{aligned}$$

or $\quad I_x = 0.323 \text{ kg}\cdot\text{m}^2$ ◀

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4 + (I_y)_5 + (I_y)_6$$

Now $\qquad (I_y)_1 = 0, \quad (I_y)_2 = (I_y)_6, \quad \text{and} \quad (I_y)_4 = (I_y)_5$

Then
$$\begin{aligned} I_y &= 2\left[\frac{1}{3}(0.0672 \text{ kg})(1.2 \text{ m})^2\right] + [0 + (0.0672 \text{ kg})(1.2 \text{ m})^2] \\ &\quad + 2\left[\frac{1}{12}(0.0672 \text{ kg})(1.2 \text{ m})^2 + (0.0672 \text{ kg})(1.2^2 + 0.6^2) \text{ m}^2\right] \\ &= [2(0.03226) + (0.09677) + 2(0.00806 + 0.12096)] \text{ kg}\cdot\text{m}^2 \\ &= [2(0.03226) + (0.09677) + 2(0.12902)] \text{ kg}\cdot\text{m}^2 \\ &= 0.41933 \text{ kg}\cdot\text{m}^2 \end{aligned}$$

or $\quad I_y = 0.419 \text{ kg}\cdot\text{m}^2$ ◀

Symmetry implies $\qquad I_y = I_z \qquad\qquad I_z = 0.419 \text{ kg}\cdot\text{m}^2$ ◀

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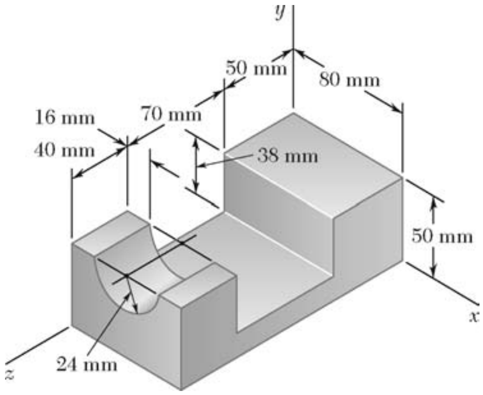
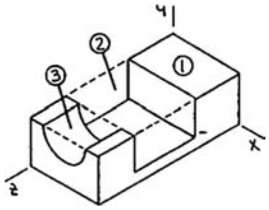


PROBLEM 9.149

Determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the steel fixture shown. (The density of steel is 7850 kg/m^3.)

SOLUTION

First compute the mass of each component. We have



$$m = \rho V$$

Then

$$m_1 = 7850 \text{ kg/m}^3 \times (0.08 \times 0.05 \times 0.16) \text{ m}^3 = 5.02400 \text{ kg}$$

$$m_2 = 7850 \text{ kg/m}^3 \times (0.08 \times 0.038 \times 0.07) \text{ m}^3 = 1.67048 \text{ kg}$$

$$m_3 = 7850 \text{ kg/m}^3 \times \left(\frac{\pi}{2} \times 0.024^2 \times 0.04\right) \text{m}^3 = 0.28410 \text{ kg}$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry. Now

$$\bar{y}_2 = \left(0.05 - \frac{0.038}{2}\right) \text{m} = 0.031 \text{ m}$$

$$\bar{y}_3 = \left(0.05 - \frac{4 \times 0.024}{3\pi}\right) \text{m} = 0.039814 \text{ m}$$

and then

	m, kg	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$m\bar{x}\bar{y}$, kg·m^2	$m\bar{y}\bar{z}$, kg·m^2	$m\bar{z}\bar{x}$, kg·m^2
1	5.02400	0.04	0.025	0.08	5.0240×10^{-3}	10.0480×10^{-3}	16.0768×10^{-3}
2	1.67048	0.04	0.031	0.085	2.0714×10^{-3}	4.4017×10^{-3}	5.6796×10^{-3}
3	0.28410	0.04	0.039814	0.14	0.4524×10^{-3}	1.5836×10^{-3}	1.5910×10^{-3}

Finally,

$$I_{xy} = (I_{xy})_1 - (I_{xy})_2 - (I_{xy})_3 = [(0 + 5.0240) - (0 + 2.0714) - (0 + 0.4524)] \times 10^{-3}$$

$$= 2.5002 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \qquad \text{or} \quad I_{xy} = 2.50 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

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PROBLEM 9.149 (Continued)

$$I_{yz} = (I_{yz})_1 - (I_{yz})_2 - (I_{yz})_3 = [(0+10.0480) - (0+4.4017) - (0+1.5836)]\times 10^{-3}$$
$$= 4.0627\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $I_{yz} = 4.06\times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

$$I_{zx} = (I_{zx})_1 - (I_{zx})_2 - (I_{zx})_3 = [(0+16.0768) - (0+5.6796) - (0+1.5910)]\times 10^{-3}$$
$$= 8.8062\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $I_{zx} = 8.81\times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

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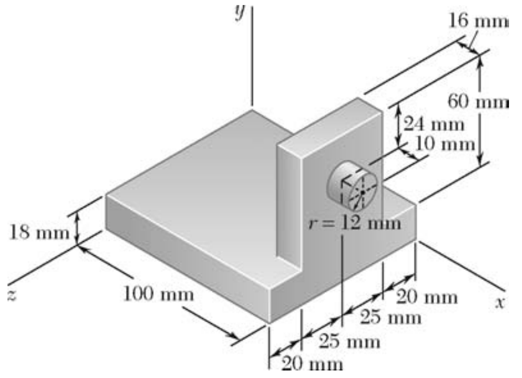
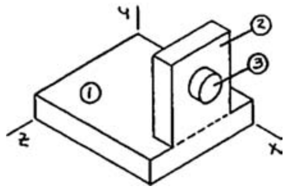


PROBLEM 9.150

Determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the steel machine element shown. (The density of steel is 7850 kg/m^3.)

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V$$

Then

$$m_1 = 7850 \text{ kg/m}^3 \times (0.1 \times 0.018 \times 0.09) \text{ m}^3 = 1.27170 \text{ kg}$$

$$m_2 = 7850 \text{ kg/m}^3 \times (0.016 \times 0.06 \times 0.05) \text{ m}^3 = 0.37680 \text{ kg}$$

$$m_3 = 7850 \text{ kg/m}^3 \times (\pi \times 0.012^2 \times 0.01) \text{ m}^3 = 0.03551 \text{ kg}$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry. Now

	m, kg	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$m\bar{x}\bar{y}$, kg·m^2	$m\bar{y}\bar{z}$, kg·m^2	$m\bar{z}\bar{x}$, kg·m^2
1	1.27170	0.05	0.009	0.045	0.57227×10^{-3}	0.51504×10^{-3}	2.86133×10^{-3}
2	0.37680	0.092	0.048	0.045	1.66395×10^{-3}	0.81389×10^{-3}	1.55995×10^{-3}
3	0.03551	0.105	0.054	0.045	0.20134×10^{-3}	0.08629×10^{-3}	0.16778×10^{-3}
Σ					2.43756×10^{-3}	1.41522×10^{-3}	4.58906×10^{-3}

Then

$$I_{xy} = \Sigma(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\bar{y}) \quad \text{or} \quad I_{xy} = 2.44 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

$$I_{yz} = \Sigma(\cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\bar{z}) \quad \text{or} \quad I_{yz} = 1.415 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

$$I_{zx} = \Sigma(\cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\bar{x}) \quad \text{or} \quad I_{zx} = 4.59 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

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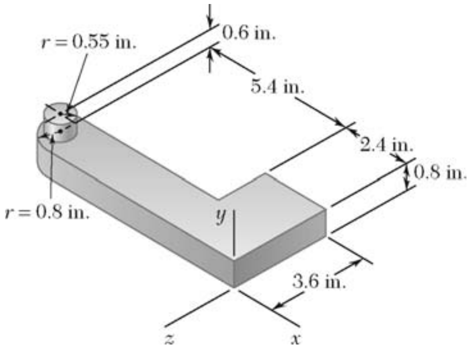


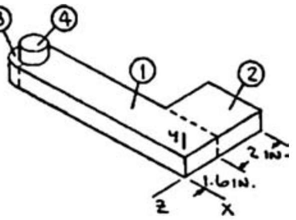
PROBLEM 9.151

Determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the cast aluminum machine component shown. (The specific weight of aluminum is 0.100 lb/in.^{3})

SOLUTION

First compute the mass of each component. We have

$$m = \rho_{AL} V = \frac{\gamma_{AL}}{g} V$$



Then

$$m_1 = \frac{0.100 \text{ lb/in.}^3}{32.2 \text{ ft/s}^2} \times (7.8 \times 0.8 \times 1.6) \text{ in.}^3$$

$$= 31.0062 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_2 = \frac{0.100 \text{ lb/in.}^3}{32.2 \text{ ft/s}^2} \times (2.4 \times 0.8 \times 2) \text{ in.}^3$$

$$= 11.9255 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_3 = \frac{0.100 \text{ lb/in.}^3}{32.2 \text{ ft/s}^2} \times \left[\frac{\pi}{2}(0.8)^2 \times 0.8\right] \text{in.}^3 = 2.4977 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_4 = \frac{0.100 \text{ lb/in.}^3}{32.2 \text{ ft/s}^2} \times [\pi(0.55)^2 \times 0.6] \text{in.}^3 = 1.7708 \text{ lb} \cdot \text{s}^2/\text{ft}$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry. Now

$$\bar{x}_3 = -\left(7.8 + \frac{4 \times 0.8}{3\pi}\right) \text{in.} = -8.13953 \text{ in.}$$

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PROBLEM 9.151 (Continued)

and then

	m, $\text{lb}\cdot\text{s}^2/\text{ft}$	$\bar{x}$, ft	$\bar{y}$, ft	$\bar{z}$, ft	$m\bar{x}\,\bar{y}$, $\text{lb}\cdot\text{ft}\cdot\text{s}^2$	$m\bar{y}\,\bar{z}$, $\text{lb}\cdot\text{ft}\cdot\text{s}^2$	$m\bar{z}\,\bar{x}$, $\text{lb}\cdot\text{ft}\cdot\text{s}^2$
1	31.0062×10^{-3}	$-\frac{3.9}{12}$	$\frac{0.4}{12}$	$-\frac{0.8}{12}$	-335.901×10^{-6}	-68.903×10^{-6}	671.801×10^{-6}
2	11.9255×10^{-3}	$-\frac{1.2}{12}$	$\frac{0.4}{12}$	$-\frac{2.6}{12}$	-39.752×10^{-6}	-86.129×10^{-6}	258.386×10^{-6}
3	2.4977×10^{-3}	$-\frac{8.13953}{12}$	$\frac{0.4}{12}$	$-\frac{0.8}{12}$	-56.473×10^{-6}	-5.550×10^{-6}	112.945×10^{-6}
4	1.7708×10^{-3}	$-\frac{7.8}{12}$	$\frac{1.1}{12}$	$-\frac{0.8}{12}$	-105.511×10^{-6}	-10.822×10^{-6}	76.735×10^{-6}
Σ					-537.637×10^{-6}	-171.404×10^{-6}	1119.867×10^{-6}

Then

$$I_{xy} = \Sigma(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y}) \quad \text{or} \quad I_{xy} = -538\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \ \blacktriangleleft$$

$$I_{yz} = \Sigma(\cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z}) \quad \text{or} \quad I_{yz} = -171.4\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \ \blacktriangleleft$$

$$I_{zx} = \Sigma(\cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x}) \quad \text{or} \quad I_{zx} = 1120\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \ \blacktriangleleft$$

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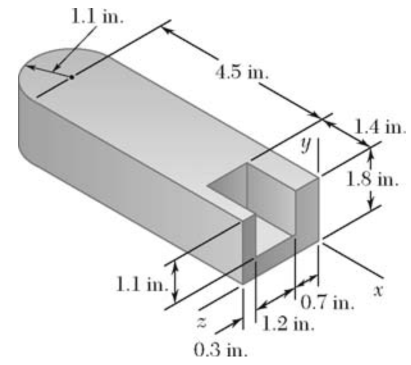
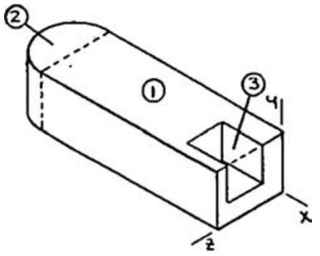


PROBLEM 9.152

Determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the cast aluminum machine component shown. (The specific weight of aluminum is 0.100 lb/in.^{3})

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{AL}} V = \frac{\gamma_{\text{AL}}}{g} V$$

Then
$$m_1 = \frac{0.100 \text{ lb/in.}^3}{32.2 \text{ ft/s}^2} \times (5.9 \times 1.8 \times 2.2) \text{ in.}^3$$

$$= 72.5590 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_2 = \frac{0.100 \text{ lb/in.}^3}{32.2 \text{ ft/s}^2} \times \left[\frac{\pi}{2} (1.1)^2 \times 1.8 \right] \text{in.}^3$$

$$= 10.6248 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_3 = \frac{0.100 \text{ lb/in.}^3}{32.2 \text{ ft/s}^2} \times (1.4 \times 1.1 \times 1.2) \text{ in.}^3$$

$$= 5.7391 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry. Now

$$\bar{x}_2 = -\left(5.9 + \frac{4 \times 1.1}{3\pi} \right) \text{in.}$$

$$= -6.36685 \text{ in.}$$

$$\bar{y}_3 = \left(1.8 - \frac{1.1}{2} \right) \text{in.}$$

$$= 1.25 \text{ in.}$$

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PROBLEM 9.152 (Continued)

and then

	m, $\text{lb}\cdot\text{s}^2/\text{ft}$	$\bar{x}$, ft	$\bar{y}$, ft	$\bar{z}$, ft	$m\bar{x}\,\bar{y}$, $\text{lb}\cdot\text{ft}\cdot\text{s}^2$	$m\bar{y}\,\bar{z}$, $\text{lb}\cdot\text{ft}\cdot\text{s}^2$	$m\bar{z}\,\bar{x}$, $\text{lb}\cdot\text{ft}\cdot\text{s}^2$
1	72.5590×10^{-3}	$-\frac{2.95}{12}$	$\frac{0.9}{12}$	$\frac{1.1}{12}$	-1.33781×10^{-3}	0.49884×10^{-3}	-1.63510×10^{-3}
2	10.6248×10^{-3}	$-\frac{6.36685}{12}$	$\frac{0.9}{12}$	$\frac{1.1}{12}$	-0.42279×10^{-3}	0.07305×10^{-3}	-0.51674×10^{-3}
3	5.7391×10^{-3}	$-\frac{0.7}{12}$	$\frac{1.25}{12}$	$\frac{1.3}{12}$	-0.03487×10^{-3}	0.06476×10^{-3}	-0.03627×10^{-3}

Finally,

$$I_{xy} = (I_{xy})_1 + (I_{xy})_2 - (I_{xy})_3$$
$$= [(0 - 1.33781) + (0 - 0.42279) - (0 - 0.03487)]\times10^{-3}$$

or $I_{xy} = -1.726\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$$I_{yz} = (I_{yz})_1 + (I_{yz})_2 - (I_{yz})_3$$
$$= [(0 + 0.49884) + (0 + 0.07305) - (0 + 0.06476)]\times10^{-3}$$

or $I_{yz} = 0.507\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$$I_{zx} = (I_{zx})_1 + (I_{zx})_2 - (I_{zx})_3$$
$$= [(0 - 1.63510) + (0 - 0.51674) - (0 - 0.3627)]\times10^{-3}$$

or $I_{zx} = -2.12\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

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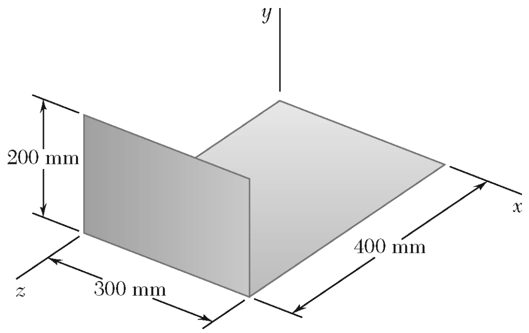
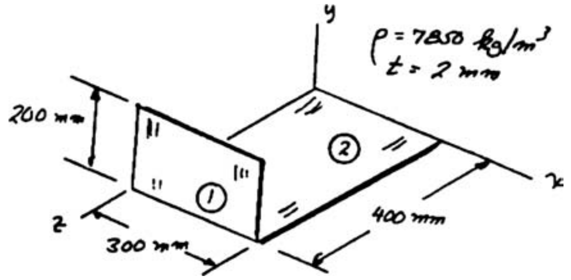


PROBLEM 9.153

A section of sheet steel 2 mm thick is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the component.

SOLUTION



$$m_1 = \rho V = 7850 \text{ kg/m}^3 (0.2 \text{ m})(0.3 \text{ m})(0.002 \text{ m}) = 0.942 \text{ kg}$$

$$m_2 = \rho V = 7850 \text{ kg/m}^3 (0.4 \text{ m})(0.3 \text{ m})(0.002 \text{ m}) = 1.884 \text{ kg}$$

For each panel the centroidal product of inertia is zero with respect to each pair of coordinate axes.

	m, kg	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$m\bar{x}\,\bar{y}$	$m\bar{y}\,\bar{z}$	$m\bar{z}\,\bar{x}$
①	0.942	0.15	0.1	0.4	$+14.13\times10^{-3}$	$+37.68\times10^{-3}$	$+56.52\times10^{-3}$
②	1.884	0.15	0	0.2	0	0	$+56.52\times10^{-3}$
Σ					$+14.13\times10^{-3}$	$+37.68\times10^{-3}$	$+113.02\times10^{-3}$

$$I_{xy} = +14.13\times10^{-3} \text{ kg}\cdot\text{m}^2 \qquad I_{xy} = +14.13 \text{ g}\cdot\text{m}^2 \blacktriangleleft$$

$$I_{yz} = +37.68\times10^{-3} \text{ kg}\cdot\text{m}^2 \qquad I_{yz} = +37.7 \text{ g}\cdot\text{m}^2 \blacktriangleleft$$

$$I_{zx} = +113.02\times10^{-3} \text{ kg}\cdot\text{m}^2 \qquad I_{zx} = +113.0 \text{ g}\cdot\text{m}^2 \blacktriangleleft$$

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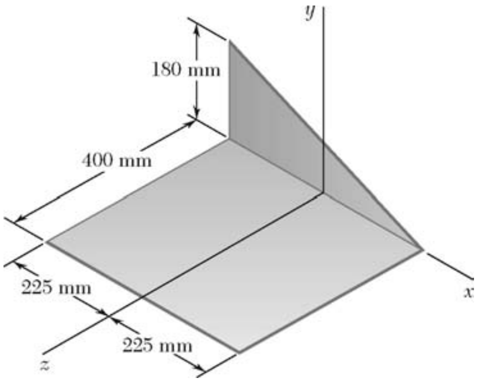
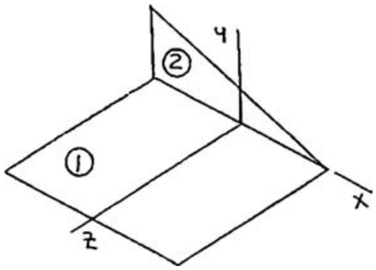


PROBLEM 9.154

A section of sheet steel 2 mm thick is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the component.

SOLUTION



First compute the mass of each component.

We have $$m = \rho_{\text{ST}} V = \rho_{\text{ST}} tA$$

Then $$m_1 = (7850\ \text{kg/m}^3)[(0.002)(0.4)(0.45)]\text{m}^3$$
$$= 2.8260\ \text{kg}$$
$$m_2 = 7850\ \text{kg/m}^3 \left[(0.002)\left(\frac{1}{2}\times 0.45\times 0.18\right)\right]\text{m}^3$$
$$= 0.63585\ \text{kg}$$

Now observe that

$$(\bar{I}_{x'y'})_1 = (\bar{I}_{y'z'})_1 = (\bar{I}_{z'x'})_1 = 0$$
$$(\bar{I}_{y'z'})_2 = (\bar{I}_{z'x'})_2 = 0$$

From Sample Problem 9.6: $$(\bar{I}_{x'y'})_{2,\text{area}} = -\frac{1}{72} b_2^2 h_2^2$$

Then $$(\bar{I}_{x'y'})_2 = \rho_{\text{ST}} t (I_{x'y'})_{2,\text{area}} = \rho_{\text{ST}} t\left(-\frac{1}{72} b_2^2 h_2^2\right) = -\frac{1}{36} m_2 b_2 h_2$$

Also $$\bar{x}_1 = \bar{y}_1 = \bar{z}_2 = 0 \qquad \bar{x}_2 = \left(-0.225 + \frac{0.45}{3}\right)\text{m} = -0.075\ \text{m}$$

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PROBLEM 9.154 (Continued)

Finally,
$$I_{xy} = \Sigma(\bar{I}_{xy} + m\bar{x}\,\bar{y}) = (0+0) + \left[-\frac{1}{36}(0.63585\text{ kg})(0.45\text{ m})(0.18\text{ m})\right.$$
$$\left. + (0.63585\text{ kg})(-0.075\text{ m})\left(\frac{0.18\text{ m}}{3}\right)\right]$$
$$= (-1.43066\times 10^{-3} - 2.8613\times 10^{-3})\text{ kg}\cdot\text{m}^2$$

or $I_{xy} = -4.29\times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

and
$$I_{yz} = \Sigma(\bar{I}_{y'z'} + m\,\bar{y}\,\bar{z}) = (0+0) + (0+0) = 0$$

or $I_{yz} = 0$ ◀

$$I_{zx} = \Sigma(\bar{I}_{z'x'} + m\,\bar{z}\,\bar{x}) = (0+0) + (0+0) = 0$$

or $I_{zx} = 0$ ◀

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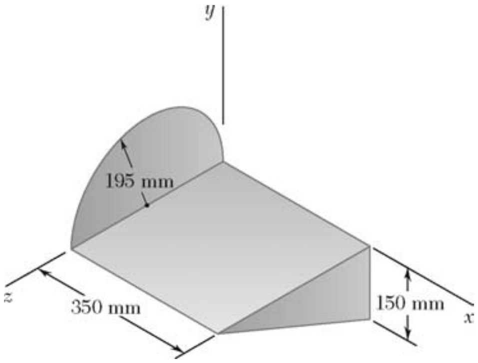
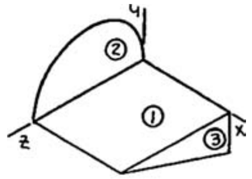


PROBLEM 9.155

A section of sheet steel 2 mm thick is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the component.

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}}V = \rho_{\text{ST}}tA$$

Then
$$m_1 = (7850\text{ kg/m}^3)(0.002\text{ m})(0.35\times 0.39)\,\text{m}^2$$
$$= 2.14305\text{ kg}$$
$$m_2 = (7850\text{ kg/m}^3)(0.002\text{ m})\left(\frac{\pi}{2}\times 0.195^2\right)\text{m}^2$$
$$= 0.93775\text{ kg}$$
$$m_3 = (7850\text{ kg/m}^3)(0.002\text{ m})\left(\frac{1}{2}\times 0.39\times 0.15\right)\text{m}^2$$
$$= 0.45923\text{ kg}$$

Now observe that because of symmetry the centroidal products of inertia of components 1 and 2 are zero and

$$(\bar{I}_{x'y'})_3 = (\bar{I}_{z'x'})_3 = 0$$

Also
$$(\bar{I}_{y'z'})_{3,\text{mass}} = \rho_{\text{ST}}t(\bar{I}_{y'z'})_{3,\text{area}}$$

Using the results of Sample Problem 9.6 and noting that the orientation of the axes corresponds to a 90° rotation, we have

$$(\bar{I}_{y'z'})_{3,\text{area}} = \frac{1}{72}b_3^2h_3^2$$

Then
$$(I_{y'z'})_3 = \rho_{\text{ST}}t\left(\frac{1}{72}b_3^2h_3^2\right) = \frac{1}{36}m_3b_3h_3$$

Also
$$\bar{y}_1 = \bar{x}_2 = 0$$
$$\bar{y}_2 = \frac{4\times 0.195}{3\pi}\text{m} = 0.082761\text{ m}$$

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PROBLEM 9.155 (Continued)

Finally,

$$I_{xy} = \Sigma(\bar{I}_{x'y'} + m\bar{x}\,\bar{y})$$

$$= (0+0) + (0+0) + \left[0 + (0.45923\text{ kg})(0.35\text{ m})\left(-\frac{0.15}{3}\text{ m}\right)\right]$$

$$= -8.0365\times10^{-3}\text{ kg}\cdot\text{m}^2 \qquad \text{or } I_{xy} = -8.04\times10^{-3}\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

$$I_{yz} = \Sigma(\bar{I}_{y'z'} + m\bar{y}\,\bar{z})$$

$$= (0+0) + [0 + (0.93775\text{ kg})(0.082761\text{ m})(0.195\text{ m})]$$

$$+\left[\frac{1}{36}(0.45923\text{ kg})(0.39\text{ m})(0.15\text{ m}) + (0.45923\text{ kg})\left(-\frac{0.15}{3}\text{ m}\right)\left(\frac{0.39}{3}\text{ m}\right)\right]$$

$$= [(15.1338) + (0.7462 - 2.9850)]\times10^{-3}\text{ kg}\cdot\text{m}^2$$

$$= (15.1338 - 2.2388)\times10^{-3}\text{ kg}\cdot\text{m}^2$$

$$= 12.8950\times10^{-3}\text{ kg}\cdot\text{m}^2 \qquad \text{or } I_{yz} = 12.90\times10^{-3}\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

$$I_{zx} = \Sigma(\bar{I}_{z'x'} + m\bar{z}\,\bar{x})$$

$$= [0 + (2.14305\text{ kg})(0.175\text{ m})(0.195\text{ m})] + (0+0)$$

$$+\left[0 + (0.45923\text{ kg})\left(\frac{0.39}{3}\text{ m}\right)(0.35\text{ m})\right]$$

$$= (73.1316 + 20.8950)\times10^{-3}\text{ kg}\cdot\text{m}^2$$

$$= 94.0266\times10^{-3}\text{ kg}\cdot\text{m}^2 \qquad \text{or } I_{zx} = 94.0\times10^{-3}\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

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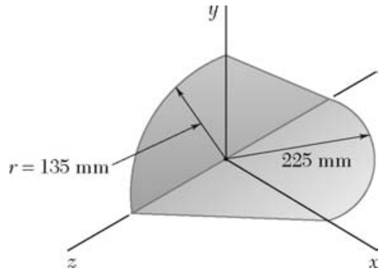
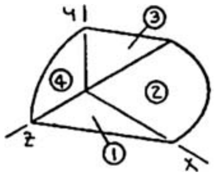


PROBLEM 9.156

A section of sheet steel 2 mm thick is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the component.

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}}V = \rho_{\text{ST}}tA$$

Then

$$m_1 = m_3 = (7850 \text{ kg/m}^3)(0.002 \text{ m})\left(\frac{1}{2}\times 0.225\times 0.135\right)\text{m}^2 = 0.23844 \text{ kg}$$

$$m_2 = (7850 \text{ kg/m}^3)(0.002 \text{ m})\left(\frac{\pi}{4}\times 0.225^2\right)\text{m}^2 = 0.62424 \text{ kg}$$

$$m_4 = (7850 \text{ kg/m}^3)(0.002 \text{ m})\left(\frac{\pi}{4}\times 0.135^2\right)\text{m}^2 = 0.22473 \text{ kg}$$

Now observe that the following centroidal products of inertia are zero because of symmetry.

$$(\bar{I}_{x'y'})_1 = (\bar{I}_{y'z'})_1 = 0 \qquad (\bar{I}_{x'y'})_2 = (\bar{I}_{y'z'})_2 = 0$$

$$(\bar{I}_{x'y'})_3 = (\bar{I}_{z'x'})_3 = 0 \qquad (\bar{I}_{x'y'})_4 = (\bar{I}_{z'x'}) = 0$$

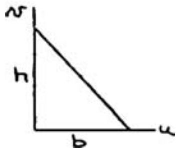
Also $\qquad \bar{y}_1 = \bar{y}_2 = 0 \qquad \bar{x}_3 = \bar{x}_4 = 0$

Now $\qquad I_{xy} = \Sigma(\bar{I}_{x'y'} + m\bar{x}\,\bar{y})$

so that $\qquad I_{xy} = 0$ ◀

Using the results of Sample Problem 9.6, we have

$$I_{uv,\text{area}} = \frac{1}{24}b^2h^2$$



Now

$$I_{uv,\text{mass}} = \rho_{\text{ST}}t\, I_{uv,\text{area}}$$

$$= \rho_{\text{ST}}t\left(\frac{1}{24}b^2h^2\right)$$

$$= \frac{1}{12}mbh$$

Thus,

$$(I_{zx})_1 = \frac{1}{12}m_1b_1h_1$$

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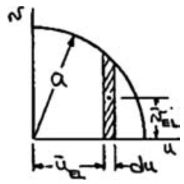
PROBLEM 9.156 (Continued)

While $$(I_{yz})_3 = -\frac{1}{12} m_3 b_3 h_3$$

because of a 90° rotation of the coordinate axes.

To determine I_{uv} for a quarter circle, we have

$$dI_{uv} = d\bar{I}_{u'v'}^{\nearrow 0} + u_{\text{EL}} v_{\text{EL}} dm$$



Where $$\bar{u}_{\text{EL}} = u \quad \bar{v}_{\text{EL}} = \frac{1}{2}v = \frac{1}{2}\sqrt{a^2 - u^2}$$

$$dm = \rho_{\text{ST}} t\, dA = \rho_{\text{ST}} tv\, du = \rho_{\text{ST}} t\sqrt{a^2 - u^2}\, du$$

Then $$I_{uv} = \int dI_{uv} = \int_0^a (u)\left(\frac{1}{2}\sqrt{a^2-u^2}\right)\left(\rho_{\text{ST}} t\sqrt{a^2-u^2}\, du\right)$$

$$= \frac{1}{2}\rho_{\text{ST}} t \int_0^a u(a^2 - u^2)\, du$$

$$= \frac{1}{2}\rho_{\text{ST}} t\left[\frac{1}{2}a^2u^2 - \frac{1}{4}u^4\right]_0^a = \frac{1}{8}\rho_{\text{ST}} t\, a^4 = \frac{1}{2\pi} ma^4$$

Thus $$(I_{zx})_2 = -\frac{1}{2\pi} m_2 a_2^2$$

because of a 90° rotation of the coordinate axes. Also

$$(I_{yz})_4 = \frac{1}{2\pi} m_4 a_4^2$$

Finally,

$$I_{yz} = \Sigma(I_{yz}) = [(\bar{I}_{y'z'})^{\nearrow 0} + m_1\bar{y}_1\bar{z}_1] + [(\bar{I}_{y'z'})^{\nearrow 0} + m_2\bar{y}_2\bar{z}_2] + (I_{yz})_3 + (I_{yz})_4$$

$$= \left[-\frac{1}{12}(0.23844 \text{ kg})(0.225 \text{ m})(0.135 \text{ m})\right] + \left[\frac{1}{2\pi}(0.22473 \text{ kg})(0.135 \text{ m})^2\right]$$

$$= (-0.60355 + 0.65185) \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

or $I_{yz} = 48.3 \times 10^{-6} \text{ kg} \cdot \text{m}^2$ ◀

$$I_{zx} = \Sigma(I_{zx}) = (I_{zx})_1 + (I_{zx})_2 + [(\bar{I}_{z'x'})_3^{\nearrow 0} + m_3\bar{z}_3\bar{x}_3] + [(\bar{I}_{z'x'})_4^{\nearrow 0} + m_4\bar{z}_4\bar{x}_4]$$

$$= \left[\frac{1}{12}(0.23844 \text{ kg})(0.225 \text{ m})(0.135 \text{ m})\right] + \left[-\frac{1}{2\pi}(0.62424 \text{ kg})(0.225 \text{ m})^2\right]$$

$$= (0.60355 - 5.02964) \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

or $I_{zx} = -4.43 \times 10^{-3} \text{ kg} \cdot \text{m}^2$ ◀

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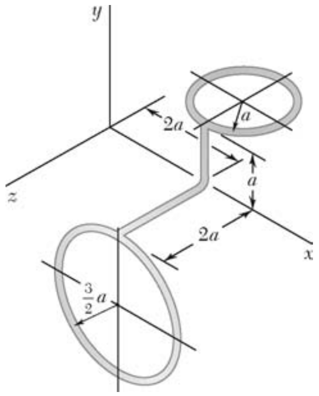
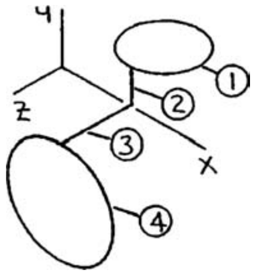


PROBLEM 9.157

Brass wire with a weight per unit length w is used to form the figure shown. Determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the wire figure.

SOLUTION

First compute the mass of each component. We have



$$m = \frac{W}{g} = \frac{1}{g}wL$$

Then
$$m_1 = \frac{W}{g}(2\pi \times a) = 2\pi\frac{w}{g}a$$

$$m_2 = \frac{w}{g}(a) = \frac{w}{g}a$$

$$m_3 = \frac{w}{g}(2a) = 2\frac{w}{g}a$$

$$m_4 = \frac{w}{g}\left(2\pi \times \frac{3}{2}a\right) = 3\pi\frac{w}{g}a$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry.

	m	$\bar{x}$	$\bar{y}$	$\bar{z}$	$m\bar{x}\,\bar{y}$	$m\bar{y}\,\bar{z}$	$m\bar{z}\,\bar{x}$
1	$2\pi\frac{w}{g}a$	$2a$	a	$-a$	$4\pi\frac{w}{g}a^3$	$-2\pi\frac{w}{g}a^3$	$-4\pi\frac{w}{g}a^3$
2	$\frac{w}{g}a$	$2a$	$\frac{1}{2}a$	0	$\frac{w}{g}a^3$	0	0
3	$2\frac{w}{g}a$	$2a$	0	a	0	0	$4\frac{w}{g}a^3$
4	$3\pi\frac{w}{g}a$	$2a$	$-\frac{3}{2}a$	$2a$	$-9\pi\frac{w}{g}a^3$	$-9\pi\frac{w}{g}a^3$	$12\pi\frac{w}{g}a^3$
Σ					$\frac{w}{g}(1-5\pi)a^3$	$-11\pi\frac{w}{g}a^3$	$4\frac{w}{g}(1+2\pi)a^3$

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PROBLEM 9.157 (Continued)

Then $I_{xy} = \Sigma(\bar{I}_{x'y'} + m\bar{x}\,\bar{y})$ (with $\bar{I}_{x'y'} = 0$) or $I_{xy} = \dfrac{w}{g}a^3(1-5\pi)$ ◀

$I_{yz} = \Sigma(\bar{I}_{y'z'} + m\bar{y}\,\bar{z})$ (with $\bar{I}_{y'z'} = 0$) or $I_{yz} = -11\pi\dfrac{w}{g}a^3$ ◀

$I_{zx} = \Sigma(\bar{I}_{z'x'} + m\bar{z}\,\bar{x})$ (with $\bar{I}_{z'x'} = 0$) or $I_{zx} = 4\dfrac{w}{g}a^3(1+2\pi)$ ◀

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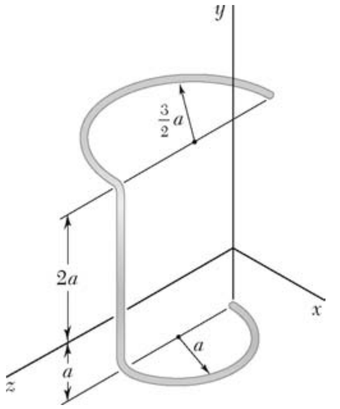
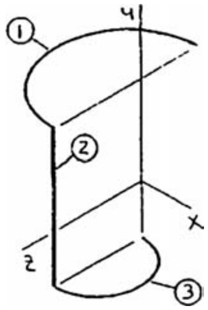


PROBLEM 9.158

Brass wire with a weight per unit length w is used to form the figure shown. Determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the wire figure.

SOLUTION

First compute the mass of each component. We have



$$m = \frac{W}{g} = \frac{1}{g}wL$$

Then

$$m_1 = \frac{W}{g}\left(\pi \times \frac{3}{2}a\right) = \frac{3}{2}\pi\frac{w}{g}a$$

$$m_2 = \frac{w}{g}(3a) = 3\frac{w}{g}a$$

$$m_3 = \frac{w}{g}(\pi \times a) = \pi\frac{w}{g}a$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry.

	m	$\bar{x}$	$\bar{y}$	$\bar{z}$	$m\bar{x}\,\bar{y}$	$m\bar{y}\,\bar{z}$	$m\bar{z}\,\bar{x}$
1	$\frac{3}{2}\pi\frac{w}{g}a$	$-\frac{2}{\pi}\left(\frac{3}{2}a\right)$	$2a$	$\frac{1}{2}a$	$-9\frac{w}{g}a^3$	$\frac{3}{2}\pi\frac{w}{g}a^3$	$-\frac{9}{4}\frac{w}{g}a^3$
2	$3\frac{w}{g}a$	0	$\frac{1}{2}a$	$2a$	0	$3\frac{w}{g}a^3$	0
3	$\pi\frac{w}{g}a$	$\frac{2}{\pi}(a)$	$-a$	a	$-2\frac{w}{g}a^3$	$-\pi\frac{w}{g}a^3$	$2\frac{w}{g}a^3$
Σ					$-11\frac{w}{g}a^3$	$\frac{w}{g}\left(\frac{\pi}{2}+3\right)a^3$	$-\frac{1}{4}\frac{w}{g}a^3$

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PROBLEM 9.158 (Continued)

Then $I_{xy} = \Sigma(\bar{I}_{x'y'}\overset{0}{\nearrow} + m\bar{x}\,\bar{y})$ or $I_{xy} = -11\dfrac{w}{g}a^3$ ◀

$I_{yz} = \Sigma(\bar{I}_{y'z'}\overset{0}{\nearrow} + m\bar{y}\,\bar{z})$ or $I_{yz} = \dfrac{1}{2}\dfrac{w}{g}a^3(\pi + b)$ ◀

$I_{zx} = \Sigma(\bar{I}_{z'x'}\overset{0}{\nearrow} + m\bar{z}\,\bar{x})$ or $I_{zx} = -\dfrac{1}{4}\dfrac{w}{g}a^3$ ◀

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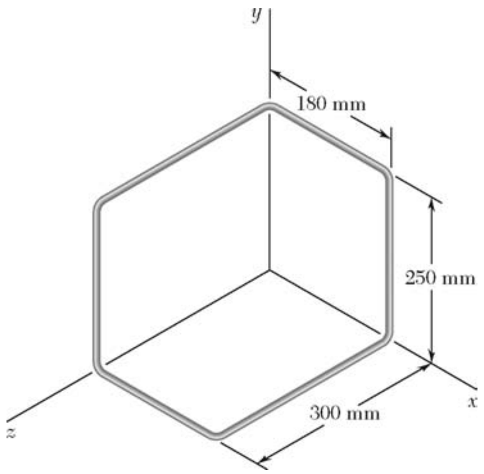
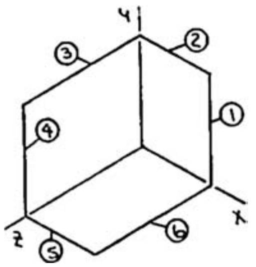


PROBLEM 9.159

The figure shown is formed of 1.5-mm-diameter aluminum wire. Knowing that the density of aluminum is 2800 kg/m^3, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the wire figure.

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{AL}} V = \rho_{\text{AL}} AL$$

Then

$$m_1 = m_4 = (2800\ \text{kg/m}^3)\left[\frac{\pi}{4}(0.0015\ \text{m})^2\right](0.25\ \text{m})$$

$$= 1.23700 \times 10^{-3}\ \text{kg}$$

$$m_2 = m_5 = (2800\ \text{kg/m}^3)\left[\frac{\pi}{4}(0.0015\ \text{m})^2\right](0.18\ \text{m})$$

$$= 0.89064 \times 10^{-3}\ \text{kg}$$

$$m_3 = m_6 = (2800\ \text{kg/m}^3)\left[\frac{\pi}{4}(0.0015\ \text{m})^2\right](0.3\ \text{m})$$

$$= 1.48440 \times 10^{-3}\ \text{kg}$$

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PROBLEM 9.159 (Continued)

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry.

	m, kg	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$m\bar{x}\,\bar{y}$, $\text{kg}\cdot\text{m}^2$	$m\bar{y}\,\bar{z}$, $\text{kg}\cdot\text{m}^2$	$m\bar{z}\,\bar{x}$, $\text{kg}\cdot\text{m}^2$
1	1.23700×10^{-3}	0.18	0.125	0	27.8325×10^{-6}	0	0
2	0.89064×10^{-3}	0.09	0.25	0	20.0394×10^{-6}	0	0
3	1.48440×10^{-3}	0	0.25	0.15	0	55.6650×10^{-6}	0
4	1.23700×10^{-3}	0	0.125	0.3	0	46.3875×10^{-6}	0
5	0.89064×10^{-3}	0.09	0	0.3	0	0	24.0473×10^{-6}
6	1.48440×10^{-3}	0.18	0	0.15	0	0	40.0788×10^{-6}
Σ					47.8719×10^{-6}	102.0525×10^{-6}	64.1261×10^{-6}

Then $\quad I_{xy} = \Sigma(\bar{I}_{x'y'}^{\;\nearrow 0} + m\bar{x}\,\bar{y})$ $\quad$ or $\quad I_{xy} = 47.9\times10^{-6}\ \text{kg}\cdot\text{m}^2$ ◀

$I_{yz} = \Sigma(\bar{I}_{y'z'}^{\;\nearrow 0} + m\bar{y}\,\bar{z})$ $\quad$ or $\quad I_{yz} = 102.1\times10^{-6}\ \text{kg}\cdot\text{m}^2$ ◀

$I_{zx} = \Sigma(\bar{I}_{z'x'}^{\;\nearrow 0} + m\bar{z}\,\bar{x})$ $\quad$ or $\quad I_{zx} = 64.1\times10^{-6}\ \text{kg}\cdot\text{m}^2$ ◀

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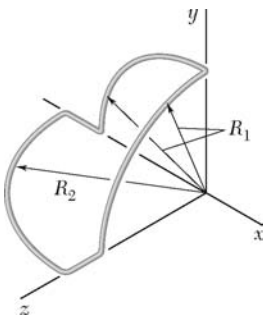
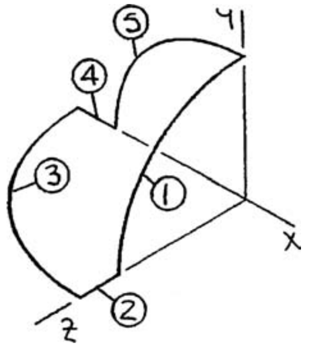


PROBLEM 9.160

Thin aluminum wire of uniform diameter is used to form the figure shown. Denoting by m' the mass per unit length of the wire, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the wire figure.

SOLUTION

First compute the mass of each component. We have



$$m = \left(\frac{m}{L}\right)L = m'L$$

Then
$$m_1 = m_5 = m'\left(\frac{\pi}{2}R_1\right) = \frac{\pi}{2}m'R_1$$
$$m_2 = m_4 = m'(R_2 - R_1)$$
$$m_3 = m'\left(\frac{\pi}{2}R_2\right) = \frac{\pi}{2}m'R_2$$

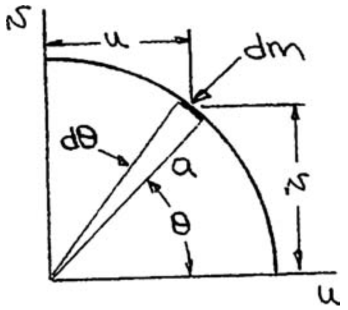
Now observe that because of symmetry the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'},$ and $\bar{I}_{z'x'},$ of components 2 and 4 are zero and

$$(\bar{I}_{x'y'})_1 = (\bar{I}_{z'x'})_1 = 0 \qquad (\bar{I}_{x'y'})_3 = (\bar{I}_{y'z'})_3 = 0$$
$$(\bar{I}_{y'z'})_5 = (\bar{I}_{z'x'})_5 = 0$$

Also
$$\bar{x}_1 = \bar{x}_2 = 0 \qquad \bar{y}_2 = \bar{y}_3 = \bar{y}_4 = 0 \qquad \bar{z}_4 = \bar{z}_5 = 0$$

Using the parallel-axis theorem [Equations (9.47)], it follows that $I_{xy} = I_{yz} = I_{zx}$ for components 2 and 4.

To determine I_{uv} for one quarter of a circular arc, we have $dI_{uv} = uvdm$



where
$$u = a\cos\theta \qquad v = a\sin\theta$$

and
$$dm = \rho dV = \rho[A(ad\theta)]$$

where A is the cross-sectional area of the wire. Now

$$m = m'\left(\frac{\pi}{2}a\right) = \rho A\left(\frac{\pi}{2}a\right)$$

so that
$$dm = m'ad\theta$$

and
$$dI_{uv} = (a\cos\theta)(a\sin\theta)(m'ad\theta)$$
$$= m'a^3 \sin\theta\cos\theta d\theta$$

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PROBLEM 9.160 (Continued)

Then

$$I_{uv} = \int dI_{uv} = \int_0^{\pi/2} m'a^3 \sin\theta\cos\theta d\theta$$

$$= m'a^3\left[\frac{1}{2}\sin^2\theta\right]_0^{\pi/2} = \frac{1}{2}m'a^3$$

Thus,

$$(I_{yz})_1 = \frac{1}{2}m'R_1^3$$

and

$$(I_{zx})_3 = -\frac{1}{2}m'R_2^3 \quad (I_{xy})_5 = -\frac{1}{2}m'R_1^3$$

because of $90°$ rotations of the coordinate axes. Finally,

$$I_{xy} = \Sigma(I_{xy}) = [(\bar{I}_{x'y'})_1^{\nearrow 0} + m_1\bar{x}_1\bar{y}_1] + [(\bar{I}_{x'y'})_3^{\nearrow 0} + m_3\bar{x}_3\bar{y}_3] + (I_{xy})_5$$

or $I_{xy} = -\frac{1}{2}m'R_1^3$ ◀

$$I_{yz} = \Sigma(I_{yz}) = (I_{yz})_1 + [(\bar{I}_{y'z'})_3^{\nearrow 0} + m_3\bar{y}_3\bar{z}_3] + [(\bar{I}_{y'z'})_5^{\nearrow 0} + m_5\bar{y}_5\bar{z}_5]$$

or $I_{yz} = \frac{1}{2}m'R_1^3$ ◀

$$I_{zx} = \Sigma(I_{zx}) = [(\bar{I}_{z'x'})_1^{\nearrow 0} + m_1\bar{z}_1\bar{x}_1] + (I_{zx})_3 + [(\bar{I}_{z'x'})_5^{\nearrow 0} + m_5\bar{z}_5\bar{x}_5]$$

or $I_{zx} = -\frac{1}{2}m'R_2^3$ ◀

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PROBLEM 9.161

Complete the derivation of Eqs. (9.47), which express the parallel-axis theorem for mass products of inertia.

SOLUTION

We have $\quad I_{xy} = \int xydm \quad I_{yz} = \int yzdm \quad I_{zx} = \int zxdm \qquad (9.45)$

and $\quad x = x' + \bar{x} \quad y = y' + \bar{y} \quad z = z' + \bar{z} \qquad (9.31)$

Consider $\quad I_{xy} = \int xydm$

Substituting for x and for y

$$I_{xy} = \int (x' + \bar{x})(y' + \bar{y})\,dm$$
$$= \int x'y'dm + \bar{y}\int x'dm + \bar{x}\int y'dm + \bar{x}\,\bar{y}\int dm$$

By definition $\quad \bar{I}_{x'y'} = \int x'y'dm$

and $\quad \int x'dm = m\bar{x}'$

$$\int y'dm = m\bar{y}'$$

However, the origin of the primed coordinate system coincides with the mass center G, so that

$$\bar{x}' = \bar{y}' = 0$$

$I_{xy} = \bar{I}_{x'y'} + m\bar{x}\,\bar{y}$ Q.E.D. ◀

The expressions for I_{yz} and I_{zx} are obtained in a similar manner.

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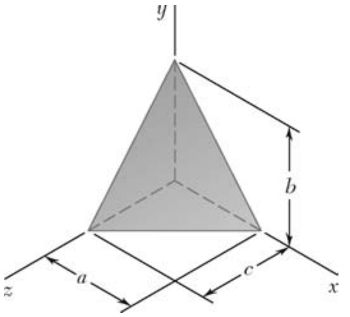
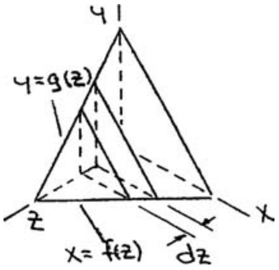


PROBLEM 9.162

For the homogeneous tetrahedron of mass m shown, (a) determine by direct integration the mass product of inertia I_{zx}, (b) deduce I_{yz} and I_{xy} from the result obtained in Part a.

SOLUTION

(a) First divide the tetrahedron into a series of thin vertical slices of thickness dz as shown.



Now $$x = -\frac{a}{c}z + a = a\left(1 - \frac{z}{c}\right)$$

and $$y = -\frac{b}{c}z + b = b\left(1 - \frac{z}{c}\right)$$

The mass dm of the slab is

$$dm = \rho dV = \rho\left(\frac{1}{2}xydz\right) = \frac{1}{2}\rho ab\left(1 - \frac{z}{c}\right)^2 dz$$

Then $$m = \int dm = \int_0^c \frac{1}{2}\rho ab\left(1 - \frac{z}{c}\right)^2 dz$$

$$= \frac{1}{2}\rho ab\left[\left(-\frac{c}{3}\right)\left(1 - \frac{z}{c}\right)^3\right]_0^c = \frac{1}{6}\rho abc$$

Now $$dI_{zx} = d\bar{I}_{z'x'} + \bar{z}_{\text{EL}}\bar{x}_{\text{EL}}dm$$

where $$d\bar{I}_{z'x'} = 0 \quad \text{(symmetry)}$$

and $$\bar{z}_{\text{EL}} = z \qquad \bar{x}_{\text{EL}} = \frac{1}{3}x = \frac{1}{3}a\left(1 - \frac{z}{c}\right)$$

Then $$I_{zx} = \int dI_{zx} = \int_0^c z\left[\frac{1}{3}a\left(1 - \frac{z}{c}\right)\right]\left[\frac{1}{2}\rho ab\left(1 - \frac{z}{c}\right)^2 dz\right]$$

$$= \frac{1}{6}\rho a^2 b\int_0^c\left(z - 3\frac{z^2}{c} + 3\frac{z^3}{c^2} - \frac{z^4}{c^3}\right)dz$$

$$= \frac{m}{c}a\left[\frac{1}{2}z^2 - \frac{z^3}{c} + \frac{3}{4}\frac{z^4}{c^2} - \frac{1}{5}\frac{z^5}{c^3}\right]_0^c$$

or $$I_{zx} = \frac{1}{20}mac \blacktriangleleft$$

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PROBLEM 9.162 (Continued)

(*b*) Because of the symmetry of the body, I_{xy} and I_{yz} can be deduced by considering the circular permutation of (x, y, z) and (a, b, c).

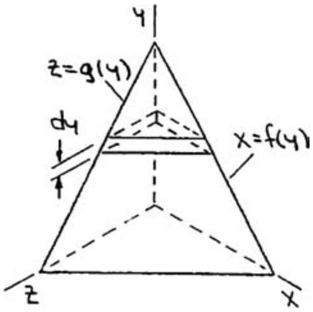
Thus,

$$I_{xy} = \frac{1}{20} mab \quad \blacktriangleleft$$

$$I_{yz} = \frac{1}{20} mbc \quad \blacktriangleleft$$

Alternative solution for Part *a*:

First divide the tetrahedron into a series of thin horizontal slices of thickness dy as shown.



Now $$x = -\frac{a}{b}y + a = a\left(1 - \frac{y}{b}\right)$$

and $$z = -\frac{c}{b}y + c = c\left(1 - \frac{y}{b}\right)$$

The mass dm of the slab is

$$dm = \rho dV = \rho\left(\frac{1}{2}xzdy\right) = \frac{1}{2}\rho ac\left(1 - \frac{y}{b}\right)^2 dy$$

Now $$dI_{zx} = \rho t dI_{zx,\text{area}}$$

where $$t = dy$$

and $dI_{zx,\text{area}} = \frac{1}{24}x^2z^2$ from the results of Sample Problem 9.6.

Then $$dIzx = \rho(dy)\left\{\frac{1}{24}\left[a\left(1 - \frac{y}{b}\right)^2\right]\left[c\left(1 - \frac{y}{b}\right)\right]^2\right\}$$

$$= \frac{1}{24}\rho a^2c^2\left(1 - \frac{y}{b}\right)^4 dy = \frac{1}{4}\frac{m}{b}ac\left(1 - \frac{y}{b}\right)^4 dy$$

Finally, $$I_{zx} = \int dI_{zx} = \int_0^b \frac{1}{4}\frac{m}{b}ac\left(1 - \frac{y}{b}\right)^4 dy$$

$$= \frac{1}{4}\frac{m}{b}ac\left[\left(-\frac{b}{5}\right)\left(1 - \frac{y}{b}\right)^5\right]_0^b$$

or $$I_{zx} = \frac{1}{20} mac \quad \blacktriangleleft$$

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PROBLEM 9.162 (Continued)

Alternative solution for Part *a*:

The equation of the included face of the tetrahedron is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

so that
$$y=b\left(1-\frac{x}{a}-\frac{z}{c}\right)$$

For an infinitesimal element of sides dx, dy, and dz:

$$dm=\rho dV=\rho dydxdz$$

From Part *a*
$$x=a\left(1-\frac{z}{c}\right)$$

Now

$$I_{zx}=\int zxdm=\int_0^c\int_0^{a(1-z/c)}\int_0^{b(1-x/a-z/c)} zx(\rho dydxdz)$$

$$=\rho\int_0^c\int_0^{a(1-z/c)} zx\left[b\left(1-\tfrac{x}{a}-\tfrac{z}{c}\right)\right]dxdz$$

$$=\rho b\int_0^c z\left[\frac{1}{2}x^2-\frac{1}{3}\frac{x^3}{a}-\frac{1}{2}\frac{z}{c}x^2\right]_0^{a(1-z/c)} dz$$

$$=\rho b\int_0^c z\left[\frac{1}{2}a^2\left(1-\frac{z}{c}\right)^2-\frac{1}{3a}a^3\left(1-\frac{z}{c}\right)^3-\frac{1}{2}\frac{z}{c}a^2\left(1-\frac{z}{c}\right)^2\right]dz$$

$$=\rho b\int_0^c \frac{1}{6}a^2z\left(1-\frac{z}{c}\right)^3 dz$$

$$=\frac{1}{6}\rho a^2b\int_0^c\left(z-3\frac{z^2}{c}+3\frac{z^3}{c^2}-\frac{z^4}{c^3}\right)dz$$

$$=\frac{m}{c}a\left[\frac{1}{2}z^2-\frac{z^3}{c}+\frac{3}{4}\frac{z^4}{c^2}-\frac{1}{5}\frac{z^5}{c^3}\right]_0^c$$

or $I_{zx}=\dfrac{1}{20}mac$ ◀

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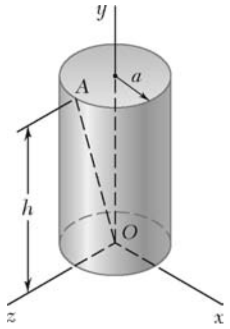
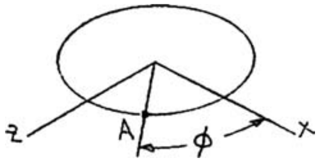


PROBLEM 9.163

The homogeneous circular cylinder shown has a mass m. Determine the mass moment of inertia of the cylinder with respect to the line joining the origin O and Point A that is located on the perimeter of the top surface of the cylinder.

SOLUTION



From Figure 9.28: $$I_y = \frac{1}{2}ma^2$$

and using the parallel-axis theorem

$$I_x = I_z = \frac{1}{12}m(3a^2 + h^2) + m\left(\frac{h}{2}\right)^2 = \frac{1}{12}m(3a^2 + 4h^2)$$

Symmetry implies $$I_{xy} = I_{yz} = I_{zx} = 0$$

For convenience, let Point A lie in the yz plane. Then

$$\lambda_{OA} = \frac{1}{\sqrt{h^2 + a^2}}(h\mathbf{j} + a\mathbf{k})$$

With the mass products of inertia equal to zero, Equation (9.46) reduces to

$$I_{OA} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 \quad (I_x\lambda_x^2 \to 0)$$

$$= \frac{1}{2}ma^2\left(\frac{h}{\sqrt{h^2 + a^2}}\right)^2 + \frac{1}{12}m(3a^2 + 4h^2)\left(\frac{a}{\sqrt{h^2 + a^2}}\right)^2$$

or $$I_{OA} = \frac{1}{12}ma^2\frac{10h^2 + 3a^2}{h^2 + a^2}$$ ◀

Note: For Point A located at an arbitrary point on the perimeter of the top surface, λ_{OA} is given by

$$\lambda_{OA} = \frac{1}{\sqrt{h^2 + a^2}}(a\cos\phi\mathbf{i} + h\mathbf{j} + a\sin\phi\mathbf{k})$$

which results in the same expression for I_{OA}.

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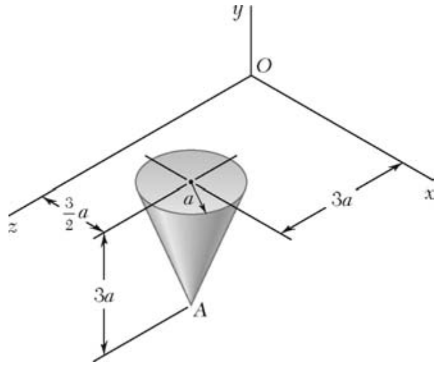

PROBLEM 9.164

The homogeneous circular cone shown has a mass m. Determine the mass moment of inertia of the cone with respect to the line joining the origin O and Point A.

SOLUTION

First note that
$$d_{OA} = \sqrt{\left(\frac{3}{2}a\right)^2 + (-3a)^2 + (3a)^2} = \frac{9}{2}a$$

Then
$$\lambda_{OA} = \frac{1}{\frac{9}{2}a}\left(\frac{3}{2}a\mathbf{i} - 3a\mathbf{j} + 3a\mathbf{k}\right) = \frac{1}{3}(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

For a rectangular coordinate system with origin at Point A and axes aligned with the given x, y, z axes, we have (using Figure 9.28)

$$I_x = I_z = \frac{3}{5}m\left[\frac{1}{4}a^2 + (3a)^2\right] \qquad I_y = \frac{3}{10}ma^2$$
$$= \frac{111}{20}ma^2$$

Also, symmetry implies
$$I_{xy} = I_{yz} = I_{zx} = 0$$

With the mass products of inertia equal to zero, Equation (9.46) reduces to

$$I_{OA} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2$$
$$= \frac{111}{20}ma^2\left(\frac{1}{3}\right)^2 + \frac{3}{10}ma^2\left(-\frac{2}{3}\right)^2 + \frac{111}{20}ma^2\left(\frac{2}{3}\right)^2$$
$$= \frac{193}{60}ma^2$$

or $I_{OA} = 3.22ma^2$ ◀

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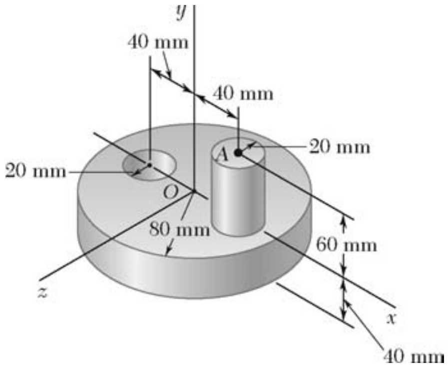
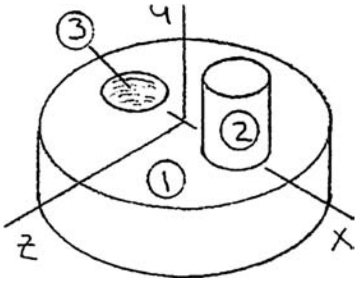


PROBLEM 9.165

Shown is the machine element of Problem 9.141. Determine its mass moment of inertia with respect to the line joining the origin *O* and Point *A*.

SOLUTION



First compute the mass of each component.

We have $$m = \rho_{\text{ST}} V = \frac{0.284\ \text{lb/in.}^3}{32.2\ \text{ft/s}^2} V = (0.008819\ \text{lb}\cdot\text{s}^2/\text{ft}\cdot\text{in.}^3)V$$

Then

$$m_1 = (7850\ \text{kg/m}^3)[\pi(0.08\ \text{m})^2(0.04\ \text{m})] = 6.31334\ \text{kg}$$
$$m_2 = (7850\ \text{kg/m}^3)[\pi(0.02\ \text{m})^2(0.06\ \text{m})] = 0.59188\ \text{kg}$$
$$m_3 = (7850\ \text{kg/m}^3)[\pi(0.02\ \text{m})^2(0.04\ \text{m})] = 0.39458\ \text{kg}$$

Symmetry implies $I_{yz} = I_{zx} = 0 \qquad (I_{xy})_1 = 0$

and $(\bar{I}_{x'y'})_2 = (\bar{I}_{x'y'})_3 = 0$

Now
$$\begin{aligned} I_{xy} &= \Sigma(\bar{I}_{x'y'} + m\bar{x}\,\bar{y}) = m_2\bar{x}_2\bar{y}_2 - m_3\bar{x}_3\bar{y}_3 \\ &= [0.59188\ \text{kg}\ (0.04\ \text{m})(0.03\ \text{m})] - [0.39458\ \text{kg}\ (-0.04\ \text{m})(-0.02\ \text{m})] \\ &= (0.71026 - 0.31566)\times 10^{-3}\ \text{kg}\cdot\text{m}^2 \\ &= 0.39460\times 10^{-3}\ \text{kg}\cdot\text{m}^2 \end{aligned}$$

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PROBLEM 9.165 (Continued)

From the solution to Problem 9.141, we have

$$I_x = 13.98800\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$
$$I_y = 20.55783\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$
$$I_z = 14.30368\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

By observation
$$\boldsymbol{\lambda}_{OA} = \frac{1}{\sqrt{13}}(2\mathbf{i} + 3\mathbf{j})$$

Substituting into Eq. (9.46)

$$I_{OA} = I_x\lambda_x^2 + I_y\lambda_y^2 + \cancelto{0}{I_z\lambda_z^2} - 2I_{xy}\lambda_x\lambda_y - \cancelto{0}{2I_{yz}\lambda_y\lambda_z} - \cancelto{0}{2I_{zx}\lambda_z\lambda_x}$$

$$= \left[(13.98800)\left(\frac{2}{\sqrt{13}}\right)^2 + (20.55783)\left(\frac{3}{\sqrt{13}}\right)^2 - 2(0.39460)\left(\frac{2}{\sqrt{13}}\right)\left(\frac{2}{\sqrt{13}}\right)\right]\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$= (4.30400 + 14.23234 - 0.36425)\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $I_{OA} = 18.17\times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

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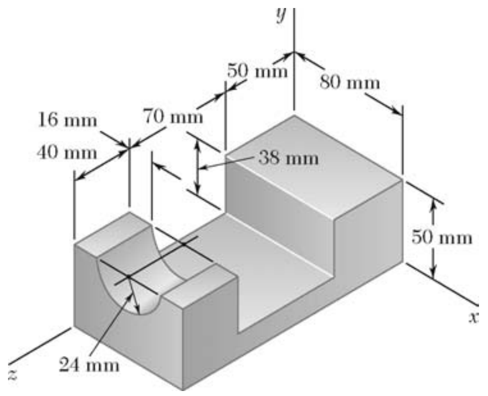

PROBLEM 9.166

Determine the mass moment of inertia of the steel fixture of Problems 9.145 and 9.149 with respect to the axis through the origin that forms equal angles with the *x*, *y*, and *z* axes.

SOLUTION

From the solutions to Problems 9.145 and 9.149, we have

Problem 9.145: $I_x = 26.4325 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$

$I_y = 31.1726 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$

$I_z = 8.5773 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$

Problem 9.149: $I_{xy} = 2.5002 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$

$I_{yz} = 4.0627 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$

$I_{zx} = 8.8062 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$

From the problem statement it follows that

$$\lambda_x = \lambda_y = \lambda_z$$

Now $\lambda_x^2 + \lambda_y^2 + \lambda_z^2 = 1 \Rightarrow 3\lambda_x^2 = 1$

or $\lambda_x = \lambda_y = \lambda_z = \dfrac{1}{\sqrt{3}}$

Substituting into Eq. (9.46)

$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

Noting that $\lambda_x^2 = \lambda_y^2 = \lambda_z^2 = \lambda_x\lambda_y = \lambda_y\lambda_z = \lambda_z\lambda_x = \dfrac{1}{3}$

We have
$$I_{OL} = \frac{1}{3}[26.4325 + 31.1726 + 8.5773 - 2(2.5002 + 4.0627 + 8.8062)] \times 10^{-3}\ \text{kg} \cdot \text{m}^2$$

or $I_{OL} = 11.81 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$ ◀

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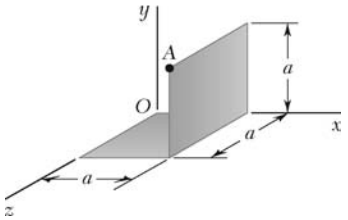
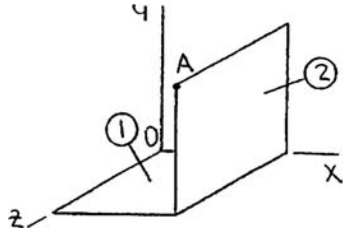


PROBLEM 9.167

The thin bent plate shown is of uniform density and weight W. Determine its mass moment of inertia with respect to the line joining the origin O and Point A.

SOLUTION



First note that $$m_1 = m_2 = \frac{1}{2}\frac{W}{g}$$

and that $$\lambda_{OA} = \frac{1}{\sqrt{3}}(\mathbf{i}+\mathbf{j}+\mathbf{k})$$

Using Figure 9.28 and the parallel-axis theorem, we have

$$\begin{aligned}
I_x &= (I_x)_1 + (I_x)_2 \\
&= \left[\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)a^2 + \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)^2\right] \\
&\quad + \left\{\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)(a^2+a^2) + \frac{1}{2}\frac{W}{g}\left[\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right]\right\} \\
&= \frac{1}{2}\frac{W}{g}\left[\left(\frac{1}{12}+\frac{1}{4}\right)a^2 + \left(\frac{1}{6}+\frac{1}{2}\right)a^2\right] = \frac{1}{2}\frac{W}{g}a^2
\end{aligned}$$

$$\begin{aligned}
I_y &= (I_y)_1 + (I_y)_2 \\
&= \left\{\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)(a^2+a^2) + \frac{1}{2}\frac{W}{g}\left[\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right]\right\} \\
&\quad + \left\{\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)a^2 + \frac{1}{2}\frac{W}{g}\left[(a)^2 + \left(\frac{a}{2}\right)^2\right]\right\} \\
&= \frac{1}{2}\frac{W}{g}\left[\left(\frac{1}{6}+\frac{1}{2}\right)a^2 + \left(\frac{1}{12}+\frac{5}{4}\right)a^2\right] = \frac{W}{g}a^2
\end{aligned}$$

$$\begin{aligned}
I_z &= (I_z)_1 + (I_z)_2 \\
&= \left[\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)a^2 + \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)^2\right] \\
&\quad + \left\{\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)a^2 + \frac{1}{2}\frac{W}{g}\left[(a)^2 + \left(\frac{a}{2}\right)^2\right]\right\} \\
&= \frac{1}{2}\frac{W}{g}\left[\left(\frac{1}{12}+\frac{1}{4}\right)a^2 + \left(\frac{1}{12}+\frac{5}{4}\right)a^2\right] = \frac{5}{6}\frac{W}{g}a^2
\end{aligned}$$

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PROBLEM 9.167 (Continued)

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'},$ and $\bar{I}_{z'x'},$ of both components are zero because of symmetry. Also, $\bar{y}_1 = 0$

Then
$$I_{xy} = \Sigma(\bar{I}_{x'y'}^{\nearrow 0} + m\bar{x}\,\bar{y}) = m_2\bar{x}_2\bar{y}_2 = \frac{1}{2}\frac{W}{g}(a)\left(\frac{a}{2}\right) = \frac{1}{4}\frac{W}{g}a^2$$

$$I_{yz} = \Sigma(\bar{I}_{y'z'}^{\nearrow 0} + m\bar{y}\,\bar{z}) = m_2\bar{y}_2\bar{z}_2 = \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right) = \frac{1}{8}\frac{W}{g}a^2$$

$$I_{zx} = \Sigma(\bar{I}_{z'x'}^{\nearrow 0} + m\bar{z}\,\bar{x}) = m_1\bar{z}_1\bar{x}_1 + m_2\bar{z}_2\bar{x}_2$$

$$= \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right) + \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)(a) = \frac{3}{8}\frac{W}{g}a^2$$

Substituting into Equation (9.46)

$$I_{OA} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

Noting that

$$\lambda_x^2 = \lambda_y^2 = \lambda_z^2 = \lambda_x\lambda_y = \lambda_y\lambda_z = \lambda_z\lambda_x = \frac{1}{3}$$

We have

$$I_{OA} = \frac{1}{3}\left[\frac{1}{2}\frac{W}{g}a^2 + \frac{W}{g}a^2 + \frac{5}{6}\frac{W}{g}a^2 - 2\left(\frac{1}{4}\frac{W}{g}a^2 + \frac{1}{8}\frac{W}{g}a^2 + \frac{3}{8}\frac{W}{g}a^2\right)\right]$$

$$= \frac{1}{3}\left[\frac{14}{6} - 2\left(\frac{3}{4}\right)\right]\frac{W}{g}a^2 \qquad \text{or} \quad I_{OA} = \frac{5}{18}\frac{W}{g}a^2 \blacktriangleleft$$

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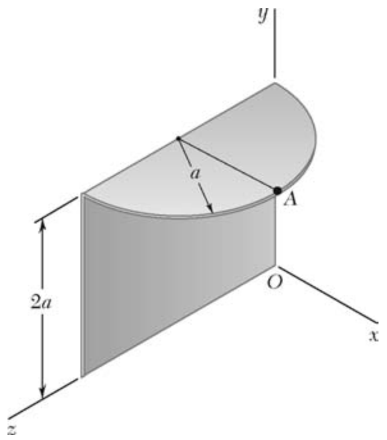
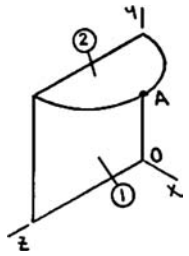


PROBLEM 9.168

A piece of sheet steel of thickness t and specific weight γ is cut and bent into the machine component shown. Determine the mass moment of inertia of the component with respect to the joining the origin O and Point A.

SOLUTION



First note that

$$\boldsymbol{\lambda}_{OA} = \frac{1}{\sqrt{6}}(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

Next compute the mass of each component. We have

$$m = \rho V = \frac{\gamma}{g}(tA)$$

Then

$$m_1 = \frac{\gamma}{g}t(2a \times 2a) = 4\frac{\gamma t}{g}a^2$$

$$m_2 = \frac{\gamma}{g}t\left(\frac{\pi}{2} \times a^2\right) = \frac{\pi}{2}\frac{\gamma t}{g}a^2$$

Using Figure 9.28 for component 1 and the equations derived above (following the solution to Problem 9.134) for a semicircular plate for component 2, we have

$$\begin{aligned}
I_x &= (I_x)_1 + (I_x)_2 \\
&= \left\{\frac{1}{12}\left(4\frac{\gamma t}{g}a^2\right)[(2a)^2 + (2a)^2] + 4\frac{\gamma t}{g}a^2(a^2 + a^2)\right\} \\
&\quad + \left\{\frac{1}{4}\left(\frac{\pi}{2}\frac{\gamma t}{g}a^2\right)a^2 + \frac{\pi}{2}\frac{\gamma t}{g}a^2[(2a)^2 + (a)^2]\right\} \\
&= 4\frac{\gamma t}{g}a^2\left(\frac{2}{3} + 2\right)a^2 + \frac{\pi}{2}\frac{\gamma t}{g}a^2\left(\frac{1}{4} + 5\right)a^2 \\
&= 18.91335\frac{\gamma t}{g}a^4
\end{aligned}$$

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PROBLEM 9.168 (Continued)

$$I_y = (I_y)_1 + (I_y)_2$$

$$= \left[\frac{1}{12}\left(4\frac{\gamma t}{g}a^2\right)(2a)^2 + 4\frac{\gamma t}{g}a^2(a^2)\right]$$

$$+\left\{\frac{\pi}{2}\frac{\gamma t}{g}a^2\left(\frac{1}{2}-\frac{16}{9\pi^2}\right)a^2 + \frac{\pi}{2}\frac{\gamma t}{g}a^2\left[\left(\frac{4a}{3\pi}\right)^2 + (a)^2\right]\right\}$$

$$= 4\frac{\gamma t}{g}a^2\left(\frac{1}{3}+1\right)a^2 + \frac{\pi}{2}\frac{\gamma t}{g}a^2\left(\frac{1}{2}-\frac{16}{9\pi^2}+\frac{16}{9\pi^2}+1\right)a^2$$

$$= 7.68953\frac{\gamma t}{g}a^4$$

$$I_z = (I_z)_1 + (I_z)_2$$

$$= \left[\frac{1}{12}\left(4\frac{\gamma t}{g}a^2\right)(2a)^2 + 4\frac{\gamma t}{g}a^2(a^2)\right]$$

$$+\left\{\frac{\pi}{2}\frac{\gamma t}{g}a^2\left(\frac{1}{4}-\frac{16}{9\pi^2}\right)a^2 + \frac{\pi}{2}\frac{\gamma t}{g}a^2\left[\left(\frac{4a}{3\pi}\right)^2 + (2a)^2\right]\right\}$$

$$= 4\frac{\gamma t}{g}a^2\left(\frac{1}{3}+1\right)a^2 + \frac{\pi}{2}\frac{\gamma t}{g}a^2\left(\frac{1}{4}-\frac{16}{9\pi^2}+\frac{16}{9\pi^2}+4\right)a^2$$

$$= 12.00922\frac{\gamma t}{g}a^4$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'},$ and $\bar{I}_{z'x'},$ of both components are zero because of symmetry. Also $\bar{x}_1 = 0$.

Then

$$I_{xy} = \Sigma(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y}) = m_2\bar{x}_2\bar{y}_2 = \frac{\pi}{2}\frac{\gamma t}{g}a^2\left(\frac{4a}{3\pi}\right)(2a)$$

$$= 1.33333\frac{\gamma t}{g}a^4$$

$$I_{yz} = \Sigma(\cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z}) = m_1\bar{y}_1\bar{z}_1 + m_2\bar{y}_2\bar{z}_2$$

$$= 4\frac{\gamma t}{g}a^2(a)(a) + \frac{\pi}{2}\frac{\gamma t}{g}a^2(2a)(a)$$

$$= 7.14159\frac{\gamma t}{g}a^4$$

$$I_{zx} = \Sigma(\cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x}) = m_2\bar{z}_2\bar{x}_2 = \frac{\pi}{2}\frac{\gamma t}{g}a^2(a)\left(\frac{4a}{3\pi}\right)$$

$$= 0.66667\frac{\gamma t}{g}a^4$$

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PROBLEM 9.168 (Continued)

Substituting into Eq. (9.46)

$$I_{OA} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

$$= 18.91335\frac{\gamma t}{g}a^4\left(\frac{1}{\sqrt{6}}\right)^2 + 7.68953\frac{\gamma t}{g}a^4\left(\frac{2}{\sqrt{6}}\right)^2$$

$$+12.00922\frac{\gamma t}{g}a^4\left(\frac{1}{\sqrt{6}}\right)^2 - 2\left(1.33333\frac{\gamma t}{g}a^4\right)\left(\frac{1}{\sqrt{6}}\right)\left(\frac{2}{\sqrt{6}}\right)$$

$$-2\left(7.14159\frac{\gamma t}{g}a^4\right)\left(\frac{2}{\sqrt{6}}\right)\left(\frac{2}{\sqrt{6}}\right) - 2\left(0.66667\frac{\gamma t}{g}a^4\right)\left(\frac{1}{\sqrt{6}}\right)\left(\frac{1}{\sqrt{6}}\right)$$

$$= (3.15223 + 5.12635 + 2.00154 - 0.88889 - 4.76106 - 0.22222)\frac{\gamma t}{g}a^4$$

or $\quad I_{OA} = 4.41\frac{\gamma t}{g}a^4$ ◀

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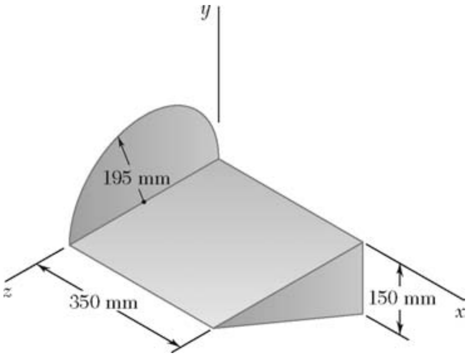

PROBLEM 9.169

Determine the mass moment of inertia of the machine component of Problems 9.136 and 9.155 with respect to the axis through the origin characterized by the unit vector $\boldsymbol{\lambda} = (-4\mathbf{i} + 8\mathbf{j} + \mathbf{k})/9$.

SOLUTION

From the solutions to Problems 9.136 and 9.155. We have

Problem 9.136:

$$I_x = 175.503 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$I_y = 308.629 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$I_z = 154.400 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

Problem 9.155:

$$I_{xy} = -8.0365 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$I_{yz} = 12.8950 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$I_{zx} = 94.0266 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

Substituting into Eq. (9.46)

$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

$$= \left[175.503\left(-\frac{4}{9}\right)^2 + 308.629\left(\frac{8}{9}\right)^2 + 154.400\left(\frac{1}{9}\right)^2 \right.$$

$$-2(-8.0365)\left(-\frac{4}{9}\right)\left(\frac{8}{9}\right) - 2(12.8950)\left(\frac{8}{9}\right)\left(\frac{1}{9}\right)$$

$$\left. -2(94.0266)\left(\frac{1}{9}\right)\left(-\frac{4}{9}\right)\right] \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$= (34.6673 + 243.855 + 1.906 - 6.350$$

$$-2.547 + 9.287) \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $\quad I_{OL} = 281 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

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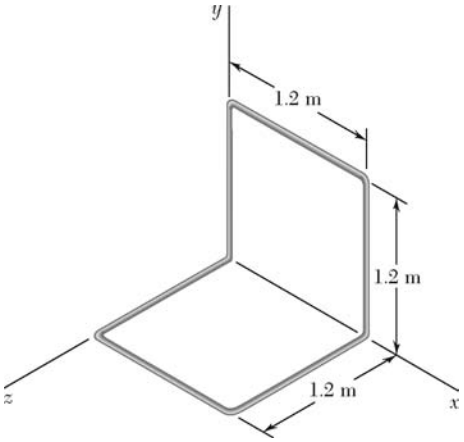
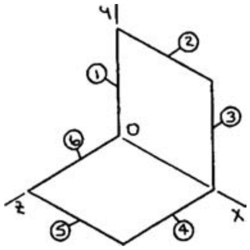


PROBLEM 9.170

For the wire figure of Problem 9.148, determine the mass moment of inertia of the figure with respect to the axis through the origin characterized by the unit vector $\boldsymbol{\lambda} = (-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})/7$.

SOLUTION



First compute the mass of each component. We have

$$m = \left(\frac{m}{L}\right)L = 0.056 \text{ kg/m} \times 1.2 \text{ m}$$
$$= 0.0672 \text{ kg}$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, for each component are zero because of symmetry.

Also

$$\bar{x}_1 = \bar{x}_6 = 0 \qquad \bar{y}_4 = \bar{y}_5 = \bar{y}_6 = 0 \qquad \bar{z}_1 = \bar{z}_2 = \bar{z}_3 = 0$$

Then $$I_{xy} = \Sigma(\bar{I}_{x'y'}^{\nearrow 0} + m\bar{x}\,\bar{y}) = m_2\bar{x}_2\bar{y}_2 + m_3\bar{x}_3\bar{y}_3$$

$$= (0.0672 \text{ kg})(0.6 \text{ m})(1.2 \text{ m}) + (0.0672 \text{ kg})(1.2 \text{ m})(0.6 \text{ m})$$
$$= 0.096768 \text{ kg}\cdot\text{m}^2$$

$$I_{yz} = \Sigma(\bar{I}_{y'z'}^{\nearrow 0} + m\bar{y}\,\bar{z}) = 0$$

$$I_{zx} = \Sigma(\bar{I}_{z'x'}^{\nearrow 0} + m\bar{z}\,\bar{x}) = m_4\bar{z}_4\bar{x}_4 + m_5\bar{z}_5\bar{x}_5$$
$$= (0.0672 \text{ kg})(0.6 \text{ m})(1.2 \text{ m}) + (0.0672 \text{ kg})(1.2 \text{ m})(0.6 \text{ m})$$
$$= 0.096768 \text{ kg}\cdot\text{m}^2$$

From the solution to Problem 9.148, we have

$$I_x = 0.32258 \text{ kg}\cdot\text{m}^2$$
$$I_y = I_z = 0.41933 \text{ kg}\cdot\text{m}^2$$

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PROBLEM 9.170 (Continued)

Substituting into Eq. (9.46)

$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2\overset{0}{\cancel{I_{yz}}}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

$$= \left[0.32258\left(-\frac{3}{7}\right)^2 + 0.41933\left(-\frac{6}{7}\right)^2 + 0.41933\left(\frac{2}{7}\right)^2 - 2(0.096768)\left(-\frac{3}{7}\right)\left(-\frac{6}{7}\right) - 2(0.096768)\left(\frac{2}{7}\right)\left(-\frac{3}{7}\right)\right]\text{kg}\cdot\text{m}^2$$

$$I_{OL} = (0.059249 + 0.30808 + 0.034231 - 0.071095 + 0.023698)\,\text{kg}\cdot\text{m}^2$$

or $\quad I_{OL} = 0.354\ \text{kg}\cdot\text{m}^2$ ◀

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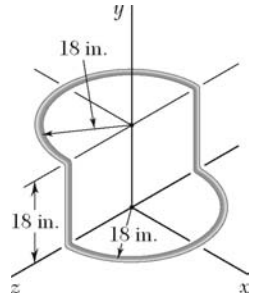
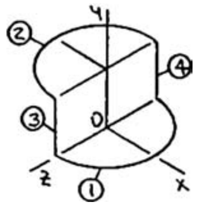


PROBLEM 9.171

For the wire figure of Problem 9.147, determine the mass moment of inertia of the figure with respect to the axis through the origin characterized by the unit vector $\lambda = (-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})/7$.

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \frac{\gamma_{\text{ST}}}{g} AL$$

Then

$$m_1 = m_2 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times \left[\frac{\pi}{4}\left(\frac{1}{8}\text{ in.}\right)^2\right] \times (\pi \times 18 \text{ in.}) \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 6.1112 \times 10^{-3} \text{ lb}\cdot\text{s}^2\text{/ft}$$

$$m_3 = m_4 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times \left[\frac{\pi}{4}\left(\frac{1}{8}\text{ in.}\right)^2\right] \times 18 \text{ in.} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 1.9453 \text{ lb}\cdot\text{s}^2\text{/ft}$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, for each component are zero because of symmetry.

Also $\qquad \bar{x}_3 = \bar{x}_4 = 0 \qquad \bar{y}_1 = 0 \qquad \bar{z}_1 = \bar{z}_2 = 0$

Then $\qquad I_{xy} = \Sigma(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y}) = m_2\bar{x}_2\bar{y}_2$

$$= (6.1112 \times 10^{-3} \text{ lb}\cdot\text{s}^2\text{/ft})\left(-\frac{2\times 18}{\pi}\text{ in.}\right)(18 \text{ in.})\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= -8.75480 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_{yz} = \Sigma(\cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z}) = m_3\bar{y}_3\bar{z}_3 + m_4\bar{y}_4\bar{z}_4$$

Now $\qquad m_3 = m_4, \qquad \bar{y}_3 = \bar{y}_4, \qquad \bar{z}_4 = -\bar{z}_3 \qquad I_{yz} = 0$

$$I_{zx} = \Sigma(\cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x}) \quad \text{or} \quad I_{zx} = 0$$

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PROBLEM 9.171 (Continued)

From the solution to Problem 9.147, we have

$$I_x = 39.1721\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$
$$I_y = 36.2542\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$
$$I_z = 30.4184\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Substituting into Eq. (9.46)

$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2\overset{0}{I_{yz}}\lambda_y\lambda_z - 2\overset{0}{I_{zx}}\lambda_z\lambda_x$$
$$= \left[39.1721\left(-\frac{3}{7}\right)^2 + 36.2542\left(-\frac{6}{7}\right)^2 + 30.4184\left(\frac{2}{7}\right)^2 - 2(-8.75480)\left(-\frac{3}{7}\right)\left(-\frac{6}{7}\right)\right]\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$
$$= (7.19488 + 26.6357 + 2.48313 + 6.43210)\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{OL} = 0.0427\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

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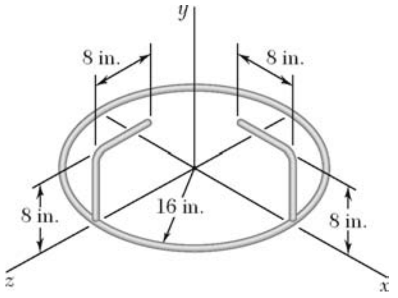
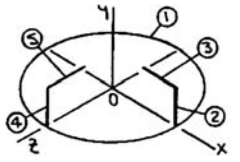


PROBLEM 9.172

For the wire figure of Problem 9.46, determine the mass moment of inertia of the figure with respect to the axis through the origin characterized by the unit vector $\boldsymbol{\lambda} = (-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})/7$.

SOLUTION



First compute the mass of each component. We have

$$m = \frac{W}{g} = \frac{1}{g}(W/L)_{\text{AL}} L$$

Then

$$m_1 = \frac{1}{32.2 \text{ ft/s}^2}(0.033 \text{ lb/ft})(2\pi \times 16 \text{ in.}) \times \frac{1 \text{ ft}}{12 \text{ in.}}$$

$$= 8.5857 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_2 = m_3 = m_4 = m_5 = \frac{1}{32.2 \text{ ft/s}^2}(0.033 \text{ lb/ft})(8 \text{ in.}) \times \frac{1 \text{ ft}}{12 \text{ in.}}$$

$$= 0.6832 \times 10^{-3} \text{ lb} \cdot \text{ft/s}^2$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry. Also

$$\bar{x}_1 = \bar{x}_4 = \bar{x}_5 = 0 \qquad \bar{y}_1 = 0 \qquad \bar{z}_1 = \bar{z}_2 = \bar{z}_3 = 0$$

Then

$$I_{xy} = \Sigma(\bar{I}_{x'y'}^{\;\nearrow 0} + m\bar{x}\,\bar{y}) = m_2\bar{x}_2\bar{y}_2 + m_3\bar{x}_3\bar{y}_3$$

$$= 0.6832 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}\left(\frac{16}{12}\text{ ft}\right)\left(\frac{4}{12}\text{ ft}\right)$$

$$+ 0.6832 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}\left(\frac{12}{12}\text{ ft}\right)\left(\frac{8}{12}\text{ ft}\right)$$

$$= (0.30364 + 0.45547) \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$= 0.75911 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

Symmetry implies

$$I_{yz} = I_{xy} \qquad I_{yz} = 0.75911 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$I_{zx} = \Sigma(\bar{I}_{z'x'}^{\;\nearrow 0} + m\bar{z}\,\bar{x}) = 0$$

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PROBLEM 9.172 (Continued)

From the solution to Problem 9.146, we have

$$I_x = I_z = 10.3642 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$
$$I_y = 19.1097 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

Substituting into Eq. (9.46)

$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2\overset{0}{\cancel{I_{zx}}}\lambda_z\lambda_x$$

$$= \left[10.3642\left(-\frac{3}{7}\right)^2 + 19.1097\left(-\frac{6}{7}\right)^2 + 10.3642\left(\frac{2}{7}\right)^2 - 2(0.75911)\left(-\frac{3}{7}\right)\left(-\frac{6}{7}\right) - 2(0.75911)\left(-\frac{6}{7}\right)\left(\frac{2}{7}\right)\right] \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$= (1.90663 + 14.03978 + 0.84606 - 0.55771 + 0.37181) \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

or $\quad I_{OL} = 16.61 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$ ◀

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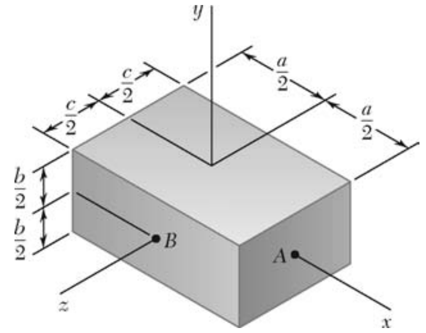
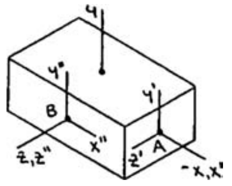


PROBLEM 9.173

For the rectangular prism shown, determine the values of the ratios b/a and c/a so that the ellipsoid of inertia of the prism is a sphere when computed (*a*) at Point *A*, (*b*) at Point *B*.

SOLUTION



(*a*) Using Figure 9.28 and the parallel-axis theorem, we have at Point *A*

$$I_{x'} = \frac{1}{12}m(b^2 + c^2)$$

$$I_{y'} = \frac{1}{12}m(a^2 + c^2) + m\left(\frac{a}{2}\right)^2$$

$$= \frac{1}{12}m(4a^2 + c^2)$$

$$I_{z'} = \frac{1}{12}m(a^2 + b^2) + m\left(\frac{a}{2}\right)^2 = \frac{1}{12}m(4a^2 + b^2)$$

Now observe that symmetry implies

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$$

Using Eq. (9.48), the equation of the ellipsoid of inertia is then

$$I_{x'}x^2 + I_{y'}y^2 + I_{z'}z^2 = 1: \quad \frac{1}{12}m(b^2 + c^2)x^2 + \frac{1}{12}m(4a^2 + c^2)y^2 + \frac{1}{12}m(4a^2 + b^2)z^2 = 1$$

For the ellipsoid to be a sphere, the coefficients must be equal. Therefore

$$\frac{1}{12}m(b^2 + c^2) = \frac{1}{12}m(4a^2 + c^2)$$

$$= \frac{1}{12}m(4a^2 + b^2)$$

Then $\qquad b^2 + c^2 = 4a^2 + c^2 \qquad$ or $\quad \dfrac{b}{a} = 2$ ◀

and $\qquad b^2 + c^2 = 4a^2 + b^2 \qquad$ or $\quad \dfrac{c}{a} = 2$ ◀

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PROBLEM 9.173 (Continued)

(*b*) Using Figure 9.28 and the parallel-axis theorem, we have at Point *B*

$$I_{x''} = \frac{1}{12}m(b^2 + c^2) + m\left(\frac{c}{2}\right)^2 = \frac{1}{12}m(b^2 + 4c^2)$$

$$I_{y''} = \frac{1}{12}m(a^2 + c^2) + m\left(\frac{c}{2}\right)^2 = \frac{1}{12}m(a^2 + 4c^2)$$

$$I_{z''} = \frac{1}{12}m(a^2 + b^2)$$

Now observe that symmetry implies

$$I_{x''y''} = I_{y''z''} = I_{z''x''} = 0$$

From Part *a* it then immediately follows that

$$\frac{1}{12}m(b^2 + 4c^2) = \frac{1}{12}m(a^2 + 4c^2) = \frac{1}{12}m(a^2 + b^2)$$

Then $b^2 + 4c^2 = a^2 + 4c^2$ or $\dfrac{b}{a} = 1$ ◀

and $b^2 + 4c^2 = a^2 + b^2$ or $\dfrac{c}{a} = \dfrac{1}{2}$ ◀

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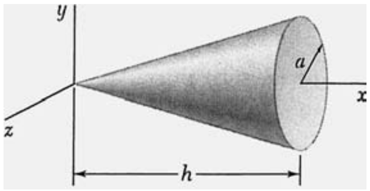
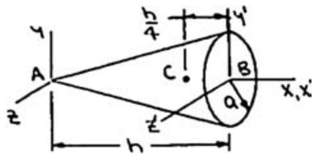


PROBLEM 9.174

For the right circular cone of Sample Problem 9.11, determine the value of the ratio *a/h* for which the ellipsoid of inertia of the cone is a sphere when computed (*a*) at the apex of the cone, (*b*) at the center of the base of the cone.

SOLUTION



(*a*) From Sample Problem 9.11, we have at the apex *A*

$$I_x = \frac{3}{10}ma^2$$

$$I_y = I_z = \frac{3}{5}m\left(\frac{1}{4}a^2 + h^2\right)$$

Now observe that symmetry implies $I_{xy} = I_{yz} = I_{zx} = 0$

Using Eq. (9.48), the equation of the ellipsoid of inertia is then

$$I_x x^2 + I_y y^2 + I_z z^2 = 1: \quad \frac{3}{10}ma^2x^2 + \frac{3}{5}m\left(\frac{1}{4}a^2 + h^2\right)y^2 + \frac{3}{5}m\left(\frac{1}{4}a^2 + h^2\right)z^2 = 1$$

For the ellipsoid to be a sphere, the coefficients must be equal. Therefore,

$$\frac{3}{10}ma^2 = \frac{3}{5}m\left(\frac{1}{4}a^2 + h^2\right) \qquad \text{or} \qquad \frac{a}{h} = 2 \blacktriangleleft$$

(*b*) From Sample Problem 9.11, we have

$$I_{x'} = \frac{3}{10}ma^2$$

and at the centroid *C* $$I_{y''} = \frac{3}{20}m\left(a^2 + \frac{1}{4}h^2\right)$$

Then $$I_{y'} = I_{z'} = \frac{3}{20}m\left(a^2 + \frac{1}{4}h^2\right) + m\left(\frac{h}{4}\right)^2 = \frac{1}{20}m(3a^2 + 2h^2)$$

Now observe that symmetry implies

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$$

From Part *a* it then immediately follows that

$$\frac{3}{10}ma^2 = \frac{1}{20}m(3a^2 + 2h^2) \qquad \text{or} \qquad \frac{a}{h} = \sqrt{\frac{2}{3}} \blacktriangleleft$$

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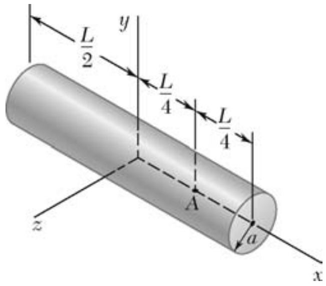
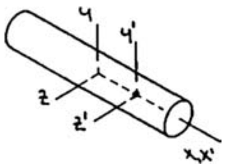


PROBLEM 9.175

For the homogeneous circular cylinder shown, of radius a and length L, determine the value of the ratio a/L for which the ellipsoid of inertia of the cylinder is a sphere when computed (a) at the centroid of the cylinder, (b) at Point A.

SOLUTION



(a) From Figure 9.28:

$$\bar{I}_x = \frac{1}{2}ma^2$$

$$\bar{I}_y = \bar{I}_z = \frac{1}{12}m(3a^2 + L^2)$$

Now observe that symmetry implies $I_{xy} = I_{yz} = I_{zx} = 0$

Using Eq. (9.48), the equation of the ellipsoid of inertia is then

$$I_x x^2 + I_y y^2 + I_z z^2 = 1: \quad \frac{1}{2}ma^2x^2 + \frac{1}{12}m(3a^2 + L^2)y^2 + \frac{1}{12}m(3a^2 + L^2) = 1$$

For the ellipsoid to be a sphere, the coefficients must be equal. Therefore

$$\frac{1}{2}ma^2 = \frac{1}{12}m(3a^2 + L^2) \qquad \text{or} \qquad \frac{a}{L} = \frac{1}{\sqrt{3}} \blacktriangleleft$$

(b) Using Figure 9.28 and the parallel-axis theorem, we have

$$I_{x'} = \frac{1}{2}ma^2$$

$$I_{y'} = I_{z'} = \frac{1}{12}m(3a^2 + L^2) + m\left(\frac{L}{4}\right)^2 = m\left(\frac{1}{4}a^2 + \frac{7}{48}L^2\right)$$

Now observe that symmetry implies

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$$

From Part a it then immediately follows that

$$\frac{1}{2}ma^2 = m\left(\frac{1}{4}a^2 + \frac{7}{48}L^2\right) \qquad \text{or} \qquad \frac{a}{L} = \sqrt{\frac{7}{12}} \blacktriangleleft$$

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PROBLEM 9.176

Given an arbitrary body and three rectangular axes *x*, *y*, and *z*, prove that the mass moment of inertia of the body with respect to any one of the three axes cannot be larger than the sum of the mass moments of inertia of the body with respect to the other two axes. That is, prove that the inequality $I_x \le I_y + I_z$ and the two similar inequalities are satisfied. Further, prove that $I_y \ge \frac{1}{2} I_x$ if the body is a homogeneous solid of revolution, where *x* is the axis of revolution and *y* is a transverse axis.

SOLUTION

(i) To prove $I_y + I_z \ge I_x$

By definition
$$I_y = \int (z^2 + x^2)dm$$
$$I_z = \int (x^2 + y^2)dm$$

Then
$$I_y + I_z = \int (z^2 + x^2)dm + \int (x^2 + y^2)dm$$
$$= \int (y^2 + z^2)dm + 2\int x^2 dm$$

Now $\int (y^2 + z^2)dm = I_x$ and $\int x^2 dm \ge 0$

$$I_y + I_z \ge I_x \qquad \text{Q.E.D.}$$

The proofs of the other two inequalities follow similar steps.

(ii) If the *x* axis is the axis of revolution, then

$$I_y = I_z$$

and from Part (i)
$$I_y + I_z \ge I_x$$

or
$$2I_y \ge I_x$$

or
$$I_y \ge \frac{1}{2} I_x \qquad \text{Q.E.D.}$$

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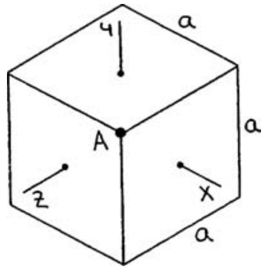
PROBLEM 9.177

Consider a cube of mass m and side a. (a) Show that the ellipsoid of inertia at the center of the cube is a sphere, and use this property to determine the moment of inertia of the cube with respect to one of its diagonals. (b) Show that the ellipsoid of inertia at one of the corners of the cube is an ellipsoid of revolution, and determine the principal moments of inertia of the cube at that point.

SOLUTION



(a) At the center of the cube have (using Figure 9.28)

$$I_x = I_y = I_z = \frac{1}{12}m(a^2 + a^2) = \frac{1}{6}ma^2$$

Now observe that symmetry implies

$$I_{xy} = I_{yz} = I_{zx} = 0$$

Using Equation (9.48), the equation of the ellipsoid of inertia is

$$\left(\frac{1}{6}ma^2\right)x^2 + \left(\frac{1}{6}ma^2\right)y^2 + \left(\frac{1}{6}ma^2\right)z^2 = 1$$

or $\quad x^2 + y^2 + z^2 = \dfrac{6}{ma^2} \;(= R^2)$ ◀

which is the equation of a sphere.

Since the ellipsoid of inertia is a sphere, the moment of inertia with respect to any axis OL through the center O of the cube must always

be the same $\left(R = \dfrac{1}{\sqrt{I_{OL}}}\right)$. $\quad I_{OL} = \dfrac{1}{6}ma^2$ ◀

(b) The above sketch of the cube is the view seen if the line of sight is along the diagonal that passes through corner A. For a rectangular coordinate system at A and with one of the coordinate axes aligned with the diagonal, an ellipsoid of inertia at A could be constructed. If the cube is then rotated 120° about the diagonal, the mass distribution will remain unchanged. Thus, the ellipsoid will also remain unchanged after it is rotated. As noted at the end of Section 9.17, this is possible only if the ellipsoid is an ellipsoid of revolution, where the diagonal is both the axis of revolution and a principal axis.

It then follows that $\quad I_{x'} = I_{OL} = \dfrac{1}{6}ma^2$ ◀

In addition, for an ellipsoid of revolution, the two transverse principal moments of inertia are equal and any axis perpendicular to the axis of revolution is a principal axis. Then, applying the parallel-axis theorem between the center of the cube and corner A for any perpendicular axis

$$I_{y'} = I_{z'} = \frac{1}{6}ma^2 + m\left(\frac{\sqrt{3}}{2}a\right)^2$$

or $\quad I_{y'} = I_{z'} = \dfrac{11}{12}ma^2$ ◀

(*Note:* Part b can also be solved using the method of Section 9.18.)

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PROBLEM 9.177 (Continued)

First note that at corner *A*

$$I_x = I_y = I_z = \frac{2}{3}ma^2$$

$$I_{xy} = I_{yz} = I_{zx} = \frac{1}{4}ma^2$$

Substituting into Equation (9.56) yields

$$k^3 - 2ma^2k^2 + \frac{55}{48}m^2a^6k - \frac{121}{864}m^3a^9 = 0$$

For which the roots are

$$k_1 = \frac{1}{6}ma^2$$

$$k_2 = k_3 = \frac{11}{12}ma^2$$

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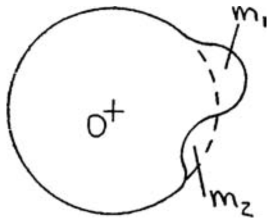
PROBLEM 9.178

Given a homogeneous body of mass m and of arbitrary shape and three rectangular axes x, y, and z with origin at O, prove that the sum $I_x + I_y + I_z$ of the mass moments of inertia of the body cannot be smaller than the similar sum computed for a sphere of the same mass and the same material centered at O. Further, using the result of Problem 9.176, prove that if the body is a solid of revolution, where x is the axis of revolution, its mass moment of inertia I_y about a transverse axis y cannot be smaller than $3ma^2/10$, where a is the radius of the sphere of the same mass and the same material.

SOLUTION



(i) Using Equation (9.30), we have

$$I_x + I_y + I_z = \int (y^2 + z^2)dm + \int (z^2 + x^2)\,dm + \int (x^2 + y^2)dm$$

$$= 2\int (x^2 + y^2 + z^2)dm$$

$$= 2\int r^2 dm$$

where r is the distance from the origin O to the element of mass dm. Now assume that the given body can be formed by adding and subtracting appropriate volumes V_1 and V_2 from a sphere of mass m and radius a which is centered at O; it then follows that

$$m_1 = m_2 \ (m_{\text{body}} = m_{\text{sphere}} = m)$$

Then

$$(I_x + I_y + I_z)_{\text{body}} = (I_x + I_y + I_z)_{\text{sphere}} + (I_x + I_y + I_z)_{V_1}$$
$$- (I_x + I_y + I_z)_{V_2}$$

or

$$(I_x + I_y + I_z)_{\text{body}} = (I_x + I_y + I_z)_{\text{sphere}} + 2\int_{m_1} r^2 dm - 2\int_{m_2} r^2 dm$$

Now, $m_1 = m_2$ and $r_1 \geq r_2$ for all elements of mass dm in volumes 1 and 2.

$$\int_{m_1} r^2 dm - \int_{m_2} r^2 dm \geq 0$$

so that

$$(I_x + I_y + I_z)_{\text{body}} \geq (I_x + I_y + I_z)_{\text{sphere}} \qquad \text{Q.E.D.}$$

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PROBLEM 9.178 (Continued)

(ii) First note from Figure 9.28 that for a sphere

$$I_x = I_y = I_z = \frac{2}{5}ma^2$$

Thus $$(I_x + I_y + I_z)_{\text{sphere}} = \frac{6}{5}ma^2$$

For a solid of revolution, where the *x* axis is the axis of revolution, we have

$$I_y = I_z$$

Then, using the results of Part (i)

$$(I_x + 2I_y)_{\text{body}} \geq \frac{6}{5}ma^2$$

From Problem 9.178 we have $$I_y \geq \frac{1}{2}I_x$$

or $$(2I_y - I_x)_{\text{body}} \geq 0$$

Adding the last two inequalities yields

$$(4I_y)_{\text{body}} \geq \frac{6}{5}ma^2$$

or $$(I_y)_{\text{body}} \geq \frac{3}{10}ma^2 \quad \text{Q.E.D.}$$

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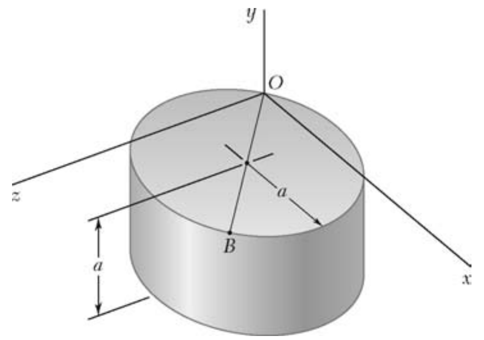
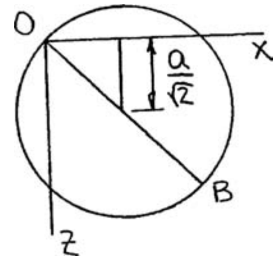


PROBLEM 9.179*

The homogeneous circular cylinder shown has a mass m, and the diameter OB of its top surface forms 45° angles with the x and z axes. (a) Determine the principal mass moments of inertia of the cylinder at the origin O. (b) Compute the angles that the principal axes of inertia at O form with the coordinate axes. (c) Sketch the cylinder, and show the orientation of the principal axes of inertia relative to the x, y, and z axes.

SOLUTION

(a) First compute the moments of inertia using Figure 9.28 and the parallel-axis theorem.



$$I_x = I_z = \frac{1}{12}m(3a^2 + a^2) + m\left[\left(\frac{a}{\sqrt{2}}\right)^2 + \left(\frac{a}{2}\right)^2\right] = \frac{13}{12}ma^2$$

$$I_y = \frac{1}{2}ma^2 + m(a)^2 = \frac{3}{2}ma^2$$

Next observe that the centroidal products of inertia are zero because of symmetry. Then

$$I_{xy} = \cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y} = m\left(\frac{a}{\sqrt{2}}\right)\left(-\frac{a}{2}\right) = -\frac{1}{2\sqrt{2}}ma^2$$

$$I_{yz} = \cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z} = m\left(-\frac{a}{2}\right)\left(\frac{a}{\sqrt{2}}\right) = -\frac{1}{2\sqrt{2}}ma^2$$

$$I_{zx} = \cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x} = m\left(\frac{a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right) = \frac{1}{2}ma^2$$

Substituting into Equation (9.56)

$$K^3 - \left(\frac{13}{12} + \frac{3}{2} + \frac{13}{12}\right)ma^2K^2$$

$$+\left[\left(\frac{13}{12}\times\frac{3}{2}\right) + \left(\frac{3}{2}\times\frac{13}{12}\right) + \left(\frac{13}{12}\times\frac{13}{12}\right) - \left(-\frac{1}{2\sqrt{2}}\right)^2 - \left(-\frac{1}{2\sqrt{2}}\right)^2 - \left(\frac{1}{2}\right)^2\right](ma^2)^2K$$

$$-\left[\left(\frac{13}{12}\times\frac{3}{2}\times\frac{13}{12}\right) - \left(\frac{13}{12}\right)\left(-\frac{1}{2\sqrt{2}}\right)^2 - \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)^2\right.$$

$$\left.-\left(\frac{13}{12}\right)\left(-\frac{1}{2\sqrt{2}}\right)^2 - 2\left(-\frac{1}{2\sqrt{2}}\right)\left(-\frac{1}{2\sqrt{2}}\right)\left(\frac{1}{2}\right)\right](ma^2)^3 = 0$$

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PROBLEM 9.179* (Continued)

Simplifying and letting $K = ma^2\zeta$ yields

$$\zeta^3 - \frac{11}{3}\zeta^2 + \frac{565}{144}\zeta - \frac{95}{96} = 0$$

Solving yields

$$\zeta_1 = 0.363383 \qquad \zeta_2 = \frac{19}{12} \qquad \zeta_3 = 1.71995$$

The principal moments of inertia are then

$K_1 = 0.363ma^2$ ◀

$K_2 = 1.583ma^2$ ◀

$K_3 = 1.720ma^2$ ◀

(*b*) To determine the direction cosines $\lambda_x, \lambda_y, \lambda_z$ of each principal axis, we use two of the equations of Equations (9.54) and (9.57).

Thus,

$$(I_x - K)\lambda_x - I_{xy}\lambda_y - I_{zx}\lambda_z = 0 \qquad (9.54a)$$

$$-I_{zx}\lambda_x - I_{yz}\lambda_y + (I_z - K)\lambda_z = 0 \qquad (9.54c)$$

$$\lambda_x^2 + \lambda_y^2 + \lambda_z^2 = 1 \qquad (9.57)$$

(*Note:* Since $I_{xy} = I_{yz}$, Equations (9.54a) and (9.54c) were chosen to simplify the "elimination" of λ_y during the solution process.)

Substituting for the moments and products of inertia in Equations (9.54a) and (9.54c)

$$\left(\frac{13}{12}ma^2 - K\right)\lambda_x - \left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_y - \left(\frac{1}{2}ma^2\right)\lambda_z = 0$$

$$-\left(\frac{1}{2}ma^2\right)\lambda_x - \left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_y + \left(\frac{13}{12}ma^2 - K\right)\lambda_z = 0$$

or

$$\left(\frac{13}{12} - \zeta\right)\lambda_x + \frac{1}{2\sqrt{2}}\lambda_y - \frac{1}{2}\lambda_z = 0 \qquad \text{(i)}$$

and

$$-\frac{1}{2}\lambda_x + \frac{1}{2\sqrt{2}}\lambda_y + \left(\frac{13}{12} - \zeta\right)\lambda_z = 0 \qquad \text{(ii)}$$

Observe that these equations will be identical, so that one will need to be replaced, if

$$\frac{13}{12} - \zeta = -\frac{1}{2} \qquad \text{or} \qquad \zeta = \frac{19}{12}$$

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PROBLEM 9.179* (Continued)

Thus, a third independent equation will be needed when the direction cosines associated with K_2 are determined. Then for K_1 and K_3

Eq. (i) through Eq. (ii): $$\left[\frac{13}{12}-\zeta-\left(-\frac{1}{2}\right)\right]\lambda_x+\left[-\frac{1}{2}-\left(\frac{13}{12}-\zeta\right)\right]\lambda_z=0$$

or $$\lambda_z=\lambda_x$$

Substituting into Eq. (i): $$\left(\frac{13}{12}-\zeta\right)\lambda_x+\frac{1}{2\sqrt{2}}\lambda_y-\frac{1}{2}\lambda_x=0$$

or $$\lambda_y=2\sqrt{2}\left(\zeta-\frac{7}{12}\right)\lambda_x$$

Substituting into Equation (9.57):

$$\lambda_x^2+\left[2\sqrt{2}\left(\zeta-\frac{7}{12}\right)\lambda_x\right]^2+(\lambda_x)^2=1$$

or $$\left[2+8\left(\zeta-\frac{7}{12}\right)^2\right]\lambda_x^2=1 \qquad \text{(iii)}$$

$\mathbf{K}_1$: Substituting the value of ζ_1 into Eq. (iii):

$$\left[2+8\left(0.363383-\frac{7}{12}\right)^2\right](\lambda_x)_1^2=1$$

or $$(\lambda_x)_1=(\lambda_z)_1=0.647249$$

and then $$(\lambda_y)_1=2\sqrt{2}\left(0.363383-\frac{7}{12}\right)(0.647249)$$

$$=-0.402662$$

$$(\theta_x)_1=(\theta_z)_1=49.7° \quad (\theta_y)_1=113.7° \ ◀$$

$\mathbf{K}_3$: Substituting the value of ζ_3 into Eq. (iii):

$$\left[2+8\left(1.71995-\frac{7}{12}\right)^2\right](\lambda_x)_3^2=1$$

or $$(\lambda_x)_3=(\lambda_z)_3=0.284726$$

and then $$(\lambda_y)_3=2\sqrt{2}\left(1.71995-\frac{7}{12}\right)(0.284726)$$

$$=0.915348$$

$$(\theta_x)_3=(\theta_z)_3=73.5° \quad (\theta_y)_3=23.7° \ ◀$$

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PROBLEM 9.179* (Continued)

$\mathbf{K}_2$: For this case, the set of equations to be solved consists of Equations (9.54a), (9.54b), and (9.57). Now

$$-I_{xy}\lambda_x + (I_y - K)\lambda_y - I_{yz}\lambda_z = 0 \qquad (9.54b)$$

Substituting for the moments and products of inertia.

$$-\left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_x + \left(\frac{3}{2}ma^2 - K\right)\lambda_y - \left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_z = 0$$

or

$$\frac{1}{2\sqrt{2}}\lambda_x + \left(\frac{3}{2} - \xi\right)\lambda_y + \frac{1}{2\sqrt{2}}\lambda_z = 0 \qquad (iv)$$

Substituting the value of ξ_2 into Eqs. (i) and (iv):

$$\left(\frac{13}{12} - \frac{19}{12}\right)(\lambda_x)_2 + \frac{1}{2\sqrt{2}}(\lambda_y)_2 - \frac{1}{2}(\lambda_z)_2 = 0$$

$$\frac{1}{2\sqrt{2}}(\lambda_x)_2 + \left(\frac{3}{2} - \frac{19}{12}\right)(\lambda_y)_2 + \frac{1}{2\sqrt{2}}(\lambda_z)_2 = 0$$

or

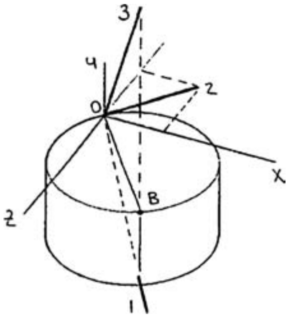
$$-(\lambda_x)_2 + \frac{1}{\sqrt{2}}(\lambda_y)_2 - (\lambda_z)_2 = 0$$

and

$$(\lambda_x)_2 - \frac{\sqrt{2}}{6}(\lambda_y)_2 + (\lambda_z)_2 = 0$$

Adding yields $\qquad (\lambda_y)_2 = 0$

and then $\qquad (\lambda_y)_2 = -(\lambda_x)_2$



Substituting into Equation (9.57)

$$(\lambda_x)_2^2 + (\lambda_y)_2^2 + (-\lambda_x)_2^2 = 1 \quad \text{(with } (\lambda_y)_2 \to 0\text{)}$$

or

$$(\lambda_x)_2 = \frac{1}{\sqrt{2}} \quad \text{and} \quad (\lambda_z)_2 = -\frac{1}{\sqrt{2}}$$

$$(\theta_x)_2 = 45.0° \quad (\theta_y)_2 = 90.0° \quad (\theta_z)_2 = 135.0° \blacktriangleleft$$

(*c*) Principal axes 1 and 3 lie in the vertical plane of symmetry passing through Points *O* and *B*. Principal axis 2 lies in the *xz* plane.

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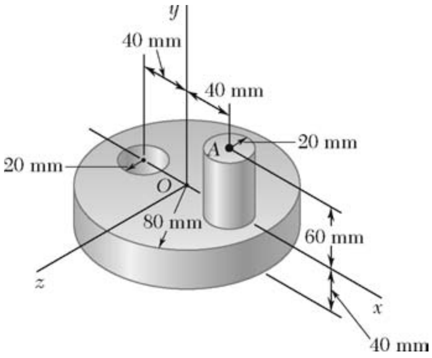

PROBLEM 9.180

For the component described in Problem 9.165, determine (*a*) the principal mass moments of inertia at the origin, (*b*) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the *x*, *y*, and *z* axes.

SOLUTION

(*a*) From the solutions to Problems 9.141 and 9.165 we have

Problem 9.141:
$$I_x = 13.98800\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$
$$I_y = 20.55783\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$
$$I_z = 14.30368\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

Problem 9.165:
$$I_{yz} = I_{zx} = 0$$
$$I_{xy} = 0.39460\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

Eq. (9.55) then becomes

$$\begin{vmatrix} I_x - K & -I_{xy} & 0 \\ -I_{xy} & I_y - K & 0 \\ 0 & 0 & I_z - K \end{vmatrix} = 0 \quad \text{or} \quad (I_x - K)(I_y - K)(I_z - K) - (I_z - K)I_{xy}^2 = 0$$

Thus
$$I_z - K = 0 \qquad I_x I_y - (I_x + I_y)K + K^2 - I_{xy}^2 = 0$$

Substituting:
$$K_1 = 14.30368\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $K_1 = 14.30\times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

and
$$(13.98800\times 10^{-3})(20.55783\times 10^{-3}) - (13.98800 + 20.55783)(10^{-3})K + K^2 - (0.39460\times 10^{-3})^2 = 0$$

or
$$K^2 - (34.54583\times 10^{-3})K + 287.4072\times 10^{-6} = 0$$

Solving yields
$$K_2 = 13.96438\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $K_2 = 13.96\times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

and $K_3 = 20.58145\times 10^{-3}\ \text{kg}\cdot\text{m}^2$ or $K_3 = 20.6\times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

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PROBLEM 9.180 (Continued)

(*b*) To determine the direction cosines $\lambda_x, \lambda_y, \lambda_z$ of each principal axis, use two of the equations of Eqs. (9.54) and Eq. (9.57). Then

K_1: Begin with Eqs. (9.54*a*) and (9.54*b*) with $I_{yz} = I_{zx} = 0$.

$$(I_x - K_1)(\lambda_x)_1 - I_{xy}(\lambda_y)_1 = 0$$
$$-I_{xy}(\lambda_x)_1 + (I_y - K_1)(\lambda_y)_1 = 0$$

Substituting:

$$[(13.98800 - 14.30368)\times 10^{-3}](\lambda_x)_1 - (0.39460\times 10^{-3})(\lambda_y)_1 = 0$$
$$-(0.39460\times 10^{-3})(\lambda_x)_1 + [(20.55783 - 14.30368)\times 10^{-3}](\lambda_y)_1 = 0$$

Adding yields $$(\lambda_x)_1 = (\lambda_y)_1 = 0$$

Then using Eq. (9.57) $$\cancelto{0}{(\lambda_x)_1^2} + \cancelto{0}{(\lambda_y)_1^2} + (\lambda_z)_1^2 = 1$$

or $$(\lambda_z)_1 = 1$$

$$(\theta_x)_1 = 90.0^\circ \quad (\theta_y)_1 = 90.0^\circ \quad (\theta_z)_1 = 0^\circ \blacktriangleleft$$

K_2: Begin with Eqs. (9.54*b*) and (9.54*c*) with $I_{yz} = I_{zx} = 0$.

$$-I_{xy}(\lambda_x)_2 + (I_y - K_2)(\lambda_y)_2 = 0 \qquad \text{(i)}$$
$$(I_z - K_2)(\lambda_z)_2 = 0$$

Now $$I_z \neq K_2 \Rightarrow (\lambda_z)_2 = 0$$

Substituting into Eq. (i):

$$-(0.39460\times 10^{-3})(\lambda_x)_z + [(20.55783 - 13.96438)\times 10^{-3}](\lambda_y)_2 = 0$$

or $$(\lambda_y)_2 = 0.059847(\lambda_x)_2$$

Using Eq. (9.57): $$(\lambda_x)_2^2 + [0.05984](\lambda_x)_2]^2 + \cancelto{0}{(\lambda_z)_2^2} = 1$$

or $$(\lambda_x)_2 = 0.998214$$

and $$(\lambda_y)_2 = 0.059740$$

$$(\theta_x)_2 = 3.4^\circ \quad (\theta_y)_2 = 86.6^\circ \quad (\theta_z)_2 = 90.0^\circ \blacktriangleleft$$

K_3: Begin with Eqs. (9.54*b*) and (9.54*c*) with $I_{yz} = I_{zx} = 0$.

$$-I_{xy}(\lambda_x)_3 + (I_y - K_3)(\lambda_y)_3 = 0 \qquad \text{(ii)}$$
$$(I_z - K_3)(\lambda_z)_3 = 0$$

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PROBLEM 9.180 (Continued)

Now $\qquad I_z \neq K_3 \Rightarrow (\lambda_z)_3 = 0$

Substituting into Eq. (ii):

$$-(0.39460\times 10^{-3})(\lambda_x)_3 + [(20.55783 - 20.58145)\times 10^{-3}](\lambda_y)_3 = 0$$

or $\qquad (\lambda_y)_3 = -16.70618(\lambda_x)_3$

Using Eq. (9.57): $\qquad (\lambda_x)_3^2 + [-16.70618(\lambda_x)_3]^2 + \cancel{(\lambda_z)_3^2}^{\,0} = 1$

or $\qquad (\lambda_x)_3 = -0.059751 \quad$ (axes right-handed set $\Rightarrow$"_")

and $\qquad (\lambda_y)_3 = 0.998211$

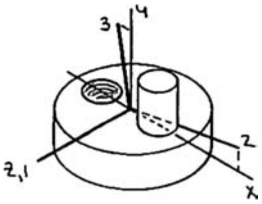
$$(\theta_x)_3 = 93.4^\circ \qquad (\theta_y)_3 = 3.43^\circ \qquad (\theta_z)_3 = 90.0^\circ \blacktriangleleft$$

Note: Since the principal axes are orthogonal and $(\theta_z)_2 = (\theta_z)_3 = 90^\circ$, it follows that

$$|(\lambda_x)_2| = |(\lambda_y)_3| \qquad |(\lambda_y)_2| = |(\lambda_z)_3|$$

The differences in the above values are due to round-off errors.

(*c*) Principal axis 1 coincides with the *z* axis, while principal axes 2 and 3 lie in the *xy* plane.



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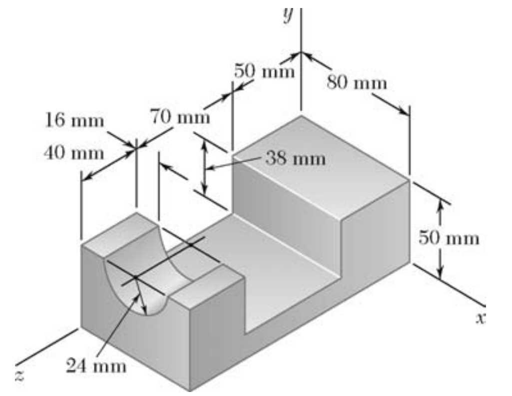

PROBLEM 9.181*

For the component described in Problems 9.145 and 9.149, determine (*a*) the principal mass moments of inertia at the origin, (*b*) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the *x*, *y*, and *z* axes.

SOLUTION

(*a*) From the solutions to Problems 9.145 and 9.149, we have

Problem 9.145: $I_x = 26.4325\times10^{-3}\ \text{kg}\cdot\text{m}^2$ Problem 9.149: $I_{xy} = 2.5002\times10^{-3}\ \text{kg}\cdot\text{m}^2$

$I_y = 31.1726\times10^{-3}\ \text{kg}\cdot\text{m}^2$ $I_{yz} = 4.0627\times10^{-3}\ \text{kg}\cdot\text{m}^2$

$I_z = 8.5773\times10^{-3}\ \text{kg}\cdot\text{m}^2$ $I_{zx} = 8.8062\times10^{-3}\ \text{kg}\cdot\text{m}^2$

Substituting into Eq. (9.56):

$$\begin{aligned}
&K^3 - [(26.4325 + 31.1726 + 8.5773)(10^{-3})]K^2\\
&\quad + [(26.4325)(31.1726) + (31.1726)(8.5773) + (8.5773)(26.4325)\\
&\quad - (2.5002)^2 - (4.0627)^2 - (8.8062)^2](10^{-6})K\\
&\quad - [(26.4325)(31.1726)(8.5773) - (26.4325)(4.0627)^2\\
&\quad - (31.1726)(8.8062)^2 - (8.5773)(2.5002)^2\\
&\quad - 2(2.5002)(4.0627)(8.8062)](10^{-9}) = 0
\end{aligned}$$

or $$K^3 - (66.1824\times10^{-3})K^2 + (1217.76\times10^{-6})K - (3981.23\times10^{-9}) = 0$$

Solving yields

$K_1 = 4.1443\times10^{-3}\ \text{kg}\cdot\text{m}^2$ or $K_1 = 4.14\times10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

$K_2 = 29.7840\times10^{-3}\ \text{kg}\cdot\text{m}^2$ or $K_2 = 29.8\times10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

$K_3 = 32.2541\times10^{-3}\ \text{kg}\cdot\text{m}^2$ or $K_3 = 32.3\times10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

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PROBLEM 9.181* (Continued)

(*b*) To determine the direction cosines $\lambda_x, \lambda_y, \lambda_z$ of each principal axis, use two of the equations of Eqs. (9.54) and Eq. (9.57). Then

$\underline{K}_1$: Begin with Eqs. (9.54*a*) and (9.54*b*).

$$(I_x - K_1)(\lambda_x)_1 - I_{xy}(\lambda_y)_1 - I_{zx}(\lambda_z)_1 = 0$$
$$I_{xy}(\lambda_x)_1 + (I_y - K_1)(\lambda_y)_1 - I_{yz}(\lambda_z)_1 = 0$$

Substituting:

$$[(26.4325 - 4.1443)(10^{-3})](\lambda_x)_1 - (2.5002\times10^{-3})(\lambda_y)_1 - (8.8062\times10^{-3})(\lambda_z)_1 = 0$$

$$-(2.5002\times10^{-3})(\lambda_x)_1 + [(31.1726 - 4.1443)(10^{-3})](\lambda_y)_1 - (4.0627\times10^{-3})(\lambda_z)_1 = 0$$

Simplifying

$$8.9146(\lambda_x)_1 - (\lambda_y)_1 - 3.5222(\lambda_z)_1 = 0$$
$$-0.0925(\lambda_x)_1 + (\lambda_y)_1 - 0.1503(\lambda_z)_1 = 0$$

Adding and solving for $(\lambda_z)_1$:

$$(\lambda_z)_1 = 2.4022(\lambda_x)_1$$

and then

$$(\lambda_y)_1 = [8.9146 - 3.5222(2.4022)](\lambda_x)_1$$
$$= 0.45357(\lambda_x)_1$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_1^2 + [0.45357(\lambda_x)_1]^2 + [2.4022(\lambda_x)_1]^2 = 1$$

or $(\lambda_x)_1 = 0.37861$

and $(\lambda_y)_1 = 0.17173$ $(\lambda_z)_1 = 0.90950$

$(\theta_x)_1 = 67.8°$ $(\theta_y)_1 = 80.1°$ $(\theta_z)_1 = 24.6°$ ◀

$\underline{K}_2$: Begin with Eqs. (9.54*a*) and (9.54*b*).

$$(I_x - K_2)(\lambda_x)_2 - I_{xy}(\lambda_y)_2 - I_{zx}(\lambda_z)_2 = 0$$
$$-I_{xy}(\lambda_x)_2 + (I_y - K_2)(\lambda_y)_2 - I_{yz}(\lambda_z)_2 = 0$$

Substituting:

$$[(26.4325 - 29.7840)(10^{-3})](\lambda_x)_2 - (2.5002\times10^{-3})(\lambda_y)_2 - (8.8062\times10^{-3})(\lambda_z)_2 = 0$$

$$-(2.5002\times10^{-3})(\lambda_x)_2 + [(31.1726 - 29.7840)(10^{-3})](\lambda_y)_2 - (4.0627\times10^{-3})(\lambda_z)_2 = 0$$

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PROBLEM 9.181* (Continued)

Simplifying

$$-1.3405(\lambda_x)_2 - (\lambda_y)_2 - 3.5222(\lambda_z)_2 = 0$$
$$-1.8005(\lambda_x)_2 + (\lambda_y)_2 - 2.9258(\lambda_z)_2 = 0$$

Adding and solving for $(\lambda_z)_2$:

$$(\lambda_z)_2 = -0.48713(\lambda_x)_2$$

and then

$$(\lambda_y)_2 = [-1.3405 - 3.5222(-0.48713)](\lambda_x)_2$$
$$= 0.37527(\lambda_x)_2$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_2^2 + [0.37527(\lambda_x)_2]^2 + [-0.48713(\lambda_x)_2]^2 = 1$$

or $(\lambda_x)_2 = 0.85184$

and $(\lambda_y)_2 = 0.31967$ $\quad (\lambda_z)_2 = -0.41496$

$$(\theta_x)_1 = 31.6° \quad (\theta_y)_2 = 71.4° \quad (\theta_z)_2 = 114.5° \blacktriangleleft$$

$\underline{K}_3$: Begin with Eqs. (9.54*a*) and (9.54*b*).

$$(I_x - K_3)(\lambda_x)_3 - I_{xy}(\lambda_y)_3 - I_{zx}(\lambda_z)_3 = 0$$
$$-I_{xy}(\lambda_x)_3 + (I_y - K_3)(\lambda_y)_3 - I_{yz}(\lambda_z)_3 = 0$$

Substituting:

$$[(26.4325 - 32.2541)(10^{-3})](\lambda_x)_3 - (2.5002\times10^{-3})(\lambda_y)_3 - (8.8062\times10^{-3})(\lambda_z)_3 = 0$$

$$-(2.5002\times10^{-3})(\lambda_x)_3 + [(31.1726 - 32.2541)(10^{-3})](\lambda_y)_3 - (4.0627\times10^{-3})(\lambda_z)_3 = 0$$

Simplifying

$$-2.3285(\lambda_x)_3 - (\lambda_y)_3 - 3.5222(\lambda_z)_3 = 0$$
$$2.3118(\lambda_x)_3 + (\lambda_y)_3 + 3.7565(\lambda_z)_3 = 0$$

Adding and solving for $(\lambda_z)_3$:

$$(\lambda_z)_3 = 0.071276(\lambda_x)_3$$

and then

$$(\lambda_y)_3 = [-2.3285 - 3.5222(0.071276)](\lambda_x)_3$$
$$= -2.5795(\lambda_x)_3$$

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PROBLEM 9.181* (Continued)

Now substitute into Eq. (9.57):

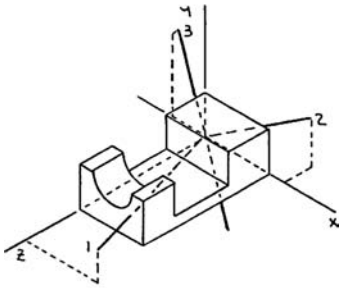
$$(\lambda_x)_3^2 + [-2.5795(\lambda_x)_3]^2 + [0.071276(\lambda_x)_3]^2 = 1 \qquad \text{(i)}$$

or $(\lambda_x)_3 = 0.36134$

and $(\lambda_y)_3 = 0.93208$ $\quad (\lambda_z)_3 = 0.025755$

$(\theta_x)_3 = 68.8° \quad (\theta_y)_3 = 158.8° \quad (\theta_z)_3 = 88.5°$ ◀

(*c*) *Note:* Principal axis 3 has been labeled so that the principal axes form a right-handed set. To obtain the direction cosines corresponding to the labeled axis, the negative root of Eq. (i) must be chosen; that is, $(\lambda_x)_3 = -0.36134$.



Then $(\theta_x)_3 = 111.2° \quad (\theta_y)_3 = 21.2° \quad (\theta_z)_3 = 91.5°$ ◀

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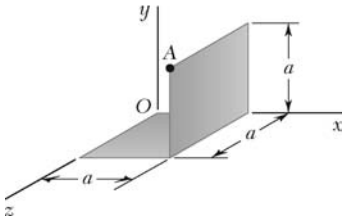

PROBLEM 9.182*

For the component described in Problem 9.167, determine (*a*) the principal mass moments of inertia at the origin, (*b*) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the *x*, *y*, and *z* axes.

SOLUTION

(*a*) From the solution of Problem 9.167, we have

$$I_x = \frac{1}{2}\frac{W}{g}a^2 \qquad I_{xy} = \frac{1}{4}\frac{W}{g}a^2$$

$$I_y = \frac{W}{g}a^2 \qquad I_{yz} = \frac{1}{8}\frac{W}{g}a^2$$

$$I_z = \frac{5}{6}\frac{W}{g}a^2 \qquad I_{zx} = \frac{3}{8}\frac{W}{g}a^2$$

Substituting into Eq. (9.56):

$$K^3 - \left[\left(\frac{1}{2}+1+\frac{5}{6}\right)\left(\frac{W}{g}a^2\right)\right]K^2$$
$$+\left[\left(\frac{1}{2}\right)(1)+(1)\left(\frac{5}{6}\right)+\left(\frac{5}{6}\right)\left(\frac{1}{2}\right)-\left(\frac{1}{4}\right)^2-\left(\frac{1}{8}\right)^2-\left(\frac{3}{8}\right)^2\right]\left(\frac{W}{g}a^2\right)^2 K$$
$$-\left[\left(\frac{1}{2}\right)(1)\left(\frac{5}{6}\right)-\left(\frac{1}{2}\right)\left(\frac{1}{8}\right)^2-(1)\left(\frac{3}{8}\right)^2-\left(\frac{5}{6}\right)\left(\frac{1}{4}\right)^2-2\left(\frac{1}{4}\right)\left(\frac{1}{8}\right)\left(\frac{3}{8}\right)\right]\left(\frac{W}{g}a^2\right)^3 = 0$$

Simplifying and letting $K = \frac{W}{g}a^2 K$ yields

$$K^3 - 2.33333K^2 + 1.53125K - 0.192708 = 0$$

Solving yields

$$K_1 = 0.163917 \quad K_2 = 1.05402 \quad K_3 = 1.11539$$

The principal moments of inertia are then $K_1 = 0.1639\frac{W}{g}a^2$ ◀

$K_2 = 1.054\frac{W}{g}a^2$ ◀

$K_3 = 1.115\frac{W}{g}a^2$ ◀

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PROBLEM 9.182* (Continued)

(*b*) To determine the direction cosines $\lambda_x,\ \lambda_y,\ \lambda_z$ of each principal axis, use two of the equations of Eqs. (9.54) and Eq. (9.57). Then

$\underline{K_1}$: Begin with Eqs. (9.54*a*) and (9.54*b*).

$$(I_x - K_1)(\lambda_x)_1 - I_{xy}(\lambda_y)_1 - I_{zx}(\lambda_z)_1 = 0$$
$$-I_{xy}(\lambda_x)_1 + (I_y - K_2)(\lambda_y)_1 - I_{yz}(\lambda_z)_1 = 0$$

Substituting

$$\left[\left(\frac{1}{2} - 0.163917\right)\left(\frac{W}{g}a^2\right)\right](\lambda_x)_1 - \left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_y)_1 - \left(\frac{3}{8}\frac{W}{g}a^2\right)(\lambda_z)_1 = 0$$

$$-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_x)_1 + \left[(1 - 0.163917)\left(\frac{W}{g}a^2\right)\right](\lambda_y)_1 - \left(\frac{1}{8}\frac{W}{g}a^2\right)(\lambda_z)_1 = 0$$

Simplifying

$$1.34433(\lambda_x)_1 - (\lambda_y)_1 - 1.5(\lambda_z)_1 = 0$$

$$-0.299013(\lambda_x)_1 + (\lambda_y)_1 - 0.149507(\lambda_z)_1 = 0$$

Adding and solving for $(\lambda_z)_1$:

$$(\lambda_z)_1 = 0.633715(\lambda_x)_1$$

and then

$$(\lambda_y)_1 = [1.34433 - 1.5(0.633715)](\lambda_x)_1$$
$$= 0.393758(\lambda_x)_1$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_1^2 + [0.393758(\lambda_x)_1]^2 + (0.633715(\lambda_x)_1]^2 = 1$$

or $$(\lambda_x)_1 = 0.801504$$

and $$(\lambda_y)_1 = 0.315599 \qquad (\lambda_z)_1 = 0.507925$$

$$(\theta_x)_1 = 36.7° \quad (\theta_y)_1 = 71.6° \quad (\theta_z)_1 = 59.5° \ ◀$$

$\underline{K_2}$: Begin with Eqs. (9.54*a*) and (9.54*b*):

$$(I_x - k_2)(\lambda_x)_2 - I_{xy}(\lambda_y)_2 - I_{zx}(\lambda_z)_2 = 0$$
$$-I_{xy}(\lambda_x)_2 + (I_y - k_2)(\lambda_y)_2 - I_{yz}(\lambda_z)_2 = 0$$

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PROBLEM 9.182* (Continued)

Substituting

$$\left[\left(\frac{1}{2}-1.05402\right)\left(\frac{W}{g}a^2\right)\right](\lambda_x)_2-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_y)_2-\left(\frac{3}{8}\frac{W}{g}a^2\right)(\lambda_z)_2=0$$

$$-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_x)_2+\left[(1-1.05402)\left(\frac{W}{g}a^2\right)\right](\lambda_y)_2-\left(\frac{1}{8}\frac{W}{g}a^2\right)(\lambda_z)_2=0$$

Simplifying

$$-2.21608(\lambda_x)_2-(\lambda_y)_2-1.5(\lambda_z)_2=0$$

$$4.62792(\lambda_x)_2+(\lambda_y)_2+2.31396(\lambda_z)_2=0$$

Adding and solving for $(\lambda_z)_2$

$$(\lambda_z)_2=-2.96309(\lambda_x)_2$$

and then

$$(\lambda_y)_2=[-2.21608-1.5(-2.96309)](\lambda_x)_2$$
$$=2.22856(\lambda_x)_2$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_2^2+[2.22856(\lambda_x)_2]^2+[-2.96309(\lambda_x)_2]^2=1$$

or

$$(\lambda_x)_2=0.260410$$

and

$$(\lambda_y)_2=0.580339 \qquad (\lambda_z)_2=-0.771618$$

$$(\theta_x)_2=74.9° \quad (\theta_y)_2=54.5° \quad (\theta_z)_2=140.5° \blacktriangleleft$$

$\underline{K_3}$: Begin with Eqs. (9.54*a*) and (9.54*b*):

$$(I_x-K_3)(\lambda_x)_3-I_{xy}(\lambda_y)_3-I_{zx}(\lambda_z)_3=0$$
$$-I_{xy}(\lambda_x)_3+(I_y-K_3)(\lambda_y)_3-I_{yz}(\lambda_z)_3=0$$

Substituting

$$\left[\left(\frac{1}{2}-1.11539\right)\left(\frac{W}{g}a^2\right)\right](\lambda_x)_3-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_y)_3-\left(\frac{3}{8}\frac{W}{g}a^2\right)(\lambda_z)_3=0$$

$$-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_x)_3+\left[(1-1.11539)\left(\frac{W}{g}a^2\right)\right](\lambda_y)_3-\left(\frac{1}{8}\frac{W}{g}a^2\right)(\lambda_z)_3=0$$

Simplifying

$$-2.46156(\lambda_x)_3-(\lambda_y)_3-1.5(\lambda_z)_3=0$$
$$2.16657(\lambda_x)_3+(\lambda_y)_3+1.08328(\lambda_z)_3=0$$

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PROBLEM 9.182* (Continued)

Adding and solving for $(\lambda_z)_3$

$$(\lambda_z)_3 = -0.707885(\lambda_x)_3$$

and then
$$(\lambda_y)_3 = [-2.46156 - 1.5(-0.707885)](\lambda_x)_3$$
$$= -1.39973(\lambda_x)_3$$

Now substitute into Eq. (9.57):

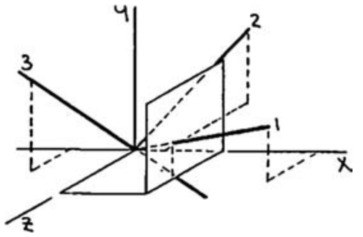
$$(\lambda_x)_3^2 + [-1.39973(\lambda_x)_3]^2 + [-0.707885(\lambda_x)_3]^2 = 1 \quad \text{(i)}$$

or
$$(\lambda_x)_3 = 0.537577$$

and
$$(\lambda_y)_3 = -0.752463 \qquad (\lambda_z)_3 = -0.380543$$

$$(\theta_x)_3 = 57.5° \quad (\theta_y)_3 = 138.8° \quad (\theta_z)_3 = 112.4° \blacktriangleleft$$

(*c*) *Note:* Principal axis 3 has been labeled so that the principal axes form a right-handed set. To obtain the direction cosines corresponding to the labeled axis, the negative root of Eq. (i) must be chosen; that is, $(\lambda_x)_3 = -0.537577$



Then
$$(\theta_x)_3 = 122.5° \quad (\theta_y)_3 = 41.2° \quad (\theta_z)_3 = 67.6° \blacktriangleleft$$

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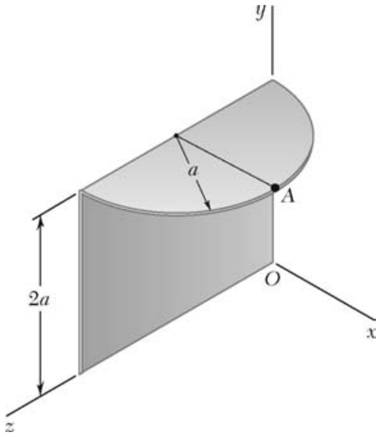

PROBLEM 9.183*

For the component described in Problem 9.168, determine (*a*) the principal mass moments of inertia at the origin, (*b*) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the *x*, *y*, and *z* axes.

SOLUTION

(*a*) From the solution to Problem 9.168, we have

$$I_x = 18.91335\frac{\gamma t}{g}a^4 \qquad I_{xy} = 1.33333\frac{\gamma t}{g}a^4$$

$$I_y = 7.68953\frac{\gamma t}{g}a^4 \qquad I_{yz} = 7.14159\frac{\gamma t}{g}a^4$$

$$I_z = 12.00922\frac{\gamma t}{g}a^4 \qquad I_{zx} = 0.66667\frac{\gamma t}{g}a^4$$

Substituting into Eq. (9.56):

$$K^3 - \left[(18.91335 + 7.68953 + 12.00922)\left(\frac{\gamma t}{g}a^4\right)\right]K^2$$
$$+ [(18.91335)(7.68953) + (7.68953)(12.00922) + (12.00922)(18.91335)$$
$$-(1.33333)^2 - (7.14159)^2 + (0.66667)^2]\left(\frac{\gamma t}{g}a^4\right)^2 K$$
$$-[(18.91335)(7.68953)(12.00922) - (18.91335)(7.14159)^2$$
$$-(7.68953)(0.66667)^2 - (12.00922)(1.33333)^2$$
$$-2(1.33333)(7.14159)(0.66667)]\left(\frac{\gamma t}{g}a^4\right)^3 = 0$$

Simplifying and letting

$$K = \frac{\gamma t}{g}a^4 K \quad \text{yields}$$

$$K^3 - 38.61210K^2 + 411.69009K - 744.47027 = 0$$

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PROBLEM 9.183* (Continued)

Solving yields

$$K_1 = 2.25890 \quad K_2 = 17.27274 \quad K_3 = 19.08046$$

The principal moments of inertia are then $K_1 = 2.26\dfrac{\gamma t}{g}a^4$ ◀

$$K_2 = 17.27\frac{\gamma t}{g}a^4 \blacktriangleleft$$

$$K_3 = 19.08\frac{\gamma t}{g}a^4 \blacktriangleleft$$

(*b*) To determine the direction cosines λ_x, λ_y, λ_z of each principal axis, use two of the equations of Eqs. (9.54) and Eq. (9.57). Then

$\underline{K_1}$: Begin with Eqs. (9.54*a*) and (9.54*b*):

$$(I_x - K_1)(\lambda_x)_1 - I_{xy}(\lambda_y)_1 - I_{zx}(\lambda_z)_1 = 0$$
$$-I_{xy}(\lambda_x)_1 + (I_y - K_2)(\lambda_y)_1 - I_{yz}(\lambda_z)_1 = 0$$

Substituting

$$\left[(18.91335 - 2.25890)\left(\frac{\gamma t}{g}a^4\right)\right](\lambda_x)_1 - \left(1.33333\frac{\gamma t}{a}a^4\right)(\lambda_y)_1 - \left(0.66667\frac{\gamma t}{g}a^4\right)(\lambda_z)_1 = 0$$

$$-\left(1.33333\frac{\gamma t}{g}a^4\right)(\lambda_x)_1 + \left[(7.68953 - 2.25890)\left(\frac{\gamma t}{g}a^4\right)\right](\lambda_y)_1 - \left(7.14159\frac{\gamma t}{g}a^4\right)(\lambda_z)_1 = 0$$

Simplifying

$$12.49087(\lambda_x)_1 - (\lambda_y)_1 - 0.5(\lambda_z)_1 = 0$$
$$-0.24552(\lambda_x)_1 + (\lambda_y)_1 - 1.31506(\lambda_z)_1 = 0$$

Adding and solving for $(\lambda_z)_1$

$$(\lambda_z)_1 = 6.74653(\lambda_x)_1$$

and then

$$(\lambda_y)_1 = [12.49087 - (0.5)(6.74653)](\lambda_x)_1$$
$$= 9.11761(\lambda_x)_1$$

Now substitute into Eq. (9.57): $(\lambda_x)_1^2 + [9.11761(\lambda_x)_1]^2 + [6.74653(\lambda_x)_1]^2 = 1$

or $(\lambda_x)_1 = 0.087825$

and $(\lambda_y)_1 = 0.80075 \qquad (\lambda_z)_1 = 0.59251$

$$(\theta_x)_1 = 85.0° \quad (\theta_y)_1 = 36.8° \quad (\theta_z)_1 = 53.7° \blacktriangleleft$$

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PROBLEM 9.183* (Continued)

$\underline{K}_2$: Begin with Eqs. (9.54*a*) and (9.54*b*):

$$(I_x - K_2)(\lambda_x)_2 - I_{xy}(\lambda_y)_2 - I_{zx}(\lambda_z)_2 = 0$$
$$-I_{xy}(\lambda_x)_2 + (I_y - K_2)(\lambda_y)_2 - I_{yz}(\lambda_z)_2 = 0$$

Substituting

$$[(18.91335 - 17.27274)\left(\frac{\gamma t}{g}a^4\right)(\lambda_x)_2 - \left(1.33333\frac{\gamma t}{g}a^4\right)(\lambda_y)_2 - \left(0.66667\frac{\gamma t}{g}a^4\right)(\lambda_z)_2 = 0$$
$$-\left(1.33333\frac{\gamma t}{g}a^4\right)(\lambda_x)_2 + \left[(7.68953 - 17.27274)\left(\frac{\gamma t}{g}a^4\right)\right](\lambda_y)_2 - \left(7.14159\frac{\gamma t}{g}a^4\right)(\lambda_z)_2 = 0$$

Simplifying

$$1.23046(\lambda_x)_2 - (\lambda_y)_2 - 0.5(\lambda_z)_2 = 0$$
$$0.13913(\lambda_x)_2 + (\lambda_y)_2 + 0.74522(\lambda_z)_2 = 0$$

Adding and solving for $(\lambda_z)_2$

$$(\lambda_z)_2 = -5.58515(\lambda_x)_2$$

and then
$$(\lambda_y)_2 = [1.23046 - (0.5)(-5.58515)](\lambda_x)_2$$
$$= 4.02304\,(\lambda_x)_2$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_2^2 + [4.02304(\lambda_x)_2]^2 + [-5.58515(\lambda_x)_2]^2 = 1$$

or
$$(\lambda_x)_2 = 0.14377$$

and
$$(\lambda_y)_2 = 0.57839 \qquad (\lambda_z)_2 = -0.80298$$

$$(\theta_x)_2 = 81.7° \quad (\theta_y)_2 = 54.7° \quad (\theta_z)_2 = 143.4° \blacktriangleleft$$

$\underline{K}_3$: Begin with Eqs. (9.54*a*) and (9.54*b*):

$$(I_x - K_3)(\lambda_x)_3 - I_{xy}(\lambda_y)_3 - I_{zx}(\lambda_z)_3 = 0$$
$$-I_{xy}(\lambda_x)_3 + (I_y - K_3)(\lambda_y)_3 - I_{yz}(\lambda_y)_3 = 0$$

Substituting

$$[(18.91335 - 19.08046)\left(\frac{\gamma t}{g}a^4\right)(\lambda_x)_3 - \left(1.33333\frac{\gamma t}{g}a^4\right)(\lambda_y)_3 - \left(0.66667\frac{\gamma t}{g}a^4\right)(\lambda_z)_3 = 0$$
$$-\left(1.33333\frac{\gamma t}{g}a^4\right)(\lambda_x)_3 + \left[(7.68953 - 19.08046)\left(\frac{\gamma t}{g}a^4\right)\right](\lambda_y)_3 - \left(7.14159\frac{\gamma t}{g}a^4\right)(\lambda_z)_3 = 0$$

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PROBLEM 9.183* (Continued)

Simplifying

$$-0.12533(\lambda_x)_3 - (\lambda_y)_3 - 0.5(\lambda_z)_3 = 0$$
$$0.11705(\lambda_x)_3 + (\lambda_y)_3 + 0.62695(\lambda_z)_3 = 0$$

Adding and solving for $(\lambda_z)_3$

$$(\lambda_z)_3 = 0.06522(\lambda_x)_3$$

and then

$$(\lambda_y)_3 = [-0.12533 - (0.5)(0.06522)](\lambda_x)_3$$
$$= -0.15794(\lambda_x)_3$$

Now substitute into Eq. (9.57):

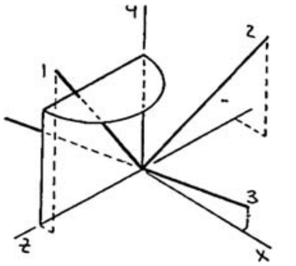
$$(\lambda_x)_3^2 + [-0.15794(\lambda_x)_3]^2 + [0.06522(\lambda_x]_3]^2 = 1 \quad \text{(i)}$$

or $(\lambda_x)_3 = 0.98571$

and $(\lambda_y)_3 = -0.15568 \qquad (\lambda_z)_3 = 0.06429$

$(\theta_x)_3 = 9.7° \quad (\theta_y)_3 = 99.0° \quad (\theta_z)_3 = 86.3°$ ◀

(*c*) *Note:* Principal axis 3 has been labeled so that the principal axes form a right-handed set. To obtain the direction cosines corresponding to the labeled axis, the negative root of Eq. (i) must be chosen; that is, $(\lambda_x)_3 = -0.98571$.



Then $(\theta_x)_3 = 170.3° \quad (\theta_y)_3 = 81.0° \quad (\theta_z)_3 = 93.7°$ ◀

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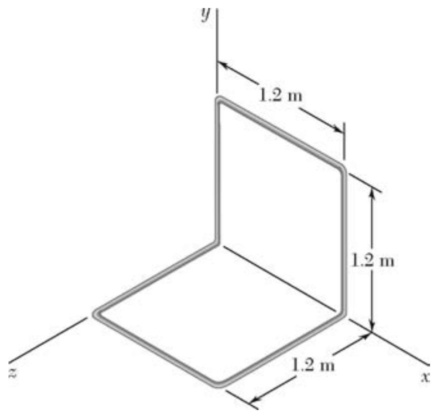

PROBLEM 9.184*

For the component described in Problems 9.148 and 9.170, determine (*a*) the principal mass moments of inertia at the origin, (*b*) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the *x*, *y*, and *z* axes.

SOLUTION

(*a*) From the solutions to Problems 9.148 and 9.170. We have

$$I_x = 0.32258 \text{ kg}\cdot\text{m}^2 \qquad I_y = I_z = 0.41933 \text{ kg}\cdot\text{m}^2$$

$$I_{xy} = I_{zx} = 0.096768 \text{ kg}\cdot\text{m}^2 \qquad I_{yz} = 0$$

Substituting into Eq. (9.56) and using

$$I_y = I_z \qquad I_{xy} = I_{zx} \qquad I_{yz} = 0$$

$$\begin{aligned} &K^3 - [0.32258 + 2(0.41933)]K^2 \\ &\quad + [2(0.32258)(0.41933) + (0.41933)^2 - 2(0.096768)^2]K \\ &\quad - [(0.32258)(0.41933)^2 - 2(0.41933)(0.096768)^2] = 0 \end{aligned}$$

Simplifying

$$K^3 - 1.16124K^2 + 0.42764K - 0.048869 = 0$$

Solving yields

$$K_1 = 0.22583 \text{ kg}\cdot\text{m}^2 \qquad \text{or} \qquad K_1 = 0.226 \text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

$$K_2 = 0.41920 \text{ kg}\cdot\text{m}^2 \qquad \text{or} \qquad K_2 = 0.419 \text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

$$K_3 = 0.51621 \text{ kg}\cdot\text{m}^2 \qquad \text{or} \qquad K_3 = 0.516 \text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

(*b*) To determine the direction cosines $\lambda_x, \lambda_y, \lambda_z$ of each principal axis, use two of the equations of Eqs. (9.54) and (9.57). Then

$\underline{K_1}$: Begin with Eqs. (9.54*b*) and (9.54*c*):

$$-I_{xy}(\lambda_x)_1 + (I_y - K_1)(\lambda_y)_1 - \cancelto{0}{I_{yz}}(\lambda_z)_3 = 0$$

$$-I_{zx}(\lambda_x)_1 - \cancelto{0}{I_{yz}}(\lambda_y)_1 + (I_z - K_1)(\lambda_z)_3 = 0$$

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PROBLEM 9.184* (Continued)

Substituting

$$-(0.096768)(\lambda_x)_1 + (0.41933 - 0.22583)(\lambda_y)_1 = 0$$

$$-(0.096768)(\lambda_x)_1 + (0.41933 - 0.22583)(\lambda_z)_1 = 0$$

Simplifying yields

$$(\lambda_y)_1 = (\lambda_z)_1 = 0.50009(\lambda_x)_1$$

Now substitute into Eq. (9.54):

$$(\lambda_x)^2 + 2[0.50009(\lambda_x)_1]^2 = 1$$

or $$(\lambda_x)_1 = 0.81645$$

and $$(\lambda_y)_1 = (\lambda_z)_1 = 0.40830$$

$$(\theta_x)_1 = 35.3° \quad (\theta_y)_1 = (\theta_z)_1 = 65.9° \blacktriangleleft$$

$\underline{K_2}$: Begin with Eqs. (9.54*a*) and (9.54*b*)

$$(I_x - K_2)(\lambda_x)_2 - I_{xy}(\lambda_y)_2 - I_{zx}(\lambda_z)_2 = 0$$

$$-I_{xy}(\lambda_x)_2 + (I_y - K_2)(\lambda_y)_2 - \overset{0}{\cancel{I_{yz}}}(\lambda_z)_2 = 0$$

Substituting

$$(0.32258 - 0.41920)(\lambda_x)_2 - (0.096768)(\lambda_y)_2 - (0.096768)(\lambda_z)_2 = 0 \qquad \text{(i)}$$

$$-(0.96768)(\lambda_x)_2 + (0.41933 - 0.41920)(\lambda_y)_2 = 0 \qquad \text{(ii)}$$

Eq. (ii) $\Rightarrow$ $(\lambda_x)_2 = 0$

and then Eq. (i) $\Rightarrow$ $(\lambda_z)_2 = -(\lambda_y)_2$

Now substitute into Eq. (9.57):

$$\overset{0}{\cancel{(\lambda_x)_2^2}} + (\lambda_y)_2^2 + [-(\lambda_y)_2]^2 = 1$$

or $$(\lambda_y)_2 = \frac{1}{\sqrt{2}}$$

and $$(\lambda_z)_2 = -\frac{1}{\sqrt{2}}$$

$$(\theta_x)_2 = 90° \quad (\theta_y)_2 = 45° \quad (\theta_z)_2 = 135° \blacktriangleleft$$

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PROBLEM 9.184* (Continued)

$\underline{K_3}$: Begin with Eqs. (9.54*b*) and (9.54*c*):

$$I_{xy}(\lambda_x)_3 + (I_y - K_3)(\lambda_y)_3 + \cancelto{0}{I_{yz}}(\lambda_z)_3 = 0$$

$$-I_{zx}(\lambda_x)_3 - \cancelto{0}{I_{yz}}(\lambda_y)_3 + (I_z - K_3)(\lambda_z)_3 = 0$$

Substituting

$$-(0.096768)(\lambda_x)_3 + (0.41933 - 0.51621)(\lambda_y)_3 = 0$$

$$-(0.096768)(\lambda_x)_3 + (0.41933 - 0.51621)(\lambda_z)_3 = 0$$

Simplifying yields

$$(\lambda_y)_3 = (\lambda_z)_3 = -(\lambda_x)_3$$

Now substitute into Eq. (9.57):

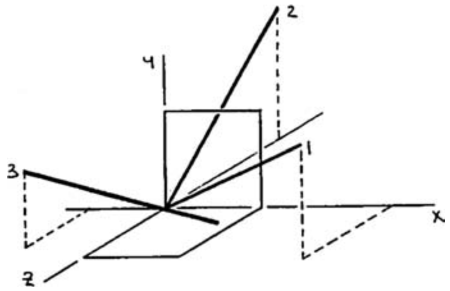
$$(\lambda_x)_3^2 + 2[-(\lambda_x)_3]^2 = 1 \qquad \text{(i)}$$

or $$(\lambda_x)_3 = \frac{1}{\sqrt{3}}$$

and $$(\lambda_y)_3 = (\lambda_z) = -\frac{1}{\sqrt{3}}$$

$$(\theta_x)_3 = 54.7° \quad (\theta_y)_3 = (\theta_z)_3 = 125.3° \blacktriangleleft$$

(*c*)



Note: Principal axis 3 has been labeled so that the principal axes form a right-handed set. To obtain the direction cosines corresponding to the labeled axis, the negative root of Eq. (i) must be chosen;

That is, $$(\lambda_x)_3 = -\frac{1}{\sqrt{3}}$$

Then $$(\theta_x)_3 = 125.3° \qquad (\theta_y)_3 = (\theta_z)_3 = 54.7° \blacktriangleleft$$

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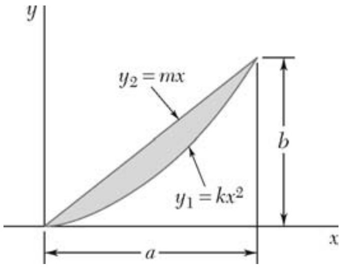
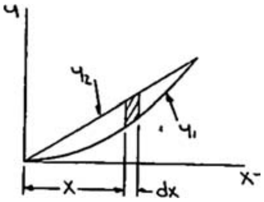


PROBLEM 9.185

Determine by direct integration the moments of inertia of the shaded area with respect to the *x* and *y* axes.

SOLUTION



At $\qquad x_1 = a, \quad y_1 = y_2 = b$

$$y_1: \quad b = ka^2 \quad \text{or} \quad k = \frac{b}{a^2}$$

$$y_2: \quad b = ma \quad \text{or} \quad m = \frac{b}{a}$$

Then $\qquad y_1 = \frac{b}{a^2}x^2$

$$y_2 = \frac{b}{a}x$$

Now $\qquad dI_x = \frac{1}{3}\left(y_2^3 - y_1^3\right)dx$

$$= \frac{1}{3}\left(\frac{b^3}{a^3}x^3 - \frac{b^3}{a^6}x^6\right)dx$$

Then $\qquad I_x = \int dI_x = \int_0^a \frac{b^3}{3}\left(\frac{1}{a^3}x^3 - \frac{1}{a^6}x^6\right)dx$

$$= \frac{b^3}{3}\left[\frac{1}{4a^3}x^4 - \frac{1}{7a^6}x^7\right]_0^a$$

or $\quad I_x = \frac{1}{28}ab^3$ ◀

Also $\qquad dI_y = x^2 dA = x^2[(y_2 - y_1)dx = x^2\left[\left(\frac{b}{a}x - \frac{b}{a^2}x^2\right)dx\right]$

Then $\qquad I_y = \int dI_y = \int_0^a b\left(\frac{1}{a}x^3 - \frac{1}{a^2}x^4\right)dx$

$$= b\left[\frac{1}{4a}x^4 - \frac{1}{5a^2}x^5\right]_0^a$$

or $\quad I_y = \frac{1}{20}a^3b$ ◀

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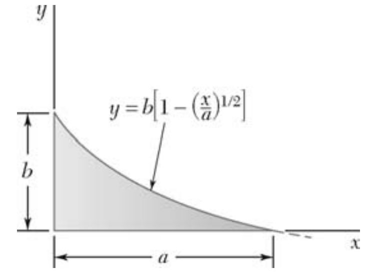
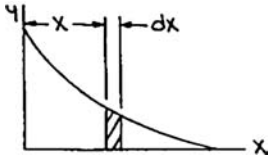


PROBLEM 9.186

Determine the moments of inertia and the radii of gyration of the shaded area shown with respect to the *x* and *y* axes.

SOLUTION

We have $dA = y\,dx = b\left[1-\left(\frac{x}{a}\right)^{1/2}\right]dx$



Then $A = \int dA = \int_0^a b\left[1-\left(\frac{x}{a}\right)^{1/2}\right]dx$

$$= b\left[x - \frac{2}{3\sqrt{a}}x^{3/2}\right]_0^a = \frac{1}{3}ab$$

Now $dI_x = \frac{1}{3}y^3 dx = \frac{1}{3}\left\{b\left[1-\left(\frac{x}{a}\right)^{1/2}\right]\right\}^3 dx$

$$= \frac{b^3}{3}\left[1-3\left(\frac{x}{a}\right)^{1/2}+3\left(\frac{x}{a}\right)-\left(\frac{x}{a}\right)^{3/2}\right]dx$$

Then $I_x = \int dI_x = \int_0^a \frac{b^3}{3}\left[1-3\left(\frac{x}{a}\right)^{1/2}+3\left(\frac{x}{a}\right)-\left(\frac{x}{a}\right)^{3/2}\right]dx$

$$= \frac{b^3}{3}\left[x - \frac{2}{\sqrt{a}}x^{3/2} + \frac{3}{2a}x^2 + \frac{2}{5a^{3/2}}x^{5/2}\right]_0^a$$

or $I_x = \frac{1}{30}ab^3$ ◀

and $k_x^2 = \frac{I_x}{A} = \frac{\frac{1}{30}ab^3}{\frac{1}{3}ab}$

or $k_x = \frac{b}{\sqrt{10}}$ ◀

Also $dI_y = x^2 dA = x^2(y\,dx) = x^2\left\{b\left[1-\left(\frac{x}{a}\right)^{1/2}\right]dx\right\}$

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PROBLEM 9.186 (Continued)

Then $$I_y = \int dI_y = \int_0^a b\left(x^2 - \frac{1}{\sqrt{a}}x^{5/2}\right)dx = b\left[\frac{1}{3}x^3 - \frac{2}{7\sqrt{a}}x^{7/2}\right]_0^a$$

or $$I_y = \frac{1}{21}a^3b \blacktriangleleft$$

and $$k_y^2 = \frac{I_y}{A} = \frac{\frac{1}{21}a^3b}{\frac{1}{3}ab}$$

or $$k_y = \frac{a}{\sqrt{7}} \blacktriangleleft$$

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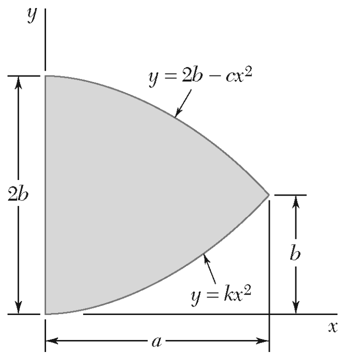
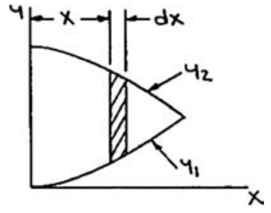


PROBLEM 9.187

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the y axis.

SOLUTION



At $x = a, \quad y_1 = y_2 = b$: $\quad y_1: \quad b = ka^2 \quad \text{or} \quad k = \dfrac{b}{a^2}$

$$y_2: \quad b = 2b - ca^2 \quad \text{or} \quad c = \frac{b}{a^2}$$

Then

$$y_1 = \frac{b}{a^2}x^2$$

$$y_2 = b\left(2 - \frac{x^2}{a^2}\right)$$

Now

$$dA = (y_2 - y_1)\,dx$$

$$= \left[b\left(2 - \frac{x^2}{a^2}\right) - \frac{b}{a^2}x^2\right]dx$$

$$= \frac{2b}{a^2}(a^2 - x^2)\,dx$$

Then

$$A = \int dA = \int_0^a \frac{2b}{a^2}(a^2 - x^2)dx$$

$$= \frac{2b}{a^2}\left[a^2x - \frac{1}{3}x^3\right]_0^a$$

$$= \frac{4}{3}ab$$

Now

$$dI_y = x^2 dA = x^2\left[\frac{2b}{a^2}(a^2 - x^2)\,dx\right]$$

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PROBLEM 9.187 (Continued)

Then $$I_y = \int dI_y = \int_0^a \frac{2b}{a^2} x^2 (a^2 - x^2) dx$$

$$= \frac{2b}{a^2} \left[\frac{1}{3} a^2 x^3 - \frac{1}{5} x^5 \right]_0^a$$

or $$I_y = \frac{4}{15} a^3 b \blacktriangleleft$$

and $$k_y^2 = \frac{I_y}{A} = \frac{\frac{4}{15} a^3 b}{\frac{4}{3} ab} = \frac{1}{5} a^2$$

or $$k_y = \frac{a}{\sqrt{5}} \blacktriangleleft$$

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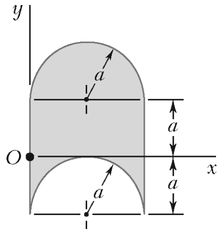
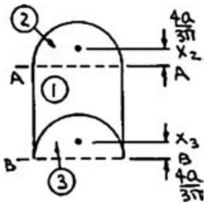


PROBLEM 9.188

Determine the moments of inertia of the shaded area shown with respect to the x and y axes.

SOLUTION

We have $$I_x = (I_x)_1 + (I_x)_2 - (I_x)_3$$



where $$(I_x)_1 = \frac{1}{12}(2a)(2a)^3 = \frac{4}{3}a^4$$

Now $$(I_{AA})_2 = (I_{BB})_3 = \frac{\pi}{8}a^4$$

and $$(I_{AA})_2 = (\bar{I}_{x_2})_2 + Ad^2$$

or $$(\bar{I}_{x_2})_2 = (\bar{I}_{x_3})_3 = \frac{\pi}{8}a^4 - \left(\frac{\pi}{2}a^2\right)\left(\frac{4a}{3\pi}\right)^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)a_1^4$$

Then $$(I_x)_2 = (\bar{I}_{x_2})_2 + Ad_2^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)a^4 + \left(\frac{\pi}{2}a^2\right)\left(a + \frac{4a}{3\pi}\right)^2$$

$$= \left(\frac{4}{3} + \frac{5\pi}{8}\right)a^4$$

$$(I_x)_3 = (\bar{I}_{x_3})_3 + Ad_3^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)a^4 + \left(\frac{\pi}{2}a^2\right)\left(a - \frac{4a}{3\pi}\right)^2$$

$$= \left(-\frac{4}{3} + \frac{5\pi}{8}\right)a^4$$

Finally $$I_x = \frac{4}{3}a^4 + \left[\left(\frac{4}{3} + \frac{5\pi}{8}\right)a^4\right] - \left[\left(-\frac{4}{3} + \frac{5\pi}{8}\right)a^4\right]$$

$$I_x = 4a^4 \blacktriangleleft$$

Also $$I_y = (I_y)_1 + (I_y)_2 - (I_y)_3$$

Where $$(I_y)_1 = \frac{1}{12}(2a)(2a)^3 + (2a)^2(a)^2 = \frac{16}{3}a^4$$

$$(I_y)_2 = (I_y)_3 \left[= \frac{\pi}{8}a^4 + \left(\frac{\pi}{2}a^2\right)(a)^2\right]$$

$$I_y = \frac{16}{3}a^4 \blacktriangleleft$$

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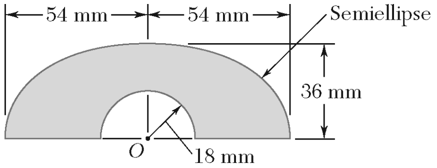
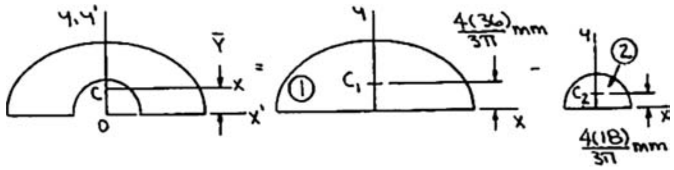


PROBLEM 9.189

Determine the polar moment of inertia of the area shown with respect to (*a*) Point *O*, (*b*) the centroid of the area.

SOLUTION

First locate centroid *C* of the area.



	A, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$\frac{\pi}{2}(54)(36) = 3053.6$	$\frac{48}{\pi} = 15.2789$	46,656
2	$-\frac{\pi}{2}(18)^2 = -508.9$	$\frac{24}{\pi} = 7.6394$	−3888
Σ	2544.7		42,768

Then $\bar{Y}\Sigma A = \Sigma\bar{y}A$: $\bar{Y}(2544.7\text{ mm}^2) = 42,768\text{ mm}^3$

or $\bar{Y} = 16.8067\text{ mm}$ ◁

(*a*)

$$J_O = (J_O)_1 - (J_O)_2$$
$$= \frac{\pi}{8}(54\text{ mm})(36\text{ mm})[(54\text{ mm})^2 + (36\text{ mm})^2] - \frac{\pi}{4}(18\text{ mm})^4$$
$$= (3.2155\times10^6 - 0.0824\times10^6)\text{ mm}^4$$
$$= 3.1331\times10^6\text{ mm}^4$$

or $J_O = 3.13\times10^6\text{ mm}^4$ ◀

(*b*)

$$J_O = \bar{J}_C + A(\bar{Y})^2$$

or

$$\bar{J}_C = 3.1331\times10^6\text{ mm}^4 - (2544.7\text{ mm}^2)(16.8067\text{ mm})^2$$

or $\bar{J}_C = 2.41\times10^6\text{ mm}^4$ ◀

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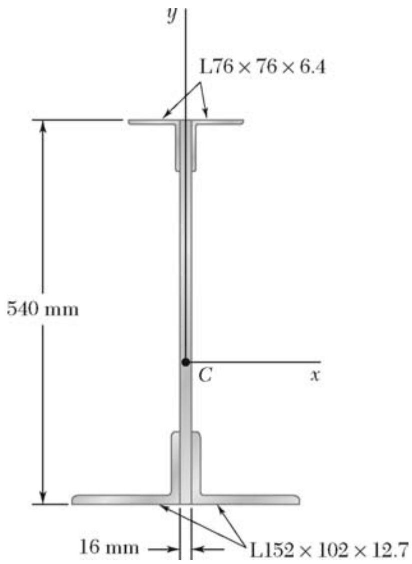
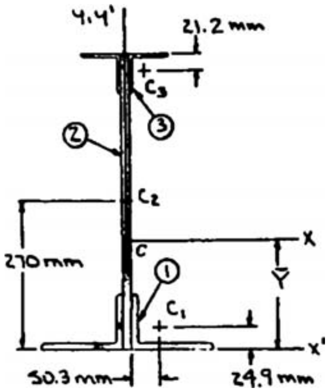


PROBLEM 9.190

To form an unsymmetrical girder, two L76 × 76 × 6.4-mm angles and two L152 × 102 × 12.7-mm angles are welded to a 16-mm steel plate as shown. Determine the moments of inertia of the combined section with respect to its centroidal *x* and *y* axes.

SOLUTION



L76×76×6.4: $A = 929\text{ mm}^2$

$$\bar{I}_x \times \bar{I}_y = 0.512\times10^6\text{ mm}^4$$

L152×102×12.7: $A = 3060\text{ mm}^2$

$$\bar{I}_x = 2.59\times10^6\text{ mm}^4$$

$$\bar{I}_y = 7.20\times10^6\text{ mm}^4$$

First locate centroid *C* of the section:

	A, mm^2	$\bar{y}$	$\bar{y}A$, mm^3
1	2(3060) = 6120	24.9	152,388
2	(16)(540) = 8640	270	2,332,800
3	2(929) = 1858	518.8	963,930
Σ	16,618		3,449,118

Then $$\bar{Y}\Sigma A = \Sigma \bar{y} A$$

or $$\bar{Y}(16,618\text{ mm}^2) = 3,449,118$$

or $$\bar{Y} = 207.553\text{ mm}$$

Now $$\bar{I}_x = 2(I_x)_1 + (I_x)_2 + 2(I_x)_3$$

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PROBLEM 9.190 (Continued)

where $(I_x)_1 = \bar{I}_x + Ad^2 = 2.59\times 10^6 \text{ mm}^4 + (3060 \text{ mm}^2)[(207.553 - 24.9) \text{ mm}]^2$

$= 104.678\times 10^6 \text{ mm}^4$

$$(I_x)_2 = \bar{I}_x + Ad^2 = \frac{1}{12}(16 \text{ mm})(540 \text{ mm})^3 + (16 \text{ mm})(540 \text{ mm})[(270 - 207.553) \text{ mm}]^2$$

$= 243.645\times 10^6 \text{ mm}^4$

$$(I_x)_3 = \bar{I}_x + Ad^2 = 0.512\times 10^6 \text{ mm}^4 + (929 \text{ mm}^2[(518.8 - 207.553) \text{ mm}]^2$$

$= 90.5086\times 10^6 \text{ mm}^4$

Then $\bar{I}_x = [2(104.678) + 243.645 + 2(90.5086)]\times 10^6 \text{ mm}^4$

or $\bar{I}_x = 634\times 10^6 \text{ mm}^4$ ◀

Also $\bar{I}_y = 2(I_y)_1 + (\bar{I}_y)_2 + 2(I_y)_3$

where $(I_y)_1 = \bar{I}_y + Ad^2 = 7.20\times 10^6 \text{ mm}^4 + (3060 \text{ mm}^2)[(8 + 50.3) \text{ mm}]^2$

$= 17.6006\times 10^6 \text{ mm}^4$

$$(\bar{I}_y)_2 = \frac{1}{12}(540 \text{ mm})(16 \text{ mm})^3$$

$= 0.1843\times 10^6 \text{ mm}^4$

$(I_y)_3 = \bar{I}_y + Ad^2 = 0.512\times 10^6 \text{ mm}^4 + (929 \text{ mm}^2)[(8 + 21.2) \text{ mm}]^2$

$= 1.3041\times 10^6 \text{ mm}^4$

Then $\bar{I}_y = [2(17.6006) + 0.1843 + 2(1.3041)]\times 10^6 \text{ mm}^4$

or $\bar{I}_y = 38.0\times 10^6 \text{ mm}^4$ ◀

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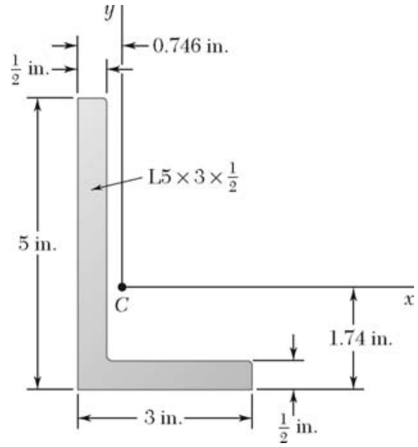
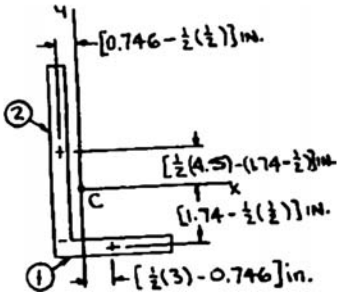


PROBLEM 9.191

Using the parallel-axis theorem, determine the product of inertia of the L5 × 3 × $\frac{1}{2}$-in. angle cross section shown with respect to the centroidal x and y axes.

SOLUTION



We have

$$\bar{I}_{xy} = (I_{xy})_1 + (I_{xy})_2$$

For each rectangle:

$$I_{xy} = \bar{I}_{x'y'} + \bar{x}\bar{y}A$$

and $\quad \bar{I}_{x'y'} = 0 \text{ (symmetry)}$

Thus $\quad I_{xy} = \Sigma \bar{x}\bar{y}A$

	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}\bar{y}A$, in.^{4}
1	$3\times\frac{1}{2}=1.5$	0.754	−1.49	−1.68519
2	$4.5\times\frac{1}{2}=2.25$	−0.496	1.01	−1.12716
Σ				−2.81235

$\bar{I}_{xy} = -2.81 \text{ in.}^4$ ◀

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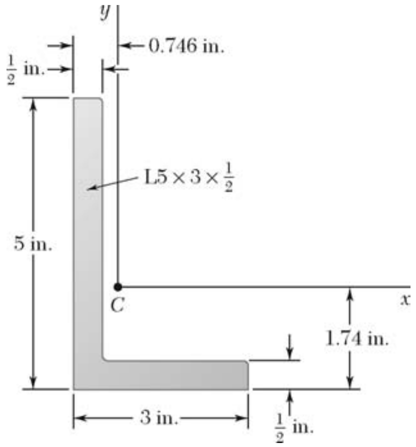
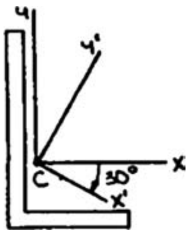


PROBLEM 9.192

For the L5 × 3 × $\frac{1}{2}$-in. angle cross section shown, use Mohr's circle to determine (*a*) the moments of inertia and the product of inertia with respect to new centroidal axes obtained by rotating the *x* and *y* axes 30° clockwise, (*b*) the orientation of the principal axes through the centroid and the corresponding values of the moments of inertia.

SOLUTION

From Figure 9.13*a*:



$$\bar{I}_x = 9.43 \text{ in.}^4 \qquad \bar{I}_y = 2.55 \text{ in.}^4$$

From the solution to Problem 9.191

$$\bar{I}_{xy} = -2.81235 \text{ in.}^4$$

The Mohr's circle is defined by the diameter *XY*, where

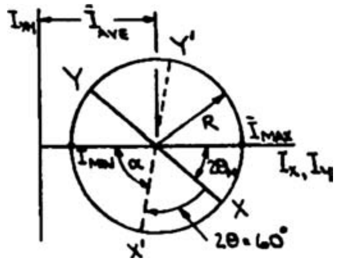
$$X(9.43 - 2.81235), \qquad Y(2.55, 2.81235)$$

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(9.43 + 2.55) = 5.99 \text{ in.}^4$$

and $$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2} = \sqrt{\left[\frac{1}{2}(9.43 - 2.55)\right]^2 + (-2.81235)^2}$$

$$= 4.4433 \text{ in.}^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(-2.81235)}{9.43 - 2.55}$$

$$= 0.81754$$

or $$2\theta_m = 39.267°$$

and $$\theta_m = 19.6335°$$

Then $$\alpha = 180° - (39.267° + 60°)$$

$$= 80.733°$$

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PROBLEM 9.192 (Continued)

(*a*) We have $\bar{I}_{x'} = \bar{I}_{ave} - R\cos\alpha = 5.99 - 4.4433\cos 80.733°$

or $\bar{I}_{x'} = 5.27 \text{ in.}^4$ ◀

$\bar{I}_{y'} = \bar{I}_{ave} + R\cos\alpha = 5.99 + 4.4433\cos 80.733°$

or $\bar{I}_{y'} = 6.71 \text{ in.}^4$ ◀

$\bar{I}_{x'y'} = -R\sin\alpha = -4.4433\sin 80.733°$

or $\bar{I}_{x'y'} = -4.39 \text{ in.}^4$ ◀

(*b*) First observe that the principal axes are obtained by rotating the *xy* axes through

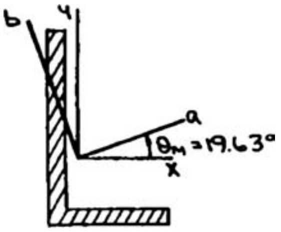
19.63° counterclockwise ◀

about *C*.

Now $\bar{I}_{max,\,min} = \bar{I}_{ave} \pm R = 5.99 \pm 4.4433$

or $\bar{I}_{max} = 10.43 \text{ in.}^4$ ◀

$\bar{I}_{min} = 1.547 \text{ in.}^4$ ◀



From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{max}$ and the *b* axis corresponds to $\bar{I}_{min}$.

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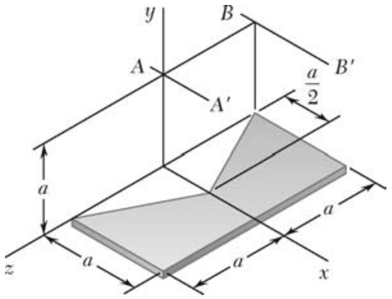
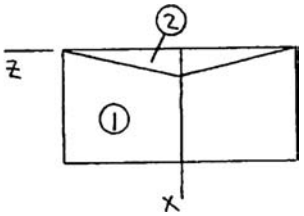


PROBLEM 9.193

A piece of thin, uniform sheet metal is cut to form the machine component shown. Denoting the mass of the component by m, determine its mass moment of inertia with respect to (*a*) the x axis, (*b*) the y axis.

SOLUTION



First note $\quad \text{mass} = m = \rho V = \rho t A$

$$= \rho t\left[(2a)(a) - \frac{1}{2}(2a)\left(\frac{a}{2}\right)\right]$$

$$= \frac{3}{2}\rho t a^2$$

Also $\quad I_{\text{mass}} = \rho t I_{\text{area}}$

$$= \frac{2m}{3a^2} I_{\text{area}}$$

(*a*) Now $\quad \bar{I}_{x,\text{area}} = (I_x)_{1,\text{area}} - 2(I_x)_{2,\text{area}}$

$$= \frac{1}{12}(a)(2a)^3 - 2\left[\frac{1}{12}\left(\frac{a}{2}\right)(a)^3\right]$$

$$= \frac{7}{12}a^4$$

Then $\quad \bar{I}_{x,\text{mass}} = \frac{2m}{3a^2} \times \frac{7}{12}a^4$

or $\quad \bar{I}_x = \frac{7}{18}ma^2$ ◀

(*b*) We have $\quad \bar{I}_{z,\text{area}} = (I_z)_{1,\text{area}} - 2(I_z)_{2,\text{area}}$

$$= \frac{1}{3}(2a)(a)^3 - 2\left[\frac{1}{12}(a)\left(\frac{a}{2}\right)^3\right]$$

$$= \frac{31}{48}a^4$$

Then $\quad I_{z,\text{mass}} = \frac{2m}{3a^2} \times \frac{31}{48}a^4$

$$= \frac{31}{72}ma^2$$

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PROBLEM 9.193 (Continued)

Finally,

$$I_{y,\text{mass}} = \bar{I}_{x,\text{mass}} + I_{z,\text{mass}}$$

$$= \frac{7}{18}ma^2 + \frac{31}{72}ma^2$$

$$= \frac{59}{72}ma^2$$

or $I_y = 0.819ma^2$ ◀

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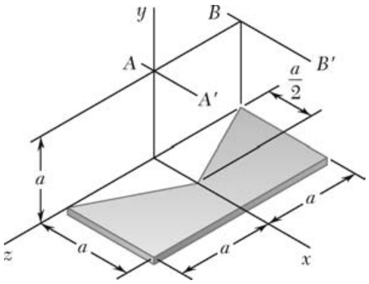

PROBLEM 9.194

A piece of thin, uniform sheet metal is cut to form the machine component shown. Denoting the mass of the component by *m*, determine its mass moment of inertia with respect to (*a*) the axis *AA*′, (*b*) the axis *BB*′, where the *AA*′ and *BB*′ axes are parallel to the *x* axis and lie in a plane parallel to and at a distance *a* above the *xz* plane.

SOLUTION

First note that the *x* axis is a centroidal axis so that

$$I = \bar{I}_{x,\text{mass}} + md^2$$

and that from the solution to Problem 9.115,

$$\bar{I}_{x,\text{mass}} = \frac{7}{18}ma^2$$

(*a*) We have

$$I_{AA',\text{mass}} = \frac{7}{18}ma^2 + m(a)^2$$

or $I_{AA'} = 1.389ma^2$ ◀

(*b*) We have

$$I_{BB',\text{mass}} = \frac{7}{18}ma^2 + m(a\sqrt{2})^2$$

or $I_{BB'} = 2.39ma^2$ ◀

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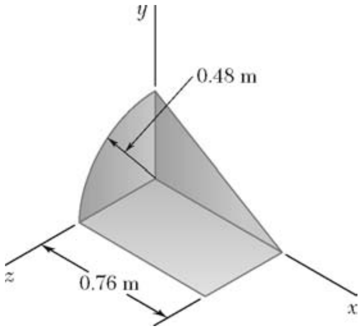
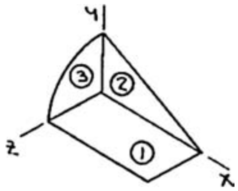


PROBLEM 9.195

A 2-mm thick piece of sheet steel is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3, determine the mass moment of inertia of the component with respect to each of the coordinate axes.

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \rho_{\text{ST}} t A$$

Then

$$m_1 = (7850 \text{ kg/m}^3)(0.002 \text{ m})(0.76 \times 0.48) \text{ m}^2$$
$$= 5.72736 \text{ kg}$$

$$m_2 = (7850 \text{ kg/m}^3)(0.002 \text{ m})\left(\frac{1}{2} \times 0.76 \times 0.48\right) \text{m}^2$$
$$= 2.86368 \text{ kg}$$

$$m_3 = (7850 \text{ kg/m}^3)(0.002 \text{ m})\left(\frac{\pi}{4} \times 0.48^2\right) \text{m}^2$$
$$= 2.84101 \text{ kg}$$

Using Figure 9.28 for component 1 and the equations derived above for components 2 and 3, we have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$
$$= \left[\frac{1}{12}(5.72736 \text{ kg})(0.48 \text{ m})^2 + (5.72736 \text{ kg})\left(\frac{0.48}{2} \text{ m}\right)^2\right]$$
$$+ \left[\frac{1}{18}(2.86368 \text{ kg})(0.48 \text{ m})^2 + (2.86368 \text{ kg})\left(\frac{0.48}{3} \text{ m}\right)^2\right] + \left[\frac{1}{2}(2.84101 \text{ kg})(0.48 \text{ m})^2\right]$$
$$= [(0.109965 + 0.329896) + (0.036655 + 0.073310) + (0.327284)] \text{ kg} \cdot \text{m}^2$$
$$= (0.439861 + 0.109965 + 0.327284) \text{ kg} \cdot \text{m}^2$$

or $\quad I_x = 0.877 \text{ kg} \cdot \text{m}^2$ ◀

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PROBLEM 9.195 (Continued)

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

$$= \left\{\frac{1}{12}(5.72736\text{ kg})(0.76^2 + 0.48^2)\text{ m}^2 + (5.72736\text{ kg})\left[\left(\frac{0.76}{2}\right)^2 + \left(\frac{0.48}{2}\right)^2\right]\text{m}^2\right\}$$

$$+ \left[\frac{1}{18}(2.86368\text{ kg})(0.76\text{ m})^2 + (2.86368\text{ kg})\left(\frac{0.76}{3}\text{ m}\right)^2\right] + \left[\frac{1}{4}(2.84101\text{ kg})(0.48\text{ m})^2\right]$$

$$= [(0.385642 + 1.156927) + (0.091892 + 0.183785) + (0.163642)]\text{ kg}\cdot\text{m}^2$$

$$= (1.542590 + 0.275677 + 0.163642)\text{ kg}\cdot\text{m}^2$$

or $I_y = 1.982\text{ kg}\cdot\text{m}^2$ ◀

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3$$

$$= \left[\frac{1}{12}(5.72736\text{ kg})(0.76\text{ m})^2 + (5.72736\text{ kg})\left(\frac{0.76}{2}\text{ m}\right)^2\right]$$

$$+ \left\{\frac{1}{18}(2.86368\text{ kg})(0.76^2 + 0.48^2)\text{ m}^2 + (2.86368\text{ kg})\left[\left(\frac{0.76}{3}\right)^2 + \left(\frac{0.48}{3}\right)^2\right]\text{m}^2\right\}$$

$$+ \left[\frac{1}{4}(2.84101\text{ kg})(0.48\text{ m})^2\right]$$

$$I_z = [(0.275677 + 0.827031) + (0.128548 + 0.257095) + (0.163642)]\text{ kg}\cdot\text{m}^2$$

$$= (1.102708 + 0.385643 + 0.163642)\text{ kg}\cdot\text{m}^2$$

or $I_z = 1.652\text{ kg}\cdot\text{m}^2$ ◀

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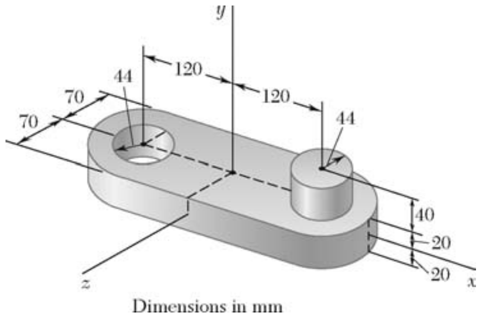
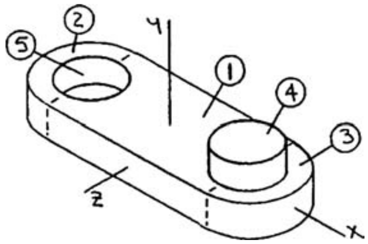


PROBLEM 9.196

Determine the mass moments of inertia and the radii of gyration of the steel machine element shown with respect to the *x* and *y* axes. (The density of steel is 7850 kg/m^3.)

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V$$

Then

$$m_1 = (7850 \text{ kg/m}^3)(0.24 \times 0.04 \times 0.14) \text{ m}^3$$
$$= 10.5504 \text{ kg}$$

$$m_2 = m_3 = (7850 \text{ kg/m}^3)\left[\frac{\pi}{2}(0.07)^2 \times 0.04\right] \text{m}^3 = 2.41683 \text{ kg}$$

$$m_4 = m_5 = (7850 \text{ kg/m}^3)[\pi(0.044)^2 \times (0.04)] \text{ m}^3 = 1.90979 \text{ kg}$$

Using Figure 9.28 for components 1, 4, and 5 and the equations derived above (before the solution to Problem 9.144) for a semicylinder, we have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 - (I_x)_5 \quad \text{where} \quad (I_x)_2 = (I_x)_3$$
$$= \left[\frac{1}{12}(10.5504 \text{ kg})(0.04^2 + 0.14^2) \text{ m}^2\right] + 2\left\{\frac{1}{12}(2.41683 \text{ kg})\left[3(0.07 \text{ m})^2 + (0.04 \text{ m})^2\right]\right\}$$
$$+ \left\{\frac{1}{12}(1.90979 \text{ kg})\left[3(0.044 \text{ m})^2 + (0.04 \text{ m})^2\right] + (1.90979 \text{ kg})(0.04 \text{ m})^2\right\}$$
$$- \left\{\frac{1}{12}(1.90979 \text{ kg})\left[3(0.044 \text{ m})^2 + (0.04 \text{ m})^2\right]\right\}$$
$$= [(0.0186390) + 2(0.0032829) + (0.0011790 + 0.0030557) - (0.0011790)] \text{ kg} \cdot \text{m}^2$$
$$= 0.0282605 \text{ kg} \cdot \text{m}^2$$

or $\quad I_x = 28.3 \times 10^{-3} \text{ kg} \cdot \text{m}^2$ ◀

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PROBLEM 9.196 (Continued)

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4 - (I_y)_5$$

where $$(I_y)_2 = (I_y)_3 \quad (I_y)_4 = |(I_y)_5|$$

Then $$I_y = \left[\frac{1}{12}(10.5504 \text{ kg})(0.24^2 + 0.14^2) \text{ m}^2\right]$$

$$+2\left[(2.41683 \text{ kg})\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)(0.07 \text{ m}^2) + (2.41683 \text{ kg})\left(0.12 + \frac{4\times 0.07}{3\pi}\right)^2 \text{ m}^2\right]$$

$$= [(0.0678742) + 2(0.0037881 + 0.0541678)] \text{ kg}\cdot\text{m}^2$$

$$= 0.1837860 \text{ kg}\cdot\text{m}^2$$

or $I_y = 183.8 \times 10^{-3} \text{ kg}\cdot\text{m}^2$ ◀

Also $$m = m_1 + m_2 + m_3 + m_4 - m_5 \text{ where } m_2 = m_3,\ m_4 = |m_5|$$

$$= (10.5504 + 2\times 2.41683) \text{ kg} = 15.38406 \text{ kg}$$

Then $$k_x^2 = \frac{I_x}{m} = \frac{0.0282605 \text{ kg}\cdot\text{m}^2}{15.38406 \text{ kg}}$$

or $k_x = 42.9 \text{ mm}$ ◀

and $$k_y^2 = \frac{I_y}{m} = \frac{0.1837860 \text{ kg}\cdot\text{m}^2}{15.38406 \text{ kg}}$$

or $k_y = 109.3 \text{ mm}$ ◀

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CHAPTER 10

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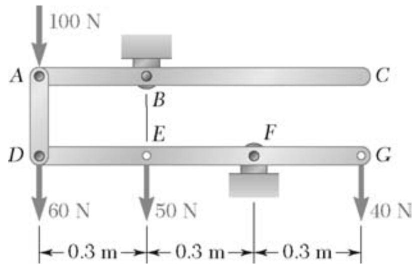
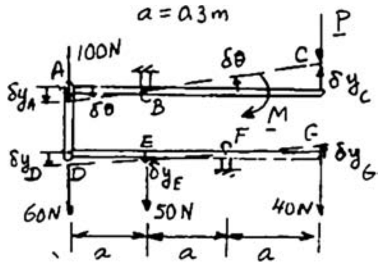


PROBLEM 10.1

Determine the vertical force **P** that must be applied at C to maintain the equilibrium of the linkage.

SOLUTION

Assume $\delta y_A \downarrow$



$$\delta y_C = 2\delta y_A \uparrow$$

$$\delta y_D = \delta y_A \downarrow$$

$$\delta y_E = \frac{1}{2}\delta y_D = \frac{1}{2}\delta y_A \downarrow$$

$$\delta y_G = \delta y_E = \frac{1}{2}\delta y_A \uparrow$$

Also $$\delta\theta = \frac{\delta y_A}{a} \circlearrowright$$

<u>Virtual Work</u>: We apply both P and M to member ABC.

$$\delta U = 100\delta y_A - P\delta y_C - M\,\delta\theta + 60\delta y_D + 50\delta y_E - 40\delta y_G = 0$$

$$100\delta y_A - P(2\delta y_A) - M\left(\frac{\delta y_A}{a}\right) + 60\delta y_A + 50\left(\frac{1}{2}\delta y_A\right) - 40\left(\frac{1}{2}\delta y_A\right) = 0$$

$$100 - 2P - \frac{M}{a} + 60 + 25 - 20 = 0$$

$$2P + \frac{M}{a} = 165 \text{ N} \qquad (1)$$

Now from Eq. (1) for $M = 0$ $\qquad 2P = 165$ N $\qquad$ $\mathbf{P} = 82.5 \text{ N}\downarrow$ ◀

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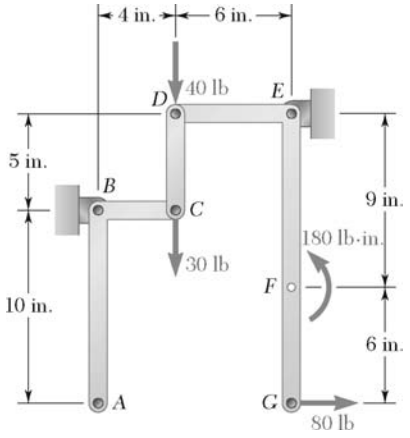
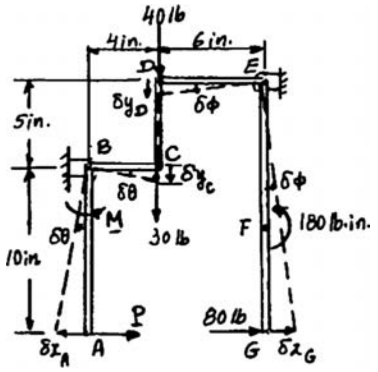


PROBLEM 10.2

Determine the horizontal force **P** that must be applied at A to maintain the equilibrium of the linkage.

SOLUTION

Assume $\delta\theta$ ↻



$$\delta x_A = 10\delta\theta \leftarrow$$

$$\delta y_C = 4\delta\theta \downarrow$$

$$\delta y_D = \delta y_C = 4\delta\theta \downarrow$$

$$\delta\phi = \frac{\delta y_D}{6} = \frac{2}{3}\delta\theta \circlearrowright$$

$$\delta x_G = 15\delta\phi = 15\left(\frac{2}{3}\delta\theta\right) = 10\delta\theta \rightarrow$$

Virtual Work: We shall assume that a force **P** and a couple **M** are applied to member ABC as shown.

$$\delta U = -P\delta x_A - M\delta\theta + 30\delta y_C + 40\delta y_D + 180\delta\phi + 80\delta x_G = 0$$

$$-P(10\delta\theta) - M\delta\theta + 30(4\delta\theta) + 40(4\delta\theta) + 180\left(\frac{2}{3}\delta\theta\right) + 80(10\delta\theta) = 0$$

$$-10P - M + 120 + 160 + 120 + 800 = 0$$

$$(10\text{ in.})P + M = 1200\text{ lb}\cdot\text{in.} \qquad (1)$$

Making $M = 0$ in Eq. (1): $P = +120.0$ lb $\qquad$ $\mathbf{P} = 120.0\text{ lb} \rightarrow$ ◀

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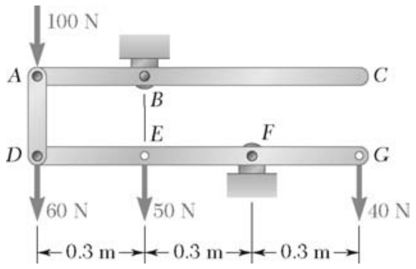


PROBLEM 10.3

Determine the couple **M** that must be applied to member *ABC* to maintain the equilibrium of the linkage.

SOLUTION

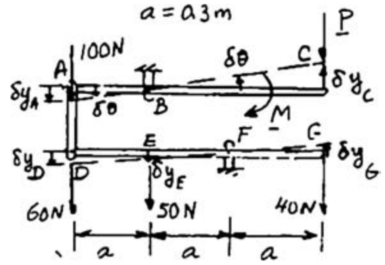
Assume $\delta y_A \downarrow$

$$\delta y_C = 2\delta y_A \uparrow$$

$$\delta y_D = \delta y_A \downarrow$$

$$\delta y_E = \frac{1}{2}\delta y_D = \frac{1}{2}\delta y_A \downarrow$$

$$\delta y_G = \delta y_E = \frac{1}{2}\delta y_A \uparrow$$



Also $$\delta\theta = \frac{\delta y_A}{a} \circlearrowright$$

<u>Virtual Work</u>: We apply both *P* and *M* to member *ABC*.

$$\delta U = 100\delta y_A - P\delta y_C - M\delta\theta + 60\delta y_D + 50\delta y_E - 40\delta y_G = 0$$

$$100\delta y_A - P(2\delta y_A) - M\left(\frac{\delta y_A}{a}\right) + 60\delta y_A + 50\left(\frac{1}{2}\delta y_A\right) - 40\left(\frac{1}{2}\delta y_A\right) = 0$$

$$100 - 2P - \frac{M}{a} + 60 + 25 - 20 = 0$$

$$2P + \frac{M}{a} = 165 \text{ N} \qquad (1)$$

Now from Eq. (1) for $P = 0$ and $a = 0.3$

$$\frac{M}{0.3 \text{ m}} = 165 \text{ N}$$

$\mathbf{M} = 49.5 \text{ N}\cdot\text{m} \circlearrowright$ ◀

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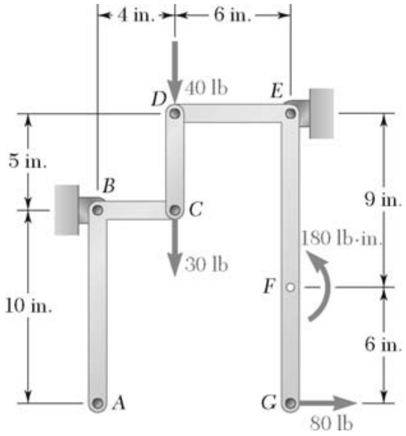
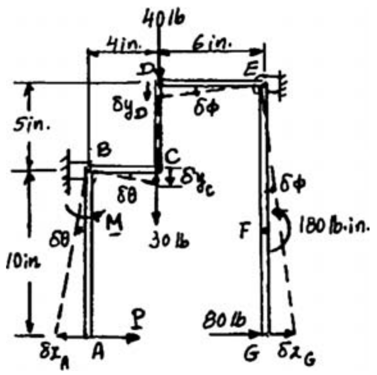


PROBLEM 10.4

Determine the couple **M** that must be applied to member *ABC* to maintain the equilibrium of the linkage.

SOLUTION

Assume $\delta\theta$ ↻



$$\delta x_A = 10\delta\theta \leftarrow$$

$$\delta y_C = 4\delta\theta \downarrow$$

$$\delta y_D = \delta y_C = 4\delta\theta \downarrow$$

$$\delta\phi = \frac{\delta y_D}{6} = \frac{2}{3}\delta\theta \circlearrowright$$

$$\delta x_G = 15\delta\phi$$

$$= 15\left(\frac{2}{3}\delta\theta\right)$$

$$= 10\delta\theta \rightarrow$$

Virtual Work: We shall assume that a force **P** and a couple **M** are applied to member *ABC* as shown.

$$\delta U = -P\delta x_A - M\,\delta\theta + 30\delta y_C + 40\delta y_D + 180\delta\phi + 80\delta x_G = 0$$

$$-P(10\delta\theta) - M\,\delta\theta + 30(4\delta\theta) + 40(4\delta\theta) + 180\left(\frac{2}{3}\delta\theta\right) + 80(10\delta\theta) = 0$$

$$-10P - M + 120 + 160 + 120 + 800 = 0$$

$$(10 \text{ in.})P + M = 1200 \text{ lb}\cdot\text{in.} \qquad (1)$$

Now from Eq. (1) for $P = 0$ $\quad$ $\mathbf{M} = 1200 \text{ lb}\cdot\text{in.}$ ◀

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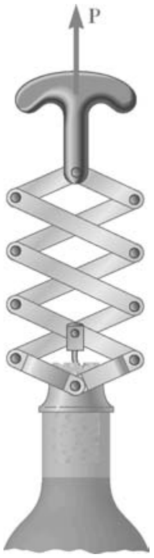


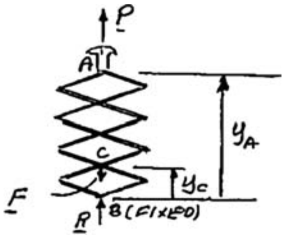
PROBLEM 10.5

Knowing that the maximum friction force exerted by the bottle on the cork is 60 lb, determine (*a*) the force **P** that must be applied to the corkscrew to open the bottle, (*b*) the maximum force exerted by the base of the corkscrew on the top of the bottle.

SOLUTION

From sketch $y_A = 4y_C$

Thus $\delta y_A = 4\delta y_C$



(*a*) Virtual Work: $\delta U = 0\text{:}\quad P\delta y_A - F\delta y_C = 0$

$$P(4\delta y_C) - F\delta y_C = 0$$

$$P = \frac{1}{4}F$$

$$F = 60\text{ lb:}\quad P = \frac{1}{4}(60\text{ lb}) = 15\text{ lb}$$

$\mathbf{P} = 15.00\text{ lb}\uparrow$ ◀

(*b*) Free body: Corkscrew

$$+\uparrow \Sigma F_y = 0 \quad R + P - F = 0; \quad R + 15\text{ lb} - 60\text{ lb} = 0$$

On corkscrew: $\mathbf{R} = 45\text{ lb}\uparrow$, On battle: $\mathbf{R} = 45.0\text{ lb}\downarrow$ ◀

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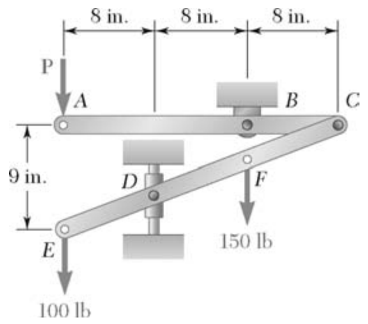
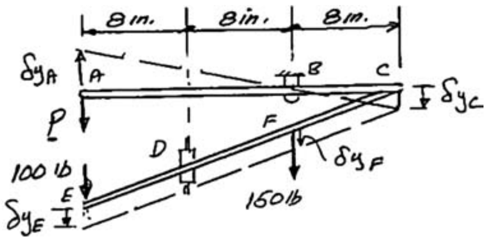


PROBLEM 10.6

The two-bar linkage shown is supported by a pin and bracket at *B* and a collar at *D* that slides freely on a vertical rod. Determine the force **P** required to maintain the equilibrium of the linkage.

SOLUTION



Assume $\delta y_A \uparrow$:

$$\delta y_C = \frac{8\text{ in.}}{16\text{ in.}}\delta y_A; \quad \delta y_C = \frac{1}{2}\delta y_A \downarrow$$

Since bar *CD* move in translation

$$\delta y_E = \delta y_F = \delta y_C$$

or

$$\delta y_E = \delta y_F = \frac{1}{2} y_A \downarrow$$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_A + (100\text{ lb})\delta y_E + (150\text{ lb})\delta y_F = 0$$

$$-P\delta y_A + 100\left(\frac{1}{2}\delta y_A\right) + 150\left(\frac{1}{2}\delta y_A\right) = 0$$

$$P = 125\text{ lb}$$

$$\mathbf{P} = 125.0\text{ lb} \downarrow \blacktriangleleft$$

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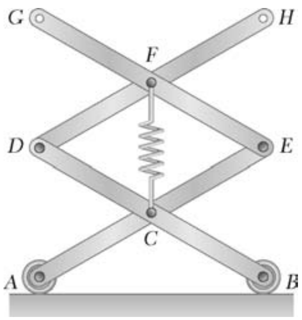
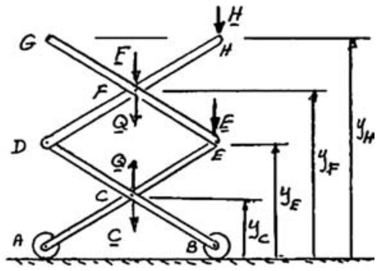


PROBLEM 10.7

A spring of constant 15 kN/m connects Points C and F of the linkage shown. Neglecting the weight of the spring and linkage, determine the force in the spring and the vertical motion of Point G when a vertical downward 120-N force is applied (*a*) at Point C, (*b*) at Points C and H.

SOLUTION



$$y_G = 4y_C$$
$$y_H = 4y_C \quad \delta y_H = 4\delta y_C$$
$$y_F = 3y_C \quad \delta y_F = 3\delta y_C$$
$$y_E = 2y_C \quad \delta y_E = 2\delta y_C$$

For spring:

$$\Delta = y_F - y_C$$

Q = Force in spring (assumed in tension)

$$Q = +k\Delta = k(y_F - y_C) = k(3y_C - y_C) = 2ky_C \qquad (1)$$

(*a*) $\mathbf{C} = 120 \text{ N}, \quad \mathbf{E} = \mathbf{F} = \mathbf{H} = 0$

<u>Virtual Work</u>:

$$\delta U = 0: \quad -(120 \text{ N})\delta y_C + Q\delta y_C - Q\delta y_F = 0$$

$$-120\delta y_C + Q\delta y_C - Q(3\delta y_C) = 0$$

$$Q = -60 \text{ N} \qquad Q = 60.0 \text{ N} \quad \text{C} \blacktriangleleft$$

Eq. (1): $Q = 2ky_C, \quad -60 \text{ N} = 2(15 \text{ kN/m})y_C, \quad y_C = -2 \text{ mm}$

At Point G: $y_G = 4y_C = 4(-2 \text{ mm}) = -8 \text{ mm}$ $\qquad \mathbf{y}_G = 8.00 \text{ mm}\downarrow \blacktriangleleft$

(*b*) $\mathbf{C} = \mathbf{H} = 120 \text{ N}, \mathbf{E} = \mathbf{F} = 0$

<u>Virtual Work</u>:

$$\delta U = 0: \quad -(120 \text{ N})\delta y_C - (120 \text{ N})y_H + Q\delta y_C - Q\delta y_F = 0$$

$$-120\delta y_C - 120(4\delta y_C) + Q\delta y_C - Q(3\delta y_C) = 0$$

$$Q = -300 \text{ N} \qquad Q = 300 \text{ N} \quad \text{C} \blacktriangleleft$$

Eq. (1): $Q = 2ky_C \quad -300 \text{ N} = 2(15 \text{ kN/m})y_C, \; y_C = -10 \text{ mm}$

At Point G: $y_G = 4y_C = 4(-10 \text{ mm}) = -40 \text{ mm}$ $\qquad \mathbf{y}_G = 40.0 \text{ mm}\downarrow \blacktriangleleft$

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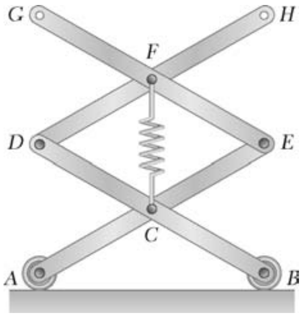
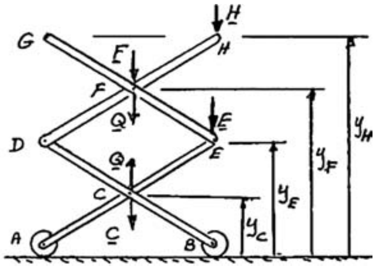


PROBLEM 10.8

A spring of constant 15 kN/m connects Points *C* and *F* of the linkage shown. Neglecting the weight of the spring and linkage, determine the force in the spring and the vertical motion of Point *G* when a vertical downward 120-N force is applied (*a*) at Point *E*, (*b*) at Points *E* and *F*.

SOLUTION



$$y_G = 4y_C$$

$$y_H = 4y_C \quad \delta y_H = 4\delta y_C$$

$$y_F = 3y_C \quad \delta y_F = 3\delta y_C$$

$$y_E = 2y_C \quad \delta y_E = 2\delta y_C$$

For spring: $\Delta = y_F - y_C$

Q = Force in spring (assumed in tension)

$$\mathbf{Q} = +k\Delta = k(y_F - y_C) = k(3y_C - y_C) = 2ky_C \qquad (1)$$

(*a*) $\mathbf{E} = 120\text{ N}, \quad \mathbf{C} = \mathbf{F} = \mathbf{H} = 0$

Virtual Work: $\delta U = 0: \quad -(120\text{ N})\delta y_E + Q\delta y_C - Q\delta y_F = 0$

$$-120(2\delta y_C) + Q\delta y_C - Q(3\delta y_C) = 0$$

$Q = -120$ N $\qquad Q = 120.0$ N C ◀

Eq. (1): $Q = 2ky_C, \quad -120\text{ N} = 2(15\text{ kN/m})y_C, \quad y_C = -4\text{ mm}$

At Point *G*: $y_G = 4y_C = 4(-4\text{ mm}) = 16\text{ mm}$ $\qquad \mathbf{y}_G = 16.00$ mm ↓ ◀

(*b*) $\mathbf{E} = \mathbf{F} = 120\text{ N}, \mathbf{C} = \mathbf{H} = 0$

Virtual Work:

$$\delta U = 0: \quad -(120\text{ N})\delta y_E - (120\text{ N})\delta y_F + Q\delta y_C - Q\delta y_F = 0$$

$$-120(2\delta y_C) - 120(3\delta y_C) + Q\delta y_C - Q(3\delta y_C) = 0$$

$Q = -300$ N $\qquad Q = 300$ N C ◀

Eq. (1): $Q = 2ky_C, \quad -300\text{ N} = 2(15\text{ kN/m})y_C, \quad y_C = -10\text{ mm}$

At Point *G*: $y_G = 4y_C = 4(-10\text{ mm}) = -40\text{ mm}$ $\qquad \mathbf{y}_G = 40.0$ mm ↓ ◀

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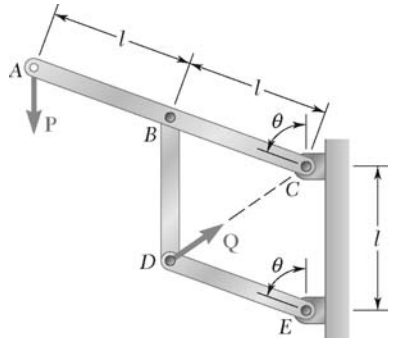
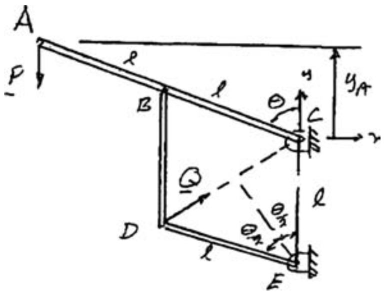


PROBLEM 10.9

Knowing that the line of action of the force **Q** passes through Point *C*, derive an expression for the magnitude of **Q** required to maintain equilibrium.

SOLUTION



We have $$y_A = 2l\cos\theta; \quad \delta y_A = -2l\sin\theta\,\delta\theta$$

$$CD = 2l\sin\frac{\theta}{2}; \quad \delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_A - Q\delta(CD) = 0$$

$$-P(-2l\sin\theta\,\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

$$Q = 2P\frac{\sin\theta}{\cos(\theta/2)} \blacktriangleleft$$

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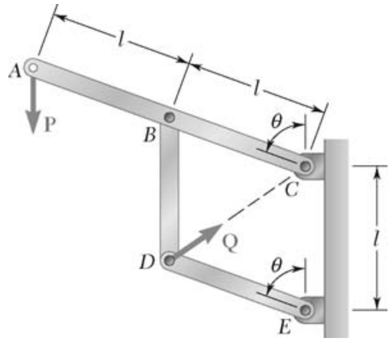
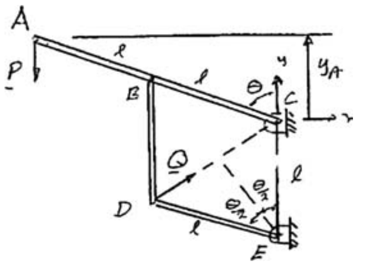


PROBLEM 10.10

Solve Problem 10.9 assuming that the force **P** applied at Point *A* acts horizontally to the left.

PROBLEM 10.9 Knowing that the line of action of the force **Q** passes through Point *C*, derive an expression for the magnitude of **Q** required to maintain equilibrium.

SOLUTION



We have

$$x_A = 2l\cos\theta; \quad \delta x_A = 2l\cos\theta\,\delta\theta$$

$$CD = 2l\sin\frac{\theta}{2}; \quad \delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P\delta x_A - Q\delta(CD) = 0$$

$$P(2l\cos\theta\,\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0 \qquad Q = 2P\frac{\cos\theta}{\cos(\theta/2)} \blacktriangleleft$$

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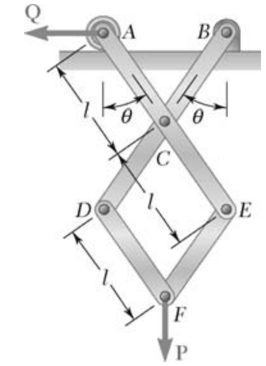
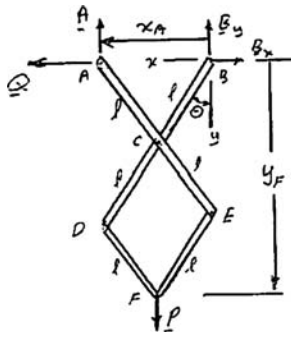


PROBLEM 10.11

The mechanism shown is acted upon by the force **P** derive an expression for the magnitude of the force **Q** required to maintain equilibrium.

SOLUTION



Virtual Work:

We have $x_A = 2l\sin\theta$

$$\delta x_A = 2l\cos\theta\delta\theta$$

and $y_F = 3l\cos\theta$

$$\delta y_F = -3l\sin\theta\delta\theta$$

Virtual Work: $\delta U = 0\text{:}\quad Q\delta x_A + P\delta y_F = 0$

$$Q(2l\cos\theta\delta\theta) + P(-3l\sin\theta\delta\theta) = 0$$

$$Q = \frac{3}{2}P\tan\theta \blacktriangleleft$$

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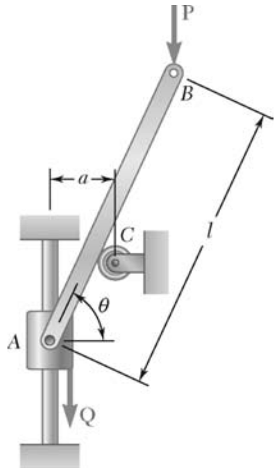
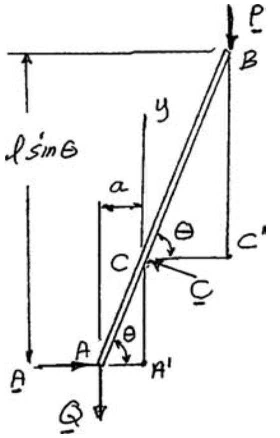


PROBLEM 10.12

The slender rod *AB* is attached to a collar *A* and rests on a small wheel at *C*. Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the rod.

SOLUTION



For $\Delta AA'C$:

$$A'C = a\tan\theta$$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta CC'B$:

$$BC' = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$y_B = BC' = l\sin\theta - a\tan\theta$$

$$\delta y_B = l\cos\theta\delta\theta - \frac{a}{\cos^2\theta}\delta\theta$$

Virtual Work:

$$\delta U = 0:\quad Q\delta y_A - P\delta y_B = 0$$

$$-Q\left(-\frac{a}{\cos^2\theta}\right)\delta\theta - P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)\delta\theta = 0$$

$$Q\left(\frac{a}{\cos^2\theta}\right) = P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)$$

$$Q = P\left(\frac{l}{a}\cos^3\theta - 1\right) \blacktriangleleft$$

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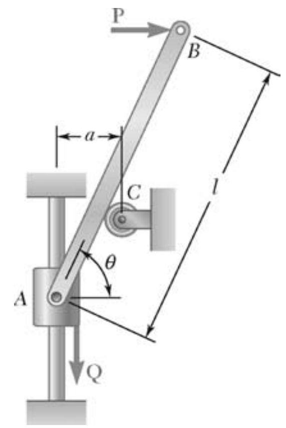
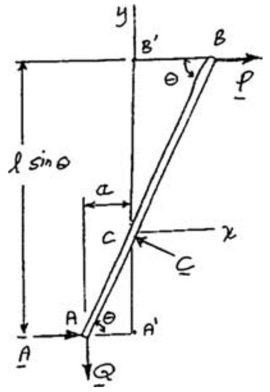


PROBLEM 10.13

The slender rod AB is attached to a collar A and rests on a small wheel at C. Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the rod.

SOLUTION



For $\Delta AA'C$:

$$A'C = a\tan\theta$$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta BB'C$:

$$B'C = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$BB' = \frac{B'C}{\tan\theta} = \frac{l\sin\theta - a\tan\theta}{\tan\theta}$$

$$x_B = BB' = l\cos\theta - a$$

$$\delta x_B = -l\sin\theta\delta\theta$$

<u>Virtual Work</u>:

$$\delta U = 0: \quad P\delta x_B - Q\delta y_A = 0$$

$$P(-l\sin\theta\delta\theta) - Q\left(-\frac{a}{\cos^2\theta}\delta\theta\right) = 0$$

or $\qquad Pl\sin\theta\cos^2\theta = Qa \qquad Q = P\frac{1}{a}\sin\theta\cos^2\theta$ ◀

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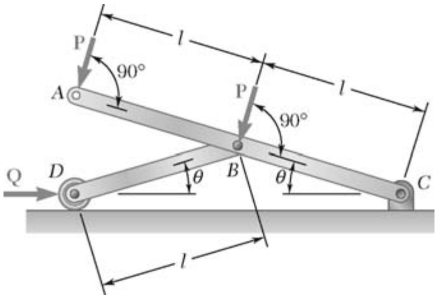
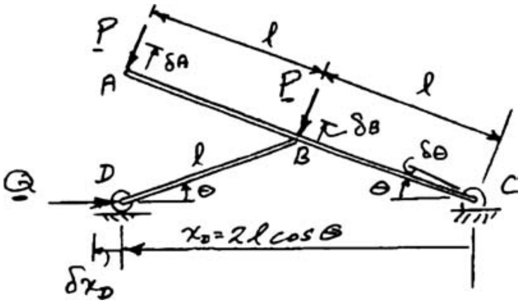


PROBLEM 10.14

Derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the mechanism shown.

SOLUTION



We have $x_D = 2l\cos\theta$ so that $\delta x_D = -2l\sin\theta\delta\theta$

$$\delta A = 2l\delta\theta$$

$$\delta B = l\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -Q\delta x_D - P\delta A - P\delta B = 0$$

$$-Q(-2l\sin\theta\delta\theta) - P(2l\delta\theta) - P(l\delta\theta) = 0$$

$$2Ql\sin\theta - 3Pl = 0 \qquad Q = \frac{3}{2}\frac{P}{\sin\theta} \blacktriangleleft$$

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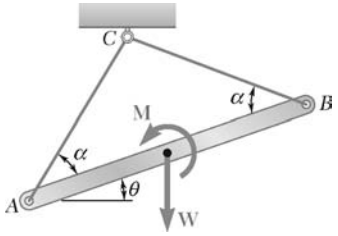
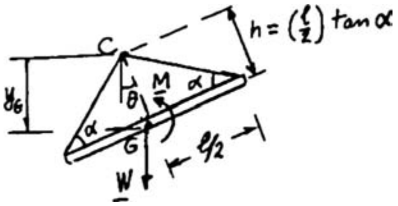


PROBLEM 10.15

A uniform rod *AB* of length *l* and weight *W* is suspended from two cords *AC* and *BC* of equal length. Derive an expression for the magnitude of the couple **M** required to maintain equilibrium of the rod in the position shown.

SOLUTION



$$y_G = h\cos\theta = \frac{1}{2}l\tan\alpha\cos\theta$$

$$\delta y_G = -\frac{1}{2}l\tan\alpha\sin\theta\delta\theta$$

<u>Virtual Work</u>:

$$\delta U = W\delta y_G + M\delta\theta = 0$$

$$W\left(-\frac{1}{2}l\tan\alpha\sin\theta\delta\theta\right) + M\delta\theta = 0$$

$$M = \frac{1}{2}Wl\tan\alpha\sin\theta \quad ◀$$

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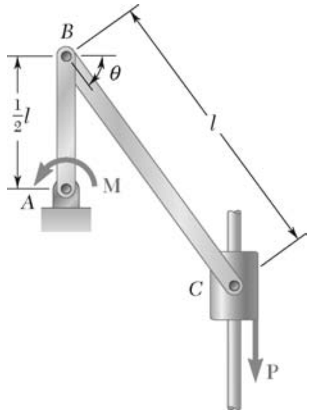
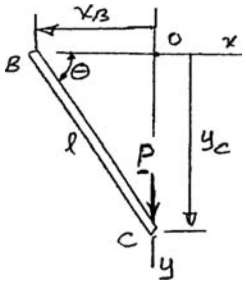


PROBLEM 10.16

Derive an expression for the magnitude of the couple **M** required to maintain the equilibrium of the linkage shown.

SOLUTION



We have $x_B = l\cos\theta$

$$\delta x_B = -l\sin\theta\delta\theta \qquad (1)$$

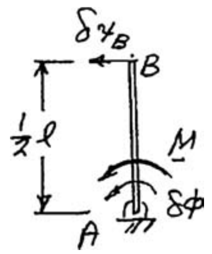
$$y_C = l\sin\theta$$

$$\delta y_C = l\cos\theta\delta\theta$$

Now $$\delta x_B = \frac{1}{2}l\delta\theta$$

Substituting from Equation (1)

$$-l\sin\theta\delta\theta = \frac{1}{2}l\delta\phi$$



or $$\delta\phi = -2\sin\theta\delta\theta$$

Virtual Work: $\delta U = 0$: $\quad M\delta\varphi + P\delta y_C = 0$

$$M(-2\sin\theta\delta\theta) + P(l\cos\theta\delta\theta) = 0$$

or $$M = \frac{1}{2}Pl\frac{\cos\theta}{\sin\theta} \qquad M = \frac{Pl}{2\tan\theta} \blacktriangleleft$$

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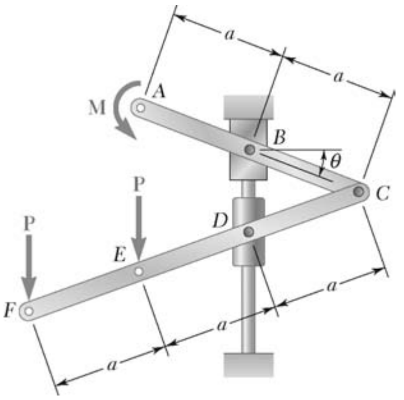
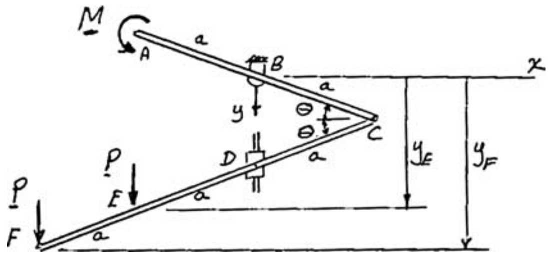


PROBLEM 10.17

Derive an expression for the magnitude of the couple **M** required to maintain the equilibrium of the linkage shown.

SOLUTION



$$y_E = 3a\sin\theta \quad \delta y_E = 3a\cos\theta\delta\theta$$
$$y_F = 4a\sin\theta \quad \delta y_F = 4a\cos\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -M\delta\theta + P\delta y_E + P\delta y_F = 0$$

$$-M\delta\theta + P(3a\cos\theta\delta\theta) + P(4a\cos\theta\delta\theta) = 0 \qquad M = 7Pa\cos\theta \blacktriangleleft$$

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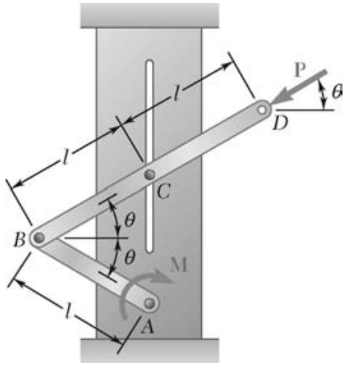
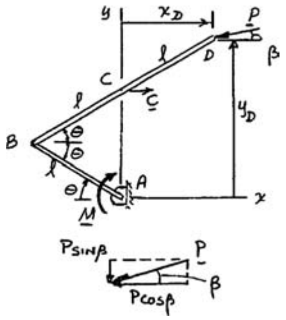


PROBLEM 10.18

The pin at C is attached to member BCD and can slide along a slot cut in the fixed plate shown. Neglecting the effect of friction, derive an expression for the magnitude of the couple **M** required to maintain equilibrium when the force **P** that acts at D is directed (a) as shown, (b) vertically downward, (c) horizontally to the right.

SOLUTION



We have $x_D = l\cos\theta$

$$\delta x_D = -l\sin\theta\delta\theta$$

$$y_D = 3l\sin\theta$$

$$\delta y_D = 3l\cos\theta\delta\theta$$

<u>Virtual Work</u>: $\delta U = 0$: $M\delta\theta - (P\cos\beta)\delta x_D - (P\sin\beta)\delta y_D = 0$

$$M\delta\theta - (P\cos\beta)(-l\sin\theta\delta\theta) - (P\sin\beta)(3l\cos\theta\delta\theta) = 0$$

$$M = Pl(3\sin\beta\cos\theta - \cos\beta\sin\theta) \quad (1)$$

(a) For **P** directed along BCD, $\beta = \theta$

Equation (1): $M = Pl(3\sin\theta\cos\theta - \cos\theta\sin\theta)$

$M = Pl(2\sin\theta\cos\theta)$ $\qquad M = Pl\sin 2\theta$ ◀

(b) For **P** directed ↓, $\beta = 90°$

Equation (1): $M = Pl(3\sin 90°\cos\theta - \cos 90°\sin\theta)$ $\qquad M = 3Pl\cos\theta$ ◀

(c) For **P** directed →, $\beta = 180°$

Equation (1): $M = Pl(3\sin 180°\cos\theta - \cos 180°\sin\theta)$ $\qquad M = Pl\sin\theta$ ◀

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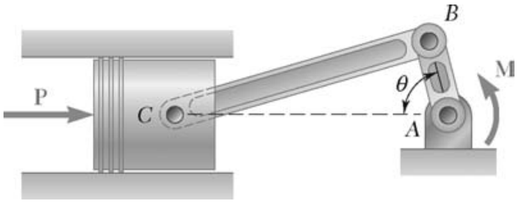
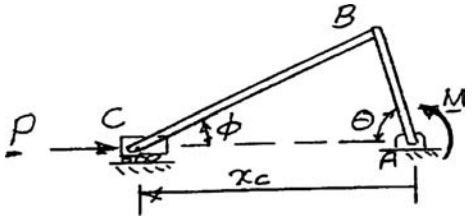
PROBLEM 10.19

A 4-kN force **P** is applied as shown to the piston of the engine system. Knowing that $AB = 50$ mm and $BC = 200$ mm, determine the couple **M** required to maintain the equilibrium of the system when (*a*) $\theta = 30°$, (*b*) $\theta = 150°$.



SOLUTION

Analysis of the geometry:



Law of sines

$$\frac{\sin\phi}{AB} = \frac{\sin\theta}{BC}$$

$$\sin\phi = \frac{AB}{BC}\sin\theta \qquad (1)$$

Now

$$x_C = AB\cos\theta + BC\cos\phi$$

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\delta\phi \qquad (2)$$

Now, from Equation (1) $\qquad \cos\phi\delta\phi = \frac{AB}{BC}\cos\theta\delta\theta$

or

$$\delta\phi = \frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta \qquad (3)$$

From Equation (2)

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\left(\frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta\right)$$

or

$$\delta x_C = -\frac{AB}{\cos\phi}(\sin\theta\cos\phi + \sin\phi\cos\theta)\delta\theta$$

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PROBLEM 10.19 (Continued)

Then $$\delta x_C = -\frac{AB\sin(\theta+\phi)}{\cos\phi}\delta\theta$$

Virtual Work: $$\delta U = 0: \quad -P\delta x_C - M\,\delta\theta = 0$$

$$-P\left[-\frac{AB\sin(\theta+\phi)}{\cos\phi}\delta\theta\right] - M\,\delta\theta = 0$$

Thus, $$M = AB\frac{\sin(\theta+\phi)}{\cos\phi}P \qquad (4)$$

(*a*) $P = 4\text{ kN}, \quad \theta = 30°$

Eq. (1): $$\sin\phi = \frac{50\text{ mm}}{200\text{ mm}}\sin 30° \qquad \phi = 7.181°$$

Eq. (4): $$M = (0.05\text{ m})\frac{\sin(30°+7.181°)}{\cos 7.818°}(4\text{ kN}) \qquad \mathbf{M} = 121.8\text{ N}\cdot\text{m} ◀$$

(*b*) $P = 4\text{ kN}, \quad \theta = 150°$

Eq. (1): $$\sin\phi = \frac{50\text{ mm}}{200\text{ mm}}\sin 160° \qquad \phi = 7.181°$$

Eq. (4): $$M = (0.05\text{ m})\frac{\sin(150°+7.181°)}{\cos 7.181°}(4\text{ kN}) \qquad \mathbf{M} = 78.2\text{ N}\cdot\text{m} ◀$$

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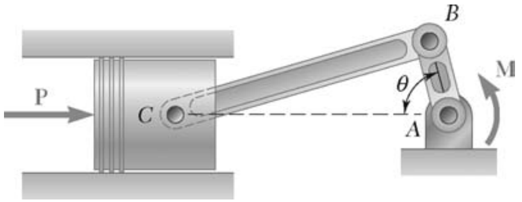
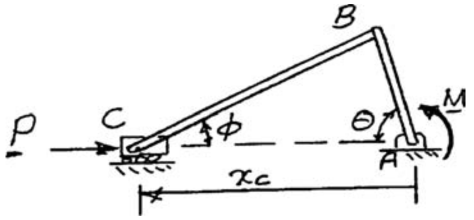
PROBLEM 10.20

A couple **M** of magnitude 100 N · m is applied as shown to the crank of the engine system. Knowing that $AB = 50$ mm and $BC = 200$ mm, determine the force **P** required to maintain the equilibrium of the system when (*a*) $\theta = 60°$, (*b*) $\theta = 120°$.



SOLUTION

Analysis of the geometry:



Law of sines

$$\frac{\sin\phi}{AB} = \frac{\sin\theta}{BC}$$

$$\sin\phi = \frac{AB}{BC}\sin\theta \qquad (1)$$

Now

$$x_C = AB\cos\theta + BC\cos\phi$$

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\delta\phi \qquad (2)$$

Now, from Equation (1) $\quad \cos\phi\delta\phi = \frac{AB}{BC}\cos\theta\delta\theta$

or

$$\delta\phi = \frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta \qquad (3)$$

From Equation (2)

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\left(\frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta\right)$$

or

$$\delta x_C = -\frac{AB}{\cos\phi}(\sin\theta\cos\phi + \sin\phi\cos\theta)\delta\theta$$

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PROBLEM 10.20 (Continued)

Then $$\delta x_C = -\frac{AB\sin(\theta+\phi)}{\cos\phi}\delta\theta$$

Virtual Work: $$\delta U = 0: \quad -P\delta x_C - M\,\delta\theta = 0$$

$$-P\left[-\frac{AB\sin(\theta+\phi)}{\cos\phi}\delta\theta\right] - M\,\delta\theta = 0$$

Thus, $$M = AB\frac{\sin(\theta+\phi)}{\cos\phi}P \qquad (4)$$

(*a*) $$M = 100\ \text{N}\cdot\text{m}, \quad \theta = 60°$$

Eq. (1): $$\sin\phi = \frac{50\ \text{mm}}{200\ \text{mm}}\sin 60° \qquad \phi = 12.504°$$

Eq. (4): $$100\ \text{N}\cdot\text{m} = (0.05\ \text{m})\frac{\sin(60° + 12.504°)}{\cos 12.504°}P$$

$$P = 2047\ \text{N}$$ **P** = 2.05 kN ⟶ ◀

(*b*) $$M = 100\ \text{N}\cdot\text{m}, \quad \theta = 120°$$

Eq. (1): $$\sin\phi = \frac{50\ \text{mm}}{200\ \text{mm}}\sin 120° \qquad \phi = 12.504°$$

Eq. (4): $$100\ \text{N}\cdot\text{m} = (0.05\ \text{m})\frac{\sin(120° + 12.504°)}{\cos 12.504°}P$$

$$P = 2649\ \text{N}$$ **P** = 2.65 kN ⟶ ◀

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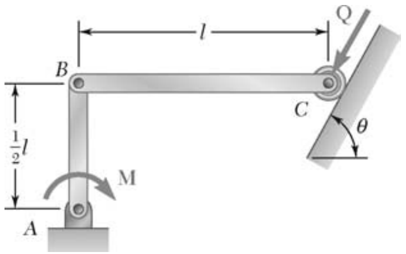
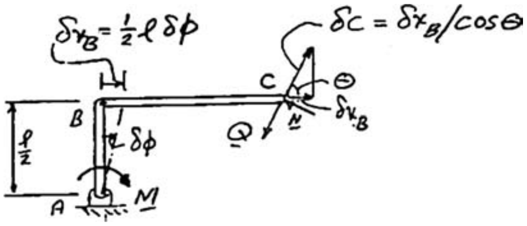


PROBLEM 10.21

For the linkage shown, determine the couple **M** required for equilibrium when $l = 1.8$ ft, $Q = 40$ lb, and $\theta = 65°$.

SOLUTION



$$\delta C = \frac{\frac{1}{2} l \delta\phi}{\cos\theta}$$

Virtual Work:

$$\delta U = 0: \quad M\delta\phi - Q\delta C = 0$$

$$M\delta\phi - Q\left(\frac{1}{2}\frac{l}{\cos\theta}\right)\delta\phi = 0$$

$$M = \frac{1}{2}\frac{Ql}{\cos\theta}$$

Data:

$$M = \frac{1}{2}\frac{(40 \text{ lb})(1.8 \text{ ft})}{\cos 65°} = 85.18 \text{ lb}\cdot\text{ft}$$

$\mathbf{M} = 85.2 \text{ lb}\cdot\text{ft}$ ◀

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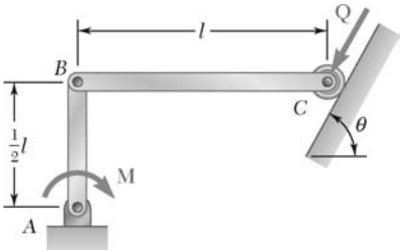
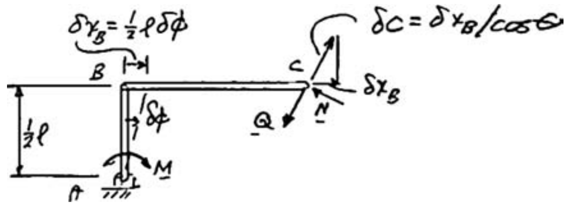


PROBLEM 10.22

For the linkage shown, determine the force **Q** required for equilibrium when $l = 18$ in., $M = 600$ lb · in., and $\theta = 70°$.

SOLUTION



$$\delta C = \frac{1}{2}\frac{l\delta\phi}{\cos\theta}$$

Virtual Work:

$$\delta U = 0: \qquad M\,\delta\phi - Q\delta C = 0$$

$$M\,\delta\phi - Q\left(\frac{1}{2}\frac{l}{\cos\theta}\right)\delta\phi = 0$$

$$Q = \frac{2M\cos\theta}{l}$$

Data:

$$Q = \frac{2(600\text{ lb}\cdot\text{in.})\cos 70°}{18\text{ in.}} = 22.801\text{ lb}$$

Q = 22.8 lb ↗ 70.0° ◀

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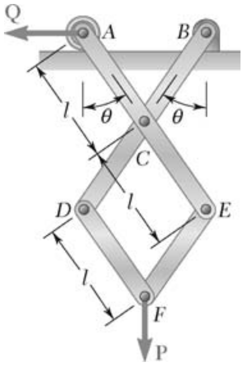


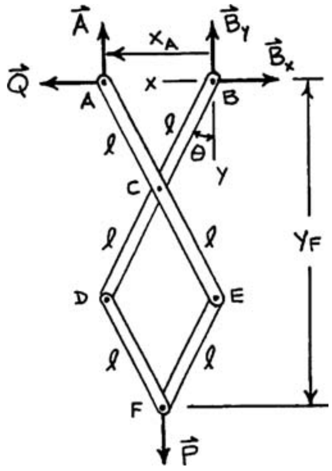
PROBLEM 10.23

Determine the value of θ corresponding to the equilibrium position of the mechanism of Problem 10.11 when $P = 45$ lb and $Q = 160$ lb.

PROBLEM 10.11 The mechanism shown is acted upon by the force **P** derive an expression for the magnitude of the force **Q** required to maintain equilibrium.

SOLUTION

<u>Virtual Work</u>:



$$x_A = 2l\sin\theta$$

$$\delta x_A = 2l\cos\theta\,\delta\theta$$

$$y_F = 3l\cos\theta$$

$$\delta y_F = -3l\sin\theta\,\delta\theta$$

$$\delta U = 0: \quad Q\delta x_A + P\delta y_F = 0$$

$$Q(2l\cos\theta\,\delta\theta) + P(-3l\sin\theta\,\delta\theta) = 0$$

$$Q = \frac{3}{2}P\tan\theta$$

<u>Data</u>: $P = 45$ lb $\quad Q = 160$ lb

$$(160\text{ lb}) = \frac{3}{2}(45\text{ lb})\tan\theta$$

$\theta = 67.1°$ ◀

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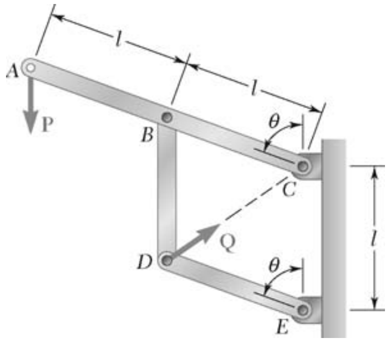
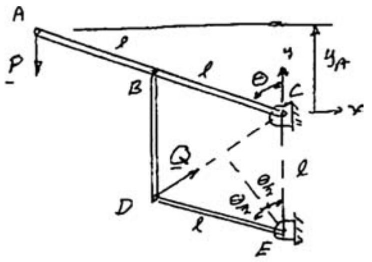


PROBLEM 10.24

Determine the value of θ corresponding to the equilibrium position of the mechanism of Problem 10.9 when $P = 80$ N and $Q = 100$ N.

SOLUTION

From geometry



$$y_A = 2l\cos\theta, \quad \delta y_A = -2l\sin\theta\,\delta\theta$$

$$CD = 2l\sin\frac{\theta}{2}, \quad \delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_A - Q\delta(CD) = 0$$

$$-P(-2l\sin\theta\,\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

or

$$Q = 2P\frac{\sin\theta}{\cos\left(\frac{\theta}{2}\right)}$$

With

$$P = 80\text{ N}, \quad Q = 100\text{ N}$$

$$(100\text{ N}) = 2(80\text{ N})\frac{\sin\theta}{\cos\left(\frac{\theta}{2}\right)}$$

$$\frac{\sin\theta}{\cos\left(\frac{\theta}{2}\right)} = 0.625$$

or

$$\frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = 0.625$$

$$\theta = 36.42° \qquad \theta = 36.4° \blacktriangleleft$$

(Additional solutions discarded as not applicable are $\theta = \pm 180°$)

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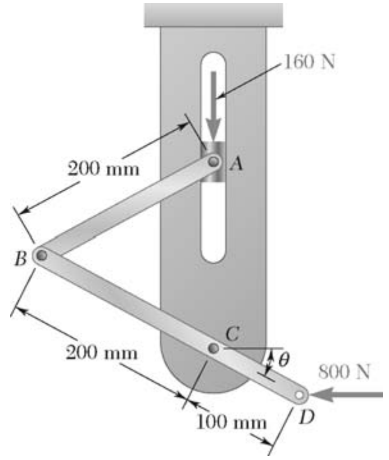
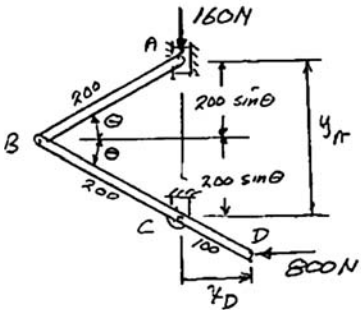


PROBLEM 10.25

Rod *AB* is attached to a block at *A* that can slide freely in the vertical slot shown. Neglecting the effect of friction and the weights of the rods, determine the value of θ corresponding to equilibrium.

SOLUTION

Dimensions in mm.



$$y_A = 400\sin\theta$$

$$\delta y_A = 400\cos\theta\,\delta\theta$$

$$x_D = 100\cos\theta$$

$$\delta x_D = -100\sin\theta\,\delta\theta$$

<u>Virtual Work</u>: $\delta U = -(160\text{ N})\delta y_A - (800\text{ N})\delta x_D = 0$

$$-(160)(400\cos\theta\,\delta\theta) - (800)(-100\sin\theta\,\delta\theta) = 0$$

$$\frac{\sin\theta}{\cos\theta} = 0.8; \quad \tan\theta = 0.8$$

$\theta = 38.7°$ ◀

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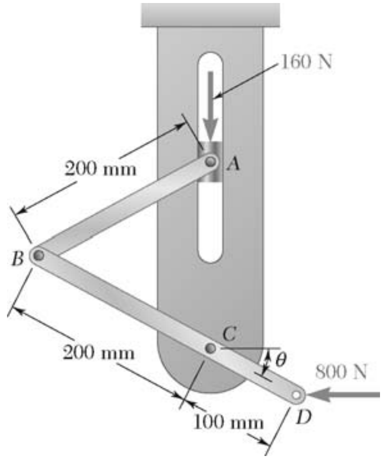
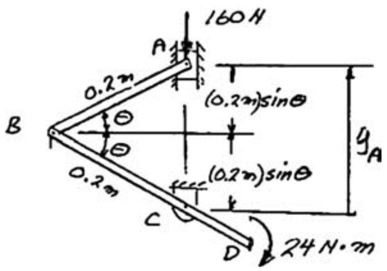


PROBLEM 10.26

Solve Problem 10.25 assuming that the 800-N force is replaced by a $24\text{-N}\cdot\text{m}$ clockwise couple applied at *D*.

PROBLEM 10.25 Rod *AB* is attached to a block at *A* that can slide freely in the vertical slot shown. Neglecting the effect of friction and the weights of the rods, determine the value of θ corresponding to equilibrium.

SOLUTION



$$y_A = (0.4\text{ m})\sin\theta$$

$$\delta y_A = (0.4\text{ m})\cos\theta\,\delta\theta$$

Virtual Work: $\delta U = -(160\text{ N})\delta y_A + (24\text{ N}\cdot\text{m})\delta\theta = 0$

$$-(160\text{ N})(0.4\text{ m})\cos\theta\,\delta\theta + (24\text{ N}\cdot\text{m})\delta\theta = 0$$

$\cos\theta = 0.375$ $\qquad\theta = 68.0°$ ◀

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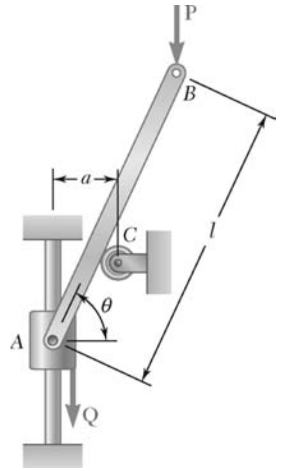
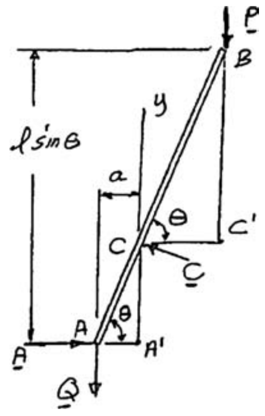


PROBLEM 10.27

Determine the value of θ corresponding to the equilibrium position of the rod of Problem 10.12 when $l = 30$ in., $a = 5$ in., $P = 25$ lb, and $Q = 40$ lb.

PROBLEM 10.12 The slender rod *AB* is attached to a collar *A* and rests on a small wheel at *C*. Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the rod.

SOLUTION



For $\Delta AA'C$: $A'C = a\tan\theta$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta CC'B$:

$$BC' = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$y_B = BC' = l\sin\theta - a\tan\theta$$

$$\delta y_B = l\cos\theta\,\delta\theta - \frac{a}{\cos^2\theta}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -Q\delta y_A - P\delta y_B = 0$$

$$-Q\left(-\frac{a}{\cos^2\theta}\right)\delta\theta - P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)\delta\theta = 0$$

$$Q\left(\frac{a}{\cos^2\theta}\right) = P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)$$

or $$Q = P\left(\frac{l}{a}\cos^3\theta - 1\right)$$

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PROBLEM 10.27 (Continued)

with $l = 30\text{ in.}, a = 5\text{ in.}, P = 25\text{ lb, and } Q = 40\text{ lb}$

$$(40\text{ lb}) = (25\text{ lb})\left(\frac{30\text{ in.}}{5\text{ in.}}\cos^3\theta - 1\right)$$

or $\cos^3\theta = 0.4333$ $\theta = 40.8°$ ◀

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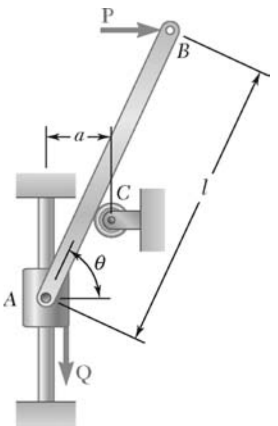
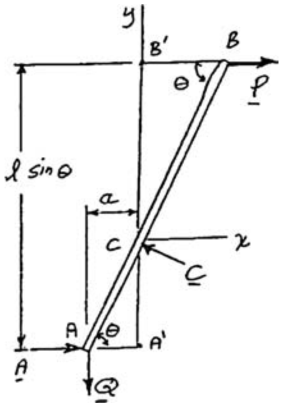


PROBLEM 10.28

Determine the values of θ corresponding to the equilibrium position of the rod of Problem 10.13 when $l = 600$ mm, $a = 100$ mm, $P = 50$ N, and $Q = 90$ N.

PROBLEM 10.13 The slender rod *AB* is attached to a collar *A* and rests on a small wheel at *C*. Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the rod.

SOLUTION



For $\Delta AA'C$:

$$A'C = a\tan\theta$$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta BB'C$:

$$B'C = l\sin\theta - A'C$$
$$= l\sin\theta - a\tan\theta$$

$$BB' = \frac{B'C}{\tan\theta} = \frac{l\sin\theta - a\tan\theta}{\tan\theta}$$

$$x_B = BB' = l\cos\theta - a$$

$$\delta x_B = -l\sin\theta\,\delta\theta$$

<u>Virtual Work</u>:

$$\delta U = 0: \quad P\delta x_B - Q\delta y_A = 0$$

$$P(-l\sin\theta\,\delta\theta) - Q\left(-\frac{a}{\cos^2\theta}\delta\theta\right) = 0$$

$$Pl\sin\theta\cos^2\theta = Qa$$

or

$$Q = P\frac{l}{a}\sin\theta\cos^2\theta$$

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PROBLEM 10.28 (Continued)

with $l = 600\text{ mm},\ a = 100\text{ mm},\ P = 50\text{ N},\text{ and } Q = 90\text{ N}$

$$90\text{ N} = (50\text{ N})\frac{600\text{ mm}}{100\text{ mm}}\sin\theta\cos^2\theta$$

or $\sin\theta\cos^2\theta = 0.300$

Solving numerically $\theta = 19.81°\text{ and } 51.9°$ ◀

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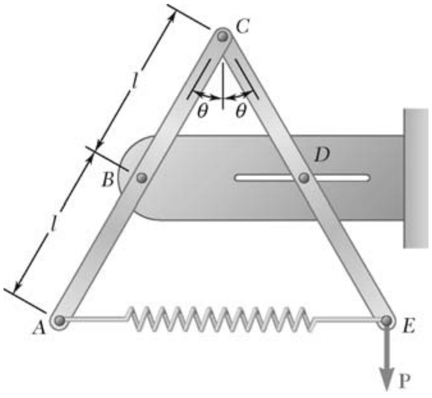
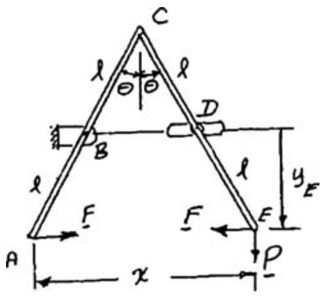


PROBLEM 10.29

Two rods *AC* and *CE* are connected by a pin at *C* and by a spring *AE*. The constant of the spring is *k*, and the spring is unstretched when $\theta = 30°$. For the loading shown, derive an equation in *P*, *θ*, *l*, and *k* that must be satisfied when the system is in equilibrium.

SOLUTION



$$y_E = l\cos\theta$$

$$\delta y_E = -l\sin\theta\,\delta\theta$$

<u>Spring</u>:

Unstretched length = 2*l*

$$x = 2(2l\sin\theta) = 4l\sin\theta$$

$$\delta x = 4l\cos\theta\,\delta\theta$$

$$F = k(x - 2l)$$

$$F = k\left(4l\sin\theta - 2l\right)$$

<u>Virtual Work</u>:

$$\delta U = 0: \quad P\delta y_E - F\delta x = 0$$

$$P(-l\sin\theta\,\delta\theta) - k(4l\sin\theta - 2l)(4l\cos\theta\,\delta\theta) = 0$$

$$-P\sin\theta - 8kl(2\sin\theta - 1)\cos\theta = 0$$

or

$$\frac{P}{8kl} = (1 - 2\sin\theta)\frac{\cos\theta}{\sin\theta}$$

$$\frac{P}{8kl} = \frac{1 - 2\sin\theta}{\tan\theta} \blacktriangleleft$$

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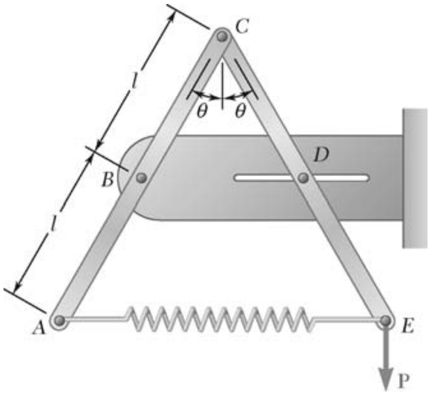

PROBLEM 10.30

Two rods *AC* and *CE* are connected by a pin at *C* and by a spring *AE*. The constant of the spring is 1.5 lb/in., and the spring is unstretched when $\theta = 30°$. Knowing that $l = 10$ in. and neglecting the weight of the rods, determine the value of θ corresponding to equilibrium when $P = 40$ lb.

SOLUTION

From the analysis of Problem 10.29

$$\frac{P}{8kl} = \frac{1 - 2\sin\theta}{\tan\theta}$$

Then with

$$P = 160 \text{ N}, l = 0.2 \text{ m, and } k = 300 \text{ N/m}$$

$$\frac{160 \text{ N}}{8(300 \text{ N/m})(0.2 \text{ m})} = \frac{1 - 2\sin\theta}{\tan\theta}$$

or

$$\frac{1 - 2\sin\theta}{\tan\theta} = \frac{1}{3} = 0.3333$$

Solving numerically $\theta = 25.0°$ ◀

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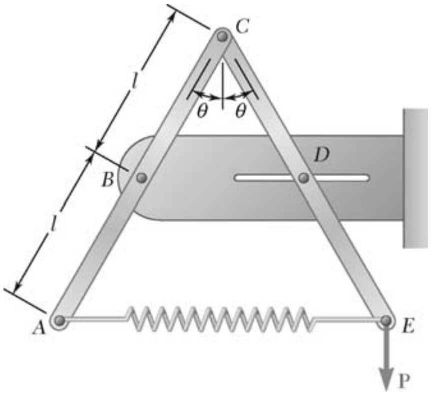


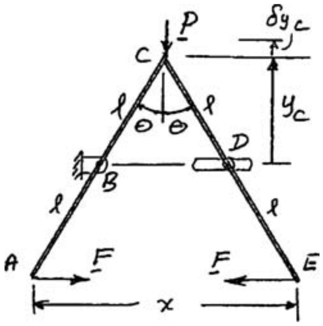
PROBLEM 10.31

Solve Problem 10.30 assuming that force **P** is moved to C and acts vertically downward.

PROBLEM 10.30 Two rods AC and CE are connected by a pin at C and by a spring AE. The constant of the spring is 1.5 lb/in., and the spring is unstretched when $\theta = 30°$. Knowing that $l = 10$ in. and neglecting the weight of the rods, determine the value of θ corresponding to equilibrium when $P = 40$ lb.

SOLUTION

$$y_C = l\cos\theta, \quad \delta y_C = -l\sin\theta\delta\theta$$



Spring:

$$\text{Unstretched length} = 2l$$

$$x = 2(2l\sin\theta) = 4l\sin\theta$$

$$\delta x = 4l\cos\theta\delta\theta$$

$$F = k(x - 2l)$$

$$F = k(4l\sin\theta - 2l)$$

<u>Virtual Work</u>:

$$\delta U = 0: \quad -P\delta y_C - F\delta x$$

$$-P(-l\sin\theta\delta\theta) - k(4l\sin\theta - 2l)(4l\cos\theta\delta\theta) = 0$$

$$P\sin\theta - 8kl(2\sin\theta - 1)\cos\theta = 0$$

or $$\frac{P}{8kl} = (2\sin\theta - 1)\frac{\cos\theta}{\sin\theta}$$

with $$l = 200 \text{ mm}, \; k = 300 \text{ N/m}, \text{ and } P = 160 \text{ N}$$

$$\frac{(160 \text{ N})}{8(300 \text{ N/m})(0.2)} = (2\sin\theta - 1)\frac{\cos\theta}{\sin\theta}$$

or $$(2\sin\theta - 1)\frac{\cos\theta}{\sin\theta} = \frac{1}{3}$$

Solving numerically $\theta = 39.65°$

and $\theta = 68.96°$ $\quad\theta = 39.7°$ ◀

and $\quad\theta = 69.0°$ ◀

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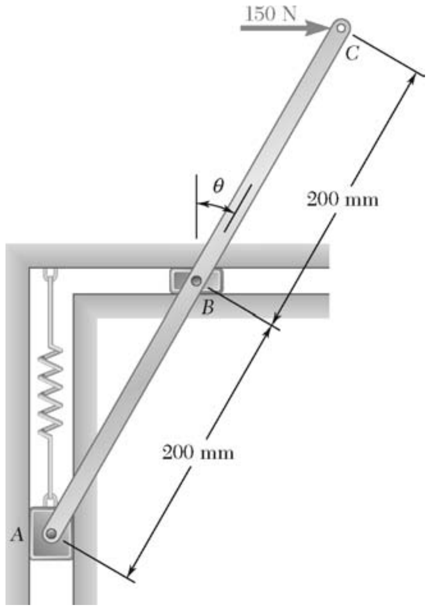
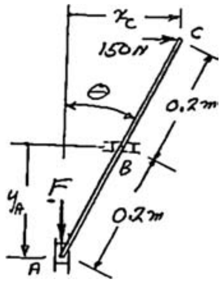


PROBLEM 10.32

Rod ABC is attached to blocks A and B that can move freely in the guides shown. The constant of the spring attached at A is $k = 3$ kN/m, and the spring is unstretched when the rod is vertical. For the loading shown, determine the value of θ corresponding to equilibrium.

SOLUTION



$$x_C = (0.4 \text{ m})\sin\theta$$
$$\delta x_C = 0.4\cos\theta\delta\theta$$
$$y_A = (0.2 \text{ m})\cos\theta$$
$$\delta y_A = -0.2\sin\theta\delta\theta$$

Spring:

Unstretched length $= 0.2$ m

$$F = k(0.2 \text{ m} - y_A) = k(0.2 - 0.2\cos\theta)$$
$$= (300 \text{ N/m})(0.2)(1-\cos\theta)$$
$$F = 600(1-\cos\theta)$$

Virtual Work: $\delta U = 0$: $(150 \text{ N})\delta x_C + F\delta y_A = 0$

$$150(0.4\cos\theta\delta\theta) + 600(1-\cos\theta)(-0.2\sin\theta\delta\theta) = 0$$

$$\frac{150(0.4)}{600(0.2)} = \frac{1}{2}: \quad \frac{1}{2} = (1-\cos\theta)\tan\theta$$

Solve by trial and error: $\theta = 57.2°$ ◀

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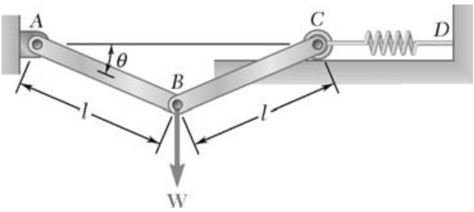
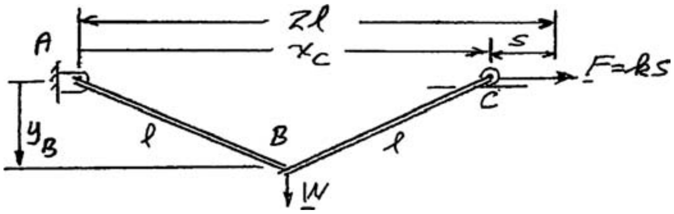


PROBLEM 10.33

A load **W** of magnitude 600 N is applied to the linkage at *B*. The constant of the spring is $k = 2.5$ kN/m, and the spring is unstretched when *AB* and *BC* are horizontal. Neglecting the weight of the linkage and knowing that $l = 300$ mm, determine the value of θ corresponding to equilibrium.

SOLUTION



$$x_C = 2l\cos\theta \quad \delta x_C = -2l\sin\theta\delta\theta$$
$$y_B = l\sin\theta \quad \delta y_B = l\cos\theta\delta\theta$$
$$F = ks = k(2l - x_C) = 2kl(1-\cos\theta)$$

Virtual Work: $\delta U = 0$: $F\delta x_C + W\delta y_B = 0$

$$2kl(1-\cos\theta)(-2l\sin\theta\delta\theta) + W(l\cos\theta\delta\theta) = 0$$

$$4kl^2(1-\cos\theta)\sin\theta = Wl\cos\theta$$

or $$(1-\cos\theta)\tan\theta = \frac{W}{4kl}$$

Given: $l = 0.3$ m, $W = 600$ N, $k = 2500$ N/m

Then $$(1-\cos\theta)\tan\theta = \frac{600 \text{ N}}{4(2500 \text{ N/m})(0.3 \text{ m})}$$

or $$(1-\cos\theta)\tan\theta = 0.2$$

Solving numerically $\theta = 40.22°$ $\qquad \theta = 40.2°$ ◀

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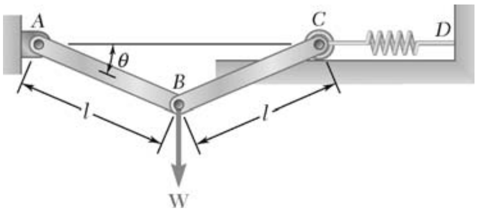
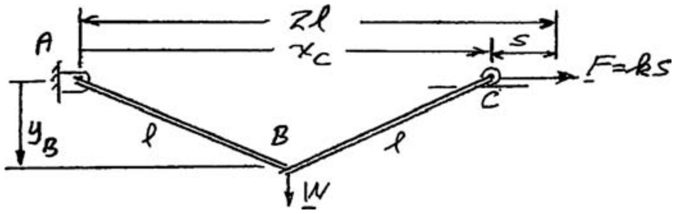


PROBLEM 10.34

A vertical load **W** is applied to the linkage at *B*. The constant of the spring is *k*, and the spring is unstretched when *AB* and *BC* are horizontal. Neglecting the weight of the linkage, derive an equation in θ, *W*, *l*, and *k* that must be satisfied when the linkage is in equilibrium.

SOLUTION



$$x_C = 2l\cos\theta \quad \delta x_C = -2l\sin\theta\delta\theta$$
$$y_B = l\sin\theta \quad \delta y_B = l\cos\theta\delta\theta$$
$$F = ks = k(2l - x_C) = 2kl(1-\cos\theta)$$

Virtual Work: $\delta U = 0$: $F\delta x_C + W\delta y_B = 0$

$$2kl(1-\cos\theta)(-2l\sin\theta\delta\theta) + W(l\cos\theta\delta\theta) = 0$$

$$4kl^2(1-\cos\theta)\sin\theta = Wl\cos\theta$$

or $$(1-\cos\theta)\tan\theta = \frac{W}{4kl}$$

From above $$(1-\cos\theta)\tan\theta = \frac{W}{4kl} \blacktriangleleft$$

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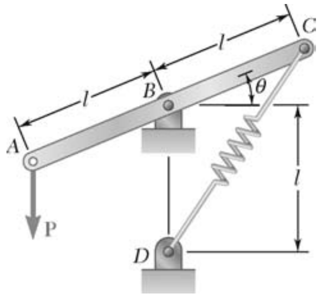
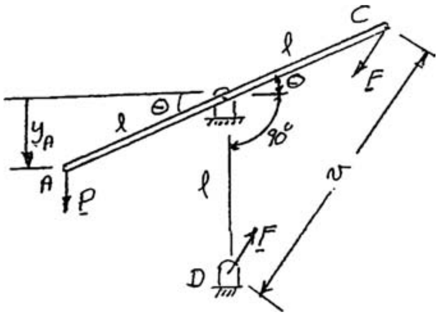


PROBLEM 10.35

Knowing that the constant of spring *CD* is *k* and that the spring is unstretched when rod *ABC* is horizontal, determine the value of θ corresponding to equilibrium for the data indicated.

$$P = 300\text{ N},\quad l = 400\text{ mm},\quad k = 5\text{ kN/m}.$$

SOLUTION



$$y_A = l\sin\theta$$

$$\delta y_A = l\cos\theta\delta\theta$$

Spring: $\quad v = CD$

Unstretched when $\quad \theta = 0$

so that $\quad v_0 = \sqrt{2}l$

For θ: $\quad v = 2l\sin\left(\frac{90° + \theta}{2}\right)$

$$\delta v = l\cos\left(45° + \frac{\theta}{2}\right)\delta\theta$$

Stretched length: $\quad s = v - v_0 = 2l\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}l$

Then $\quad F = ks = kl\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]$

<u>Virtual Work</u>:

$$\delta U = 0:\quad P\delta y_A - F\delta v = 0$$

$$Pl\cos\theta\delta\theta - kl\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]l\cos\left(45° + \frac{\theta}{2}\right)\delta\theta = 0$$

or $\quad \frac{P}{kl} = \frac{1}{\cos\theta}\left[2\sin\left(45° + \frac{\theta}{2}\right)\cos\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right)\right]$

$$= \frac{1}{\cos\theta}\left[2\sin\left(45° + \frac{\theta}{2}\right)\cos\left(45° + \frac{\theta}{2}\right)\cos\theta - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right)\right]$$

$$= 1 - \sqrt{2}\frac{\cos\left(45° \frac{\theta}{2}\right)}{\cos\theta}$$

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PROBLEM 10.35 (Continued)

Now, with $P = 300\text{ N}, \quad l = 400\text{ mm}, \quad \text{and} \quad k = 5\text{ kN/m}$

$$\frac{(300\text{ N})}{(5000\text{ N/m})(0.4\text{ m})} = 1 - \sqrt{2}\,\frac{\cos\left(45^\circ + \frac{\theta}{2}\right)}{\cos\theta}$$

or

$$\frac{\cos\left(45^\circ + \frac{\theta}{2}\right)}{\cos\theta} = 0.60104$$

Solving numerically $\theta = 22.6^\circ$ ◀

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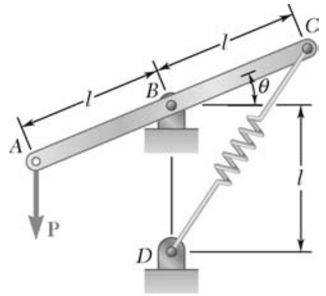

PROBLEM 10.36

Knowing that the constant of spring CD is k and that the spring is unstretched when rod ABC is horizontal, determine the value of θ corresponding to equilibrium for the data indicated.

$$P = 75 \text{ lb}, \quad l = 15 \text{ in.}, \quad k = 20 \text{ lb/in.}$$

SOLUTION

From the analysis of Problem 10.35, we have

$$\frac{P}{kl} = 1 - \sqrt{2}\,\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta}$$

with $P = 75 \text{ lb}, \quad l = 15 \text{ in.} \quad \text{and} \quad k = 20 \text{ lb/in.}$

$$\frac{(75 \text{ lb})}{(20 \text{ lb/in.})(15 \text{ in.})} = 1 - \sqrt{2}\,\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta}$$

or

$$\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = 0.53033$$

Solving numerically $\theta = 51.1°$ ◀

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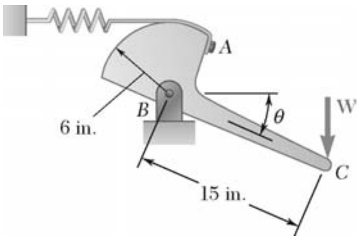
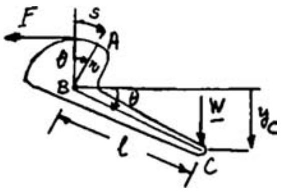


PROBLEM 10.37

A load **W** of magnitude 72 lb is applied to the mechanism at *C*. Neglecting the weight of the mechanism, determine the value of θ corresponding to equilibrium. The constant of the spring is $k = 20$ lb/in., and the spring is unstretched when $\theta = 0$.

SOLUTION



$$s = r\theta$$

$$\delta s = r\delta\theta$$

$$F = ks = kr\theta$$

$$y_C = l\sin\theta$$

$$\delta y_C = l\cos\theta\delta\theta$$

Virtual Work: $\delta U = -F\delta s + W\delta y_C = 0$

$$-kr\theta(r\delta\theta) + W(l\cos\theta\delta\theta) = 0$$

$$\frac{\theta_C}{\cos\theta} = \frac{Wl}{kr^2} = \frac{(72\text{ lb})(15\text{ in.})}{(20\text{ lb/in.})(6\text{ in.})^2} = 1.500$$

Solving by trial and error: $\theta = 0.91486$ rad $\qquad \theta = 52.4° \blacktriangleleft$

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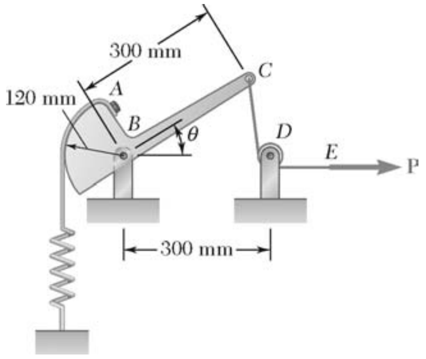


PROBLEM 10.38

A force **P** of magnitude 240 N is applied to end *E* of cable *CDE*, which passes under pulley *D* and is attached to the mechanism at *C*. Neglecting the weight of the mechanism and the radius of the pulley, determine the value of θ corresponding to equilibrium. The constant of the spring is $k = 4$ kN/m, and the spring is unstretched when $\theta = 90°$.

SOLUTION

$$s = r\left(\frac{\pi}{2} - \theta\right)$$

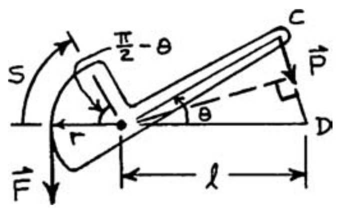
$$\delta s = -r\delta\theta$$

$$F = ks = kr\left(\frac{\pi}{2} - \theta\right)$$

$$CD = 2l\sin\frac{\theta}{2}$$

$$\delta(CD) = 2l\cos\frac{\theta}{2}\left(\frac{1}{2}\delta\theta\right)$$

$$= l\cos\frac{\theta}{2}\delta\theta$$



Virtual Work:

Since **F** tends to decrease *s* and **P** tends to decrease *CD*, we have

$$\delta U = -F\delta s - P\delta(CD) = 0$$

$$-kr\left(\frac{\pi}{2} - \theta\right)(-r\delta\theta) - P\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

$$\frac{\frac{\pi}{2} - \theta}{\cos\frac{\theta}{2}} = \frac{pl}{kr^2} = \frac{(240\text{ N})(0.3\text{ m})}{(4000\text{ N/m})(0.12\text{ m})^2} = 1.25$$

Solving by trial and error: $\theta = 0.33868$ rad $\qquad \theta = 19.40°$ ◀

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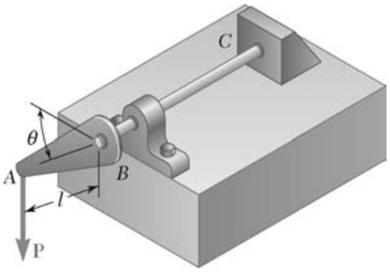
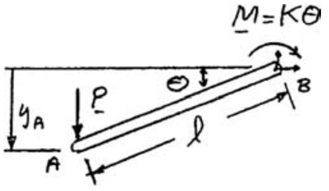


PROBLEM 10.39

The lever *AB* is attached to the horizontal shaft *BC* that passes through a bearing and is welded to a fixed support at *C*. The torsional spring constant of the shaft *BC* is *K*; that is, a couple of magnitude *K* is required to rotate end *B* through 1 rad. Knowing that the shaft is untwisted when *AB* is horizontal, determine the value of θ corresponding to the position of equilibrium when $P = 100$ N, $l = 250$ mm, and $K = 12.5\ \text{N}\cdot\text{m/rad}$.

SOLUTION



We have

$$y_A = l\sin\theta$$

$$\delta y_A = l\cos\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P\delta y_A - M\delta\theta = 0$$

$$Pl\cos\theta\delta\theta - K\theta\delta\theta = 0$$

or

$$\frac{\theta}{\cos\theta} = \frac{Pl}{K} \qquad (1)$$

with $P = 100$ N, $l = 250$ mm and $K = 12.5\ \text{N}\cdot\text{m/rad}$

$$\frac{\theta}{\cos\theta} = \frac{(100\ \text{N})(0.250\ \text{m})}{12.5\ \text{N}\cdot\text{m/rad}}$$

or

$$\frac{\theta}{\cos\theta} = 2.0000$$

Solving numerically $\theta = 59.0°$ ◀

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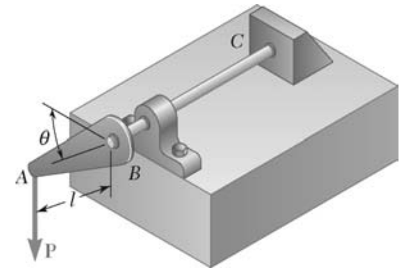
PROBLEM 10.40

Solve Problem 10.39 assuming that $P = 350$ N, $l = 250$ mm, and $K = 12.5$ N · m/rad. Obtain answers in each of the following quadrants: $0 < \theta < 90°$, $270° < \theta < 360°$, $360° < \theta < 450°$.



PROBLEM 10.39 The lever *AB* is attached to the horizontal shaft *BC* that passes through a bearing and is welded to a fixed support at *C*. The torsional spring constant of the shaft *BC* is *K*; that is, a couple of magnitude *K* is required to rotate end *B* through 1 rad. Knowing that the shaft is untwisted when *AB* is horizontal, determine the value of θ corresponding to the position of equilibrium when $P = 100$ N, $l = 250$ mm, and $K = 12.5$ N · m/rad.

SOLUTION

Using Equation (1) of Problem 10.39 and

$$P = 350 \text{ N}, \quad l = 250 \text{ mm} \quad \text{and} \quad K = 12.5 \text{ N} \cdot \text{m/rad}$$

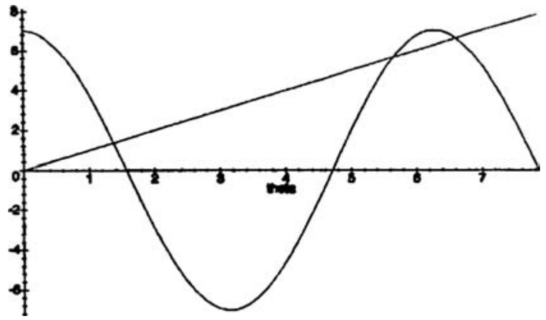
We have
$$\frac{\theta}{\cos\theta} = \frac{(350 \text{ N})(0.250 \text{ m})}{12.5 \text{ N} \cdot \text{m/rad}}$$

or
$$\frac{\theta}{\cos\theta} = 7 \quad \text{or} \quad \theta = 7\cos\theta \qquad (1)$$

The solutions to this equation can be shown graphically using any appropriate graphing tool, such as Maple, with the command: plot ({theta, 7 * cos(theta)}, $t = 0.5 *$ Pi/2);

Thus, we plot $y = \theta$ and $y = 7\cos\theta$ in the range

$$0 \le \theta \le \frac{5\pi}{2}$$



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PROBLEM 10.40 (Continued)

We observe that there are three points of intersection, which implies that Equation (1) has three roots in the specified range of θ.

$$0 \le \theta \le 90°\left(\frac{\pi}{2}\right); \quad \theta = 1.37333 \text{ rad}, \quad \theta = 78.69° \qquad \theta = 78.7° \blacktriangleleft$$

$$270 \le \theta \le 360°\left(\frac{3\pi}{2} \le \theta \le 2\pi\right); \quad \theta = 5.65222 \text{ rad}, \quad \theta = 323.85° \qquad \theta = 324° \blacktriangleleft$$

$$360 \le \theta \le 450°\left(2\pi \le \theta \le \frac{5\pi}{2}\right); \quad \theta = 6.61597 \text{ rad}, \quad \theta = 379.07 \qquad \theta = 379° \blacktriangleleft$$

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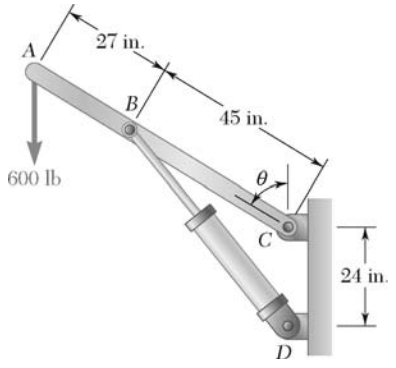
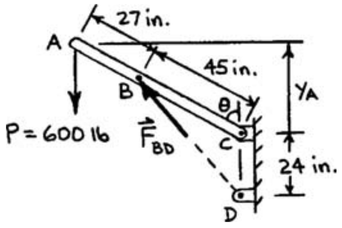


PROBLEM 10.41

The position of boom *ABC* is controlled by the hydraulic cylinder *BD*. For the loading shown, determine the force exerted by the hydraulic cylinder on pin *B* when $\theta = 65°$.

SOLUTION

We have



$$y_A = (72\text{ in.})\cos\theta \quad \delta y_A = -72\sin\theta\delta\theta$$

$$(BD)^2 = (BC)^2 + (CD)^2 - 2(BC)(CD)\cos(180° - \theta)$$

$$= (45)^2 + (24)^2 + 2(45)(24)\cos\theta$$

$$(BD)^2 = 2601 + 2160\cos\theta \qquad (1)$$

Differentiating:

$$2(BD)\delta(BD) = -2160\sin\theta\delta\theta$$

$$\delta(BD) = -\frac{1080}{BD}\sin\theta\delta\theta \qquad (2)$$

<u>Virtual Work</u>: Noting that **P** tends to decrease y_A and $\mathbf{F}_{BD}$ tends to increase *BD*, write

$$\delta U = -P\delta y_A + F_{BD}\delta(BD) = 0$$

$$-P(-72\sin\theta\delta\theta) + F_{BD}\left(-\frac{1080}{BD}\sin\theta\delta\theta\right) = 0$$

$$F_{BD} = \frac{1}{15}(BD)P$$

or, since

$$P = 600\text{ lb:} \quad F_{BD} = \frac{600}{15}(BD) = (40\text{ lb})(BD) \qquad (3)$$

Making $\theta = 65°$ in Eq. (1), we have

$$(BD)^2 = 2601 + 2160\cos 65° = 3513.9$$

$$BD = 59.278$$

Carrying into Eq. (3):

$$F_{BD} = (40\text{ lb})(59.278) = 2371.1\text{ lb}$$

$F_{BD} = 2370$ lb ◀

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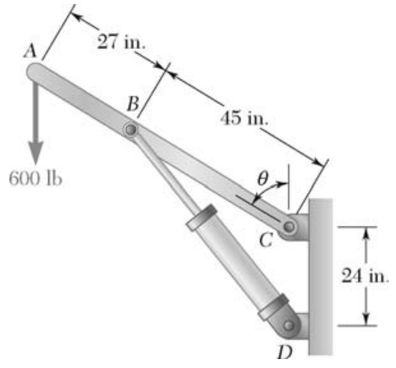

PROBLEM 10.42

The position of boom *ABC* is controlled by the hydraulic cylinder *BD*. For the loading shown, (*a*) express the force exerted by the hydraulic cylinder on pin *B* as a function of the length *BD*, (*b*) determine the smallest possible value of the angle θ if the maximum force that the cylinder can exert on pin *B* is 2.5 kips.

SOLUTION

(*a*) See solution of Problem 10.41 for the derivation of Eq. (3):

$$F_{BD} = (40\text{ lb})(BD) \blacktriangleleft$$

(*b*) For $(F_{BD})_{\max} = 2.5\text{ kips} = 2500\text{ lb}$, we have

$$2500\text{ lb} = (40\text{ lb})(BD)$$
$$BD = 62.5$$

Carrying this value into Eq. (1) of Problem 10.41, write

$$(BD)^2 = 2601 + 2160\cos\theta$$
$$(62.5)^2 = 2601 + 2160\cos\theta$$
$$\cos\theta = 0.60428$$

$$\theta = 52.8° \blacktriangleleft$$

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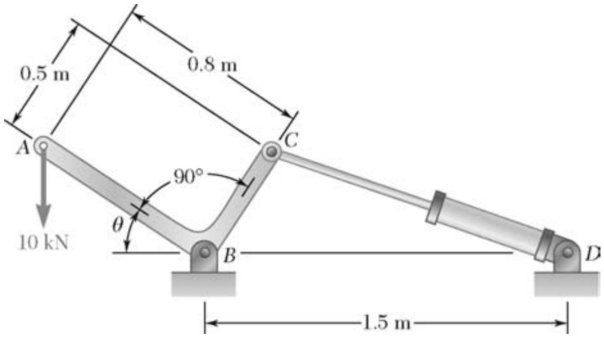
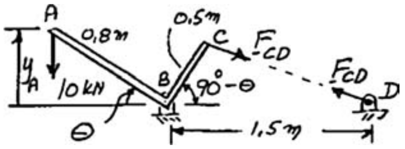


PROBLEM 10.43

The position of member *ABC* is controlled by the hydraulic cylinder *CD*. For the loading shown, determine the force exerted by the hydraulic cylinder on pin *C* when $\theta = 55°$.

SOLUTION



$$y_A = (0.8\text{ m})\sin\theta$$

$$\delta y_A = 0.8\cos\theta\delta\theta$$

$$CD^2 = BC^2 + BD^2 - 2(BC)(BD)\cos(90° - \theta)$$

$$CD^2 = 0.5^2 + 1.5^2 - 2(0.5)(1.5)\sin\theta$$

$$CD^2 = 2.5 - 1.5\sin\theta \qquad (1)$$

$$2(CD)(\delta_{CD}) = -1.5\cos\theta\delta\theta \quad \delta_{CD} = -\frac{3\cos\theta}{4CD}\delta\theta$$

<u>Virtual Work</u>:

$$\delta U = 0: \quad -(10\text{ kN})\delta y_A - F_{CD}\delta_{CD} = 0$$

$$-10(0.8\cos\theta\delta\theta) - F_{CD}\left(-\frac{3\cos\theta}{4CD}\delta\theta\right) = 0$$

$$F_{CD} = \frac{32}{3}CD \qquad (2)$$

For $\theta = 55°$:

Eq. (1): $CD^2 = 2.5 - 1.5\sin 55° = 1.2713; \quad CD = 1.1275\text{ m}$

Eq. (2): $F_{CD} = \frac{32}{3}CD = \frac{32}{3}(1.1275) = 12.027\text{ kN}$

$\mathbf{F}_{CD} = 12.03\text{ kN}$ ↘ ◀

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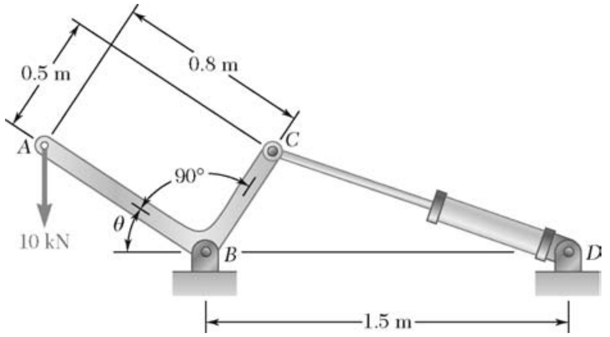
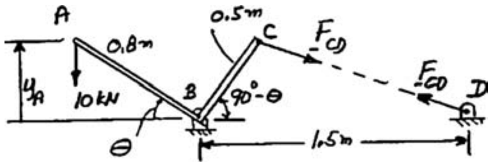


PROBLEM 10.44

The position of member *ABC* is controlled by the hydraulic cylinder *CD*. Determine the angle θ knowing that the hydraulic cylinder exerts a 15-kN force on pin *C*.

SOLUTION



$$y_A = (0.8 \text{ m})\sin\theta$$
$$\delta y_A = 0.8\cos\theta\delta\theta$$
$$CD^2 = BC^2 + BD^2 - 2(BC)(BD)\cos(90° - \theta)$$
$$CD^2 = 0.5^2 + 1.5^2 - 2(0.5)(1.5)\sin\theta$$
$$CD^2 = 2.5 - 1.5\sin\theta \qquad (1)$$

$$2(CD)(\delta_{CD}) = -1.5\cos\theta\delta\theta; \qquad \delta_{CD} = -\frac{3\cos\theta}{4CD}\delta\theta$$

Virtual Work: $\delta U = 0$: $\quad -(10 \text{ kN})\delta y_A - F_{CD}\delta_{CD} = 0$

$$-10(0.8\cos\theta\delta\theta) - F_{CD}\left(-\frac{3\cos\theta}{4CD}\delta\theta\right) = 0$$

$$F_{CD} = \frac{32}{3}CD \qquad (2)$$

For $F_{CD} = 15 \text{ kN}$:

Eq. (2): $\quad 15 \text{ kN} = \frac{32}{3}CD$: $\qquad CD = \frac{45}{32} = 1.40625 \text{ m}$

Eq. (1): $\quad (1.40625)^2 = 2.5 - 1.5\sin\theta$: $\qquad \sin\theta = 0.34831$

$$\theta = 20.38°$$

$\theta = 20.4°$ ◀

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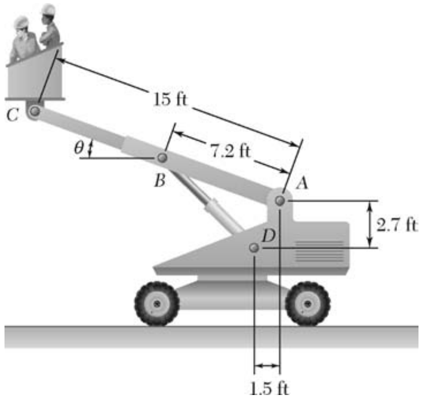
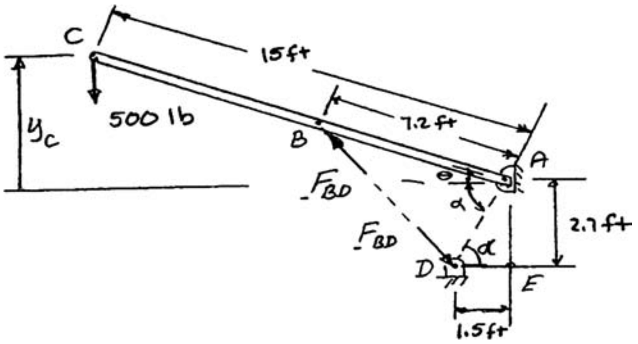


PROBLEM 10.45

The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The workers and the platform together weigh 500 lb and their combined center of gravity is located directly above *C*. For the position when $\theta = 20°$, determine the force exerted on pin *B* by the single hydraulic cylinder *BD*.

SOLUTION



In ΔADE:

$$\tan\alpha = \frac{AE}{DE} = \frac{2.7 \text{ ft}}{1.5 \text{ ft}}$$

$$\alpha = 60.945°$$

$$AD = \frac{2.7 \text{ ft}}{\sin 60.945°} = 3.0887 \text{ m}$$

From the geometry:

$$y_C = (15 \text{ ft})\sin\theta$$

$$\delta y_C = (15 \text{ ft})\cos\theta\delta\theta$$

Then, in triangle *BAD*: Angle $BAD = \alpha + \theta$

Law of cosines:

$$BD^2 = AB^2 + AD^2 - 2(AB)(AD)\cos(\alpha + \theta)$$

or

$$BD^2 = (7.2 \text{ ft})^2 + (3.0887 \text{ ft})^2 - 2(7.2 \text{ ft})(3.0887 \text{ ft})\cos(\alpha + \theta)$$

$$BD^2 = 61.380 \text{ ft}^2 - (44.477\cos(\alpha + \theta))\text{ ft}^2 \quad (1)$$

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PROBLEM 10.45 (Continued)

And then

$$2(BD)(\delta BD) = (44.477\sin(\alpha+\theta))\delta\theta$$

$$\delta BD = \frac{44.477\sin(\alpha+\theta)}{2(BD)}\delta\theta$$

Virtual Work: $\delta U = 0:\ \ -P\delta y_C + F_{BD}\delta BD = 0$

Substituting $$-(500\text{ lb})(15\text{ ft})\cos\theta\delta\theta + F_{BD}\left[\frac{(44.477\text{ ft}^2)\sin(\alpha+\theta)}{2(BD)}\delta\theta\right] = 0$$

or $$F_{BD} = \left[337.25\frac{\cos\theta}{\sin(\alpha+\theta)}BD\right]\text{lb/ft} \qquad (2)$$

Now, with $\theta = 20°$ and $\alpha = 60.945°$

Equation (1):

$$BD^2 = 61.380 - 44.477\cos(60.945° + 20°)$$

$$BD^2 = 54.380$$

$$BD = 7.3743\text{ ft}$$

Equation (2):

$$F_{BD} = \left[337.25\frac{\cos 20°}{\sin(60.945° + 20°)}(7.3743\text{ ft})\right]\text{lb/ft}$$

or $F_{BD} = 2366\text{ lb}$ $\mathbf{F}_{BD} = 2.370\text{ lb}$ ↖ ◀

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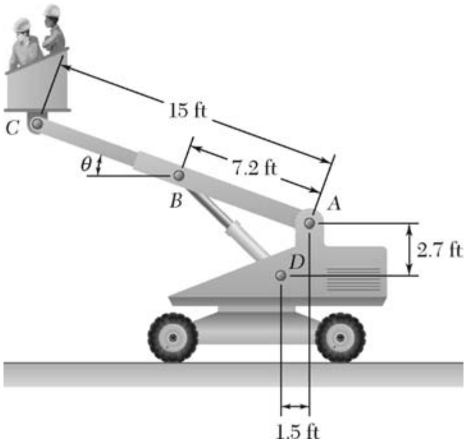

PROBLEM 10.46

Solve Problem 10.45 assuming that the workers are lowered to a point near the ground so that $\theta = -20°$.

PROBLEM 10.45 The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The workers and the platform together weigh 500 lb and their combined center of gravity is located directly above *C*. For the position when $\theta = 20°$, determine the force exerted on pin *B* by the single hydraulic cylinder *BD*.

SOLUTION

Using the figure and analysis of Problem 10.45, including Equations (1) and (2), and with $\theta = -20°$, we have

Equation (1): $BD^2 = 61.380 - 44.477\cos(60.945° - 20°)$

$$BD^2 = 27.785$$

$$BD = 5.2711 \text{ ft}$$

Equation (2): $F_{BD} = 337.25\dfrac{\cos(-20°)}{\sin(60.945° - 20°)}(5.2711)$

$F_{BD} = 2549 \text{ lb}$ or $\mathbf{F}_{BD} = 2550 \text{ lb}$ ◀

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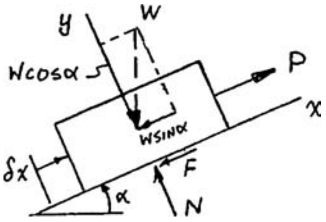
PROBLEM 10.47

A block of weight W is pulled up a plane forming an angle α with the horizontal by a force **P** directed along the plane. If μ is the coefficient of friction between the block and the plane, derive an expression for the mechanical efficiency of the system. Show that the mechanical efficiency cannot exceed $\frac{1}{2}$ if the block is to remain in place when the force **P** is removed.

SOLUTION



$$\text{Input work} = P\delta x$$
$$\text{Output work} = (W\sin\alpha)\delta x$$

Efficiency:

$$\eta = \frac{W\sin\alpha\delta x}{P\delta x} \quad \text{or} \quad \eta = \frac{W\sin\alpha}{P} \qquad (1)$$

$$+\nearrow \Sigma F_x = 0: \quad P - F - W\sin\alpha = 0 \quad \text{or} \quad P = W\sin\alpha + F \qquad (2)$$

$$+\nwarrow \Sigma F_y = 0: \quad N - W\cos\alpha = 0 \quad \text{or} \quad N = W\cos\alpha$$

$$F = \mu N = \mu W\cos\alpha$$

Equation (2): $$P = W\sin\alpha + \mu W\cos\alpha = W(\sin\alpha + \mu\cos\alpha)$$

Equation (1): $$\eta = \frac{W\sin\alpha}{W(\sin\alpha + \mu\cos\alpha)} \quad \text{or} \quad \eta = \frac{1}{1+\mu\cot\alpha}$$ ◀

If block is to remain in place when $P = 0$, we know (see Chapter 8) that $\phi_s \geq \alpha$ or, since

$$\mu = \tan\phi_s, \quad \mu \geq \tan\alpha$$

Multiply by $\cot\alpha$: $$\mu\cot\alpha \geq \tan\alpha\cot\alpha = 1$$

Add 1 to each side: $$1 + \mu\cot\alpha \geq 2$$

Recalling the expression for η, we find $$\eta \leq \frac{1}{2}$$ ◀

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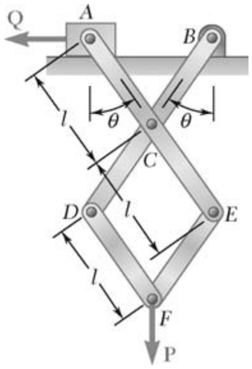


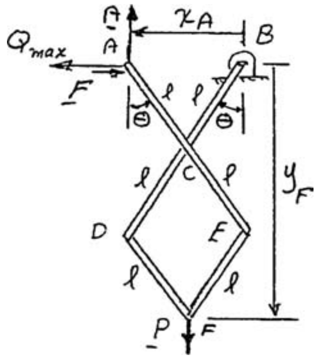
PROBLEM 10.48

Denoting by μ_s the coefficient of static friction between the block attached to rod *ACE* and the horizontal surface, derive expressions in terms of *P*, μ_s, and θ for the largest and smallest magnitude of the force **Q** for which equilibrium is maintained.

SOLUTION

For the linkage:

$$+\circlearrowleft \Sigma M_B = 0: \quad -x_A + \frac{x_A}{2}P = 0 \quad \text{or} \quad \mathbf{A} = \frac{P}{2}\uparrow$$



Then: $$F = \mu_s A = \mu_s \frac{P}{2} = \frac{1}{2}\mu_s P$$

Now $$x_A = 2l\sin\theta$$
$$\delta x_A = 2l\cos\theta\,\delta\theta$$

and $$y_F = 3l\cos\theta$$
$$\delta y_F = -3l\sin\theta\,\delta\theta$$

<u>Virtual Work</u>: $$\delta U = 0: \quad (Q_{\max} - F)\delta x_A + P\delta y_F = 0$$

$$\left(Q_{\max} - \frac{1}{2}\mu_s P\right)(2l\cos\theta\,\delta\theta) + P(-3l\sin\theta\,\delta\theta) = 0$$

or $$Q_{\max} = \frac{3}{2}P\tan\theta + \frac{1}{2}\mu_s P$$

$$Q_{\max} = \frac{P}{2}(3\tan\theta + \mu_s) \blacktriangleleft$$

For $Q_{\min}$, motion of *A* impends to the right and **F** acts to the left. We change μ_s to $-\mu_s$ and find

$$Q_{\min} = \frac{P}{2}(3\tan\theta - \mu_s) \blacktriangleleft$$

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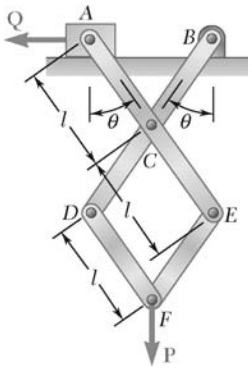

PROBLEM 10.49

Knowing that the coefficient of static friction between the block attached to rod *ACE* and the horizontal surface is 0.15, determine the magnitude of the largest and smallest force **Q** for which equilibrium is maintained when $\theta = 30°$, $l = 0.2$ m, and $P = 40$ N.

SOLUTION

Using the results of Problem 10.48 with

$$\theta = 30°$$
$$l = 0.2 \text{ m}$$
$$P = 40 \text{ N}, \quad \text{and} \quad \mu_s = 0.15$$

We have

$$Q_{\max} = \frac{P}{2}(3\tan\theta + \mu_s)$$
$$= \frac{(40 \text{ N})}{2}(3\tan 30° + 0.15)$$
$$= 37.64 \text{ N}$$

$Q_{\max} = 37.6$ N ◀

and

$$Q_{\min} = \frac{P}{2}(3\tan\theta - \mu_s)$$
$$= \frac{(40 \text{ N})}{2}(3\tan 30° - 0.15)$$
$$= 31.64 \text{ N}$$

$Q_{\min} = 31.6$ N ◀

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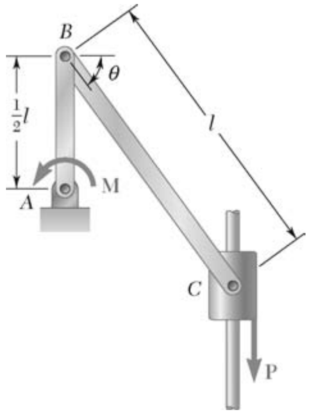
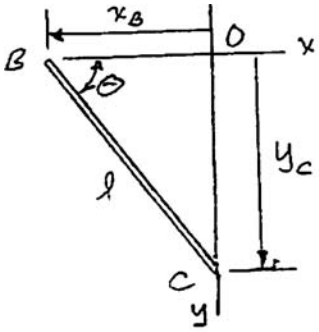


PROBLEM 10.50

Denoting by μ_s the coefficient of static friction between collar C and the vertical rod, derive an expression for the magnitude of the largest couple **M** for which equilibrium is maintained in the position shown. Explain what happens if $\mu_s \geq \tan\theta$.

SOLUTION



Member BC: We have

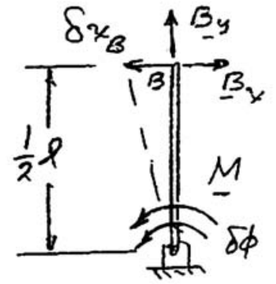
$$x_B = l\cos\theta$$

$$\delta x_B = -l\sin\theta\delta\theta \quad (1)$$

and

$$y_C = l\sin\theta$$

$$\delta y_C = l\cos\theta\delta\theta \quad (2)$$



Member AB: We have

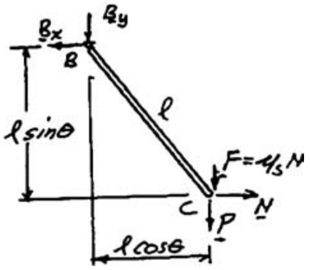
$$\delta x_B = \frac{1}{2}l\delta\phi$$

Substituting from Equation (1),

$$-l\sin\theta\delta\theta = \frac{1}{2}l\delta\phi$$

or

$$\delta\phi = -2\sin\theta\delta\theta \quad (3)$$



Free body of rod BC

For $M_{\max}$, motion of collar C impends upward

$$+\circlearrowleft \Sigma M_B = 0: \quad Nl\sin\theta - (P + \mu_s N)(l\cos\theta) = 0$$

$$N\tan\theta - \mu_s N = P$$

$$N = \frac{P}{\tan\theta - \mu_s}$$

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PROBLEM 10.50 (Continued)

Virtual Work:

$$\delta U = 0: \quad M\delta\phi + (P + \mu_s N)\delta y_C = 0$$

$$M(-2\sin\theta\delta\theta) + (P + \mu_s N)l\cos\theta\delta\theta = 0$$

$$M_{\max} = \frac{(P + \mu_s N)}{2\tan\theta}l = \frac{P + \mu_s \frac{P}{\tan\theta - \mu_s}}{2\tan\theta}l$$

$$\text{or} \quad M_{\max} = \frac{Pl}{2(\tan\theta - \mu_s)} \blacktriangleleft$$

If $\mu_s = \tan\theta$, $M = \infty$, system becomes *self-locking*

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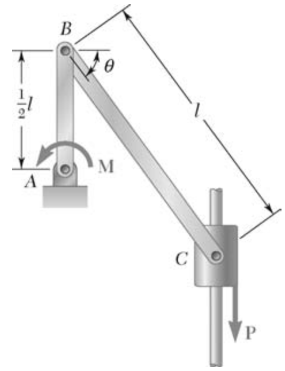

PROBLEM 10.51

Knowing that the coefficient of static friction between collar C and the vertical rod is 0.40, determine the magnitude of the largest and smallest couple **M** for which equilibrium is maintained in the position shown, when $\theta = 35°$, $l = 600$ mm, and $P = 300$ N.

SOLUTION

From the analysis of Problem 10.50, we have

$$M_{\max} = \frac{Pl}{2(\tan\theta + \mu_s)}$$

With $\theta = 35°, \quad l = 0.6 \text{ m}, \quad P = 300 \text{ N}$

$$M_{\max} = \frac{(300 \text{ N})(0.6 \text{ m})}{2(\tan 35° - 0.4)}$$

$$= 299.80 \text{ N}\cdot\text{m} \qquad M_{\max} = 300 \text{ N}\cdot\text{m} \blacktriangleleft$$

For $M_{\min}$, motion of C impends downward and F acts upward. The equations of Problem 10.50 can still be used if we replace μ_s by $-\mu_s$. Then

$$M_{\min} = \frac{Pl}{2(\tan\theta + \mu_s)}$$

Substituting,

$$M_{\min} = \frac{(300 \text{ N})(0.6 \text{ m})}{2(\tan 35° + 0.4)}$$

$$= 81.803 \text{ N}\cdot\text{m} \qquad M_{\min} = 81.8 \text{ N}\cdot\text{m} \blacktriangleleft$$

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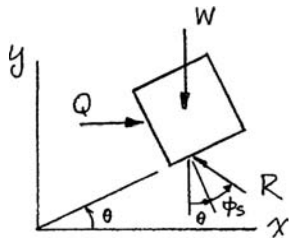
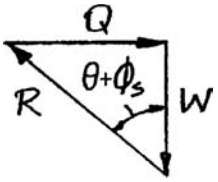
PROBLEM 10.52

Derive an expression for the mechanical efficiency of the jack discussed in Section 8.6. Show that if the jack is to be self-locking, the mechanical efficiency cannot exceed $\frac{1}{2}$.

SOLUTION

Recall Figure 8.9*a*. Draw force triangle





$$Q = W\tan(\theta + \phi_s)$$

$$y = x\tan\theta \text{ so that } \delta y = \delta x \tan\theta$$

$$\text{Input work} = Q\delta x = W\tan(\theta + \phi_s)\delta x$$

$$\text{Output work} = W\delta y = W(\delta x)\tan\theta$$

Efficiency:

$$\eta = \frac{W\tan\theta\delta x}{W\tan(\theta+\phi_s)\delta x}; \qquad \eta = \frac{\tan\theta}{\tan(\theta+\phi_s)} \blacktriangleleft$$

From Page 432, we know the jack is self-locking if

$$\phi_s \geq \theta$$

Then

$$\theta + \phi_s \geq 2\theta$$

so that

$$\tan(\theta+\phi_s) \geq \tan 2\theta$$

From above

$$\eta = \frac{\tan\theta}{\tan(\theta+\phi_s)}$$

It then follows that

$$\eta \leq \frac{\tan\theta}{\tan 2\theta}$$

But

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Then

$$\eta \leq \frac{\tan\theta(1-\tan^2\theta)}{2\tan\theta} = \frac{1-\tan^2\theta}{2} \qquad \eta \leq \frac{1}{2} \blacktriangleleft$$

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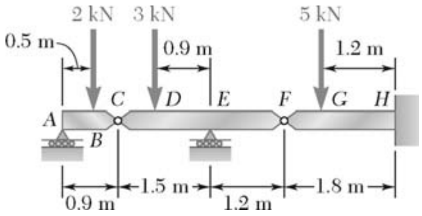
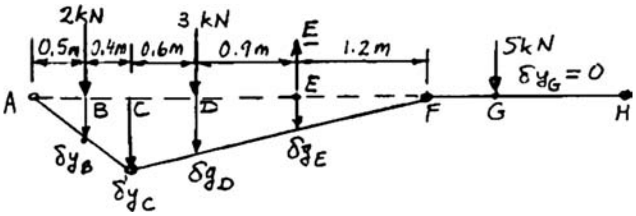


PROBLEM 10.53

Using the method of virtual work, determine the reaction at E.

SOLUTION

We release the support at E and assume a virtual displacement δy_E for Point E.



From similar triangles:

$$\delta y_D = \frac{2.1}{1.2}\delta y_E = 1.75\delta y_E$$

$$\delta y_C = \frac{2.7}{1.2}\delta y_E = 2.25\delta y_E$$

$$\delta y_B = \frac{0.5}{0.9}\delta y_C = \frac{0.5}{0.9}(2.25\delta y_E) = 1.25\delta y_E$$

Virtual Work:

$$\delta U = (2\text{ kN})\delta y_B + (3\text{ kN})\delta y_D - E\delta y_E = 0$$

$$2(1.25\delta y_E) + 3(1.75\delta y_E) - E\delta y_E = 0$$

$E = +7.75$ kN $\quad\quad$ $\mathbf{E} = 7.75$ kN ↑ ◀

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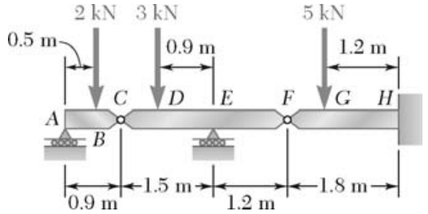
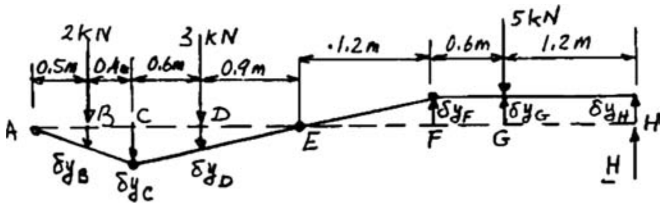


PROBLEM 10.54

Using the method of virtual work, determine separately the force and couple representing the reaction at *H*.

SOLUTION

<u>Force at *H*</u>. We give a vertical virtual displacement δy_H to Point *H*, keeping member *FH* horizontal.



From the geometry of the diagram:

$$\delta y_F = \delta y_G = \delta y_H$$

$$\delta y_D = \frac{0.9}{1.2}\delta y_F = \frac{0.9}{1.2}\delta y_H = 0.75\delta y_H$$

$$\delta y_C = \frac{1.5}{0.9}\delta y_D = \frac{1.5}{0.9}(0.75\delta y_H) = 1.25\delta y_H$$

$$\delta y_B = \frac{0.5}{0.9}\delta y_C = \frac{0.5}{0.9}(1.25\delta y_H) = 0.69444\delta y_H$$

<u>Virtual Work</u>:

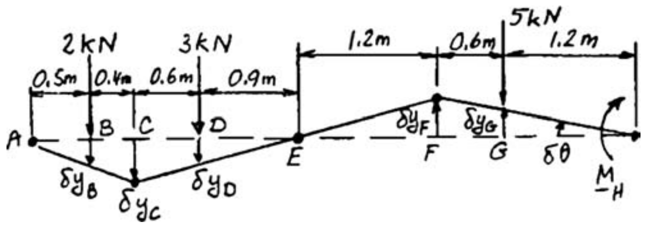
$$\delta U = (2\text{ kN})\delta y_B + (3\text{ kN})\delta y_D - (5\text{ kN})\delta y_G + H\delta y_H = 0$$

$$2(0.69444\delta y_H) + 3(0.75\delta y_H) - 5\delta y_H + H\delta y_H = 0$$

$$H = +1.3611\text{ kN}$$

$$\mathbf{H} = 1.361\text{ kN}\uparrow \blacktriangleleft$$

<u>Couple at *H*</u>. We rotate beam *FH* through $\delta\theta$ about Point *H*.



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PROBLEM 10.54 (Continued)

From the geometry of the diagram:

$$\delta y_G = 1.2\delta\theta \qquad \delta y_F = 1.8\delta\theta$$

$$\delta y_D = \frac{0.9}{1.2}\delta y_F = \frac{0.9}{1.2}(1.8\delta\theta) = 1.35\delta\theta$$

$$\delta y_C = \frac{1.5}{0.9}\delta y_D = \frac{1.5}{0.9}(1.35\delta\theta) = 2.25\delta\theta$$

$$\delta y_B = \frac{0.5}{0.9}\delta y_C = \frac{0.5}{0.9}(2.25\delta\theta) = 1.25\delta\theta$$

<u>Virtual Work</u>:

$$\delta U = (2\text{ kN})\delta y_B + (3\text{ kN})\delta y_D - (5\text{ kN})\delta y_G + M_H\delta\theta = 0$$

$$2(1.25\delta\theta) + 3(1.35\delta\theta) - 5(1.2\delta\theta) + M_H\delta\theta = 0$$

$$M_H = -0.550\text{ kN}\cdot\text{m} \qquad \mathbf{M}_H = 550\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

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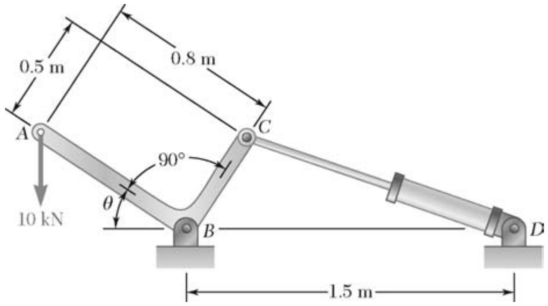
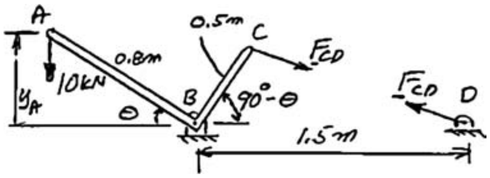


PROBLEM 10.55

Referring to Problem 10.43 and using the value found for the force exerted by the hydraulic cylinder *CD*, determine the change in the length of *CD* required to raise the 10-kN load by 15 mm.

PROBLEM 10.43 The position of member *ABC* is controlled by the hydraulic cylinder *CD*. For the loading shown, determine the force exerted by the hydraulic cylinder on pin *C* when $\theta = 55°$.

SOLUTION



Virtual Work: Assume both δy_A and δ_{CD} increase

$$\delta U = 0: \quad -(10 \text{ kN})\delta y_A - F_{CD}\delta_{CD} = 0$$

Substitute: $\delta y_A = 15$ mm and $F_{CD} = 12.03$ kN

$$-(10 \text{ kN})(15 \text{ mm}) - (12.03 \text{ kN})\delta_{CD} = 0$$

$$\delta_{CD} = -12.47 \text{ mm}$$

The negative sign indicates that *CD* shortened $\qquad \delta_{CD} = 12.47$ mm shorter ◀

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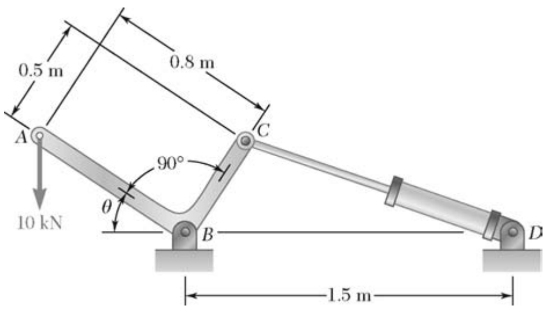
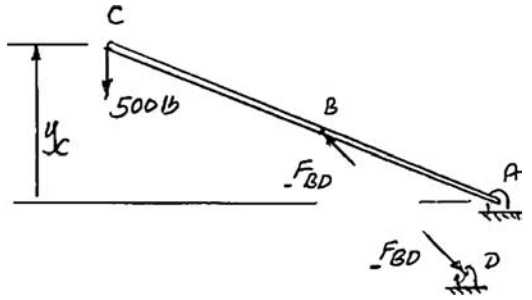


PROBLEM 10.56

Referring to Problem 10.45 and using the value found for the force exerted by the hydraulic cylinder *BD*, determine the change in the length of *BD* required to raise the platform attached at *C* by 2.5 in.

PROBLEM 10.45 The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The workers and the platform together weigh 500 lb and their combined center of gravity is located directly above *C*. For the position when $\theta = 20°$, determine the force exerted on pin *B* by the single hydraulic cylinder *BD*.

SOLUTION



Virtual Work: Assume both δy_C and δ_{BD} increase

$$\delta U = 0: \quad -(500 \text{ lb})\delta y_C + F_{BD}\delta_{BD} = 0$$

Substitute: $\delta y_C = 2.5 \text{ in.} \quad \text{and} \quad F_{BD} = 2370 \text{ lb}$

$$-(500 \text{ lb})(2.5 \text{ in.}) + (2370 \text{ lb})\delta_{BD} = 0$$

$$\delta_{BD} = +0.527 \text{ in.}$$

The positive sign indicates that cylinder *BD* increases in length $\delta_{BD} = 0.527$ in. longer ◀

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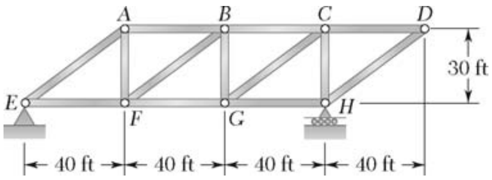
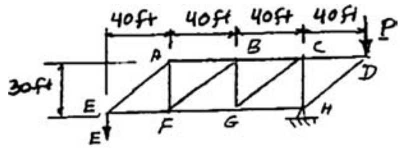
PROBLEM 10.57



Determine the vertical movement of joint D if the length of member BF is increased by 1.5 in. (*Hint:* Apply a vertical load at joint D, and, using the methods of Chapter 6, compute the force exerted by member BF on joints B and F. Then apply the method of virtual work for a virtual displacement resulting in the specified increase in length of member BF. This method should be used only for small changes in the lengths of members.)

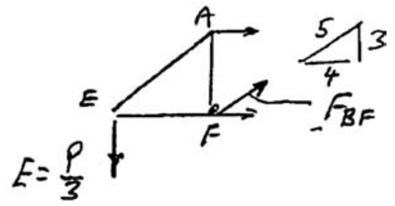
SOLUTION

Apply vertical load P at D.



$$+\circlearrowleft \Sigma M_H = 0: \quad -P(40 \text{ ft}) + E(120 \text{ ft}) = 0$$

$$\mathbf{E} = \frac{P}{3}\downarrow$$



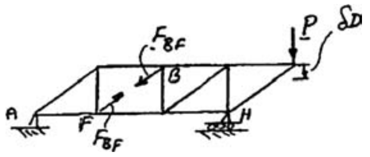
$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BF} - \frac{P}{3} = 0$$

$$F_{BF} = \frac{5}{9}P$$

Virtual Work:

We remove member BF and replace it with forces $\mathbf{F}_{BF}$ and $-\mathbf{F}_{BF}$ at pins F and B, respectively. Denoting the virtual displacements of Points B and F as $\delta \mathbf{r}_B$ and $\delta \mathbf{r}_F$, respectively, and noting that $\mathbf{P}$ and $\overrightarrow{\delta D}$ have the same direction, we have

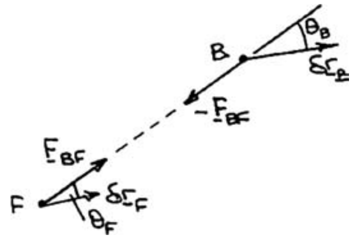
Virtual Work:



$$\delta U = 0: \qquad P\delta D + \mathbf{F}_{BF} \cdot \delta \mathbf{r}_F + (-\mathbf{F}_{BF}) \cdot \delta \mathbf{r}_B = 0$$

$$P\delta D + F_{BF}\delta r_F \cos\theta_F - F_{BF}\delta r_B \cos\theta_B = 0$$

$$P\delta D - F_{BF}(\delta r_B \cos\theta_B - \delta r_F \cos\theta_F) = 0$$



where $(\delta r_B \cos\theta_B - \delta r_F \cos\theta_F) = \delta_{BF}$, which is the change in length of member BF. Thus,

$$P\delta D - F_{BF}\delta_{BF} = 0$$

$$P\delta D - \left(\frac{5}{9}P\right)(1.5 \text{ in.}) = 0$$

$$\delta D = 0.833 \text{ in.}$$

$\delta D = 0.833$ in.$\downarrow$ ◀

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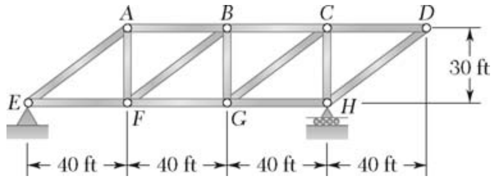
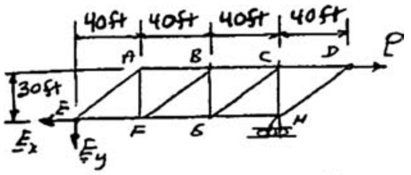


PROBLEM 10.58

Determine the horizontal movement of joint D if the length of member BF is increased by 1.5 in. (See the hint for Problem 10.57.)

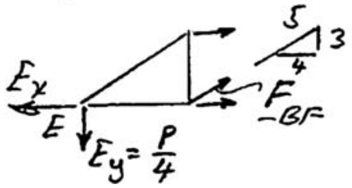
SOLUTION

Apply horizontal load P at D.



$$+\circlearrowleft \Sigma M_H = 0: \quad P(30 \text{ ft}) - E_y(120 \text{ ft}) = 0$$

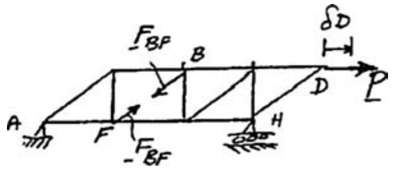
$$\mathbf{E}_y = \frac{P}{4} \downarrow$$



$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BF} - \frac{P}{4} = 0$$

$$F_{BF} = \frac{5}{12}P$$

We remove member BF and replace it with forces $\mathbf{F}_{BF}$ and $-\mathbf{F}_{BF}$ at pins F and B, respectively. Denoting the virtual displacements of Points B and F as $\delta\mathbf{r}_B$ and $\delta\mathbf{r}_F$, respectively, and noting that $\mathbf{P}$ and $\overrightarrow{\delta D}$ have the same direction, we have

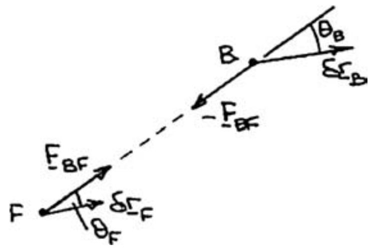


Virtual Work:

$$\delta U = 0: \qquad P\delta D + \mathbf{F}_{BF} \cdot \delta\mathbf{r}_F + (-\mathbf{F}_{BF}) \cdot \delta\mathbf{r}_B = 0$$

$$P\delta D + F_{BF}\delta r_F \cos\theta_F - F_{BF}\delta r_B \cos\theta_B = 0$$

$$P\delta D - F_{BF}(\delta r_B \cos\theta_B - \delta r_F \cos\theta_F) = 0$$



where $(\delta r_B \cos\theta_B - \delta r_F \cos\theta_F) = \delta_{BF}$, which is the change in length of member BF. Thus,

$$P\delta D - F_{BF}\delta_{BF} = 0$$

$$P\delta D - \left(\frac{5}{9}P\right)(1.5 \text{ in.}) = 0$$

$$\delta D = 0.625 \text{ in.}$$

$\delta D = 0.625$ in. $\longrightarrow$ ◀

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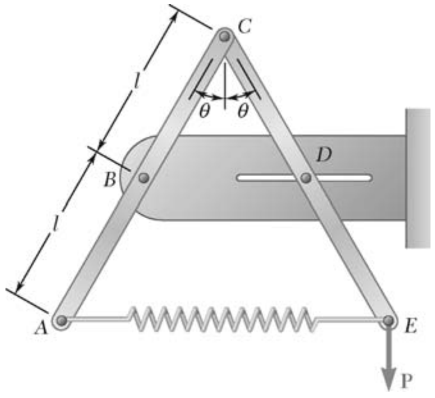
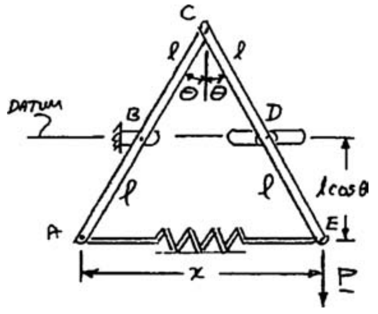


PROBLEM 10.59

Using the method of Section 10.8, solve Problem 10.29.

PROBLEM 10.29 Two rods *AC* and *CE* are connected by a pin at *C* and by a spring *AE*. The constant of the spring is *k*, and the spring is unstretched when $\theta = 30°$. For the loading shown, derive an equation in *P*, *θ*, *l*, and *k* that must be satisfied when the system is in equilibrium.

SOLUTION



Spring: $AE = x = 2(2l\sin\theta) = 4l\sin\theta$

Unstretched length: $x_0 = 4l\sin 30° = 2l$

Deflection of spring

$$s = x - x_0$$

$$s = 2l(2\sin\theta - 1)$$

$$V = \frac{1}{2}ks^2 + Py_E$$

$$V = \frac{1}{2}k\left[2l(2\sin\theta - 1)\right]^2 + P(-l\cos\theta)$$

$$\frac{dV}{d\theta} = 4kl^2(2\sin\theta - 1)2\cos\theta + Pl\sin\theta = 0$$

$$(2\sin\theta - 1)\frac{\cos\theta}{\sin\theta} + \frac{P}{8kl} = 0$$

$$\frac{P}{8kl} = \frac{1 - 2\sin\theta}{\tan\theta} \blacktriangleleft$$

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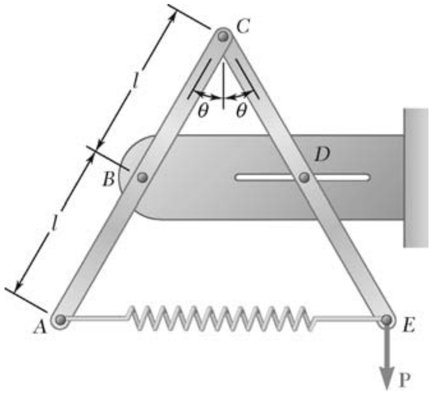

PROBLEM 10.60

Using the method of Section 10.8, solve Problem 10.30.

PROBLEM 10.30 Two rods *AC* and *CE* are connected by a pin at *C* and by a spring *AE*. The constant of the spring is 1.5 lb/in., and the spring is unstretched when $\theta = 30°$. Knowing that $l = 10$ in. and neglecting the weight of the rods, determine the value of θ corresponding to equilibrium when $P = 40$ lb.

SOLUTION

Using the result of Problem 10.59, with

$$P = 40 \text{ lb}$$

$$l = 10 \text{ in.} \quad \text{and} \quad k = 1.5 \text{ lb/in.}$$

$$\frac{P}{8kl} = \frac{1 - 2\sin\theta}{\tan\theta}$$

or

$$\frac{1 - 2\sin\theta}{\tan\theta} = \frac{40 \text{ lb}}{8(1.5 \text{ lb/in.})(10 \text{ in.})} = \frac{1}{3}$$

Solving numerically, $\theta = 25.0°$ ◀

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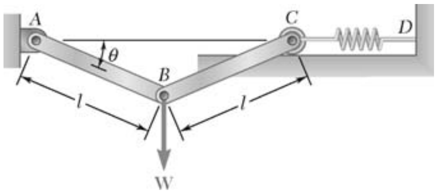
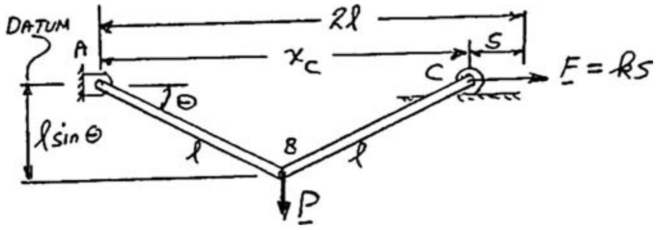


PROBLEM 10.61

Using the method of Section 10.8, solve Problem 10.33.

PROBLEM 10.33 A load **W** of magnitude 600 N is applied to the linkage at *B*. The constant of the spring is $k = 2.5$ kN/m, and the spring is unstretched when *AB* and *BC* are horizontal. Neglecting the weight of the linkage and knowing that $l = 300$ mm, determine the value of θ corresponding to equilibrium.

SOLUTION



$$P = 600 \text{ N}$$

$$l = 0.3 \text{ m}, \quad \text{and} \quad k = 2500 \text{ N} \cdot \text{m}$$

We have

$$(l - \cos\theta)\tan\theta = \frac{600 \text{ N}}{4(2500 \text{ N/m})(0.3 \text{ m})}$$

$$= 0.2$$

Solving numerically $\theta = 40.2°$ ◀

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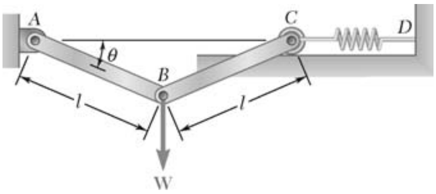
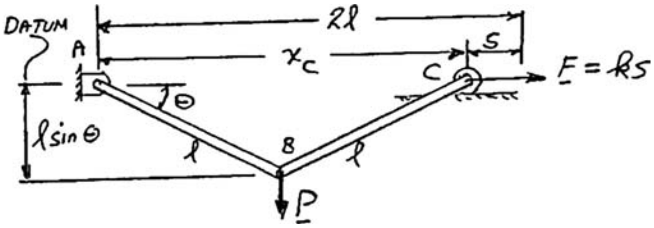


PROBLEM 10.62

Using the method of Section 10.8, solve Problem 10.34.

PROBLEM 10.34 A vertical load **W** is applied to the linkage at *B*. The constant of the spring is *k*, and the spring is unstretched when *AB* and *BC* are horizontal. Neglecting the weight of the linkage, derive an equation in *θ*, *W*, *l*, and *k* that must be satisfied when the linkage is in equilibrium.

SOLUTION



$$V = \frac{1}{2}ks^2 + Py_B$$

$$V = \frac{1}{2}k(2i - x_C)^2 + Py_B$$

$$x_C = 2l\cos\theta \quad \text{and} \quad y_B = -l\sin\theta$$

Thus

$$V = \frac{1}{2}k(2l - 2l\cos\theta)^2 - Pl\sin\theta$$

$$= 2kl^2(1 - \cos\theta)^2 - Pl\sin\theta$$

$$\frac{dV}{d\theta} = 2kl^2\,2(1 - \cos\theta)\sin\theta - Pl\cos\theta = 0$$

$$(1 - \cos\theta)\tan\theta = \frac{P}{4kl} \blacktriangleleft$$

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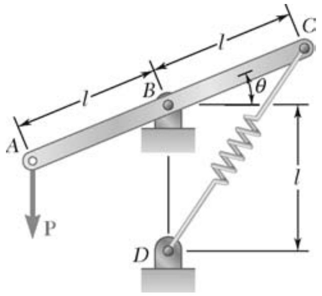


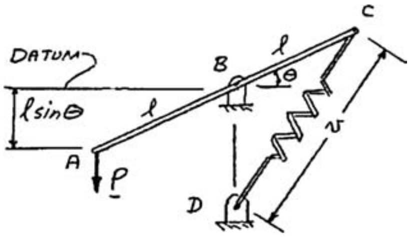
PROBLEM 10.63

Using the method of Section 10.8, solve Problem 10.35.

PROBLEM 10.35 Knowing that the constant of spring *CD* is *k* and that the spring is unstretched when rod *ABC* is horizontal, determine the value of θ corresponding to equilibrium for the data indicated.

$$P = 300 \text{ N}, \quad l = 400 \text{ mm}, \quad k = 5 \text{ kN/m}.$$

SOLUTION



Spring $\quad v = 2l\sin\left(\frac{90° + \theta}{2}\right)$

$$v = 2l\sin\left(45° + \frac{\theta}{2}\right)$$

Unstretched $(\theta = 0)$ $\quad v_0 = 2l\sin 45° = \sqrt{2}l$

Deflection of spring $\quad s = v - v_0 = 2l\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}l$

$$V = \frac{1}{2}ks^2 + Py_A = \frac{1}{2}kl^2\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]^2 + P(-l\sin\theta)$$

$$\frac{dV}{d\theta} = kl^2\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]\cos\left(45° + \frac{\theta}{2}\right) - Pl\cos\theta = 0$$

$$\left[2\sin\left(45° + \frac{\theta}{2}\right)\cos\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right)\right] = \frac{P}{kl}\cos\theta$$

$$\cos\theta - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right) = \frac{P}{kl}\cos\theta$$

Divide each member by cos θ $\quad 1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = \frac{P}{kl}$

Then with $P = 300$ N, $l = 0.4$ m and $k = 5000$ N/m

$$1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = \frac{300 \text{ N}}{(5000 \text{ N/m})(0.4 \text{ m})}$$

$$= 0.15$$

or $\quad \frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = 0.60104$

Solving numerically $\quad \theta = 22.6°$ ◀

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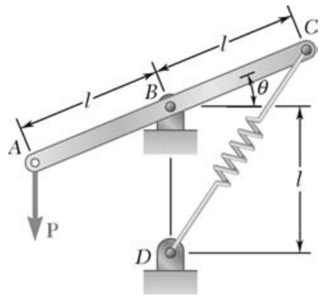

PROBLEM 10.64

Using the method of Section 10.8, solve Problem 10.36.

PROBLEM 10.36 Knowing that the constant of spring *CD* is k and that the spring is unstretched when rod *ABC* is horizontal, determine the value of θ corresponding to equilibrium for the data indicated.

$$P = 75\text{ lb},\quad l = 15\text{ in.},\quad k = 20\text{ lb/in.}$$

SOLUTION

Using the results of Problem 10.63 with $P = 75$ lb, $l = 15$ in. and $k = 20$ lb/in., we have

$$1-\sqrt{2}\,\frac{\cos\left(45^\circ+\frac{\theta}{2}\right)}{\cos\theta} = \frac{P}{kl}$$

$$= \frac{75\text{ lb}}{(20\text{ lb/in.})(15\text{ in.})}$$

$$= 0.25$$

or

$$\frac{\cos\left(45^\circ+\frac{\theta}{2}\right)}{\cos\theta} = 0.53033$$

Solving numerically $\theta = 51.058^\circ$ $\qquad \theta = 51.1^\circ$ ◀

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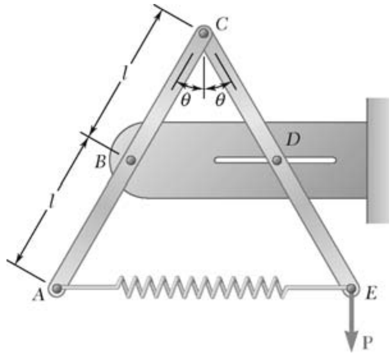
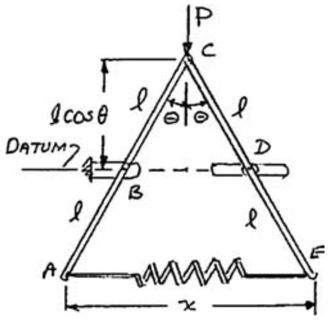


PROBLEM 10.65

Using the method of Section 10.8, solve Problem 10.31.

PROBLEM 10.31 Solve Problem 10.30 assuming that force **P** is moved to *C* and acts vertically downward.

SOLUTION



Spring: $AE = x = 2(2l\sin\theta) = 4l\sin\theta$

Unstretched length: $x_0 = 4l\sin 30° = 2l$

Deflection of spring

$$s = x - x_0$$

$$s = 2l(2\sin\theta - 1)$$

$$V = \frac{1}{2}ks^2 + Py_C$$

$$= \frac{1}{2}k[2l(2\sin\theta - 1)]^2 + P(l\cos\theta)$$

$$V = 2kl^2(2\sin\theta - 1)^2 + Pl\cos\theta$$

$$\frac{dV}{d\theta} = 4kl^2(2\sin\theta - 1)2\cos\theta - Pl\sin\theta = 0$$

$$(1 - 2\sin\theta)\frac{\cos\theta}{\sin\theta} + \frac{P}{8kl} = 0$$

$$\frac{P}{8kl} = \frac{2\sin\theta - 1}{\tan\theta}$$

with $P = 40$ lb, $l = 10$ in. and $k = 1.5$ lb/in.

We have $$\frac{(40 \text{ lb})}{8(1.5 \text{ lb/in.})(10 \text{ in.})} = \frac{2\sin\theta - 1}{\tan\theta}$$

or $$\frac{2\sin - 1}{\tan\theta} = \frac{1}{3}$$

Solving numerically $\theta = 39.65°$ and $68.96°$ $\theta = 39.7°$

$\theta = 69.0°$ ◀

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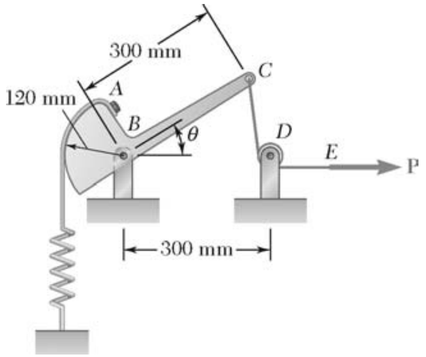
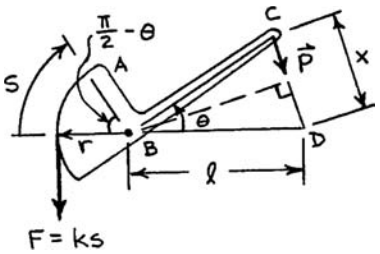


PROBLEM 10.66

Using the method of Section 10.8, solve Problem 10.38.

PROBLEM 10.38 A force **P** of magnitude 240 N is applied to end *E* of cable *CDE*, which passes under pulley *D* and is attached to the mechanism at *C*. Neglecting the weight of the mechanism and the radius of the pulley, determine the value of θ corresponding to equilibrium. The constant of the spring is $k = 4$ kN/m, and the spring is unstretched when $\theta = 90°$.

SOLUTION



For spring: $s = r\left(\dfrac{\pi}{2} - \theta\right)$

$$V_e = \frac{1}{2}ks^2 = \frac{1}{2}kr^2\left(\frac{\pi}{2} - \theta\right)^2$$

For force **P**: $x = 2l\sin\dfrac{\theta}{2}$

$$V_p = Px = 2Pl\sin\frac{\theta}{2}$$

Then $V = V_e + V_p = \dfrac{1}{2}kr^2\left(\dfrac{\pi}{2} - \theta\right)^2 + 2Pl\sin\dfrac{\theta}{2}$

$$\frac{dV}{d\theta} = -kr^2\left(\frac{\pi}{2} - \theta\right) + Pl\cos\frac{\theta}{2}$$

$$= -(4000\text{ N/m})(0.12\text{ m})^2\left(\frac{\pi}{2} - \theta\right) + (240\text{ N})(0.3\text{ m})\cos\frac{\theta}{2} = 0$$

$$\frac{\pi}{2} - \theta = 1.25\cos\frac{\theta}{2}$$

Solving by trial and error: $\theta = 0.33868$ rad $\qquad \theta = 19.40°$ ◀

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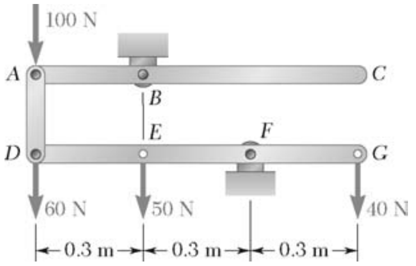
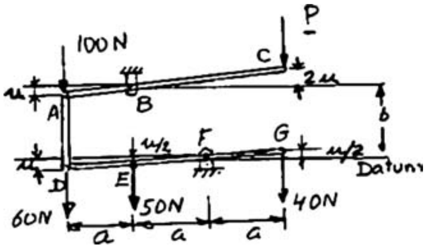


PROBLEM 10.67

Show that equilibrium is neutral in Problem 10.1.

PROBLEM 10.1 Determine the vertical force **P** that must be applied at *C* to maintain the equilibrium of the linkage.

SOLUTION



We have

$$y_A = b - u$$
$$y_C = b + 2u$$
$$y_D = -u$$
$$y_E = -\frac{1}{2}u$$
$$y_G = +\frac{1}{2}u$$

$$V = (100\text{ N})y_A + Py_C + (60\text{ N})y_D + (50\text{ N})y_E + (40\text{ N})y_G$$
$$= 100(b-u) + P(b+2u) + 60(-u) + 50\left(-\frac{1}{2}u\right) + 40\left(\frac{1}{2}u\right)$$

$$\frac{dV}{du} = -100 + 2P - 60 - 25 + 20 = 0$$
$$P = 82.5\text{ N}$$

Substituting $P = 82.5\text{ N}$ in the expression obtained for V:

$$V = (100 + 82.5)b + (-100 + 165 - 60 - 25 + 20)u$$
$$V = 182.5 \quad b = \text{constant}$$

Since V is constant, the equilibrium is neutral ◀

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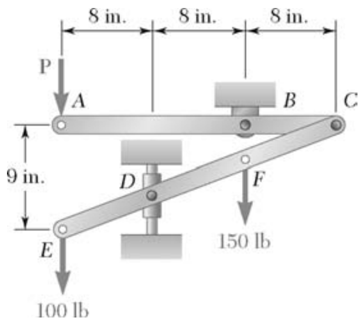
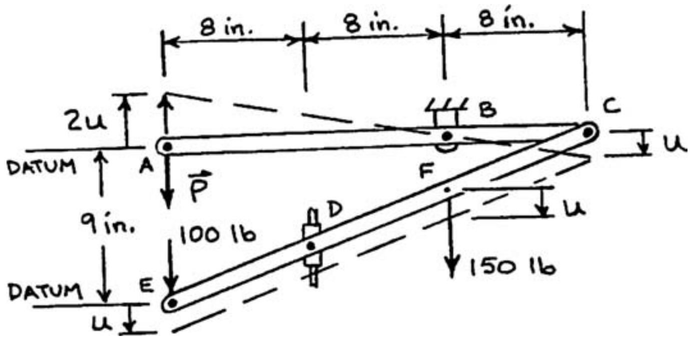


PROBLEM 10.68

Show that equilibrium is neutral in Problem 10.6.

PROBLEM 10.6 The two-bar linkage shown is supported by a pin and bracket at *B* and a collar at *D* that slides freely on a vertical rod. Determine the force **P** required to maintain the equilibrium of the linkage.

SOLUTION



$$y_A = 2u$$
$$y_E = -u$$
$$y_F = -u$$
$$V = Py_A + (100\text{ lb})y_E + (150\text{ lb})y_F$$
$$V = P(2u) + (100\text{ lb})(-u) + (150\text{ lb})(-u)$$
$$\frac{dV}{du} = 2P - 100 - 150 = 0$$
$$P = 125\text{ lb}$$

Now, substitute $P = 125$ lb in expression for *V*, making $V = 0$. Thus, *V* is constant and

equilibrium is neutral. ◀

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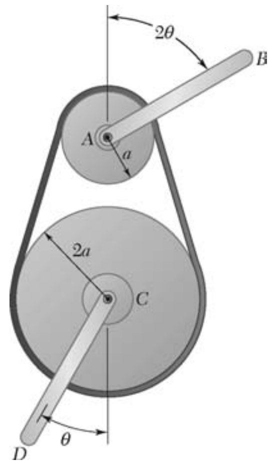
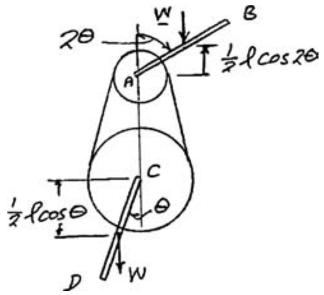


PROBLEM 10.69

Two uniform rods, each of mass m and length l, are attached to drums that are connected by a belt as shown. Assuming that no slipping occurs between the belt and the drums, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



$$W = mg$$

$$V = W\left(\frac{l}{2}\cos 2\theta\right) - W\left(\frac{l}{2}\cos\theta\right)$$

$$\frac{dV}{d\theta} = W\frac{l}{2}(-2\sin 2 + \sin\theta)$$

$$\frac{d^2V}{d\theta^2} = W\frac{1}{2}(-4\cos 2\theta - \cos\theta)$$

Equilibrium: $\dfrac{dV}{d\theta} = 0$: $\quad \dfrac{Wl}{2}(-2\sin 2\theta + \sin\theta) = 0$

or $\quad \sin\theta(-4\cos\theta + 1) = 0$

Solving $\quad \theta = 0,\ 75.5°,\ 180°,\text{ and } 284°$

Stability: $\dfrac{d^2V}{d\theta^2} = W\dfrac{l}{2}(-4\cos 2\theta - \cos\theta)$

At $\theta = 0$: $\dfrac{d^2V}{d\theta^2} = W\dfrac{l}{2}(-4-1) < 0$ $\qquad \theta = 0,$ Unstable ◀

At $\theta = 75.5°$: $\dfrac{d^2V}{d\theta^2} = W\dfrac{l}{2}(-4(-.874) - .25) > 0$ $\qquad \theta = 75.5°,$ Stable ◀

At $\theta = 180°$: $\dfrac{d^2V}{d\theta^2} = W\dfrac{l}{2}(-4+1) < 0$ $\qquad \theta = 180.0°,$ Unstable ◀

At $\theta = 284°$: $\dfrac{d^2V}{d\theta^2} = W\dfrac{l}{2}(-4(-.874) - .25) > 0$ $\qquad \theta = 284°,$ Stable ◀

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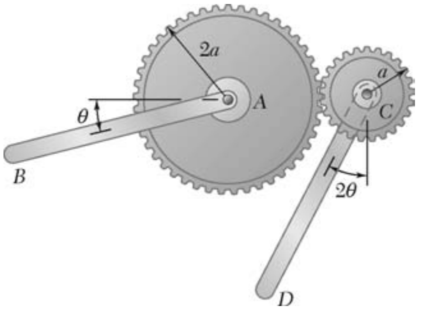


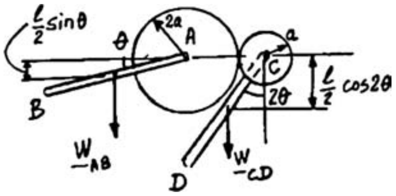
PROBLEM 10.70

Two uniform rods *AB* and *CD*, of the same length *l*, are attached to gears as shown. Knowing that rod *AB* weighs 3 lb and that rod *CD* weighs 2 lb, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

$$V = -W_{AB}\left(\frac{l}{2}\sin\theta\right) - W_{CD}\left(\frac{l}{2}\cos 2\theta\right)$$

$$\frac{dV}{d\theta} = -\frac{1}{2}W_{AB}l\cos\theta + W_{CD}l\sin 2\theta$$



Equilibrium: $$\frac{dV}{d\theta} = 0$$

$$-\frac{1}{2}W_{AB}\,l\cos\theta + W_{CD}l\sin 2\theta = 0$$

$$-W_{AB}\cos\theta + 4W_{CD}\sin\theta\cos\theta = 0$$

$$-W_{AB}\cos\theta\left(1 - \frac{W_{CD}}{W_{AB}}\sin\theta\right) = 0$$

$$\cos\theta = 0 \quad \text{and} \quad \sin\theta = \frac{W_{AB}}{4W_{CD}}$$

$$\theta = 90°, \quad \theta = 270°, \quad \theta = \sin^{-1}\frac{W_{AB}}{4W_{CD}} \qquad (1)$$

For $$W_{AB} = 3\text{ lb} \quad \text{and} \quad W_{CD} = 2\text{ lb, We have}$$

$$\theta = 90°, \quad \theta = 270°, \quad \theta = \sin^{-1}\frac{3}{8}$$

$$\theta = 90°, \quad \theta = 270°, \quad \theta = 22.0°, \quad \theta = 158.0°$$ ◀

Stability: $$\frac{d^2V}{d\theta^2} = \left(\frac{1}{2}W_{AB}\sin\theta + 2W_{CD}\cos 2\theta)\right)l \qquad (2)$$

$$\theta = 270°: \quad \frac{d^2V}{d\theta^2} = \left[\frac{1}{2}(3)\sin(270°) + 2(2)\cos(540°)\right]l = -5.5l < 0, \text{ unstable}$$ ◀

$$\theta = 22.0°: \quad \frac{d^2V}{d\theta^2} = \left[\frac{1}{2}(3)\sin 220° + 2(2)\cos 44.0°\right]l > 0 \quad \text{Stable}$$ ◀

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PROBLEM 10.70 (Continued)

$$\theta = 90°: \quad \frac{d^2V}{d\theta^2} = \left[\frac{1}{2}(3)\sin 90° + 2(2)\cos 180°\right] l = 2.5l < 0, \quad \text{Unstable} \quad \blacktriangleleft$$

$$\theta = 150.0°: \quad \frac{d^2V}{d\theta^2} = \left[\frac{1}{2}(3)\sin 158° + 2(2)\cos 316°\right] l = 3.44l > 0, \quad \text{Stable} \quad \blacktriangleleft$$

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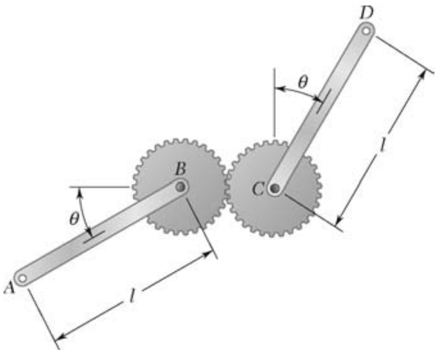


PROBLEM 10.71

Two uniform rods, each of mass *m*, are attached to gears of equal radii as shown. Determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

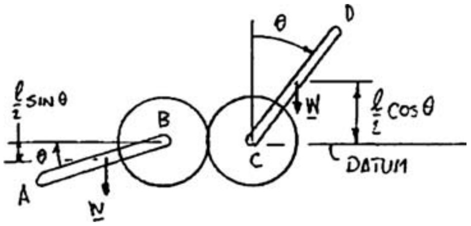
Potential energy

$$V = W\left(-\frac{l}{2}\sin\theta\right) + W\left(\frac{l}{2}\cos\theta\right) \quad W = mg$$

$$= W\frac{l}{2}(\cos\theta - \sin\theta)$$

$$\frac{dV}{d\theta} = \frac{Wl}{2}(-\sin\theta - \cos\theta)$$

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}(\sin\theta - \cos\theta)$$



For equilibrium: $\dfrac{dV}{d\theta} = 0: \quad \sin\theta = -\cos\theta$

or $\tan\theta = -1$

Thus $\theta = -45.0°$ and $\theta = 135.0°$

Stability:

At $\theta = -45.0°$:

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}[\sin(-45°) - \cos 45°]$$

$$= \frac{Wl}{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) < 0$$

$\theta = -45.0°$, Unstable ◀

At $\theta = 135.0°$:

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}(\sin 135° - \cos 135°)$$

$$= \frac{Wl}{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) > 0$$

$\theta = 135.0°$, Stable ◀

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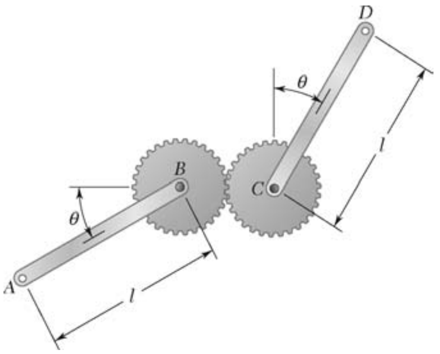


PROBLEM 10.72

Two uniform rods, *AB* and *CD*, are attached to gears of equal radii as shown. Knowing that $W_{AB} = 8$ lb and $W_{CD} = 4$ lb, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

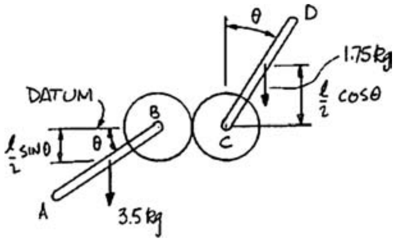
Potential energy

$$V = (3.5\text{ kg}\times 9.81\text{ m/s}^2)\left(-\frac{l}{2}\sin\theta\right) + (1.75\text{ kg}\times 9.81\text{ m/s}^2)\left(\frac{l}{2}\cos\theta\right)$$

$$= (8.5838\text{ N})l(-2\sin\theta + \cos\theta)$$

$$\frac{dV}{d\theta} = (8.5838\text{ N})l(-2\cos\theta - \sin\theta)$$

$$\frac{d^2V}{d\theta^2} = (8.5838\text{ N})l(2\sin\theta - \cos\theta)$$



Equilibrium: $\frac{dV}{d\theta} = 0$: $\quad -2\cos\theta - \sin\theta = 0$

or $\tan\theta = -2$

Thus $\theta = -63.4°$ and $116.6°$

Stability

At $\theta = -63.4°$:

$$\frac{d^2V}{d\theta^2} = (8.5838\text{ N})l[2\sin(-63.4°) - \cos(-63.4°)]$$

$$= (8.5838\text{ N})l(-1.788 - 0.448) < 0$$

$\theta = -63.4°$, Unstable ◀

At $\theta = 116.6°$:

$$\frac{d^2V}{d\theta^2} = (8.5838\text{ N})l[2\sin(116.6°) - \cos(116.6°)]$$

$$= (8.5838\text{ N})l(1.788 + 0.447) > 0$$

$\theta = 116.6°$, Stable ◀

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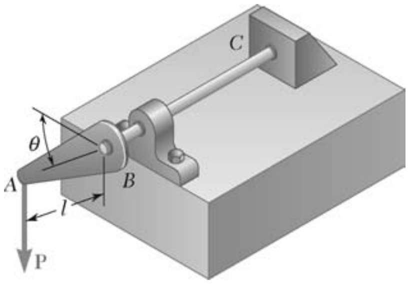


PROBLEM 10.73

Using the method of Section 10.8, solve Problem 10.39. Determine whether the equilibrium is stable, unstable, or neutral. (*Hint:* The potential energy corresponding to the couple exerted by a torsion spring is $\frac{1}{2}K\theta^2$, where K is the torsional spring constant and θ is the angle of twist.)

PROBLEM 10.39 The lever AB is attached to the horizontal shaft BC that passes through a bearing and is welded to a fixed support at C. The torsional spring constant of the shaft BC is K; that is, a couple of magnitude K is required to rotate end B through 1 rad. Knowing that the shaft is untwisted when AB is horizontal, determine the value of θ corresponding to the position of equilibrium when $P = 100$ N, $l = 250$ mm, and $K = 12.5\ \text{N}\cdot\text{m/rad}$.

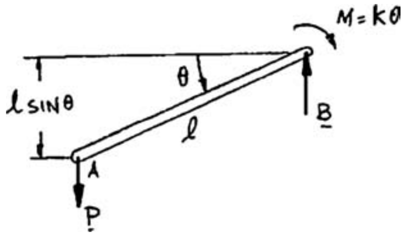
SOLUTION

Potential energy

$$V = \frac{1}{2}K\theta^2 - Pl\sin\theta$$

$$\frac{dV}{d\theta} = K\theta - Pl\cos\theta$$

$$\frac{d^2V}{d\theta^2} = K + Pl\sin\theta$$



Equilibrium:

$$\frac{dV}{d\theta} = 0: \quad \cos\theta = \frac{K}{Pl}\theta$$

For

$$P = 100\ \text{N}, \quad l = 0.25\ \text{m.}, \quad K = 12.5\ \text{N}\cdot\text{m/rad}$$

$$\cos\theta = \frac{12.5\ \text{N}\cdot\text{m/rad}}{(100)(0.25\ \text{m})}\theta$$

$$= 0.500\theta$$

Solving numerically, we obtain

$$\theta = 1.02967\ \text{rad} = 59.000°$$

$\theta = 59.0°$ ◀

Stability

$$\frac{d^2V}{d\theta^2} = (12.5\ \text{N}\cdot\text{m/rad}) + (100\ \text{N})(0.25\ \text{m})\sin 59.0° > 0$$

Stable ◀

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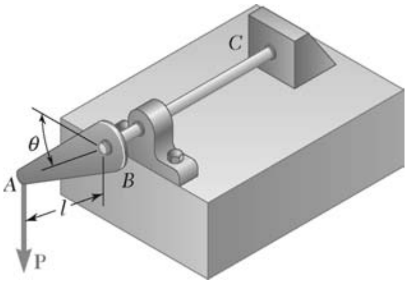
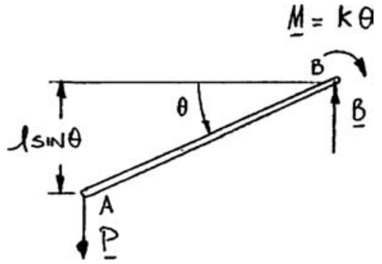


PROBLEM 10.74

In Problem 10.40, determine whether each of the positions of equilibrium is stable, unstable, or neutral. (See hint for Problem 10.73.)

PROBLEM 10.40 Solve Problem 10.39 assuming that $P = 350$ N, $l = 250$ mm, and $K = 12.5$ N·m/rad. Obtain answers in each of the following quadrants: $0 < \theta < 90°$, $270° < \theta < 360°$, $360° < \theta < 450°$.

SOLUTION



Potential energy $\quad V = \frac{1}{2}K\theta^2 - Pl\sin\theta$

$$\frac{dV}{d\theta} = K\theta - Pl\cos\theta$$

$$\frac{d^2V}{d\theta^2} = K + Pl\sin\theta$$

Equilibrium $\quad \frac{dV}{d\theta} = 0: \quad \cos\theta = \frac{K}{Pl}\theta$

For $\quad P = 350\text{ N}, \quad l = 0.250\text{ m} \quad \text{and} \quad K = 12.5\text{ N}\cdot\text{m/rad}$

$$\cos\theta = \frac{12.5\text{ N}\cdot\text{m/rad}}{(350\text{ N})(0.250\text{ m})}\theta$$

or $\quad \cos\theta = \frac{\theta}{7}$

Solving numerically $\quad \theta = 1.37333\text{ rad}, \quad 5.652\text{ rad}, \quad \text{and} \quad 6.616\text{ rad}$

or $\quad \theta = 78.7°, \; 323.8°, \; 379.1°$

Stability at $\theta = 78.7°$: $\quad \frac{d^2V}{d\theta^2} = (12.5\text{ N}\cdot\text{m/rad}) + (350\text{ N})(0.250\text{ m})\sin 78.7°$

$= 98.304 > 0 \qquad \theta = 78.7°, \text{ Stable}$ ◀

At $\theta = 323.8°$: $\quad \frac{d^2V}{d\theta^2} = (12.5\text{ N}\cdot\text{m/rad}) + (350\text{ N})(0.250\text{ m})\sin 323.8°$

$= -39.178\text{ N}\cdot\text{m} < 0 \qquad \theta = 324°, \quad \text{Unstable}$ ◀

At At $\theta = 379.1°$: $\quad \frac{d^2V}{d\theta^2} = (12.5\text{ N}\cdot\text{m/rad}) + (350\text{ N})(0.250\text{ m})\sin 379.1°$

$= 44.132\text{ N}\cdot\text{m} > 0 \qquad \theta = 379°, \quad \text{Stable}$ ◀

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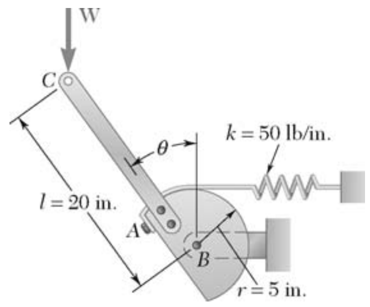
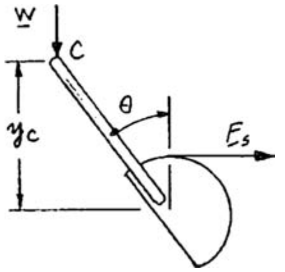


PROBLEM 10.75

A load **W** of magnitude 100 lb is applied to the mechanism at *C*. Knowing that the spring is unstretched when $\theta = 15°$, determine that value of θ corresponding to equilibrium and check that the equilibrium is stable.

SOLUTION



We have $$y_C = l\cos\theta$$

$$V = \frac{1}{2}k[r(\theta - \theta_0)]^2 + Wy_C \qquad \theta_0 = 15° = \frac{\pi}{12}\text{rad}$$

$$= \frac{1}{2}kr^2(\theta - \theta_0)^2 + Wl\cos\theta$$

$$\frac{dV}{d\theta} = kr^2(\theta - \theta_0) - Wl\sin\theta$$

Equilibrium $$\frac{dV}{d\theta} = 0: \quad kr^2(\theta - \theta_0) - wl\sin\theta = 0 \qquad (1)$$

with $W = 100$ lb, $R = 50$ lb./in., $l = 20$ in., and $r = 5$ in.

$$(50\text{ lb./in.})(25\text{ in.}^2)\left(\theta - \frac{\pi}{12}\right) - (100\text{ lb})(20\text{ in.})\sin\theta = 0$$

or $$0.625\theta - \sin\theta = 0.16362$$

Solving numerically $$\theta = 1.8145\text{ rad} = 103.97°$$

$\theta = 104.0°$ ◀

Stability $$\frac{d^2V}{d\theta^2} = kr^2 - Wl\cos\theta \qquad (2)$$

or $$= 1250 - 2000\cos\theta$$

For $\theta = 104.0°$: $$= 1734\text{ in.}\cdot\text{lb} > 0$$

Stable ◀

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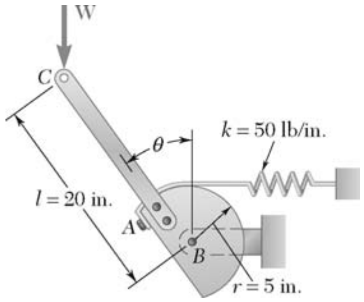

PROBLEM 10.76

A load **W** of magnitude 100 lb is applied to the mechanism at *C*. Knowing that the spring is unstretched when $\theta = 30°$, determine that value of θ corresponding to equilibrium and check that the equilibrium is stable.

SOLUTION

Using the solution of Problem 10.111, particularly Equations (1), with 15° replace by $30°\left(\frac{\pi}{6}\text{ rad}\right)$:

For equilibrium $$kr^2\left(\theta - \frac{\pi}{6}\right) - Wl\sin\theta = 0$$

With $k = 50$ lb/in., $W = 100$ lb, $r = 5$ in., and $l = 20$ in.

$$(50\text{ lb/in.})(25\text{ in.}^2)\left(\theta - \frac{\pi}{6}\right) - (100\text{ lb})(20\text{ in.})\sin\theta = 0$$

or $$1250\theta - 654.5 - 200\sin\theta = 0$$

Solving numerically, $$\theta = 1.9870\text{ rad } = 113.8°$$

$$\theta = 113.8° \blacktriangleleft$$

Stability: Equation (2), Problem 111:

$$\frac{d^2V}{d\theta^2} = kr^2 - Wl\cos\theta$$

or $$= 1250 - 2000\cos\theta$$

For $\theta = 113.8°$: $$= 2057\text{ in.}\cdot\text{lb} > 0$$ Stable $\blacktriangleleft$

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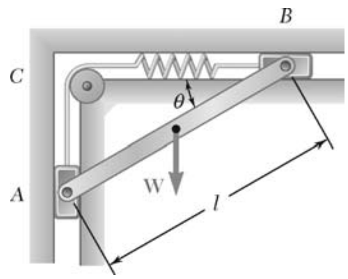
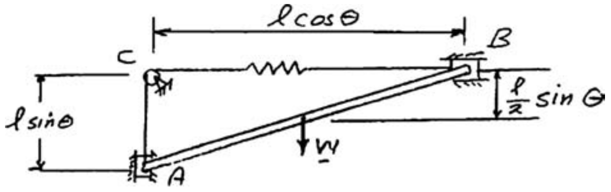


PROBLEM 10.77

A slender rod *AB*, of weight *W*, is attached to two blocks *A* and *B* that can move freely in the guides shown. The constant of the spring is *k*, and the spring is unstretched when *AB* is horizontal. Neglecting the weight of the blocks, derive an equation in θ, *W*, *l*, and *k* that must be satisfied when the rod is in equilibrium.

SOLUTION



Elongation of spring: $s = l\sin\theta + l\cos\theta - l$

$$s = l(\sin\theta + \cos\theta - 1)$$

Potential energy: $V = \frac{1}{2}ks^2 - W\frac{1}{2}\sin\theta \quad W = mg$

$$= \frac{1}{2}kl^2(\sin\theta + \cos\theta - 1)^2 - mg\frac{l}{2}\sin\theta$$

$$\frac{dV}{d\theta} = kl^2(\sin\theta + \cos\theta - 1)(\cos\theta - \sin\theta) - \frac{1}{2}mgl\cos\theta \qquad (1)$$

Equilibrium: $\frac{dV}{d\theta} = 0: \quad (\sin\theta + \cos\theta - 1)(\cos\theta - \sin\theta) - \frac{mg}{2kl}\cos\theta = 0$

$$\text{or} \quad \cos\theta\left[(\sin\theta + \cos\theta - 1)(1 - \tan\theta) - \frac{mg}{2kl}\right] = 0 \blacktriangleleft$$

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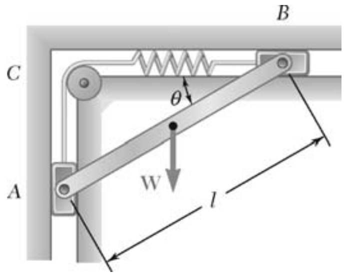

PROBLEM 10.78

A slender rod *AB*, of weight *W*, is attached to two blocks *A* and *B* that can move freely in the guides shown. Knowing that the spring is unstretched when *AB* is horizontal, determine three values of θ corresponding to equilibrium when $W = 300$ lb, $l = 16$ in., and $k = 75$ lb/in. State in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

Using the results of Problem 10.77, particularly the condition of equilibrium

$$\cos\theta\left[(\sin\theta+\cos\theta-1)(1-\tan\theta)-\frac{mg}{2kl}\right]=0$$

Now, with $W = 300$ lb, $l = 16$ in., and $k = 75$ lb/in.

$$\frac{W}{2kl}=\frac{300\text{ lb}}{(16\text{ in.})(75\text{ lb/in.})}=0.25$$

Thus: $\cos\theta\left[(\sin\theta+\cos\theta-1)(1-\tan\theta)-0.25\right]=0$

$$\cos\theta=0 \quad\text{and}\quad (\sin\theta+\cos\theta-1)(1-\tan\theta)=0.25$$

First equation yields $\theta = 90°$. Solving the second equation by trial, we find $\theta = 9.39°$ and $34.16°$
Values of θ for equilibrium are

$$\theta = 9.39°,\ 34.2°,\text{ and } 90.0°$$

Stability we differentiate Eq. (1).

$$\frac{d^2y}{ds^2}=kl^2[(\cos\theta-\sin\theta)(\cos\theta-\sin\theta)+(\sin\theta+\cos\theta-1)(-\sin\theta-\cos\theta)]+\frac{1}{2}wl\sin\theta$$

$$=kl^2\left[\cos^2\theta+\sin^2\theta-2\cos\theta\sin\theta-\sin^2\theta-\cos^2\theta-2\cos\theta\sin\theta+\sin\theta+\cos\theta+\frac{W}{2kl}\sin\theta\right]$$

$$=kl^2\left[\left(1+\frac{W}{2kl}\right)\sin\theta+\cos\theta-2\sin 2\theta\right]$$

$$\frac{d^2V}{d\theta^2}=kl^2(1.25\sin\theta+\cos\theta-2\sin 2\theta)$$

$\theta = 9.39°$: $$\frac{d^2V}{d\theta^2}=kl^2(1.25\sin 9.4+\cos 9.4-2\sin 18.8)$$

$$=kl^2(+0.55)<0$$ Stable ◀

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PROBLEM 10.78 (Continued)

$\theta = 34.2°$: $\quad \dfrac{d^2v}{d\theta^2} = kl^2(1.25\sin 34.2° + \cos 34.3° - 2\sin 68.4°)$

$= kl^2(-0.33) < 0$ Unstable ◀

$\theta = 90.0°$: $\quad \dfrac{d^2V}{d\theta^2} = kl^2(1.25\sin 90° + \cos 90° - 2\sin 180°)$

$= kl^2(1.25) > 0$ Stable ◀

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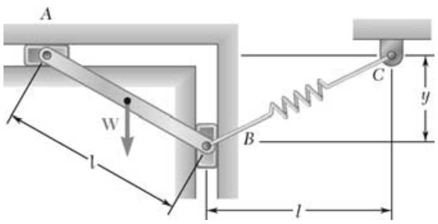


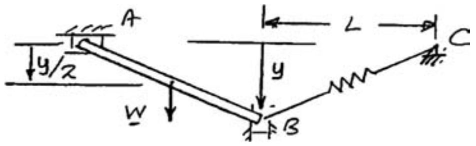
PROBLEM 10.79

A slender rod *AB*, of weight *W*, is attached to two blocks *A* and *B* that can move freely in the guides shown. Knowing that the spring is unstretched when $y = 0$, determine the value of *y* corresponding to equilibrium when $W = 80$ N, $l = 500$ mm, and $k = 600$ N/m.

SOLUTION

Deflection of spring = *s*, where $s = \sqrt{l^2 + y^2} - l$

$$\frac{ds}{dy} = \frac{y}{\sqrt{l^2 - y^2}}$$



Potential energy: $V = \frac{1}{2}ks^2 - W\frac{y}{2}$

$$\frac{dV}{dy} = ks\frac{ds}{dy} - \frac{1}{2}W$$

$$\frac{dV}{dy} = k\left(\sqrt{l^2 + y^2} - l\right)\frac{y}{\sqrt{l^2 + y^2}} - \frac{1}{2}W$$

$$= k\left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right)y - \frac{1}{2}W$$

Equilibrium $\frac{dV}{dy} = 0: \quad \left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right)y = \frac{1}{2}\frac{W}{k}$

Now $W = 80$ N, $l = 0.500$ m, and $k = 600$ N/m

Then $\left(1 - \frac{0.500 \text{ m}}{\sqrt{(0.500)^2 + y^2}}\right)y = \frac{1}{2}\frac{(80 \text{ N})}{(600 \text{ N/m})}$

or $\left(1 - \frac{0.500}{\sqrt{0.25 + y^2}}\right)y = 0.066667$

Solving numerically, $y = 0.357$ m $\qquad y = 357$ mm ◀

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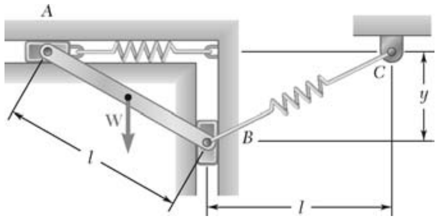
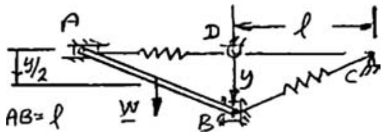


PROBLEM 10.80

Knowing that both springs are unstretched when $y = 0$, determine the value of y corresponding to equilibrium when $W = 80$ N, $l = 550$ mm, and $k = 600$ N/m.

SOLUTION



Spring deflections

$$S_{AD} = l - \sqrt{l^2 - y^2}$$

$$S_{BC} = \sqrt{l^2 + y^2} - l$$

$$V = \frac{1}{2}kS_{AD}^2 + \frac{1}{2}kS_{BC}^2 - W\frac{y}{2}$$

$$V = \frac{1}{2}k\left(l - \sqrt{l^2 - y^2}\right)^2 + \frac{1}{2}k\left(\sqrt{l^2 + y^2} - l\right)^2 - W\frac{y}{2}$$

$$\frac{dV}{dy} = k\left(l - \sqrt{l^2 - y^2}\right)\left(\frac{y}{\sqrt{l^2 - y^2}}\right) + k\left(\sqrt{l^2 + y^2} - l\right)\left(\frac{y}{\sqrt{l^2 + y^2}}\right) - \frac{W}{2}$$

$$\frac{dV}{dy} = 0: \quad \left[\left(\frac{l}{\sqrt{l^2 - y^2}} - 1\right) + \left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right)\right]y = \frac{W}{2k}$$

Data: $W = 80$ N, $l = 0.5$ m, $k = 600$ N/m

$$\left[\frac{0.5}{\sqrt{(0.5)^2 - y^2}} - \frac{0.5}{\sqrt{(0.5)^2 + y^2}}\right]y = \frac{80}{2(1200)} = 0.066667$$

Solve by trial and error: $y = 0.252$ m $\qquad y = 252$ mm ◀

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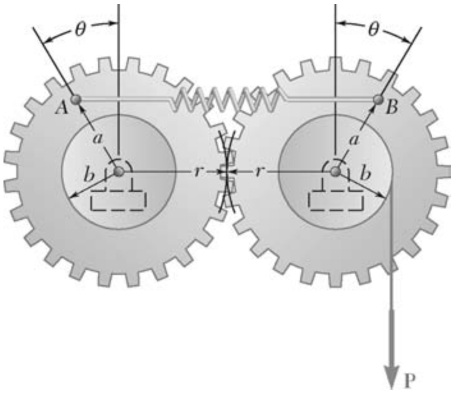
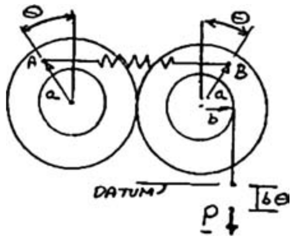


PROBLEM 10.81

A spring AB of constant k is attached to two identical gears as shown. Knowing that the spring is undeformed when $\theta = 0$, determine two values of the angle θ corresponding to equilibrium when $P = 30$ lb, $a = 4$ in., $b = 3$ in., $r = 6$ in., and $k = 5$ lb/in. State in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



Elongation of spring

$$s = 2(a\sin\theta) = 2a\sin\theta$$

$$V = \frac{1}{2}ks^2 - Pb\theta$$

$$= \frac{1}{2}k(2a\sin\theta)^2 - Pb\theta$$

$$\frac{dV}{d\theta} = 4ka^2\sin\theta\cos\theta - Pb$$

$$= 2ka^2\sin 2\theta - Pb \qquad (1)$$

Equilibrium $\quad \dfrac{dV}{d\theta} = 0: \quad \sin 2\theta = \dfrac{Pb}{2ka^2}$

$$\sin 2\theta = \frac{(30\text{ lb})(3\text{ in.})}{2(5\text{ lb/in.})(4\text{ in.})^2}; \quad \sin 2\theta = 0.5625$$

$$2\theta = 34.229° \quad \text{and} \quad 145.771°$$

$$\theta = 17.11° \quad \text{and} \quad 72.9° \blacktriangleleft$$

Stability: We differentiate Eq. (1)

$$\frac{d^2V}{d\theta^2} = 4ka^2\cos 2\theta$$

$$\theta = 17.11°: \quad \frac{d^2v}{d\theta^2} = 4ka^2\cos 34.2° = 4ka^2(0.83) > 0$$

Stable ◀

$$\theta = 72.9°: \quad \frac{d^2v}{d\theta^2} = 4ka^2\cos 145.8° = 4ka^2(-0.83) < 0$$

Unstable ◀

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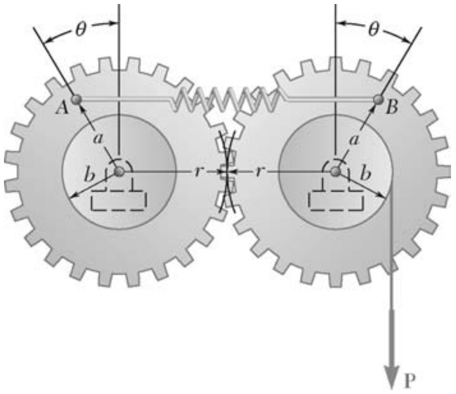
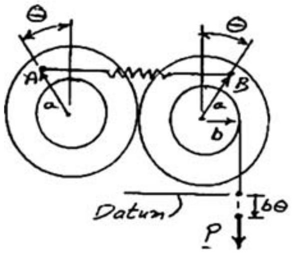


PROBLEM 10.82

A spring AB of constant k is attached to two identical gears as shown. Knowing that the spring is undeformed when $\theta = 0$, and given that $a = 60$ mm, $b = 45$ mm, $r = 90$ mm, and $k = 6$ kN/m, determine (*a*) the range of values of P for which a position of equilibrium exists, (*b*) two values of θ corresponding to equilibrium if the value of P is equal to half the upper limit of the range found in Part *a*.

SOLUTION



Elongation of spring

$$s = 2(a\sin\theta) = 2a\sin\theta$$

<u>Potential energy</u>

$$V = \frac{1}{2}ks^2 - Pb\theta = \frac{1}{2}k(2a\sin\theta)^2 - Pb\theta$$

$$V = 2ka^2\sin^2\theta - Pb\theta$$

<u>Equilibrium</u>

$$\frac{dV}{d\theta} = 0: \quad \frac{dV}{d\theta} = 4ka^2\sin\theta\cos\theta - Pb \qquad (1)$$

$$= 2ka^2\sin 2\theta - Pb = 0$$

$$\sin 2\theta = \frac{Pb}{2ka^2}; \quad \text{For} \quad P_{\max}; \ \frac{P_{\max}b}{2ka^2} = 1$$

(*a*)

$$\frac{P_{\max}(0.045\text{ m})}{2(6000\text{ N/m})(0.06\text{ m})^2} = 1$$

$P_{\max} = 960$ N ◀

(*b*) For $P = \frac{1}{2}P_{\max}$, $\quad \sin 2\theta = \frac{1}{2}; 2\theta = 30° \quad \text{and} \quad 150°$

$\theta = 15.00°$ and $75.0°$ ◀

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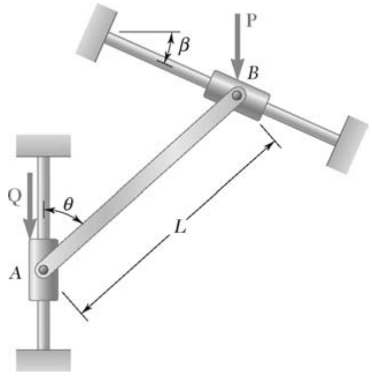
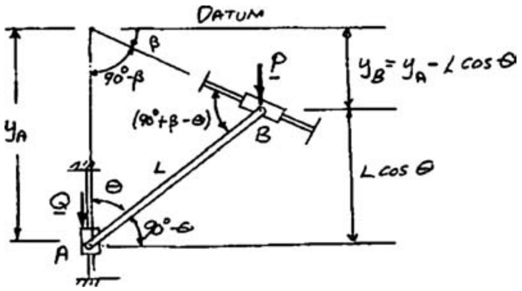


PROBLEM 10.83

A slender rod AB is attached to two collars A and B that can move freely along the guide rods shown. Knowing that $\beta = 30°$ and $P = Q = 400$ N, determine the value of the angle θ corresponding to equilibrium.

SOLUTION



Law of Sines
$$\frac{y_A}{\sin(90° + \beta - \theta)} = \frac{L}{\sin(90 - \beta)}$$

$$\frac{y_A}{\cos(\theta - \beta)} = \frac{L}{\cos\beta}$$

or
$$y_A = L\frac{\cos(\theta - \beta)}{\cos\beta}$$

From the figure:
$$y_B = L\frac{\cos(\theta - \beta)}{\cos\beta} - L\cos\theta$$

Potential Energy:
$$V = -Py_B - Qy_A = -P\left[L\frac{\cos(\theta - \beta)}{\cos\beta} - L\cos\theta\right] - QL\frac{\cos(\theta - \beta)}{\cos\beta}$$

$$\frac{dV}{d\theta} = -PL\left[-\frac{\sin(\theta - \beta)}{\cos\beta} + \sin\theta\right] + QL\frac{\sin(\theta - \beta)}{\cos\beta}$$

$$= L(P + Q)\frac{\sin(\theta - \beta)}{\cos\beta} - PL\sin\theta$$

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PROBLEM 10.83 (Continued)

Equilibrium $$\frac{dV}{d\theta}=0:\quad L(P+Q)\frac{\sin(\theta-\beta)}{\cos\beta}-PL\sin\theta=0$$

or $$(P+Q)\sin(\theta-\beta)=P\sin\theta\cos\beta$$

$$(P+Q)(\sin\theta\cos\beta-\cos\theta\sin\beta)=P\sin\theta\cos\beta$$

or $$-(P+Q)\cos\theta\sin\beta+Q\sin\theta\cos\beta=0$$

$$-\frac{P+Q}{Q}\frac{\sin\beta}{\cos\beta}+\frac{\sin\theta}{\cos\theta}=0$$

$$\tan\theta=\frac{P+Q}{Q}\tan\beta \qquad (2)$$

With $$P=Q=400\text{ N},\quad \beta=30^\circ$$

$$\tan\theta=\frac{800\text{ N}}{400\text{ N}}\tan30^\circ=1.1547 \qquad \theta=49.1^\circ \blacktriangleleft$$

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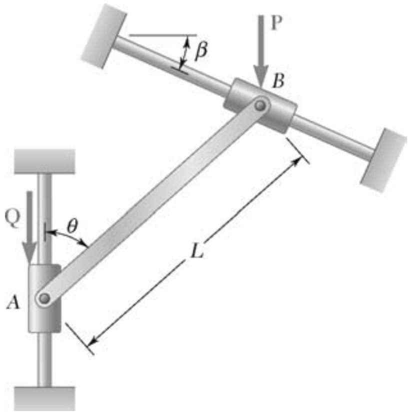

PROBLEM 10.84

A slender rod AB is attached to two collars A and B that can move freely along the guide rods shown. Knowing that $\beta = 30°$, $P = 100$ N, and $Q = 25$ N, determine the value of the angle θ corresponding to equilibrium.

SOLUTION

Using Equation (2) of Problem 10.83, with $P = 100$ N, $Q = 25$ N, and $\beta = 30°$, we have

$$\tan\theta = \frac{(100\text{ N})(25\text{ N})}{(25\text{ N})}\tan 30°$$

$$= 57.735$$

$$\theta = 89.007°$$

$\theta = 89.0°$ ◀

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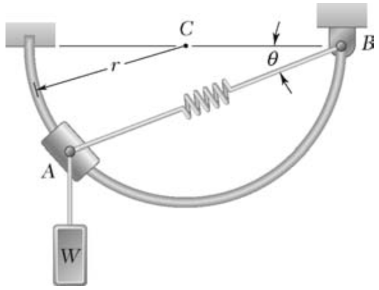
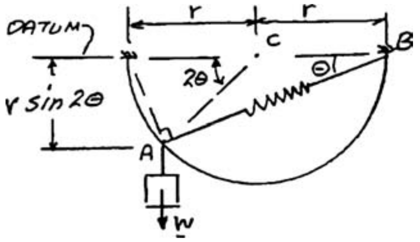


PROBLEM 10.85

Collar A can slide freely on the semicircular rod shown. Knowing that the constant of the spring is k and that the unstretched length of the spring is equal to the radius r, determine the value of θ corresponding to equilibrium when $W = 50$ lb, $r = 9$ in., and $k = 15$ lb/in.

SOLUTION



Stretch of Spring $\quad s = AB - r$

$$s = 2(r\cos\theta) - r$$

$$s = r(2\cos\theta - 1)$$

Potential Energy: $\quad V = \frac{1}{2}ks^2 - Wr\sin 2\theta \qquad W = mg$

$$V = \frac{1}{2}kr^2(2\cos\theta - 1)^2 - Wr\sin 2\theta$$

$$\frac{dV}{d\theta} = -kr^2(2\cos\theta - 1)2\sin\theta - 2Wr\cos 2\theta$$

Equilibrium $\quad \frac{dV}{d\theta} = 0: \quad -kr^2(2\cos\theta - 1)\sin\theta - Wr\cos 2\theta = 0$

$$\frac{(2\cos\theta - 1)\sin\theta}{\cos 2\theta} = -\frac{W}{kr}$$

Now $\quad \frac{W}{kr} = \frac{(50 \text{ lb})}{(15 \text{ lb/in.})(9 \text{ in.})} = 0.37037$

Then $\quad \frac{(2\cos\theta - 1)\sin\theta}{\cos 2\theta} = -0.37037$

Solving numerically, $\quad \theta = 0.95637 \text{ rad} = 54.8°$ $\qquad \theta = 54.8°$ ◀

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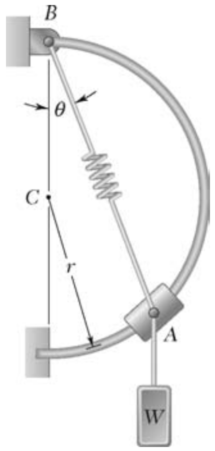
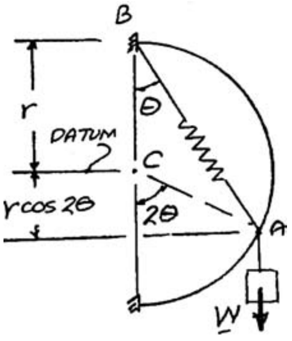


PROBLEM 10.86

Collar A can slide freely on the semicircular rod shown. Knowing that the constant of the spring is k and that the unstretched length of the spring is equal to the radius r, determine the value of θ corresponding to equilibrium when $W = 50$ lb, $r = 9$ in., and $k = 15$ lb/in.

SOLUTION

Stretch of spring



$$s = AB - r = 2(r\cos\theta) - r$$

$$s = r(2\cos\theta - 1)$$

$$V = \frac{1}{2}ks^2 - Wr\cos 2\theta$$

$$= \frac{1}{2}kr^2(2\cos\theta - 1)^2 - Wr\cos 2\theta$$

$$\frac{dV}{d\theta} = -kr^2(2\cos\theta - 1)2\sin\theta + 2Wr\sin 2\theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: \quad -kr^2(2\cos\theta - 1)\sin\theta + Wr\sin 2\theta = 0$$

$$-kr^2(2\cos\theta - 1)\sin\theta + Wr(2\sin\theta\cos\theta) = 0$$

or $$\frac{(2\cos\theta - 1)\sin\theta}{2\cos\theta} = \frac{W}{kr}$$

Now $$\frac{W}{kr} = \frac{(50 \text{ lb})}{(15 \text{ lb/in.})(9 \text{ in.})} = 0.37037$$

Then $$\frac{2\cos\theta - 1}{2\cos\theta} = 0.37037$$

Solving $\theta = 37.4°$ ◀

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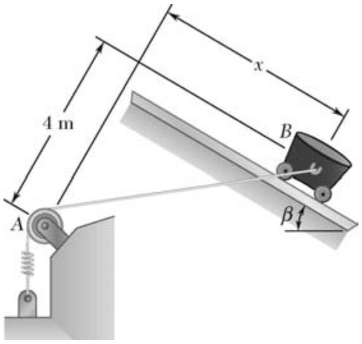
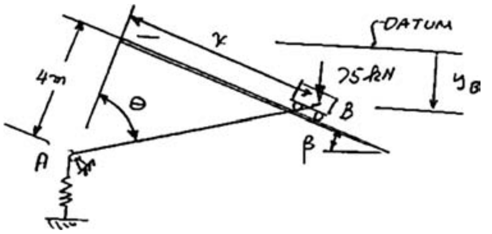


PROBLEM 10.87

Cart B, which weighs 75 kN, rolls along a sloping track that forms an angle β with the horizontal. The spring constant is 5 kN/m, and the spring is unstretched when $x = 0$. Determine the distance x corresponding to equilibrium for the angle β indicated.

Angle $\beta = 30°$.

SOLUTION



$$x = (4\text{ m})\tan\theta \qquad (1)$$

$$y_B = x\sin\beta = 4\tan\theta\sin\beta$$

$$AC = (4\text{ m})\cos\theta$$

For $x = 0$, $(AC)_0 = 4$ m

Stretch of spring. $$s = AC - (AC)_0 = \frac{4}{\cos\theta} - 4 = 4\left(\frac{1}{\cos\theta} - 1\right)$$

$$V = \frac{1}{2}ks^2 - (75\text{ kN})y_B$$

$$= \frac{1}{2}(5\text{ kN/m})\,16\left(\frac{1}{\cos\theta} - 1\right)^2 - (75\text{ kN})4\tan\theta\sin\beta$$

$$\frac{dV}{d\theta} = 80\left(\frac{1}{\cos\theta} - 1\right)\frac{\sin\theta}{\cos^2\theta} - 300\frac{\sin\beta}{\cos^2\theta}$$

<u>Equilibrium</u> $$\frac{dV}{d\theta} = 0: \quad \left(\frac{1}{\cos\theta} - 1\right)\sin\theta = 3.75\sin\beta \qquad (2)$$

<u>Given</u>: $\beta = 30°, \quad \sin\theta = 0.5$

Eq. (2): $$\left(\frac{1}{\cos\theta} - 1\right)\sin\theta = 3.75(0.5) = 1.875$$

Solve by trial and error: $\theta = 70.46°$

Eq. (1): $x = (4\text{ m})\tan 70.46°$ $\qquad x = 11.27$ m ◀

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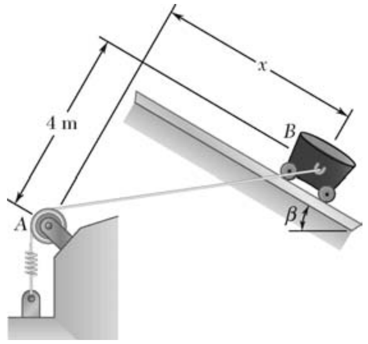
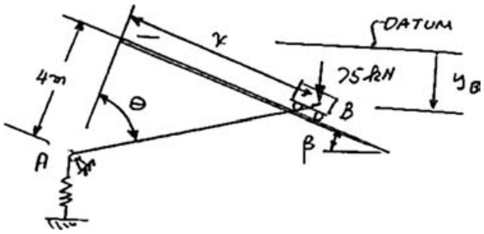


PROBLEM 10.88

Cart *B*, which weighs 75 kN, rolls along a sloping track that forms an angle β with the horizontal. The spring constant is 5 kN/m, and the spring is unstretched when $x = 0$. Determine the distance x corresponding to equilibrium for the angle β indicated.

Angle $\beta = 60°$.

SOLUTION



$$x = (4 \text{ m})\tan\theta \qquad (1)$$

$$y_B = x\sin\beta = 4\tan\theta\sin\beta$$

$$AC = (4 \text{ m})\cos\theta$$

For $x = 0$, $\qquad (AC)_0 = 4 \text{ m}$

Stretch of spring. $\qquad s = AC - (AC)_0 = \dfrac{4}{\cos\theta} - 4 = 4\left(\dfrac{1}{\cos\theta} - 1\right)$

$$V = \frac{1}{2}ks^2 - (75 \text{ kN})y_B$$

$$= \frac{1}{2}(5 \text{ kN/m})\,16\left(\frac{1}{\cos\theta} - 1\right)^2 - (75 \text{ kN})4\tan\theta\sin\beta$$

$$\frac{dV}{d\theta} = 80\left(\frac{1}{\cos\theta} - 1\right)\frac{\sin\theta}{\cos^2\theta} - 300\frac{\sin\beta}{\cos^2\theta}$$

<u>Equilibrium</u> $\qquad \dfrac{dV}{d\theta} = 0: \quad \left(\dfrac{1}{\cos\theta} - 1\right)\sin\theta = 3.75\sin\beta \qquad (2)$

<u>Given</u>: $\qquad \beta = 60°, \sin\theta = 0.86603$

Eq. (2): $\qquad \left(\dfrac{1}{\cos\theta} - 1\right)\sin\theta = 3.75(0.86603) = 3.2476$

Solve by trial and error: $\qquad \theta = 76.67°$

Eq. (1): $\qquad x = (4 \text{ m})\tan 26.67° \qquad x = 16.88 \text{ m}$ ◀

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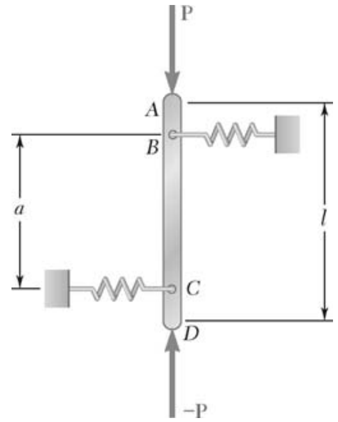


PROBLEM 10.89

A vertical bar *AD* is attached to two springs of constant k and is in equilibrium in the position shown. Determine the range of values of the magnitude *P* of two equal and opposite vertical forces **P** and **−P** for which the equilibrium position is stable if (*a*) $AB = CD,$ (*b*) $AB = 2CD.$

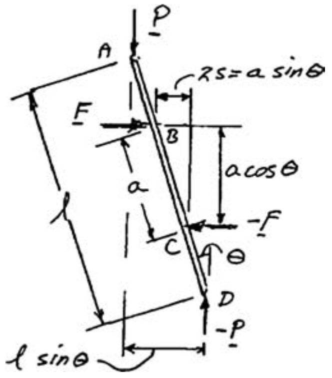
SOLUTION

For both (*a*) and (*b*): Since **P** and **−P** are vertical, they form a couple of moment

$$M_P = +Pl\sin\theta$$

The forces **F** and **−F** exerted by springs must, therefore, also form a couple, with moment

$$M_F = -Fa\cos\theta$$



We have $$dU = M_P d\theta + M_F d\theta$$
$$= (Pl\sin\theta - Fa\cos\theta)d\theta$$

but $$F = ks = k\left(\frac{1}{2}a\sin\theta\right)$$

Thus, $$dU = \left(Pl\sin\theta - \frac{1}{2}ka^2\sin\theta\cos\theta\right)d\theta$$

From Equation (10.19), page 580, we have

$$dV = -dU = -Pl\sin\theta d\theta + \frac{1}{4}ka^2\sin 2\theta d\theta$$

or $$\frac{dV}{d\theta} = -Pl\sin\theta + \frac{1}{4}ka^2\sin 2\theta$$

and $$\frac{d^2V}{d\theta^2} = -Pl\cos\theta + \frac{1}{2}ka^2\cos 2\theta \qquad (1)$$

For $\theta = 0$: $$\frac{d^2V}{d\theta^2} = -Pl + \frac{1}{2}ka^2$$

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PROBLEM 10.89 (Continued)

For Stability: $$\frac{d^2V}{d\theta^2} > 0, \quad -Pl + \frac{1}{2}ka^2 > 0$$

or (for Parts *a* and *b*) $$P < \frac{ka^2}{2l} \blacktriangleleft$$

Note: To check that equilibrium is unstable for $P = \frac{ka^2}{2l}$, we differentiate (1) twice:

$$\frac{d^3V}{d\theta^3} = +Pl\sin\theta - ka^2\sin 2\theta = 0, \quad \text{for} \quad \theta = 0,$$

$$\frac{d^4V}{d\theta^4} = Pl\cos\theta - 2ka^2\cos 2\theta$$

For $\theta = 0$ $$\frac{d^4V}{d\theta^4} = Pl - 2ka^2 = \frac{ka^2}{2} - 2ka^2 < 0$$

Thus, equilibrium is unstable when $$P = \frac{ka^2}{2l}$$

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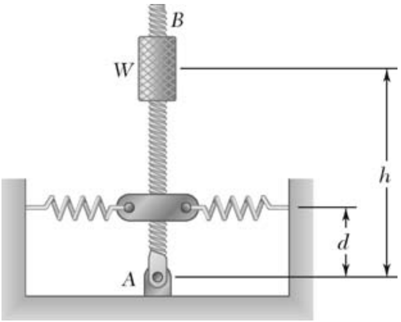
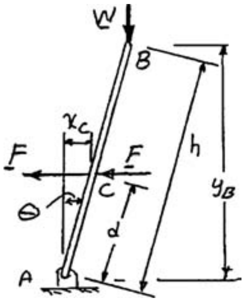


PROBLEM 10.90

Rod AB is attached to a hinge at A and to two springs, each of constant k. If $h = 25$ in., $d = 12$ in., and $W = 80$ lb, determine the range of values of k for which the equilibrium of the rod is stable in the position shown. Each spring can act in either tension or compression.

SOLUTION



We have $$x_C = d\sin\theta \qquad y_B = h\cos\theta$$

Potential Energy: $$V = 2\left(\frac{1}{2}kx_C^2 + Wy_B\right)$$

$$= kd^2\sin^2\theta + Wh\cos\theta$$

Then $$\frac{dV}{d\theta} = 2kd^2\sin\theta\cos\theta - Wh\sin\theta$$

$$= kd^2\sin 2\theta - Wh\sin\theta$$

and $$\frac{d^2V}{d\theta^2} = 2kd^2\cos 2\theta - Wh\cos\theta \qquad (1)$$

For equilibrium position $\theta = 0$ to be stable, we must have

$$\frac{d^2V}{d\theta^2} = 2kd^2 - Wh > 0$$

or $$kd^2 > \frac{1}{2}Wh \qquad (2)$$

Note: For $kd^2 = \frac{1}{2}Wh$, we have $\frac{d^2V}{d\theta^2} = 0$, so that we must determine which is the first derivative that is not equal to zero. Differentiating Equation (1), we write

$$\frac{d^3V}{d\theta^3} = -4kd^2\sin 2\theta + Wh\sin\theta = 0 \qquad \text{for } \theta = 0$$

$$\frac{d^4V}{d\theta^2} = -8kd^2\cos 2\theta + Wh\cos\theta$$

For $\theta = 0$: $$\frac{d^4V}{d\theta^4} = -8kd^2 + Wh$$

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PROBLEM 10.90 (Continued)

Since $kd^2 = \frac{1}{2}Wh$, $\frac{d^4V}{d\theta^4} = -4Wh + Wh < 0$, we conclude that the equilibrium is unstable for $kd^2 = \frac{1}{2}Wh$ and the $>$ sign in Equation (2) is correct.

With $\quad W = 80 \text{ lb}, \quad h = 25 \text{ in.}, \quad \text{and} \quad d = 12 \text{ in.}$

Equation (2) gives $$k(12 \text{ in.})^2 > \frac{1}{2}(80 \text{ lb})(25 \text{ in.})$$

or $\quad k > 6.944 \text{ lb/in.}$ $\qquad k > 6.94 \text{ lb/in.}$ ◀

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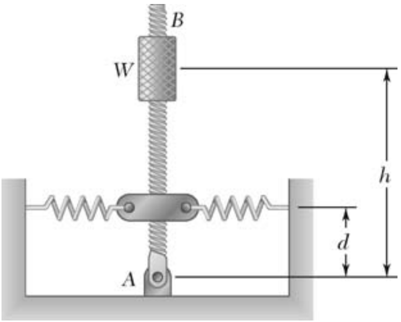

PROBLEM 10.91

Rod AB is attached to a hinge at A and to two springs, each of constant k. If $h = 45$ in., $k = 6$ lb/in., and $W = 60$ lb, determine the smallest distance d for which the equilibrium of the rod is stable in the position shown. Each spring can act in either tension or compression.

SOLUTION

Using Equation (2) of Problem 10.90 with

$$h = 45 \text{ in.}, \quad k = 6 \text{ lb/in.}, \quad \text{and} \quad W = 60 \text{ lb}$$

$$(6 \text{ lb/in.})d^2 > \frac{1}{2}(60 \text{ lb})(45 \text{ in.})$$

or

$$d^2 > 225 \text{ in.}^2$$

$$d > 15.0000 \text{ in.}$$

smallest $d = 15.00$ in. ◀

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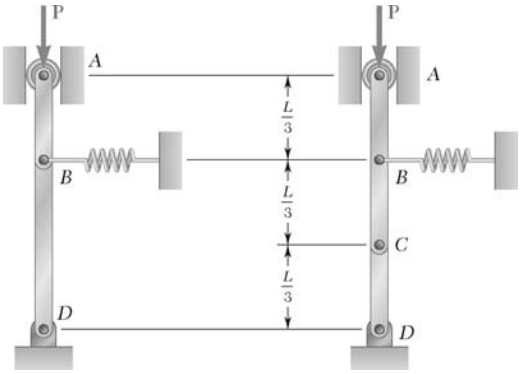
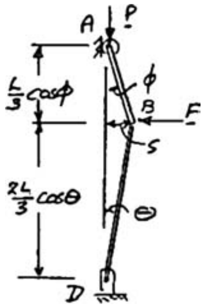


PROBLEM 10.92

Two bars are attached to a single spring of constant k that is unstretched when the bars are vertical. Determine the range of values of P for which the equilibrium of the system is stable in the position shown.

SOLUTION

$$s = \frac{L}{3}\sin\phi = \frac{2L}{3}\sin\theta$$



For small values of ϕ and θ

$$\phi = 2\theta$$

$$V = P\left(\frac{L}{3}\cos\phi + \frac{2L}{3}\cos\theta\right) + \frac{1}{2}ks^2$$

$$V = \frac{PL}{3}(\cos 2\theta + 2\cos\theta) + \frac{1}{2}k\left(\frac{2L}{3}\sin\theta\right)^2$$

$$\frac{dV}{d\theta} = \frac{PL}{3}(-2\sin 2\theta - 2\sin\theta) + \frac{2}{9}kL^2\sin\theta\cos\theta$$

$$= -\frac{PL}{3}(2\sin 2\theta + 2\sin\theta) + \frac{2}{9}kL^2\sin 2\theta$$

$$\frac{d^2V}{d\theta^2} = -\frac{PL}{3}(4\cos 2\theta + 2\cos\theta) + \frac{4}{9}kL^2\cos 2\theta$$

when $\theta = 0$: $$\frac{d^2V}{d\theta^2} = -\frac{6PL}{3} + \frac{4}{9}kL^2$$

For stability: $$\frac{d^2V}{d\theta^2} > 0, \quad -2PL + \frac{4}{9}kL^2 > 0$$

$$P < \frac{2}{9}kL \blacktriangleleft$$

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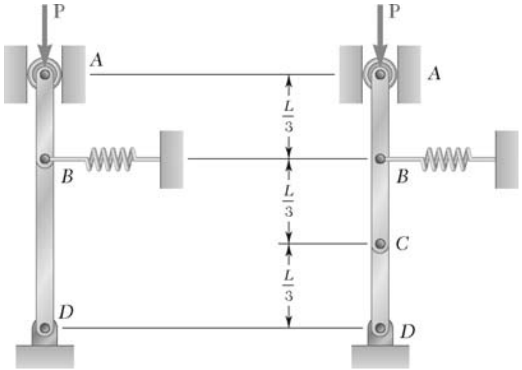


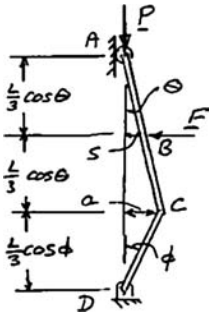
PROBLEM 10.93

Two bars are attached to a single spring of constant k that is unstretched when the bars are vertical. Determine the range of values of P for which the equilibrium of the system is stable in the position shown.

SOLUTION

$$a = \frac{2L}{3}\sin\theta = \frac{L}{3}\sin\phi$$

For small values of ϕ and θ



$$\phi = 2\theta$$

$$s = \frac{L}{3}\sin\theta$$

$$V = P\left(\frac{2L}{3}\cos\theta + \frac{L}{3}\cos\phi\right) + \frac{1}{2}ks^2$$

$$= \frac{PL}{3}(2\cos\theta + \cos 2\theta) + \frac{1}{2}k\left(\frac{L}{3}\sin\theta\right)^2$$

$$\frac{dV}{d\theta} = \frac{PL}{3}(-2\sin\theta - 2\sin 2\theta) + \frac{kL^2}{9}\sin\theta\cos\theta$$

$$\frac{dV}{d\theta} = -\frac{2PL}{3}(\sin\theta + \sin 2\theta) + \frac{kL^2}{18}\sin 2\theta$$

$$\frac{d^2V}{d\theta^2} = -\frac{2PL}{3}(\cos\theta + 2\cos 2\theta) + \frac{kL^2}{9}\cos 2\theta$$

when $\theta = 0$: $$\frac{dV^2}{d\theta^2} = -2PL + \frac{kL^2}{9}$$

For stability: $$\frac{d^2V}{d\theta^2} > 0, \quad -2PL + \frac{kL^2}{9} > 0$$

$$P < \frac{1}{18}kL \blacktriangleleft$$

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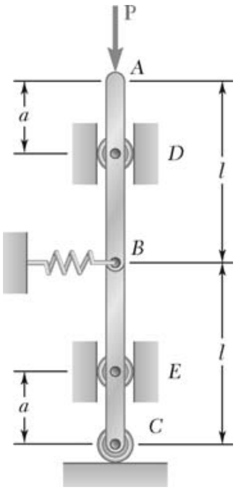
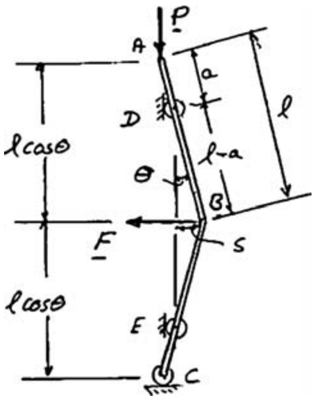


PROBLEM 10.94

Two bars *AB* and *BC* are attached to a single spring of constant *k* that is unstretched when the bars are vertical. Determine the range of values of *P* for which the equilibrium of the system is stable in the position shown.

SOLUTION



$$s = (l-a)\sin\theta$$

$$V = P(2l\cos\theta) + \frac{1}{2}ks^2$$

$$= 2Pl\cos\theta + \frac{1}{2}k(l-a)^2\sin^2\theta$$

$$\frac{dV}{d\theta} = -2Pl\sin\theta + k(l-a)^2\sin\theta\cos\theta$$

$$= -2Pl\sin\theta + \frac{1}{2}k(l-a)^2\sin 2\theta$$

$$\frac{d^2V}{d\theta^2} = -2Pl\cos\theta + k(l-a)^2\cos 2\theta \qquad (1)$$

when $\qquad \theta = 0: \quad \dfrac{d^2V}{d\theta^2} = -2Pl + k(l-a)^2$

<u>Stability</u>: $\qquad \dfrac{d^2V}{d\theta^2} > 0: \quad -2Pl + k(l-a)^2 > 0 \qquad P < \dfrac{k(l-a)^2}{2l}$ ◀

To check whether equilibrium is unstable for $P = \frac{k(l-a)^2}{2l}$, we differentiate

Eq. (1) twice:

$$\frac{d^3V}{d\theta^3} = 2Pl\sin\theta - 2k(l-a)^2\sin 2\theta = 0, \quad \text{For} \quad \theta = 0$$

$$\frac{d^4V}{d\theta^4} = 2Pl\cos\theta - 4k(l-a)^2\cos 2\theta$$

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PROBLEM 10.94 (Continued)

For $G = 0$ and $$P = \frac{k(l-a)^2}{2l}$$

$$\frac{d^4V}{d\theta^4} = 2Pl - 4k(l-a)^2$$

$$= k(l-a)^3 - 4k(l-a)^2 < 0$$

Thus equil is unstable for $$P = \frac{k(l-a)^2}{2l}$$

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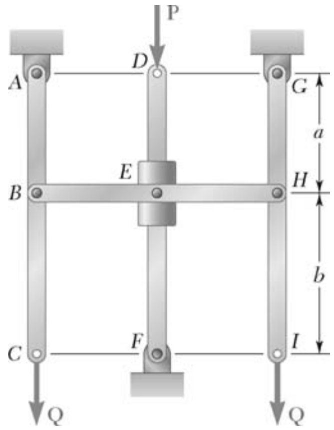
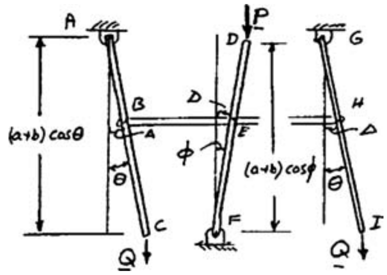


PROBLEM 10.95

The horizontal bar *BEH* is connected to three vertical bars. The collar at *E* can slide freely on bar *DF*. Determine the range of values of *Q* for which the equilibrium of the system is stable in the position shown when $a = 24$ in., $b = 20$ in., and $P = 150$ lb.

SOLUTION



First note $\quad A = a\sin\theta = b\sin\phi$

For small values of θ and ϕ: $\quad a\theta = b\phi$

or $\quad \phi = \dfrac{a}{b}\theta$

$$V = P(a+b)\cos\phi - 2Q(a+b)\cos\theta$$

$$= (a+b)\left[P\cos\left(\frac{a}{b}\theta\right) - 2Q\cos\theta\right]$$

$$\frac{dV}{d\theta} = (a+b)\left[-\frac{a}{b}P\sin\left(\frac{a}{b}\theta\right) + 2Q\sin\theta\right]$$

$$\frac{d^2V}{d\theta^2} = (a+b)\left[-\frac{a^2}{b^2}P\cos\left(\frac{a}{b}\theta\right) + 2Q\cos\theta\right]$$

when $\theta = 0$: $\quad \dfrac{d^2V}{d\theta^2} = (a+b)\left(-\dfrac{a^2}{b^2}P + 2Q\right)$

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PROBLEM 10.95 (Continued)

Stability: $$\frac{d^2V}{d\theta^2} > 0: \quad -\frac{a^2}{b^2}P + 2Q > 0$$

$$P < 2\frac{b^2}{a^2}Q \qquad (1)$$

or $$Q > \frac{a^2}{2b^2}P \qquad (2)$$

with $P = 150$ lb, $a = 24$ in., and $b = 20$ in.

Equation (1): $$Q > \frac{(24 \text{ in.})^2}{2(20 \text{ in.})^2}(150 \text{ lb}) = 108.000 \text{ lb}$$

For stability $Q > 108.0$ lb ◀

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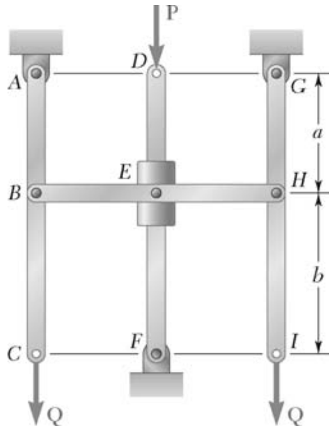

PROBLEM 10.96

The horizontal bar *BEH* is connected to three vertical bars. The collar at *E* can slide freely on bar *DF*. Determine the range of values of *P* for which the equilibrium of the system is stable in the position shown when $a = 150$ mm, $b = 200$ mm, and $Q = 45$ N.

SOLUTION

Using Equation (2) of Problem 10.95 with

$$Q = 45 \text{ N}, \quad a = 150 \text{ mm}, \quad \text{and} \quad b = 200 \text{ mm}$$

Equation (2)

$$P < 2\frac{(200 \text{ mm})^2}{(150 \text{ mm})^2}(45 \text{ N})$$

$$= 160.000 \text{ N}$$

For stability $P < 160.0 \text{ N}$ ◀

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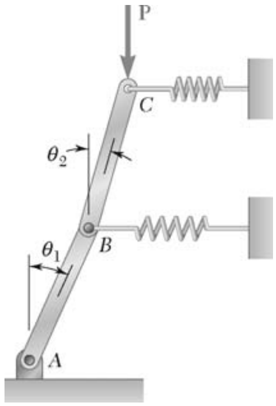
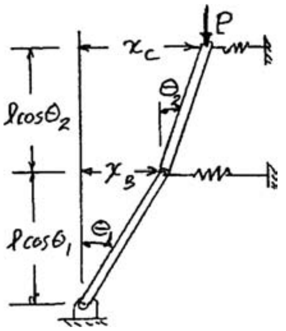


PROBLEM 10.97*

Bars *AB* and *BC*, each of length *l* and of negligible weight, are attached to two springs, each of constant *k*. The springs are undeformed, and the system is in equilibrium when $\theta_1 = \theta_2 = 0$. Determine the range of values of *P* for which the equilibrium position is stable.

SOLUTION



We have
$$x_B = l\sin\theta$$
$$x_C = l\sin\theta_1 + l\sin\theta_2$$
$$y_C = l\cos\theta_1 + l\cos\theta_2$$
$$V = Py_C + \frac{1}{2}kx_B^2 + \frac{1}{2}kx_C^2$$

or
$$V = Pl(\cos\theta_1 + \cos\theta_2) + \frac{1}{2}kl^2\left[\sin^2\theta_1 + (\sin\theta_1 + \sin\theta_2)^2\right]$$

For small values of θ_1 and θ_2:

$$\sin\theta_1 \approx \theta_1, \quad \sin\theta_2 \approx \theta_2, \quad \cos\theta_1 \approx 1 - \frac{1}{2}\theta_1^2, \quad \cos\theta_2 \approx 1 - \frac{1}{2}\theta_2^2$$

Then
$$V = Pl\left(1 - \frac{\theta_1^2}{2} + 1 - \frac{\theta_2^2}{2}\right) + \frac{1}{2}kl^2\left[\theta_1^2 + (\theta_1 + \theta_2)^2\right]$$

and
$$\frac{\partial V}{\partial \theta_1} = -Pl\theta_1 + kl^2[\theta_1 + (\theta_1 + \theta_2)]$$
$$\frac{\partial V}{\partial \theta_2} = -Pl\theta_2 + kl^2(\theta_1 + \theta_2)$$
$$\frac{\partial^2 V}{\partial \theta_1^2} = -Pl + 2kl^2 \quad \frac{\partial^2 V}{\partial \theta_2^2} = -Pl + kl^2$$
$$\frac{\partial^2 V}{\partial \theta_1 \partial \theta_2} = kl^2$$

Stability: Conditions for stability (see Page 583).

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PROBLEM 10.97* (Continued)

For $\theta_1 = \theta_2 = 0$: $\dfrac{\partial V}{\partial \theta_1} = \dfrac{\partial V}{\partial \theta_2} = 0$ (condition satisfied)

$$\left(\frac{\partial^2 V}{\partial \theta_1 \partial \theta_2}\right)^2 - \frac{\partial^2 V}{\partial \theta_1^2}\frac{\partial^2 V}{\partial \theta_2^2} < 0$$

Substituting $(kl^2)^2 - (-Pl + 2kl^2)(-Pl + kl) < 0$

$$k^2 l^4 - P^2 l^2 + 3Pkl^3 - 2k^2 l^4 < 0$$

$$P^2 - 3klP + k^2 l^2 > 0$$

Solving $P < \dfrac{3-\sqrt{5}}{2}kl$ or $P > \dfrac{3+\sqrt{5}}{2}kl$

or $P < 0.382kl$ or $P > 2.62kl$

$$\frac{\partial^2 V}{\partial \theta_1^2} > 0: \quad -Pl + 2kl^2 > 0$$

or $P < \dfrac{1}{2}kl$

$$\frac{\partial^2 V}{\partial \theta_2^2} > 0: \quad -Pl + kl^2 > 0$$

or $P < kl$

Therefore, all conditions for stable equilibrium are satisfied when $0 \le P < 0.382kl$ ◀

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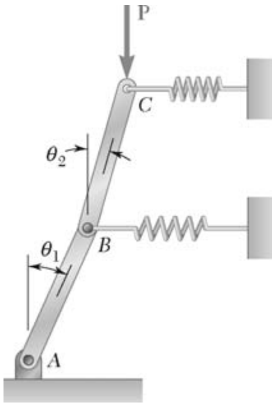

PROBLEM 10.98*

Solve Problem 10.97 knowing that $l = 800$ mm and $k = 2.5$ kN/m.

PROBLEM 10.97* Bars *AB* and *BC*, each of length l and of negligible weight, are attached to two springs, each of constant k. The springs are undeformed, and the system is in equilibrium when $\theta_1 = \theta_2 = 0$. Determine the range of values of P for which the equilibrium position is stable.

SOLUTION

From the analysis of Problem 10.97 with

$$l = 800 \text{ mm} \quad \text{and} \quad k = 2.5 \text{ kN/m}$$

$$P < 0.382kl = 0.382(2500 \text{ N/m})(0.8 \text{ m}) = 764 \text{ N}$$

$P < 764$ N ◀

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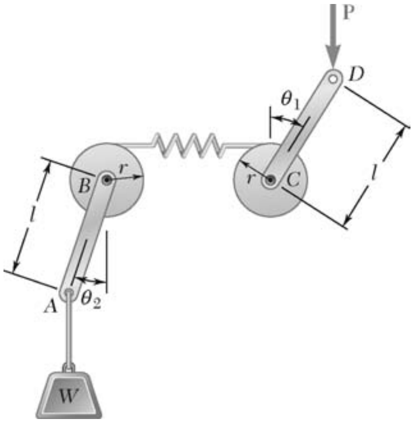
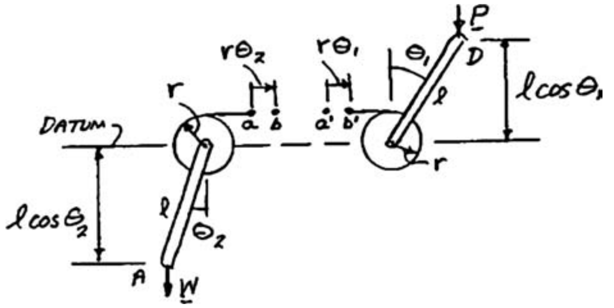


PROBLEM 10.99*

Two rods of negligible weight are attached to drums of radius *r* that are connected by a belt and spring of constant *k*. Knowing that the spring is undeformed when the rods are vertical, determine the range of values of *P* for which the equilibrium position $\theta_1 = \theta_2 = 0$ is stable.

SOLUTION



Left end of spring moves from *a* to *b*. Right end of spring moves from a' to b'. Elongation of spring

$$s = a'b' - ab = r\theta_1 - r\theta_2 = r(\theta_1 - \theta_2)$$

$$V = \frac{1}{2}ks^2 + pl\cos\theta_1 - wl\cos\theta_2$$

$$= \frac{1}{2}kr^2(\theta_1 - \theta_2)^2 + pl\cos\theta_1 - wl\cos\theta_2$$

$$\frac{\partial v}{\partial \theta_1} = kr^2(\theta_1 - \theta_2) - pl\sin\theta_1$$

$$\frac{\partial v}{\partial \theta_2} = -kr^2(\theta_1 - \theta_2) + wl\sin\theta_2$$

$$\frac{\partial^2 v}{\partial \theta_1^2} = kr^2 - pl\cos\theta_1$$

$$\frac{\partial^2 v}{\partial \theta_2^2} = kr^2 + wl\cos\theta_2$$

$$\frac{\partial^2 v}{\partial \theta_1 \partial \theta_2} = -kr^2$$

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PROBLEM 10.99* (Continued)

For $\theta_1 = \theta_2 = 0$: $\dfrac{\partial^2 v}{\partial \theta_1^2} = kr^2 - pl, \quad \dfrac{\partial^2 v}{\partial \theta_2^2} = +kr^2 + wl, \quad \dfrac{r^2 v}{\partial \theta, \partial \theta_2} = -kr^2$

Stability conditions for stability (see Page 583)

$$\left(\frac{\partial^2 v}{\partial \theta_1 \partial \theta_2}\right)^2 - \frac{\partial^2 v}{\partial \theta_1^2} \cdot \frac{\partial^2 v}{\partial \theta_2^2} < 0$$

$$(kr^2)^2 - (kr^2 - pl)(kr^2 + wl) < 0$$

$$pl(kr^2 + wl) - kr^2 wl < 0$$

$$P < \frac{wkr^2}{kr^2 + wl}; \quad P < \frac{kr^2}{l}\left(\frac{W}{\frac{kr^2}{l} + w}\right) \quad \triangleleft$$

$$\frac{\partial^2 v}{\partial \theta_1^2} >: \quad kr^2 - pl > 0; \quad P < \frac{kr^2}{l} \quad \triangleleft$$

We choose: $P < \dfrac{kr^2}{l}\left(\dfrac{W}{\frac{kr^2}{l} + W}\right)$ ◀

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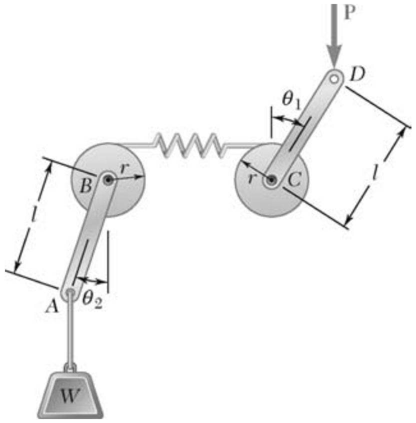

PROBLEM 10.100*

Solve Problem 10.99 knowing that k = 20 lb/in., r = 3 in., l = 6 in., and (a) W = 15 lb, (b) W = 60 lb.

PROBLEM 10.99* Two rods of negligible weight are attached to drums of radius r that are connected by a belt and spring of constant k. Knowing that the spring is undeformed when the rods are vertical, determine the range of values of P for which the equilibrium position $\theta_1 = \theta_2 = 0$ is stable.

SOLUTION

$$k = 20 \text{ lb/in.}$$
$$r = 3 \text{ in.}$$
$$l = 6 \text{ in.}$$
$$\frac{kr^2}{l} = \frac{(20 \text{ lb/in.})(3 \text{ in.})^2}{6 \text{ in.}} = 30 \text{ lb}$$

(a) $W = 15 \text{ lb}: \quad P < (30 \text{ lb})\dfrac{15 \text{ lb}}{(30 \text{ lb}) + (15 \text{ lb})}$ $\qquad$ $P < 10.00 \text{ lb}$ ◀

(b) $W = 60 \text{ lb}: \quad P < (30 \text{ lb})\dfrac{60 \text{ lb}}{(30 \text{ lb}) + (60 \text{ lb})}$ $\qquad$ $P < 20.0 \text{ lb}$ ◀

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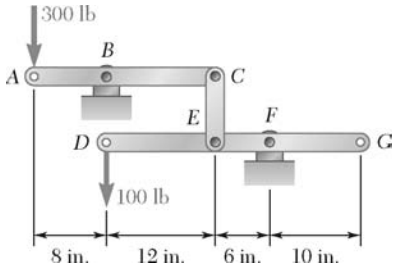
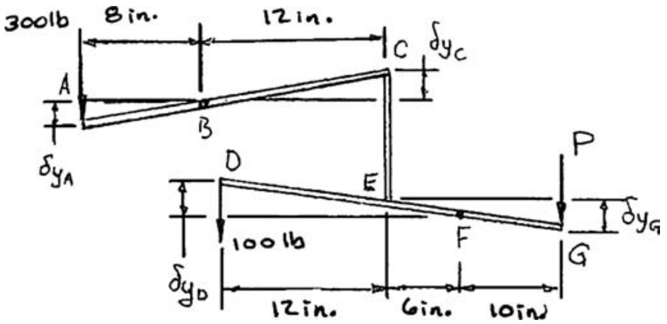


PROBLEM 10.101

Determine the vertical force **P** that must be applied at *G* to maintain the equilibrium of the linkage.

SOLUTION



Assuming $\delta y_A \downarrow$

it follows

$$\delta y_C = \frac{12}{8}\delta y_A = 1.5\delta y_A \uparrow$$

$$\delta y_E = \delta y_C = 1.5\delta y_A \uparrow$$

$$\delta y_D = \frac{18}{6}\delta y_A = 3(1.5\delta y_A) = 4.5\delta y_A \uparrow$$

$$\delta y_G = \frac{10}{6}\delta y_A = \frac{10}{6}(1.5\delta y_A) = 2.5\delta y_A \downarrow$$

Then, by virtual work

$$\delta U = 0: \quad (300 \text{ lb})\delta y_A - (100 \text{ lb})\delta y_D + P\delta y_G = 0$$

$$300\delta y_A - 100(4.5\delta y_A) + P(2.5\delta y_A) = 0$$

$$300 - 450 + 2.5P = 0$$

$$P = 60.0 \text{ lb} \qquad P = 60.0 \text{ lb} ◀$$

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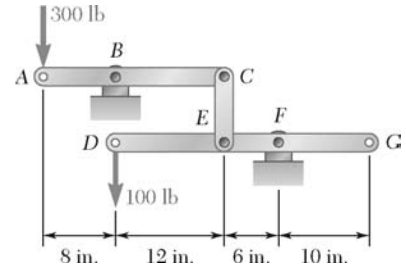
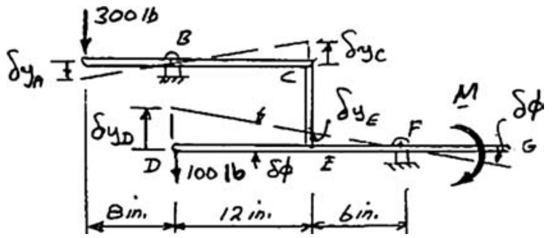


PROBLEM 10.102

Determine the couple **M** that must be applied to member *DEFG* to maintain the equilibrium of the linkage.

SOLUTION



<u>Assume</u> $\delta y_A \downarrow$:

$$\delta y_C = \frac{12}{8}\delta y_A = 1.5\delta y_A \uparrow, \qquad \delta y_E = \delta y_C = 1.5\delta y_A \uparrow$$

$$\delta y_D = \frac{18}{6}\delta y_E = 3(1.5\delta y_A) = 4.5\delta y_A \uparrow$$

$$\delta\phi = \frac{\delta y_E}{6} = \frac{1.5\delta y_A}{6} = \frac{1}{4}\delta y_A \curvearrowright$$

<u>Virtual Work</u>:

$$\delta U = 0: \qquad (300\text{ lb})\delta y_A - (100\text{ lb})\delta y_D + M\,\delta\phi = 0$$

$$300\delta y_A - 100(1.5\delta y_A) + M\left(\frac{1}{4}\delta y_A\right) = 0$$

$$300 - 450 + \frac{1}{4}M = 0$$

$$M = +600\text{ lb}\cdot\text{in.} \qquad \mathbf{M} = 600\text{ lb}\cdot\text{in.} \curvearrowright \blacktriangleleft$$

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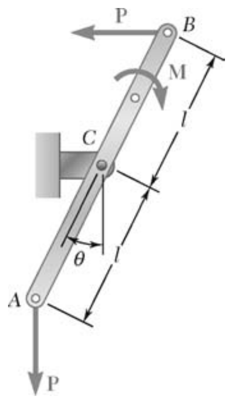
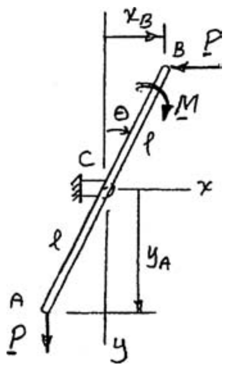


PROBLEM 10.103

Derive an expression for the magnitude of the couple **M** required to maintain the equilibrium of the linkage shown.

SOLUTION



We have

$$x_B = l\sin\theta$$
$$\delta x_B = l\cos\theta\delta\theta$$
$$y_A = l\cos\theta$$
$$\delta y_A = -l\sin\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad M\delta\theta - P\delta x_B + P\delta y_A = 0$$

$$M\delta\theta - P(l\cos\theta\delta\theta) + P(-l\sin\theta\delta\theta) = 0$$

$$M = Pl(\sin\theta + \cos\theta) \blacktriangleleft$$

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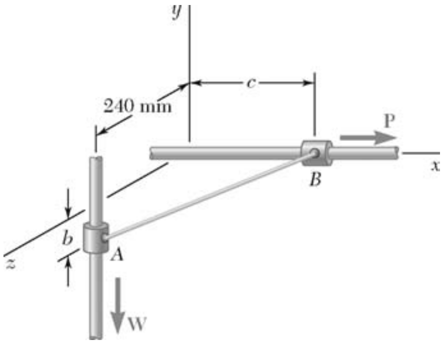

PROBLEM 10.104

Collars A and B are connected by the wire AB and can slide freely on the rods shown. Knowing that the length of the wire is 440 mm and that the weight W of collar A is 90 N, determine the magnitude of the force **P** required to maintain equilibrium of the system when (a) $c = 80$ mm, (b) $c = 280$ mm.

SOLUTION

Since $AB = 440$ mm, we have

$$(440\text{ mm})^2 = (240\text{ mm})^2 + b^2 + c^2$$

$$b^2 = (440)^2 - (240)^2 - c^2 = 136\times 10^3 - c^2$$

Differentiate:

$$2b\,\delta b = -2c\,\delta c$$

$$\delta b = -\frac{c}{b}\delta c$$

$$\delta b = -\frac{c\,\delta c}{\sqrt{136\times 10^3 - c^2}}$$

Virtual Work:

$$\delta U = 0: \quad P\delta c + W\delta b = 0$$

$$P\delta c = W\frac{c\delta c}{\sqrt{136\times 10^3 - c^2}}$$

For $W = 90$ N:

$$P = 90\frac{c}{\sqrt{136\times 10^3 - c^2}} \qquad (1)$$

(a) When $c = 80$ mm:

Eq. (1):

$$P = 90\frac{80}{\sqrt{136\times 10^3 - (80)^2}}$$

$P = 20.0$ N ◀

(b) When $c = 280$ mm:

Eq. (1)

$$P = 90\frac{280}{\sqrt{136\times 10^3 - (280)^2}}$$

$P = 105.0$ N ◀

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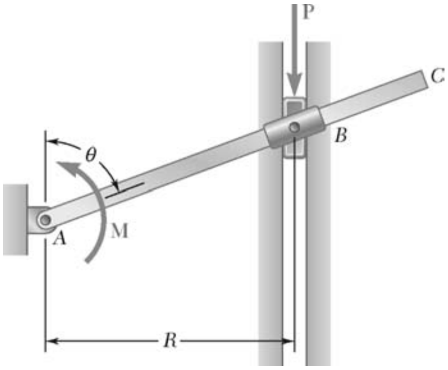
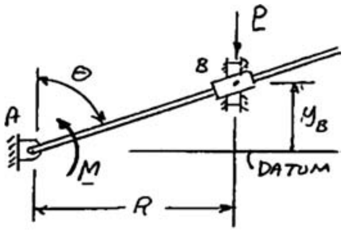


PROBLEM 10.105

Collar B can slide along rod AC and is attached by a pin to a block that can slide in the vertical slot shown. Derive an expression for the magnitude of the couple **M** required to maintain equilibrium.

SOLUTION



$$y_B = \frac{R}{\tan(90° - \theta)}$$

$$\delta y_B = \frac{-R\delta\theta}{\cos^2(90° - \theta)}$$

$$\delta y_B = \frac{-R\delta\theta}{\sin^2\theta}$$

Virtual Work: $\delta U = 0$: $\delta U = -M\,\delta\theta - P\delta y_B = 0$

$$-M\,\delta\theta + PR\frac{1}{\sin^2\theta}\delta\theta = 0$$

$$M = \frac{PR}{\sin^2\theta}$$

$M = PR\csc^2\theta$ ◀

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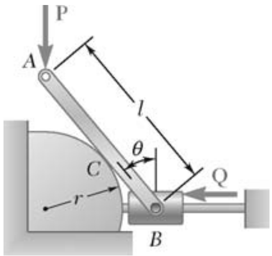
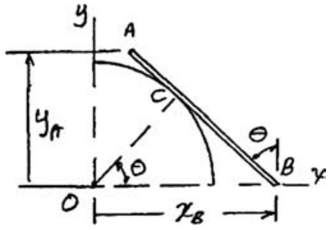
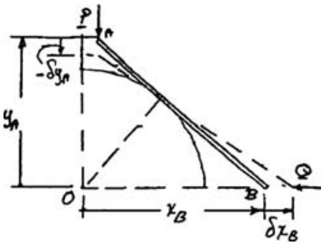


PROBLEM 10.106

A slender rod of length l is attached to a collar at B and rests on a portion of a circular cylinder of radius r. Neglecting the effect of friction, determine the value of θ corresponding to the equilibrium position of the mechanism when $l = 200$ mm, $r = 60$ mm, $P = 40$ N, and $Q = 80$ N.

SOLUTION





Geometry $\quad OC = r$

$$\cos\theta = \frac{OC}{OB} = \frac{r}{x_B}$$

$$x_B = \frac{r}{\cos\theta}$$

$$\delta x_B = \frac{r\sin\theta}{\cos^2\theta}\delta\theta$$

$$y_A = l\cos\theta$$

$$\delta y_A = -l\sin\theta\delta\theta$$

<u>Virtual Work</u>:

$$\delta U = 0: \quad P(-\delta y_A) - Q\delta x_B = 0$$

$$Pl\sin\theta\delta\theta - Q\frac{r\sin\theta}{\cos^2\theta}\delta\theta = 0$$

$$\cos^2\theta = \frac{Qr}{Pl} \qquad (1)$$

Then, with $l = 200$ mm, $r = 60$ mm, $P = 40$ N, and $Q = 80$ N

$$\cos^2\theta = \frac{(80\text{ N})(60\text{ mm})}{(40\text{ N})(200\text{ mm})} = 0.6$$

or $\quad \theta = 39.231°$ $\qquad$ $\theta = 39.2°$ ◀

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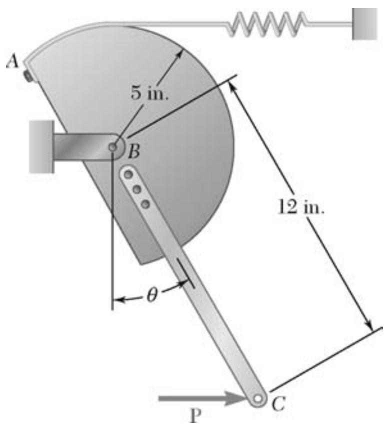
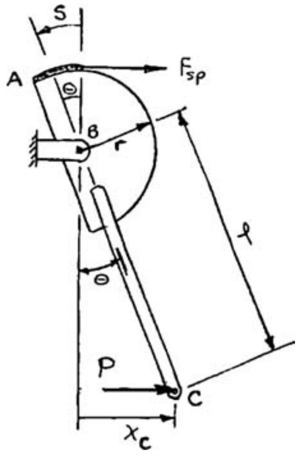


PROBLEM 10.107

A horizontal force **P** of magnitude 40 lb is applied to the mechanism at *C*. The constant of the spring is $k = 9$ lb/in., and the spring is unstretched when $\theta = 0$. Neglecting the weight of the mechanism, determine the value of θ corresponding to equilibrium.

SOLUTION



$$s = r\theta$$

$$\delta s = r\delta\theta$$

Spring is unstretched at $\theta = 0°$

$$F_{SP} = ks = kr\theta$$

$$x_C = l\sin\theta$$

$$\delta x_C = l\cos\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P\delta x_C - F_{SP}\delta s = 0$$

$$P(l\cos\theta\delta\theta) - kr\theta(r\delta\theta) = 0$$

or $$\frac{Pl}{kr^2} = \frac{\theta}{\cos\theta}$$

Thus $$\frac{(40\text{ lb})(12\text{ in.})}{(9\text{ lb/in.})(5\text{ in.})^2} = \frac{\theta}{\cos\theta}$$

or $$\frac{\theta}{\cos\theta} = 2.1333$$

$$\theta = 1.054\text{ rad} = 60.39°$$

$\theta = 60.4°$ ◀

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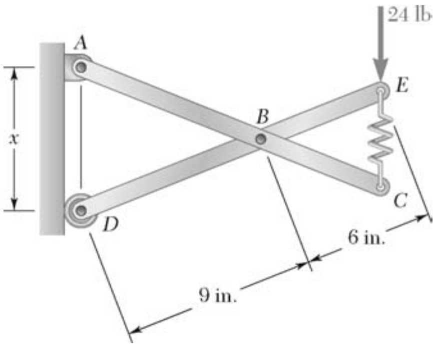
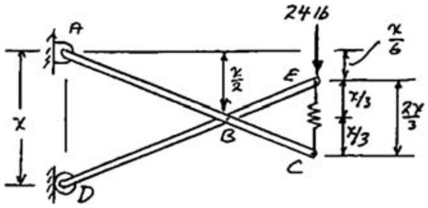


PROBLEM 10.108

Two identical rods *ABC* and *DBE* are connected by a pin at *B* and by a spring *CE*. Knowing that the spring is 4 in. long when unstretched and that the constant of the spring is 8 lb/in., determine the distance *x* corresponding to equilibrium when a 24-lb load is applied at *E* as shown.

SOLUTION



Deformation of spring

$$s = EC - 4\text{ in.} = \frac{2x}{3} - 4$$

$$V = \frac{1}{2}ks^2 - (24\text{ lb})\frac{x}{6} = \frac{1}{2}(8\text{ lb/in.})\left(\frac{2x}{3} - 4\right)^2 - 4x$$

$$\frac{dV}{dx} = 8\left(\frac{2x}{3} - 4\right)\frac{2}{3} - 4$$

<u>Equilibrium</u>: $\frac{dV}{dx} = 0 \qquad \frac{16}{3}\left(\frac{2x}{3} - 4\right) - 4 = 0$

$$\frac{2x}{3} - 4 = 4\left(\frac{3}{16}\right)$$

$$\frac{2x}{3} = 4 + \frac{3}{4}$$

$x = 7.13\text{ in.}$ ◀

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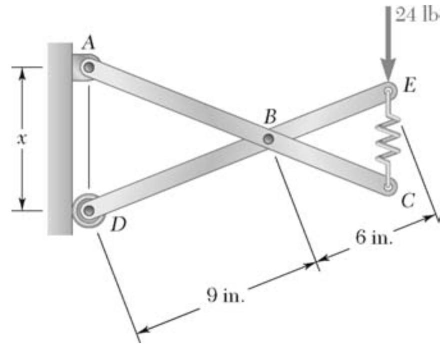
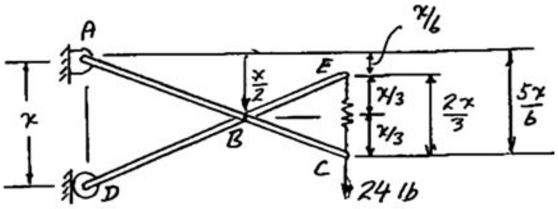


PROBLEM 10.109

Solve Problem 10.108 assuming that the 24-lb load is applied at *C* instead of *E*.

PROBLEM 10.108 Two identical rods *ABC* and *DBE* are connected by a pin at *B* and by a spring *CE*. Knowing that the spring is 4 in. long when unstretched and that the constant of the spring is 8 lb/in., determine the distance *x* corresponding to equilibrium when a 24-lb load is applied at *E* as shown.

SOLUTION



Deformation of spring

$$s = EC - 4\text{ in.} = \frac{2x}{3} - 4$$

$$V = \frac{1}{2}ks^2 - (24\text{ lb})\frac{5x}{6} = \frac{1}{2}(8\text{ lb/in.})\left(\frac{2x}{3} - 4\right)^2 - 20x$$

$$\frac{dV}{dx} = 8\left(\frac{2x}{3} - 4\right)\frac{2}{3} - 20$$

Equilibrium: $\quad \frac{dV}{dx} = 0 \qquad \frac{16}{3}\left(\frac{2x}{3} - 4\right) - 20 = 0$

$$\frac{2x}{3} - 4 = 20\left(\frac{3}{16}\right)$$

$$\frac{2x}{3} = 4 + 3.75$$

$x = 11.63$ in. ◀

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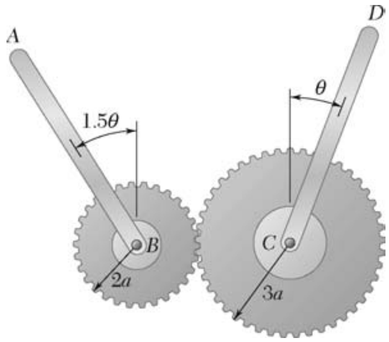
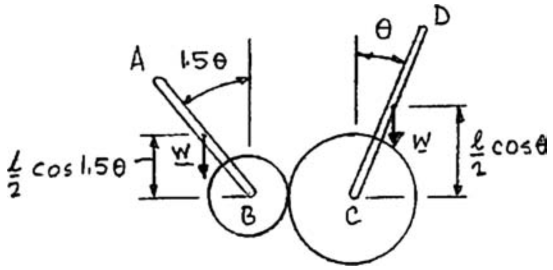


PROBLEM 10.110

Two uniform rods, each of mass *m* and length *l*, are attached to gears as shown. For the range $0 \le \theta \le 180°$, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



Potential energy $$V = W\left(\frac{l}{2}\cos 1.5\theta\right) + W\left(\frac{l}{2}\cos\theta\right) \quad W = mg$$

$$\frac{dV}{d\theta} = \frac{Wl}{2}(-1.5\sin 1.5\theta) + \frac{Wl}{2}(-\sin\theta)$$

$$= -\frac{Wl}{2}(1.5\sin 1.5\theta + \sin\theta)$$

$$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}(2.25\cos 1.5\theta + \cos\theta)$$

For equilibrium $$\frac{dV}{d\theta} = 0: \quad 1.5\sin 1.5\theta + \sin\theta = 0$$

Solutions: One solution, by inspection, is $\theta = 0$, and a second angle less than 180° can be found numerically:

$$\theta = 2.4042 \text{ rad} = 137.8°$$

Now $$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}(2.25\cos 1.5\theta + \cos\theta)$$

At $\theta = 0$: $$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}(2.25\cos 0° + \cos 0°)$$

$$= -\frac{Wl}{2}(3.25)(< 0)$$

$\theta = 0$, Unstable ◀

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PROBLEM 10.110 (Continued)

At $\theta = 137.8°$: $$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}[2.25\cos(1.5 \times 137.8°) + \cos 137.8°]$$

$$= \frac{Wl}{2}(2.75)(> 0)$$

$\theta = 137.8°$, Stable ◀

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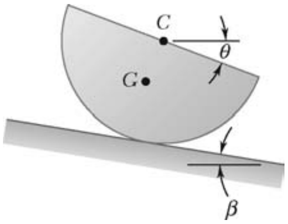
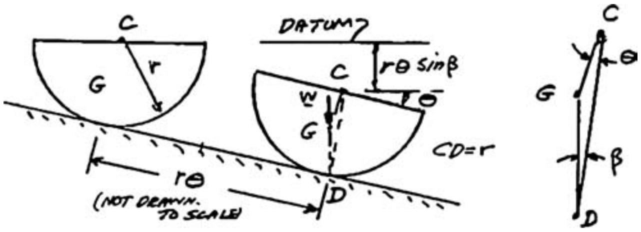


PROBLEM 10.111

A homogeneous hemisphere of radius r is placed on an incline as shown. Assuming that friction is sufficient to prevent slipping between the hemisphere and the incline, determine the angle θ corresponding to equilibrium when $\beta = 10°$.

SOLUTION



Detail

$$CG = \frac{3}{8}r$$

$$V = W(-r\theta \sin\beta - (CG)\cos\theta)$$

$$\frac{dV}{d\theta} = W\left(-r\sin\beta + \frac{3}{8}r\sin\theta\right)$$

Equilibrium: $\frac{dV}{d\theta} = 0, \quad -\sin\beta + \frac{3}{8}\sin\theta = 0$

$$\sin\beta = \frac{3}{8}\sin\theta \qquad (1)$$

For $\beta = 10°$ $\quad \sin 10° = \frac{3}{8}\sin\theta \quad \sin\theta = 0.46306, \quad \theta = 27.6°$ ◀

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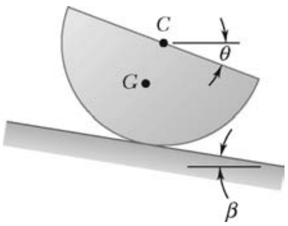
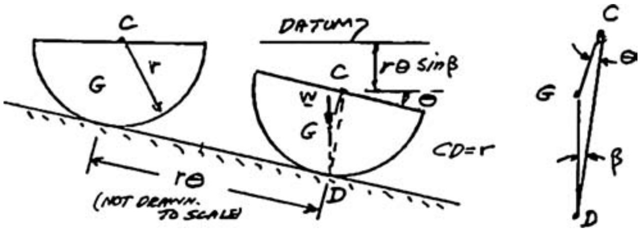


PROBLEM 10.112

A homogeneous hemisphere of radius *r* is placed on an incline as shown. Assuming that friction is sufficient to prevent slipping between the hemisphere and the incline, determine (*a*) the largest angle β for which a position of equilibrium exists, (*b*) the angle θ corresponding to equilibrium when the angle β is equal to half the value found in Part *a*.

SOLUTION



Detail

$$CG = \frac{3}{8}r$$

$$V = W(-r\theta\sin\beta - (CG)\cos\theta)$$

$$\frac{dV}{d\theta} = W\left(-r\sin\beta + \frac{3}{8}r\sin\theta\right)$$

Equilibrium: $\dfrac{dV}{d\theta} = 0, \quad -\sin\beta + \dfrac{3}{8}\sin\theta = 0$

$$\sin\beta = \frac{3}{8}\sin\theta \qquad (1)$$

(*a*) For $\beta_{\max}$, $\theta = 90°$

Eq. (1) $\quad \sin\beta_{\max} = \dfrac{3}{8}\sin 90°, \quad \sin\beta_{\max} = \dfrac{3}{8} = 22.02°$ $\qquad \beta_{\max} = 22.0°$ ◀

(*b*) When $\beta = \frac{1}{2}\beta_{\max} = 11.01°$

Eq. (1) $\quad \sin 11.01° = \dfrac{3}{8}\sin\theta; \quad \sin\theta = 0.5093$ $\qquad \theta = 30.6°$ ◀

Note: We can also use ΔCGD and law of sines to derive Eq. (1).

$$\frac{\sin\beta}{CG} = \frac{\sin\theta}{CD}; \quad \sin\beta = \frac{CG}{CD}\sin\theta; \quad \sin\beta = \frac{3}{8}\sin\theta$$

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